Physical Modeling Synthesis of Sound

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One View of Sound

Sound is a waveform, we can record it, store it, and play it back accurately

PCM playback is all we need for interactions, movies, games, etc.

But, take one visual analogy:
“IF I take lots of polaroid images, I can flip through them real fast and make any image sequence”

Interaction? We manipulate lots of PCM

Views of Sound

• Time Domain \( x(t) \)
  (from physics, and time’s arrow)
• Frequency Domain \( X(f) \)
  (from math, and perception)
• Production what caused it
• Perception our “image” of it

Views of Sound

• The Time Domain is most closely related to Production
• The Frequency Domain is most closely related to Perception

Views of Sound: Time Domain

Sound is produced modeled by physics, described by quantities of

- Force \( \text{force} = \text{mass} \times \text{acceleration} \)
- Position \( x(t) \) actually \( [x(t), y(t), z(t)] \)
- Velocity Rate of change of position \( \frac{dx}{dt} \)
- Acceleration Rate of change of velocity \( \frac{dv}{dt} \)
  (2nd derivative of position)

Examples: Mass, Spring, Damper Wave Equation

Mass/Spring/Damper

\[ F = ma = -ky - rv - mg \]
\[ ma = -ky - rv \]
(if gravity negligible)

Solution:

\[ \frac{d^2y}{dt^2} + \frac{r}{m} \frac{dy}{dt} + \frac{k}{m} y = 0 \]
2nd Order Linear Diff Eq. Solution

1) Underdamped:
y(t) = Y_0 e^{-\alpha t} \cos(\omega t)
   exp. * oscillation

2) Critically damped:
   fast exponential decay

3) Overdamped:
   slow exponential decay

The Wave Equation

\[ df = (T \sin \theta)^{x+dx} - (T \sin \theta)_x \]
   (for each dx of string)

\[ f(x+dx) = f(x) + \delta f/\delta x \ dx + ... \]
   (Taylor's series in space)

\[ F = ma = \rho \ dx \ d^2 y/dt^2 \]
   \( (\rho = \text{mass/length}) \)

Solution:
The wave equation
\[ \frac{d^2 y}{dx^2} = \frac{1}{c^2} \frac{d^2 y}{dt^2} \]
Modal Solution for Bars

- Bars are often free at one or both ends
- Spatial modal solution still holds
- Modes no longer harmonic. Stiffness of rigid bars “stretches” frequencies.
- Modes: $f$, $2.765f$, $5.404f$, $8.933f$, etc.

Modal Synthesis (Adrien)
- Impulse generator excites filters
- Filters shape spectrum, model eigenmodes
- Filter parameters can be time-varying

\[
\begin{align*}
y(n) &= g \cdot x(n); \\
y(n) &= b_1 \cdot y(n-1); \\
2y(n) &= b_2 \cdot y(n-2); \\
n++;
\end{align*}
\]

Stiffness in Bars

- Stiffness makes wave propagation frequency dependent ($c(f)$)
- Models:
  - Modal partials
  - Use all-pass phase filter to "stretch" waveguide harmonics
  - Merge waveguide with modal by modeling each mode with filter and delay

Stiffness

- All-pass waveguide (Smith & Jaffe)
  - Acoustics View: Frequency dependent propagation
  - Filter View: Stretch comb filter harmonics

- Banded waveguides (Essi)
  - Acoustics View: Wave train closures
  - Filter View: Comb filters with one resonance each

Or a purely modal model (lacks space and time)
**Tubes**
- Open or closed at either end
- Wave equation solution same as strings
- Modes always harmonic because speed of sound is constant with frequency
- Solutions:
  - Waveguide
    - \( f(x, t) \)
  - or Modal
    - Open + Closed: odd 1/4 wavelengths

**Two and Higher Dimensions**
- 2 (N) Dimensional Waveguide Meshes
- or Finite Elements and Finite Differences
  - Discretize objects into cells (elements)
  - Express interactions between them
  - Express differential equation for system
  - Solve by discrete steps in space and time
- or Modal Solution

**Finite Elements**
(with O'Brien and Essl)

**Hi-D Modal Solutions**
- Modes of Plates are inharmonic
  - Center strike round = Bessel function roots
  - Edge strike round = Square Plate Modes = \( \sqrt{t} \) factors
- Modes in higher dimensions are problematic (impossible analytically except in very simple cases)

**Where Are We So Far?**
- Physical descriptions (equations)
- Give rise to solutions:
  1. Traveling Waves
  2. Spatial/Frequency Modes
- We can solve the equations directly using
  3. Finite Elements/Meshes
- How to choose? Are there more?

**Waveguides**
- Strengths:
  - Cheap in both computation and memory
  - Parametrically meaningful, extensible for more realism
- Weaknesses:
  - Little in the real world looks, behaves, or sounds exactly like a plucked string, flute, etc.
  - Each family needs a different model
  - No general blind signal model
Modal Modeling

- **Strengths:**
  - Generic, flexible, cheap if only a few modes
  - Great for modeling struck objects of metal, glass, wood

- **Weaknesses:**
  - No inherent spatial sampling
  - No (meaningful) phase delay
  - Hard to interact directly and continuously (rubbing, damping, etc).
  - No general blind signal model (closest)

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Meshes, Finite Elements

- **Strengths**
  - (somewhat) arbitrary geometries
  - Less assumptions than parametric forms
  - Can strike, damp, rub, introduce non-linearities at arbitrary points

- **Weaknesses:**
  - Expensive
  - Don’t know all the computational solutions
  - Sampling in space/time (high Q problems)
  - Dispersion is strange (diagonals vs. not)
  - No general blind signal model

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References and Resources

**Synthesis ToolKit in C++ (STK)**
- STK: a set of classes in C++ for rapid experimentation with sound synthesis. Available for free (source, multi-platform)
  - [http://www.cs.princeton.edu/~prc](http://www.cs.princeton.edu/~prc)
  - [http://www-ccrma.stanford.edu/~gary](http://www-ccrma.stanford.edu/~gary)
  - [http://www-ccrma.stanford.edu/software/stk](http://www-ccrma.stanford.edu/software/stk)
- Based on "Unit Generators," the classical computer music/sound building blocks:
  - Oscillators, Filters, Delay Lines, etc.
  - Build your own algorithms from these

**Book on interactive sound synthesis**
- Many examples and figures from these notes

References: Waveguide & FE Modeling


References: Modal Synthesis

References: Sinusoidal Models

SMS Web site. URL: http://www.lua.upf.es/~sms.