3) *If G has no elements whose orders divide n^2-n or if G has no elements whose orders divide $n-1$ when $\bar{n}$ is an automorphism, then G is Abelian.*

It should be noted that if G is the direct product of two groups A and B, where $A(n-1)=(1)$ and $B(n)=(1)$, then $\bar{n}$ leaves A elementwise fixed and maps B into (1). Hence any group of this type admits $\bar{n}$ as an endomorphism, and some such restriction as in 3) is necessary if G is to be Abelian.

The proof of the proposition is as follows. Since $\bar{n}$ is an endomorphism $a^nb^n=(ab)^n$. If a is cancelled on the left and b on the right, then $a^{n-1}b^{n-1}=(ba)^{n-1}$. It follows that $b^{1-n}a^{1-n}=(ba)^{1-n}$ and $\overline{1-n}$ is an endomorphism (*cf.* [1]).

Then $(aa^{n-1}b^n)(a^{1-n}b^{-n}b)=(ab)^n(ab)^{1-n}=ab$, and $1=a^{n-1}b^na^{1-n}b^{-n}$, or $a^{n-1}b^n=b^na^{n-1}$. This means that nth powers commute with $(n-1)$st powers, whence $G(n^2-n)$ is Abelian (*cf.* [2] p. 29 Ex. 4).

Now the product of the two endomorphisms $\bar{n}$ by $\overline{1-n}$ is an endomorphism of G onto the Abelian group $G(n^2-n)$ with kernel $G\{n^2-n\}$. This proves the first statement of the proposition.

Statement 2) follows from the fact that when $\bar{n}$ is an automorphism, every element is an nth power, and therefore the equation $a^{n-1}b^n=b^na^{n-1}$ implies that $G(n-1)$ is in the center of the group. It follows that $\overline{1-n}$ is an endomorphism, mapping G onto the Abelian group $G(n-1)$ with kernel $G\{n-1\}$.

Statement 3) follows immediately from 1) and 2).

We are indebted to the referee for the references to the literature.

References

1. J. W. Young, On the holomorphisms of a group, Trans. Amer. Math. Soc., vol. 3, 1902, pp. 186–191.

2. Hans Zassenhaus, Group Theory (English Edition), New York, 1949.

AN ALGORITHM FOR THE EVALUATION OF FINITE TRIGONOMETRIC SERIES

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The algorithm described below enables one to obtain the simultaneous numerical evaluation of $C=\sum_0^N a_k \cos kx$ and $S=\sum_1^N a_k \sin kx$ for given a_k, $\cos x$, and $\sin x$. Tables for $\sin kx$ and $\cos kx$ are not needed and only about N multiplications and about $2N$ additions or subtractions are required, so the method may be of interest to programmers of digital computers.

The algorithm is defined by

$$U_{N+2} = U_{N+1} = 0;$$

$$U_k = a_k + 2\cos xU_{k+1} - U_{k+2}, \qquad k = N, N-1, \cdots, 1\cdot$$

$$C = a_0 + U_1 \cos x - U_2, \qquad S = U_1 \sin x.$$

To establish this result, consider

$$V_k = \sum_{j=k}^{N} a_j \sin(j-k+1)x; \qquad k = 1, \cdots, N,$$

$$V_{N+1} = V_{N+2} = 0.$$

Then

$$a_k \sin x + 2\cos x V_{k+1} - V_{k+2}$$
$$= a_k \sin x + \sum_{j=k+1}^{N} a_j[2\cos x \sin(j-k)x - \sin(j-k-1)x]$$
$$= a_k \sin x + \sum_{j=k+1}^{N} a_j \sin(j-k+1)x = V_k,$$

whence $V_k = U_k \sin x$ and, in particular, $S = V_1 = U_1 \sin x$. Furthermore

$$a_0 \sin x + V_1 \cos x - V_2 = a_0 \sin x + \sum_{j=1}^{N} a_j[\cos x \sin jx - \sin(j-1)x]$$
$$= a_0 \sin x + \sum_{j=1}^{N} a_j \cos jx \sin x = C \sin x,$$

whence $C = a_0 + U_1 \cos x - U_2$.

CLASSROOM NOTES

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A DIRECT PROOF FOR THE LEAST SQUARES SOLUTION OF A SET OF CONDITION EQUATIONS

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The problem of finding the solution of a set of m independent "condition" equations, linear in the n variables $v_1, \cdots, v_n$, $n > m$

$$(1) \qquad \sum_{j=1}^{n} a_{ij}v_j - a_{i0} = 0, \qquad i = 1, \cdots, m$$

such that $\sum_1^n v_j^2$ be a minimum is generally solved, following Lagrange, by minimizing instead, an equivalent function involving the so-called Lagrangian multipliers.

The following approach seems more direct, and generalizes a basic theorem in analytic geometry to n dimensions.

Multiplying in turn each of equations (1) by one of the m parameters