Introduction to Digital Data Acquisition:

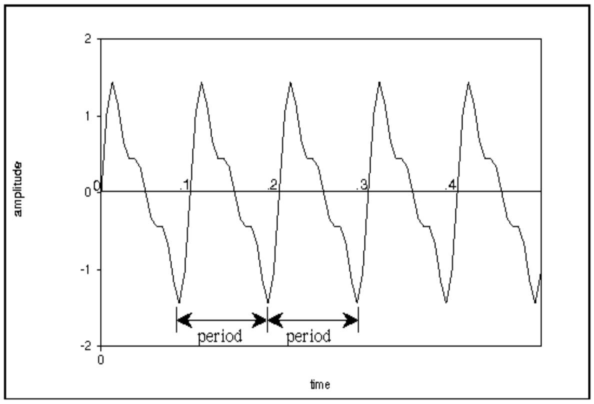
Sampling

Physical world is analog

- Digital systems need to
  - Measure analog quantities
    - Switch inputs, speech waveforms, etc
  - Control analog systems
    - Computer monitors, automotive engine control, etc
- Analog-to-digital: A/D converter (ADC)
  - Example: CD recording
- Digital-to-analog: D/A converter (DAC)
  - Example: CD playback

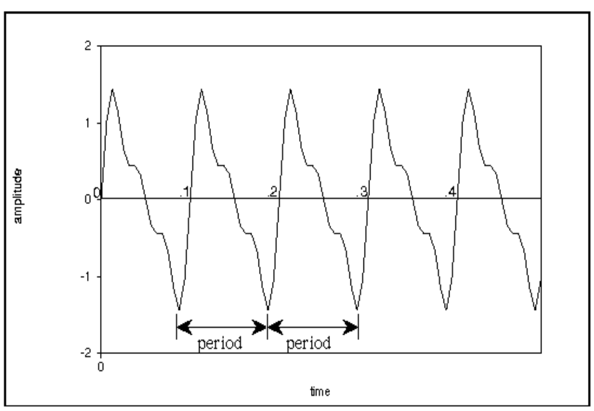
A little background

- For periodic waveforms, the duration of the waveform before it repeats is called the period of the waveform



Frequency

- the rate at which a regular vibration pattern repeats itself (frequency = 1/period)



Frequency of a Waveform

- The unit for frequency is cycles/second, also called Hertz (Hz).
- The frequency of a waveform is equal to the reciprocal of the period.

frequency = 1/period

Frequency of a Waveform

- Examples:

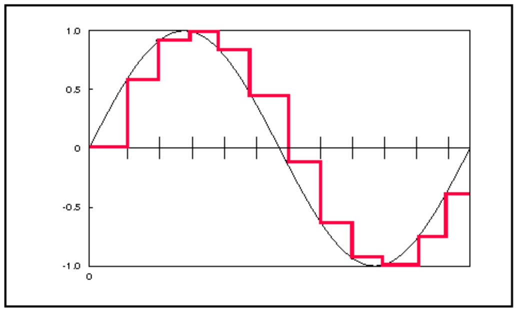
  frequency = 10 Hz
  period = .1 (1/10) seconds

  frequency = 100 Hz
  period = .01 (1/100) seconds

  frequency = 261.6 Hz (middle C)
  period = .0038226 (1/ 261.6) seconds

Waveform Sampling

- To represent waveforms in digital systems, we need to digitize or sample the waveform.



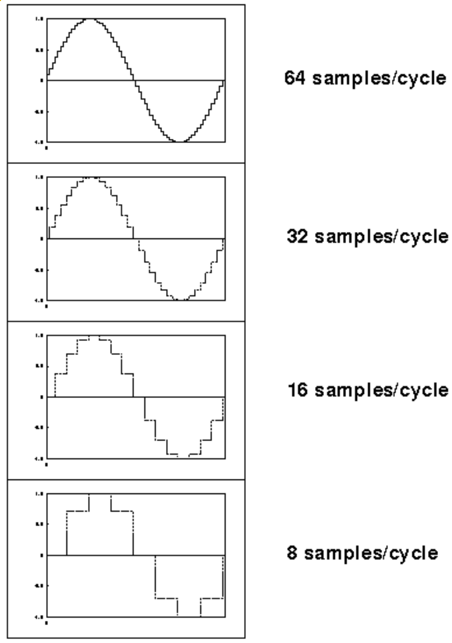
- side effects of digitization:
  - introduces some noise
  - limits the maximum upper frequency range

