Excerpt from

An Introduction to Low-Density Parity-Check Codes

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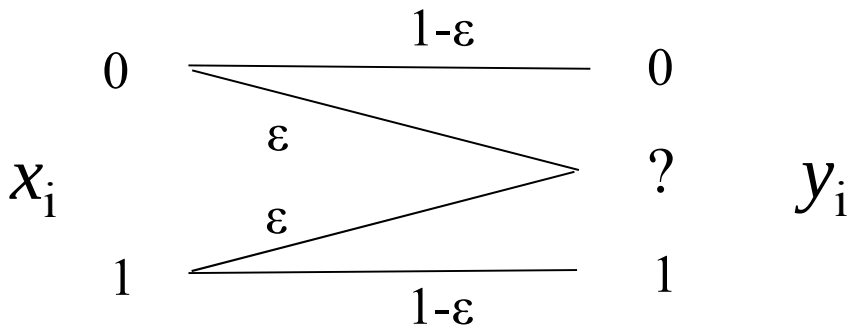
Decoding for the BEC

- Recall: Binary erasure channel, BEC(ε)



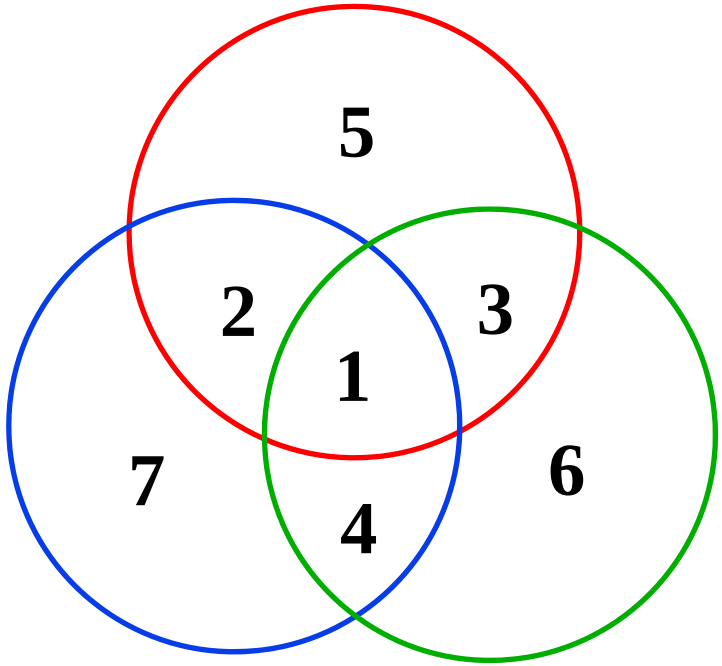
$x = (x_1, x_2, \ldots, x_n)$ transmitted codeword

$y = (y_1, y_2, \ldots, y_n)$ received word

- **Note:** if $y_i \in \{0,1\}$, then $x_i = y_i$.

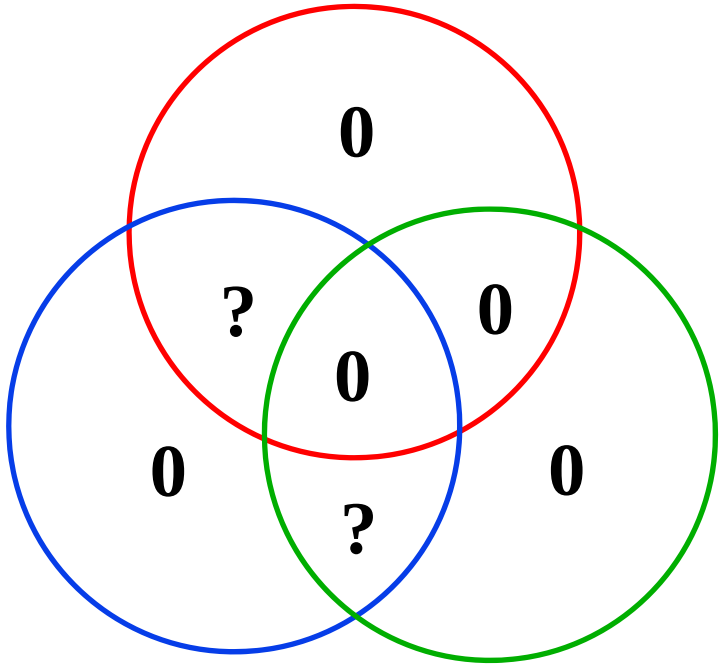
Local Decoding of Erasures

- $d_{min} = 3$, so any two erasures can be uniquely filled to get a codeword.
- Decoding can be done *locally*: Given any pattern of one or two erasures, there will always be a parity-check (circle) involving exactly one erasure.
- The parity-check represented by the circle can be used to fill in the erased bit.
- This leaves at most one more erasure. Any parity-check (circle) involving it can be used to fill it in.



