A Basic Introduction to Digital Waveguide Synthesis (for the Technically Inclined)

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1 Basics of Digital Waveguide Modeling

Digital waveguide synthesis models are *computational physical models* for certain classes of musical instruments (string, winds, brasses, etc.) which are made up of *delay lines, digital filters*, and often *nonlinear elements*. In principle, they can be viewed as a particular class of *finite difference scheme* for numerical physical modeling.

Digital waveguide models typically share the following characteristics:

- Sampled acoustic traveling waves
- Follow geometry and physical properties of a desired acoustic system
- Efficient for nearly lossless distributed wave media
  (strings, tubes, rods, membranes, plates, vocal tract,)
- *Losses* and *dispersion* are *consolidated* at sparse points along each waveguide

1.1 The Digital Waveguide

A lossless digital waveguide is a *bidirectional delay line* at some wave impedance R.

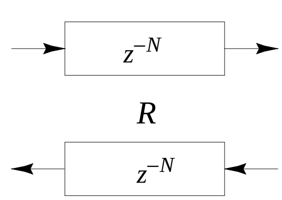

Figure 1: Sampled traveling-wave simulation for an ideal string or acoustic tube

Each delay-line element contains a sampled *traveling-wave component*. For example, the number in a delay cell may represent *pressure* in an acoustic tube or *transverse displacement* in a vibrating string.

1.2 Physical Outputs

Traveling waves are efficient for simulation, but they cannot be measured directly in the physical world.

Physical variables (force, pressure, velocity, ...) are obtained by *summing* traveling-wave components.

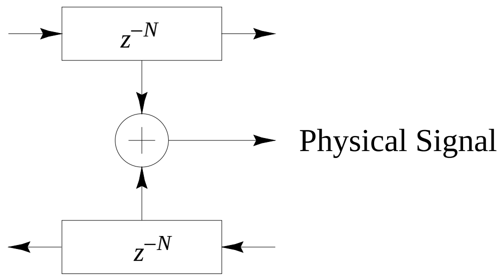

Figure 2: Extracting a physical signal from a digital waveguide.

1.3 Signal Scattering

Signal *scattering* is caused by a *change* in wave impedance R along the waveguide.

$$k_1 = \frac{R_2 - R_1}{R_2 + R_1}$$

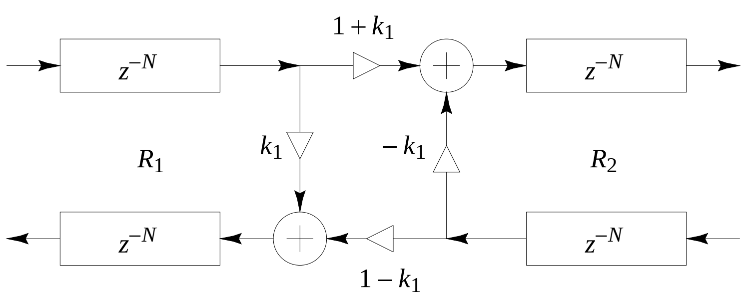

Figure 3: Signal scattering at a junction of different wave impedances R_1 and R_2. The value k_1 is called the *reflection coefficient*, and it fully characterizes the scattering junction.

2 The Rigidly Terminated Ideal String

A *rigid termination* is the simplest case of a string termination. It imposes the constraint that the string cannot move at all at the termination. Let $y(t, x)$ denote the transverse displacement of an ideal vibrating string at time t, with x denoting position along the length of the string. If we terminate a length L ideal string at $x = 0$ and $x = L$, we then have the "boundary conditions"

$$y(t, 0) \equiv 0 \qquad y(t, L) \equiv 0$$

where "≡" means "identically equal to," i.e., equal for all t.

The corresponding constraints on the sampled traveling waves are then

$$\begin{aligned} y^+(n) &= -y^-(n) \\ y^-(n + N/2) &= -y^+(n - N/2) \end{aligned}$$

where N is the time in samples to propagate from one end of the string to the other and back, or the total "string loop" delay. The loop delay is also equal to twice the number of spatial samples along the string. A digital simulation diagram for the rigidly terminated ideal string is shown in Fig. 4. A "virtual pick-up" is shown at the arbitrary location $x = \xi$.

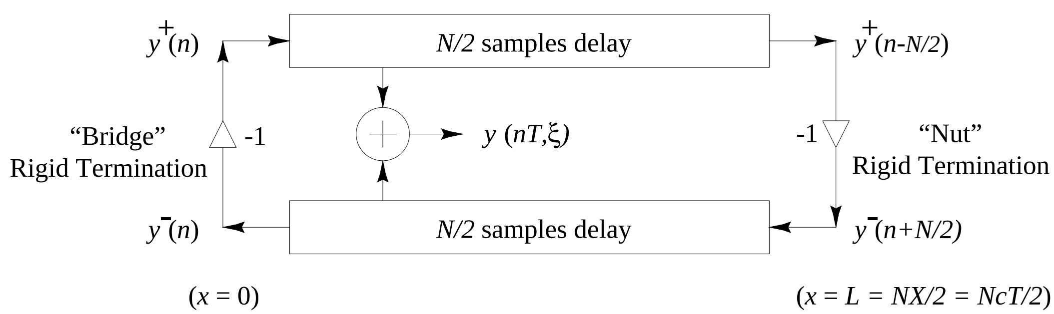

Figure 4: The rigidly terminated ideal string with position output at $x = \xi$.