Sampling Rate

- The sampling rate (SR) is the rate at which amplitude values are digitized from the original waveform.
  - CD sampling rate (high-quality):
    SR = 44,100 samples/second
  - medium-quality sampling rate:
    SR = 22,050 samples/second
  - phone sampling rate (low-quality):
    SR = 8,192 samples/second

Sampling Rate

- Higher sampling rates allow the waveform to be more accurately represented

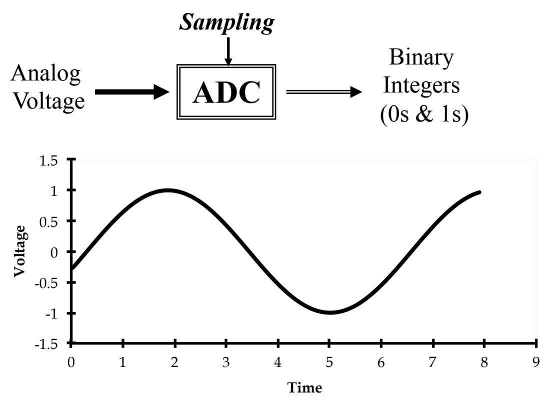


Digital Data Acquisition

- Data Representation - *Digital vs. Analog*
- Analog-to-Digital Conversion
- Number Systems
  - Binary Numbers
  - Binary Arithmetic
- Sampling & Aliasing

Analog-to-Digital Conversion

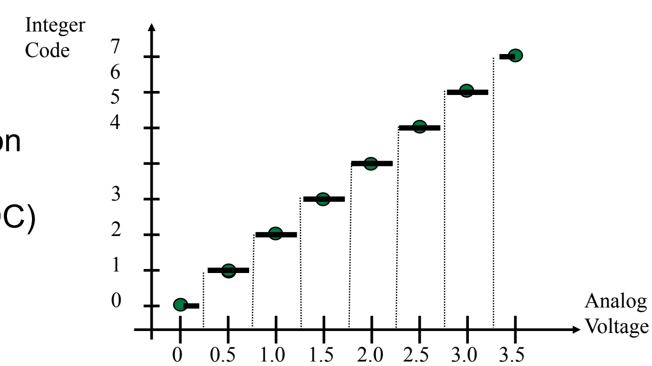
- Converts analog voltages to binary integers.



Analog-to-Digital Conversion

- **ADC calibration**

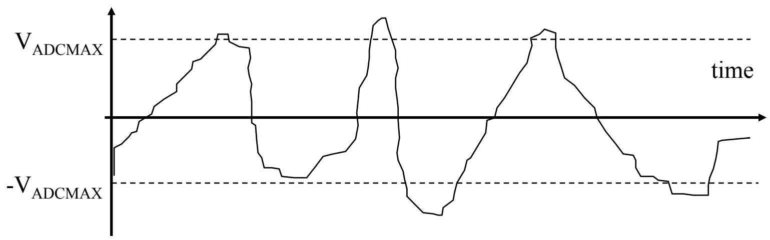
Calibration Curve
(3 bit ADC)



Analog-to-Digital Conversion

- Input Range
  - Unipolar: (0, V_{ADCMAX})
  - Bipolar: ($-V_{ADCMAX}$, $+V_{ADCMAX}$) (Nominal Range)
  - **Clipping**:

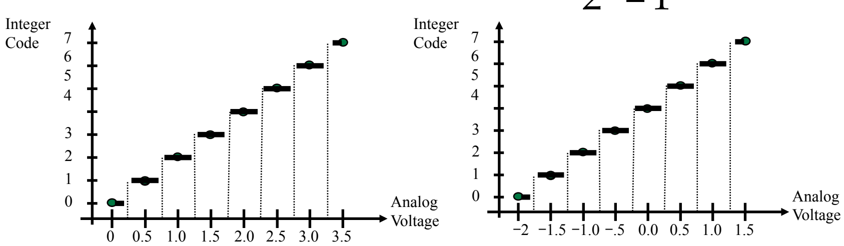
    If $|V_{IN}| > |V_{ADCMAX}|$, then $|V_{OUT}| = |V_{ADCMAX}|$