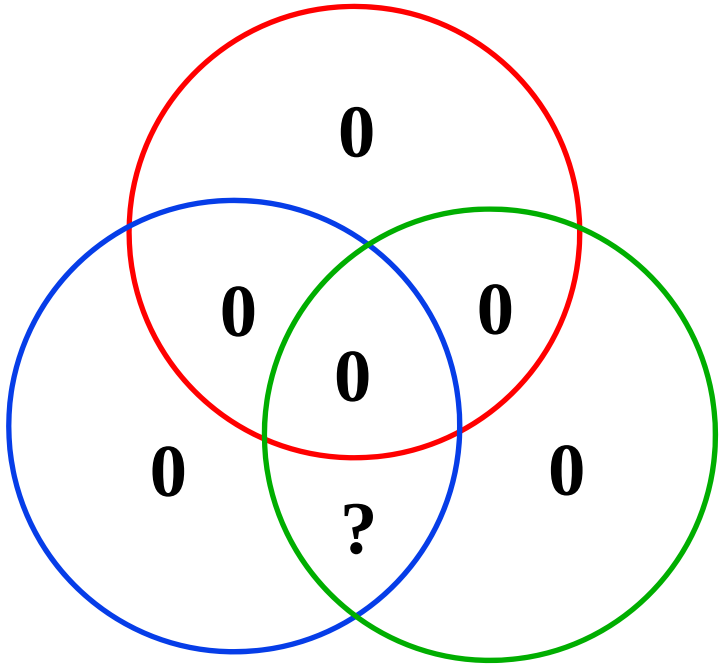
Local Decoding - Example

- All-0's codeword transmitted.
- Two erasures as shown.
- Start with either the red parity or green parity circle.
- The red parity circle requires that the erased symbol inside it be 0.



Local Decoding -Example

- Next, the green parity circle or the blue parity circle can be selected.
- Either one requires that the remaining erased symbol be 0.



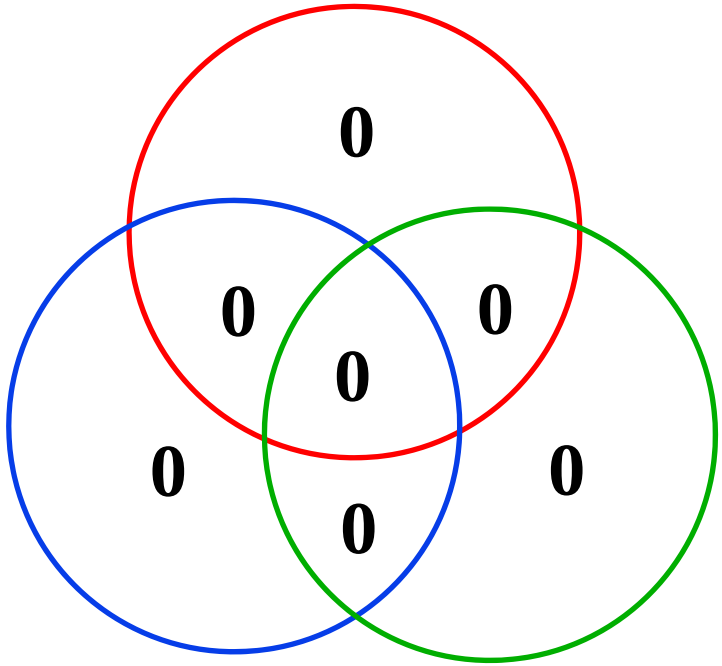
Local Decoding -Example

- Estimated codeword:

  [0 0 0 0 0 0 0]

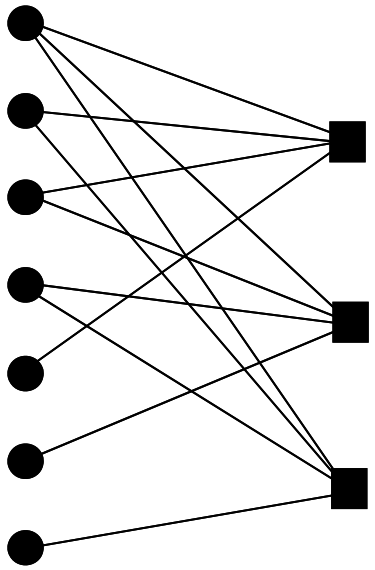
- Decoding successful!!