Note that rigid terminations reflect traveling displacement, velocity, or acceleration waves with a sign inversion. Slope or force waves reflect with no sign inversion. Since here we have displacement waves, the rigid terminations are inverting. This result may also be obtained from the reflection coefficient formula on the previous page by setting the terminating impedance to infinity in that formula.

3 Schematics for Digital Waveguide Models of Musical Instruments

Musical instruments usually also involve *nonlinear and frequency-dependent (linear) scattering junctions.*

3.1 Wind Instruments

Example of wind instruments include the clarinet, trumpet, flute, and organ pipe.

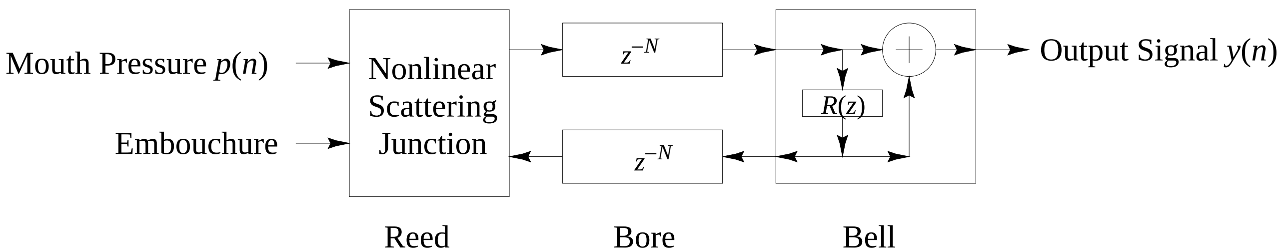

Figure 5: Digital Waveguide Woodwind Instrument.

3.2 Bowed Strings

Example of bowed-string instruments include the violin, viola, cello, and bass viol.

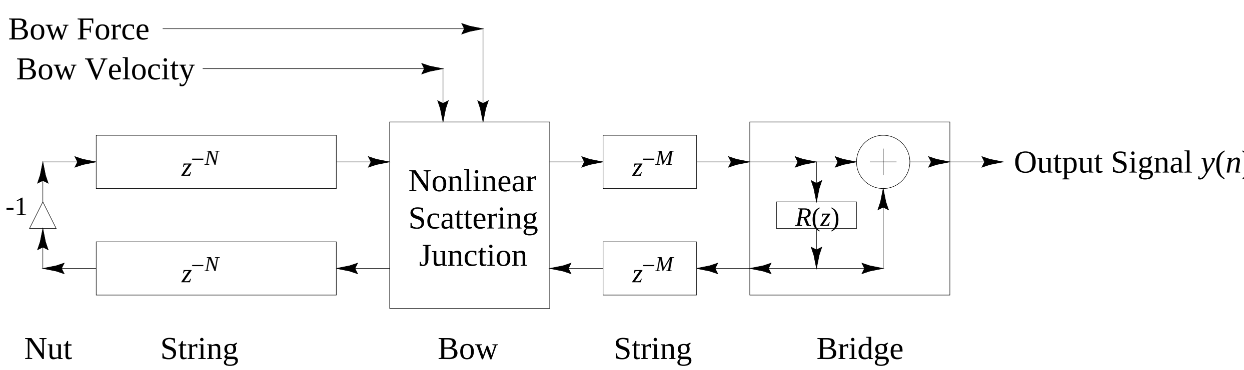

Figure 6: Digital Waveguide Bowed-String Instrument.

“A Basic Introduction to Digital Waveguide Synthesis,” J.O. Smith,
http://www-ccrma.stanford.edu/ jos/swgt/swgt.html

4 Recommended Text Books

- P.M. Morse, **Vibration and Sound**, AIP for ASA, 1976 (1st ed. 1936, 2nd ed. 1948). $18.50 from AIP:(516)349-7800x481. (Acoustics text with more musical acoustics than is typical nowadays.)
- A. Hirschberg, J. Kergomard, and G. Weinreich, eds., **Mechanics of Musical Instruments**, Springer-Verlag, 1995. (Advanced musical acoustics text.)
- N. H. Fletcher and T. D. Rossing, **The Physics of Musical Instruments**, Springer-Verlag, 1993. (Advanced musical acoustics text.)
- A. H. Benade, Fundamentals of Musical Acoustics, New York: Oxford University Press, 1976. Music ML3805.B456. (There is now a Dover version of this classic, highly readable, musical acoustics text.)
- L. Cremer, **The Physics of the Violin**, MIT Press, 1984. (Advanced musical acoustics treatise summarizing a lifetime of work in the field by a university physicist.)
- L. Kinsler, A.Frey, A. Coppens, and J. Sanders, **Fundamentals of Acoustics**, Wiley, 1982. (General acoustics text.)
- I. G. Main, **Vibrations and Waves in Physics**, Cambridge University Press, 1978. (Good basic book on traveling-wave mechanics.)
- J. D. Markel and A. H. Gray, **Linear Prediction of Speech**, New York: Springer-Verlag, 1976. (Excellent development of lattice and ladder digital filter forms. Advanced.)
- P. M. Morse and K. U. Ingard, **Theoretical Acoustics**, Princeton University Press, 1968. (Advanced acoustics text.)
- Curtis Roads, **Computer Music Tutorial**, MIT Press, 1996. (Advanced computer music.)
- Curtis Roads, ed., **The Music Machine**, MIT Press, 1989. (Anthology of papers in computer music.)