Analog-to-Digital Conversion

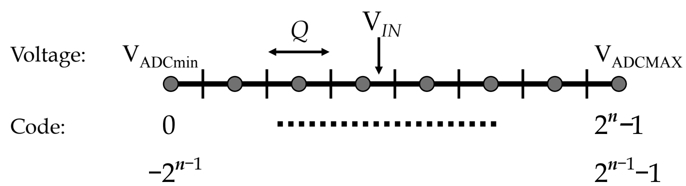
- Quantization Interval (Q)
  - n bit ADC, the input range is divided into 2^n-1 intervals.
  - 3 bit ADC:

$$Q = \frac{V_{ADCMAX} - V_{ADC\min}}{2^n - 1}$$



Analog-to-Digital Conversion

- <u>Voltage to Integer Code</u>
  - *n* bit ADC



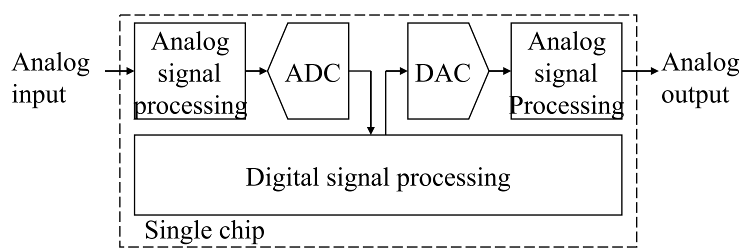
Positive Coding:

$$\text{Code} = \text{Round}\left[\frac{V_{IN} - V_{ADC\,min}}{Q}\right]$$

Positive and Negative Coding:

$$\text{Code} = \text{Round}\left[\frac{V_{IN}}{Q}\right]$$

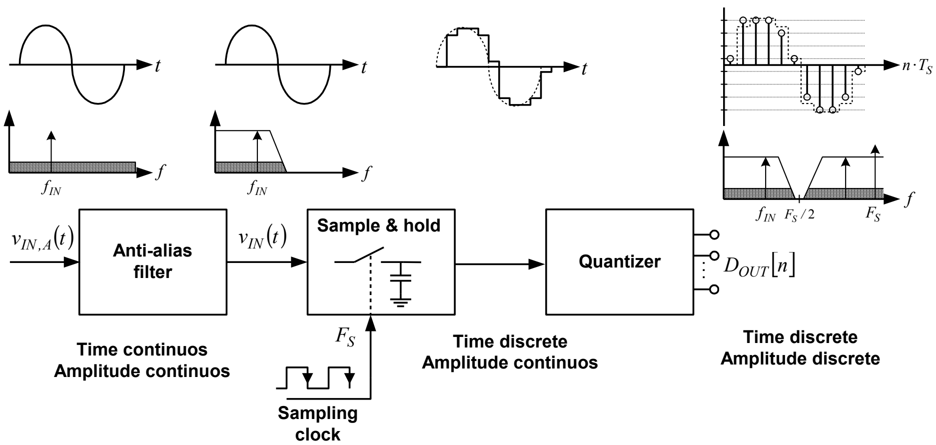
Why A/D-conversion?



- Signals are analog by nature
- ADC necessary for DSP
- Digital signal processing provides:
  - Close to infinite SNR
  - Low system cost
  - Repetitive system

- ADC bottle necks:
  - Dynamic range
  - Conversion speed
  - Power consumption

A/D-converter basics

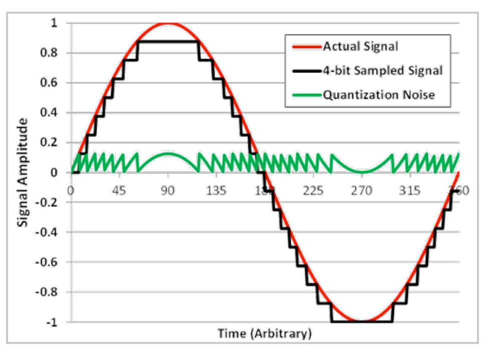


$$D_{OUT}^{ideal}[n] = G_{ideal} \cdot v_{IN}(n \cdot Ts) + q(n)$$

$$D_{OUT}^{real}[n] = G_{ideal} \cdot (1+\varepsilon) \cdot v_{IN}(n \cdot Ts) + q(n) + e_{offset}(n) + e_{noise}(n) + e_{jitter}(n) + e_{distortion}(n)$$

