



An Algorithm for the Evaluation of Finite Trigonometric Series

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Reviewed work(s):

Source: *The American Mathematical Monthly*, Vol. 65, No. 1 (Jan., 1958), pp. 34-35

Published by: [Mathematical Association of America](#)

Stable URL: <http://www.jstor.org/stable/2310304>

Accessed: 22/10/2012 01:57

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3) If G has no elements whose orders divide $n^2 - n$ or if G has no elements whose orders divide $n - 1$ when \bar{n} is an automorphism, then G is Abelian.

It should be noted that if G is the direct product of two groups A and B , where $A(n-1) = (1)$ and $B(n) = (1)$, then \bar{n} leaves A elementwise fixed and maps B into (1) . Hence any group of this type admits \bar{n} as an endomorphism, and some such restriction as in 3) is necessary if G is to be Abelian.

The proof of the proposition is as follows. Since \bar{n} is an endomorphism $a^n b^n = (ab)^n$. If a is cancelled on the left and \overline{b} on the right, then $a^{n-1} b^{n-1} = (\overline{ba})^{n-1}$. It follows that $b^{1-n} a^{1-n} = (\overline{ba})^{1-n}$ and $\overline{1-n}$ is an endomorphism (cf. [1]).

Then $(a a^{n-1} b^n)(a^{1-n} b^{-n}) = (ab)^n (ab)^{1-n} = ab$, and $1 = a^{n-1} b^n a^{1-n} b^{-n}$, or $a^{n-1} b^n = b^n a^{n-1}$. This means that n th powers commute with $(n-1)$ st powers, whence $G(n^2 - n)$ is Abelian (cf. [2] p. 29 Ex. 4).

Now the product of the two endomorphisms \bar{n} by $\overline{1-n}$ is an endomorphism of G onto the Abelian group $G(n^2 - n)$ with kernel $G\{n^2 - n\}$. This proves the first statement of the proposition.

Statement 2) follows from the fact that when \bar{n} is an automorphism, every element is an n th power, and therefore the equation $a^{n-1} b^n = b^n a^{n-1}$ implies that $G(n-1)$ is in the center of the group. It follows that $\overline{1-n}$ is an endomorphism, mapping G onto the Abelian group $G(n-1)$ with kernel $G\{n-1\}$.

Statement 3) follows immediately from 1) and 2).

We are indebted to the referee for the references to the literature.

References

1. J. W. Young, On the holomorphisms of a group, Trans. Amer. Math. Soc., vol. 3, 1902, pp. 186-191.
2. Hans Zassenhaus, Group Theory (English Edition), New York, 1949.

AN ALGORITHM FOR THE EVALUATION OF FINITE TRIGONOMETRIC SERIES

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The algorithm described below enables one to obtain the simultaneous numerical evaluation of $C = \sum_0^N a_k \cos kx$ and $S = \sum_1^N a_k \sin kx$ for given a_k , $\cos x$, and $\sin x$. Tables for $\sin kx$ and $\cos kx$ are not needed and only about N multiplications and about $2N$ additions or subtractions are required, so the method may be of interest to programmers of digital computers.

The algorithm is defined by

$$\begin{aligned} U_{N+2} &= U_{N+1} = 0; \\ U_k &= a_k + 2 \cos x U_{k+1} - U_{k+2}, \quad k = N, N-1, \dots, 1. \\ C &= a_0 + U_1 \cos x - U_2, \quad S = U_1 \sin x. \end{aligned}$$

To establish this result, consider

$$V_k = \sum_{j=k}^N a_j \sin(j - k + 1)x; \quad k = 1, \dots, N,$$

$$V_{N+1} = V_{N+2} = 0.$$

Then

$$\begin{aligned} a_k \sin x + 2 \cos x V_{k+1} - V_{k+2} \\ &= a_k \sin x + \sum_{j=k+1}^N a_j [2 \cos x \sin(j - k)x - \sin(j - k - 1)x] \\ &= a_k \sin x + \sum_{j=k+1}^N a_j \sin(j - k + 1)x = V_k, \end{aligned}$$

whence $V_k = U_k \sin x$ and, in particular, $S = V_1 = U_1 \sin x$. Furthermore

$$\begin{aligned} a_0 \sin x + V_1 \cos x - V_2 &= a_0 \sin x + \sum_{j=1}^N a_j [\cos x \sin jx - \sin(j - 1)x] \\ &= a_0 \sin x + \sum_{j=1}^N a_j \cos jx \sin x = C \sin x, \end{aligned}$$

whence $C = a_0 + U_1 \cos x - U_2$.

CLASSROOM NOTES

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A DIRECT PROOF FOR THE LEAST SQUARES SOLUTION OF A SET OF CONDITION EQUATIONS

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The problem of finding the solution of a set of m independent "condition" equations, linear in the n variables v_1, \dots, v_n , $n > m$

$$(1) \quad \sum_{j=1}^n a_{ij} v_j - a_{i0} = 0, \quad i = 1, \dots, m$$

such that $\sum_1^n v_j^2$ be a minimum is generally solved, following Lagrange, by minimizing instead, an equivalent function involving the so-called Lagrangian multipliers.

The following approach seems more direct, and generalizes a basic theorem in analytic geometry to n dimensions.

Multiplying in turn each of equations (1) by one of the m parameters