The Theory

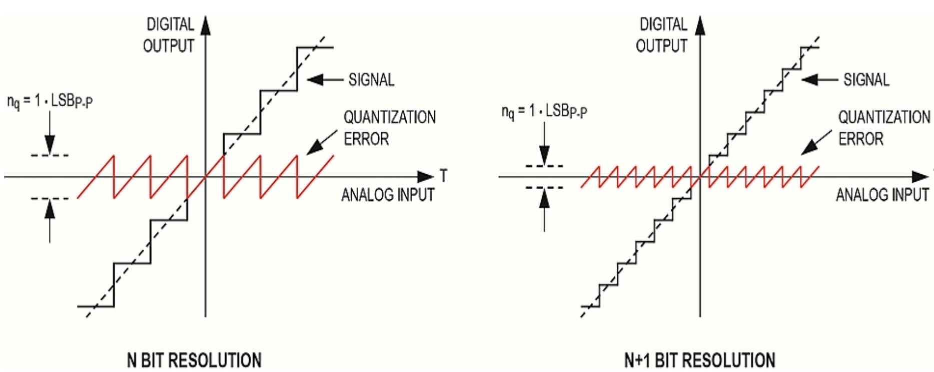
- Sampling theory is a subset of communications theory
  - Same basic math
    - Want to record signal, not noise
  - **Quantization**: Conversion from analog to discrete values
  - **Coding**: Assigning a digital word to each discrete value
    - Thermometer code, Gray code...
- Quantization adds noise
  - Analog signal is continuous
  - Digital representation is approximate
  - Difference (error) is noiselike



Some terminology

- Resolution (n): Number of states in bits
  - Example: A 3-bit A/D
- Full-scale range (FSR): The input or output voltage range
  - Example: ADC inputs outside the FSR are always 111 or 000
- Step size (Q): $\frac{\text{FSR}}{2^n}$
- RMS quantization error: $\frac{Q}{\sqrt{12}}$
  - RMS value of triangle wave

Quantization noise



•N-bit converter: $\delta = \frac{V_{FSR}}{2^N}$

Quantization noise

- Noise energy:

$$V_{Q(RMS)} = \sqrt{\frac{1}{\delta}\int_{-\delta/2}^{\delta/2} V_Q^2 \, dV_Q} = \sqrt{\frac{\delta^2}{12}}$$

- Signal energy:

$$V_{in(RMS)} = \frac{\delta \cdot 2^N}{2\sqrt{2}}$$

- SNR for ideal ADC:

$$SNR = 20\log(\frac{V_{in(RMS)}}{V_{Q(RMS)}})$$

$$SNR = 20\log(2^N \cdot \sqrt{\frac{3}{2}})$$

$$SNR = 6.02 \times N + 1.76[dB]$$

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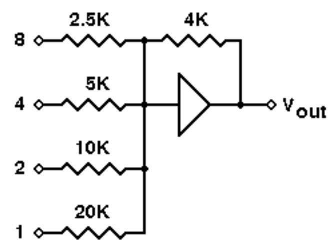
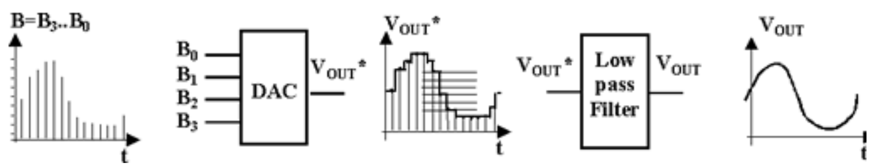
D/A converters

- Easier to design and use than A/Ds
- Types
  - Weighted current source DAC
  - R–2R DAC
  - Multiplying DAC
- Need to smooth the output

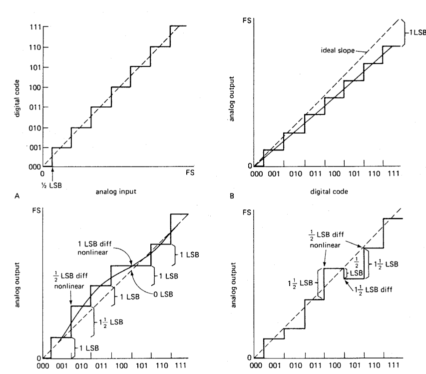


Digital to Analog Converter converts a digital signal to an analog output



You will use DACs

- DAC specs are tricky!
  - Check the errors
  - Check the settling
- Vendors use deceptive advertising
  - 16-bit DAC!!!
    - But errors may give only 12-bit accuracy
    - You have to figure this out from the specs



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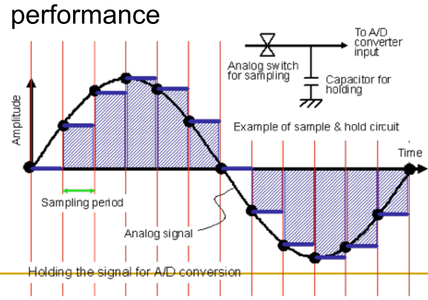
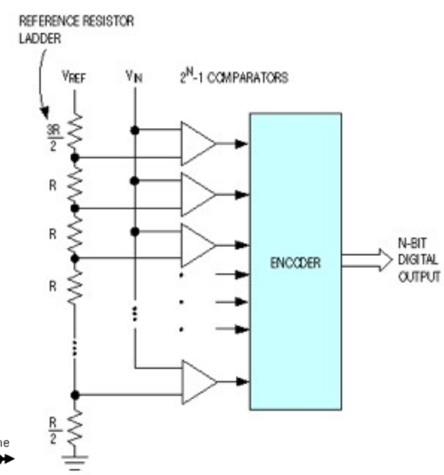
A/D converters

- ***Hard*** to design
- Contain digital parts
  - Encoders
  - FSMs
- Many types
  - Successive approximation
  - Flash
  - Pipelined-flash
  - Integrating
  - Sigma-delta
  - Charge balanced
  - Folding
  - Others

Example: Flash A/D

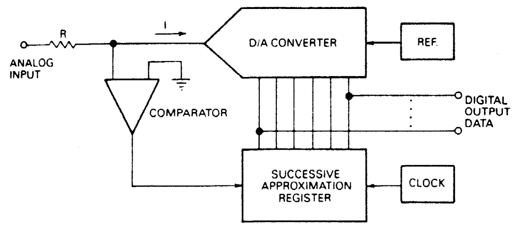
- Advantages
  - Ultra-fast
- Disadvantages
  - High power
  - Low resolution
  - Metastability
- Sample/hold improves performance



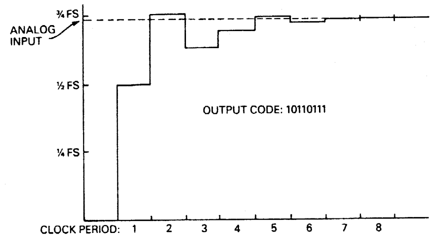


Example: Successive approximation ADC

- Advantages
  - Low power
  - High resolution
- Disadvantages
  - Slow
- Problem: DAC must settle to LSB accuracy at every step



Successive Approximation A/D Converter



D/A Output for 8-Bit Successive Approximation Conversion

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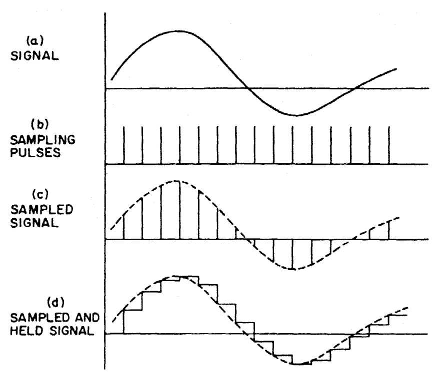
Sampling

- Quantizing a signal
  1) We sample it
  2) We encode the samples
- Questions:
  - How fast do we sample?
  - How do we so this in hardware?
  - What resolution do we need?



Signal Sampling

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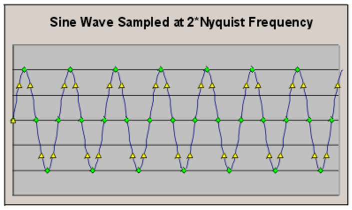
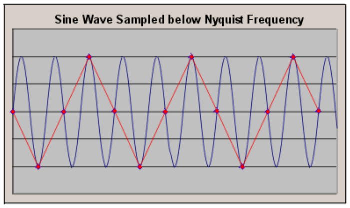
Shannon's sampling theorem

If a continuous, band-limited signal contains no frequency components higher than f_c, then we can recover the original signal without distortion if we sample at a rate of at least $2f_c$ samples/ second

- $2f_c$ is called the Nyquist rate
- Real life
  - Sample at $2.5f_c$ or faster
  - Sample clock should not be coherent with the input signal

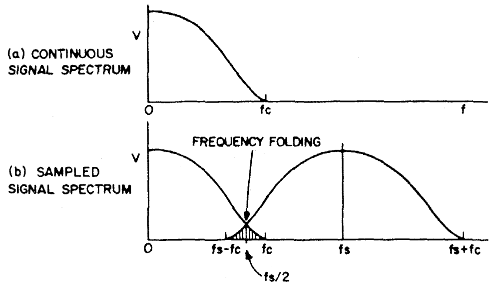




http://www.videomicroscopy.com/vancouverlecture/nyquist.htm

Frequency domain analysis

- Take the Fourier Transform of the signal
  - Shows a signal's *frequency* components
- Undersampled frequency components fold back!



Frequency Spectra Demonstrating the Sampling Theorem

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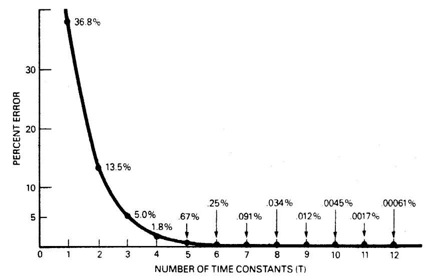
Sampling speed versus bit resolution

- Hardware issues
  - Sampling speed depends on bit resolution
    - Think time constants
    - $$\text{Settling error} = e^{-\frac{t}{\tau}}$$
- Examples:
  - 8-bit resolution takes 5.5τ
  - 12-bit resolution takes 8.3τ
  - 16-bit resolution takes 11τ



Output Settling Error as a Function of Number of Time Constants

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Nyquist–Shannon sampling theorem

- A theorem, developed by Harry Nyquist, and proven by Claude Shannon, which states that an analog signal waveform may be uniquely reconstructed, without error, from samples taken at equal time intervals.

Nyquist–Shannon sampling theorem

- The sampling rate must be equal to, or greater than, twice the highest frequency component in the analog signal.
- Stated differently:
- The highest frequency which can be accurately represented is one-half of the sampling rate.

Nyquist Theorem and Aliasing

- Nyquist Theorem:
  We can digitally represent only frequencies up to half the sampling rate.
  - Example:
    CD: SR=44,100 Hz
    Nyquist Frequency = SR/2 = 22,050 Hz
  - Example:
    SR=22,050 Hz
    Nyquist Frequency = SR/2 = 11,025 Hz

Nyquist Theorem and Aliasing

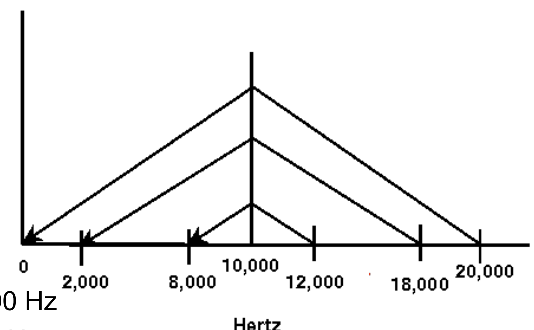
- Frequencies above Nyquist frequency "fold over" to sound like lower frequencies.
  - This foldover is called *aliasing*.

- Aliased frequency f in range [SR/2, SR] becomes f':

$$f' = |f - SR/2|$$

Nyquist Theorem and Aliasing

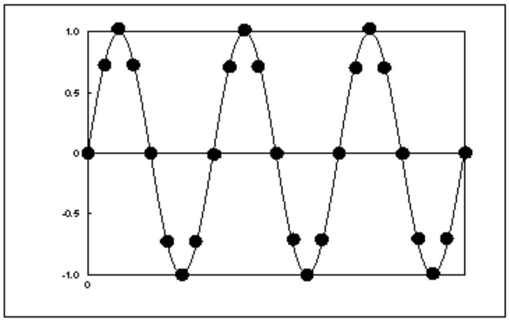
$$f' = |f - SR/2|$$



- <u>Example</u>:
  - SR = 20,000 Hz
  - Nyquist Frequency = 10,000 Hz
  - f = 12,000 Hz --> f' = 8,000 Hz
  - f = 18,000 Hz --> f' = 2,000 Hz
  - f = 20,000 Hz --> f' = 0 Hz

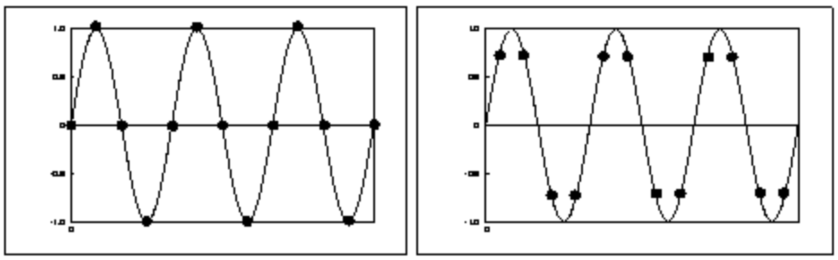
Nyquist Theorem and Aliasing

- Graphical Example 1a:
  - SR = 20,000 Hz
  - Nyquist Frequency = 10,000 Hz
  - f = 2,500 Hz (no aliasing)



Nyquist Theorem and Aliasing

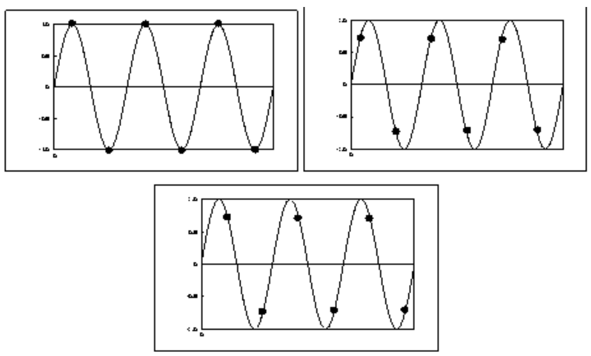
- Graphical Example 1b:
  - SR = 20,000 Hz
  - Nyquist Frequency = 10,000 Hz
  - f = 5,000 Hz (no aliasing)



(left and right figures have same frequency, but have different sampling points)

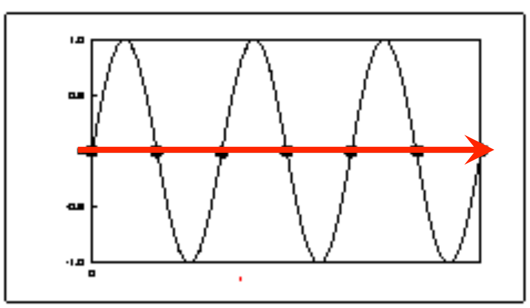
Nyquist Theorem and Aliasing

- Graphical Example 2:
  - SR = 20,000 Hz
  - Nyquist Frequency = 10,000 Hz
  - f = 10,000 Hz (no aliasing)



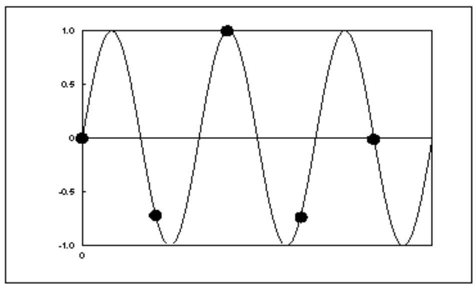
Nyquist Theorem and Aliasing

- Graphical Example 2:
  - BUT, if sample points fall on zero-crossings the sound is completely cancelled out



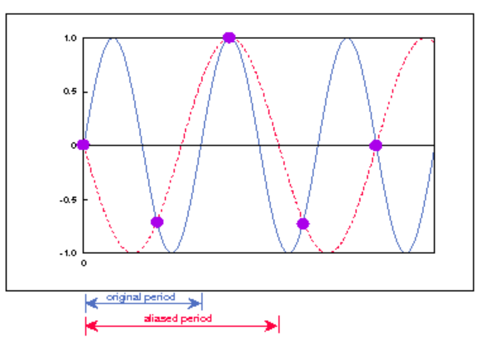
Nyquist Theorem and Aliasing

- Graphical Example 3:
  - SR = 20,000 Hz
  - Nyquist Frequency = 10,000 Hz
  - f = 12,500 Hz, f' = 7,500



Nyquist Theorem and Aliasing

- Graphical Example 3:
  - Fitting the simplest sine wave to the sampled points gives an aliased waveform (dotted line below):



Method to reduce aliasing noise

Input voltage = V

Use low pass filter to remove high frequency before sampling