- This procedure would have worked no matter which codeword was transmitted.

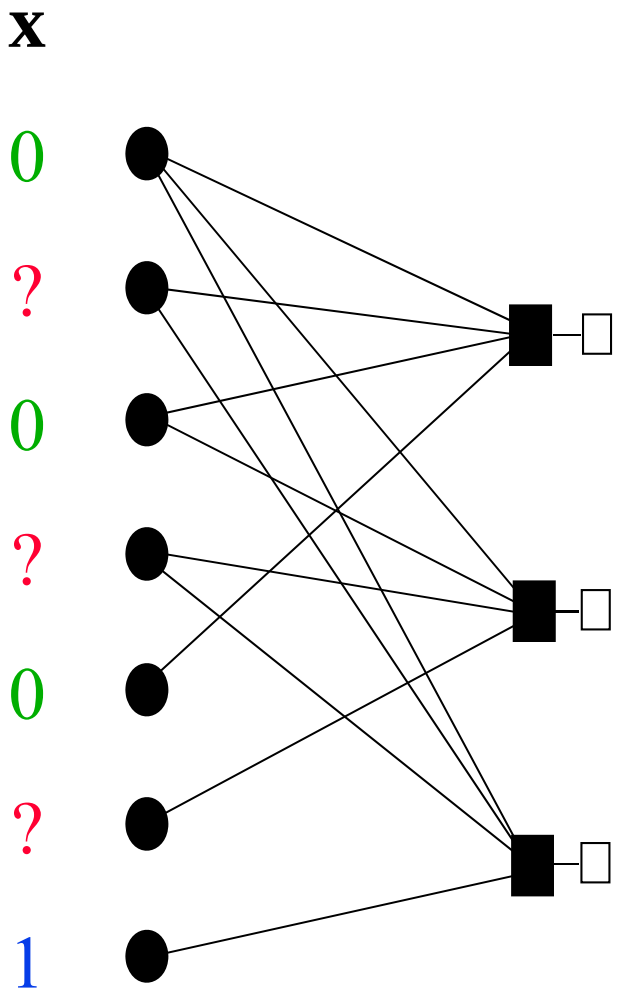


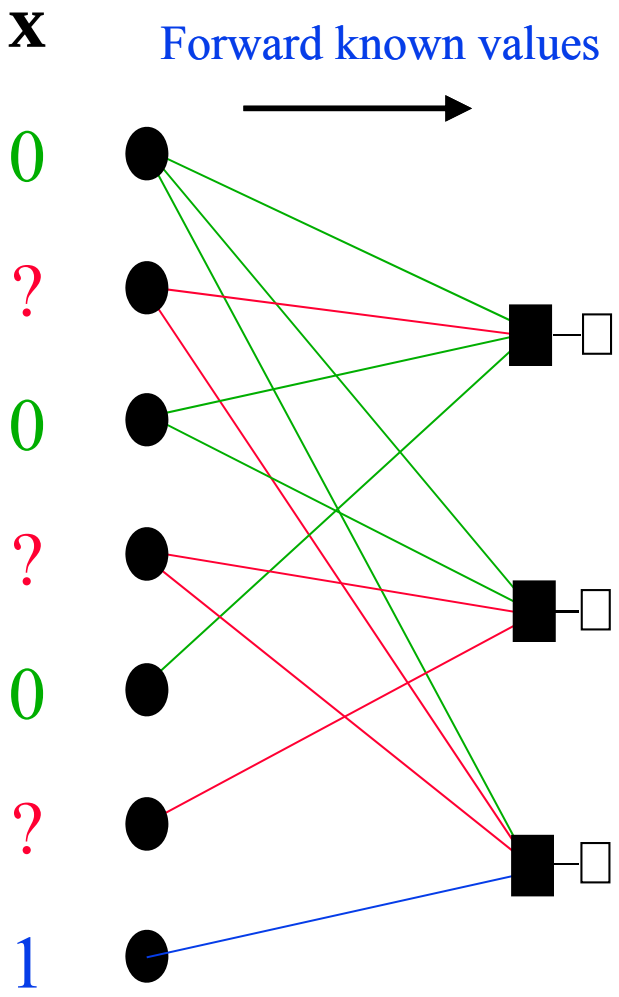
Decoding with the Tanner Graph: an a-Peeling Decoder

- Initialization:
  - Forward known variable node values along outgoing edges
  - Accumulate forwarded values at check nodes and "record" the parity
  - Delete known variable nodes and all outgoing edges

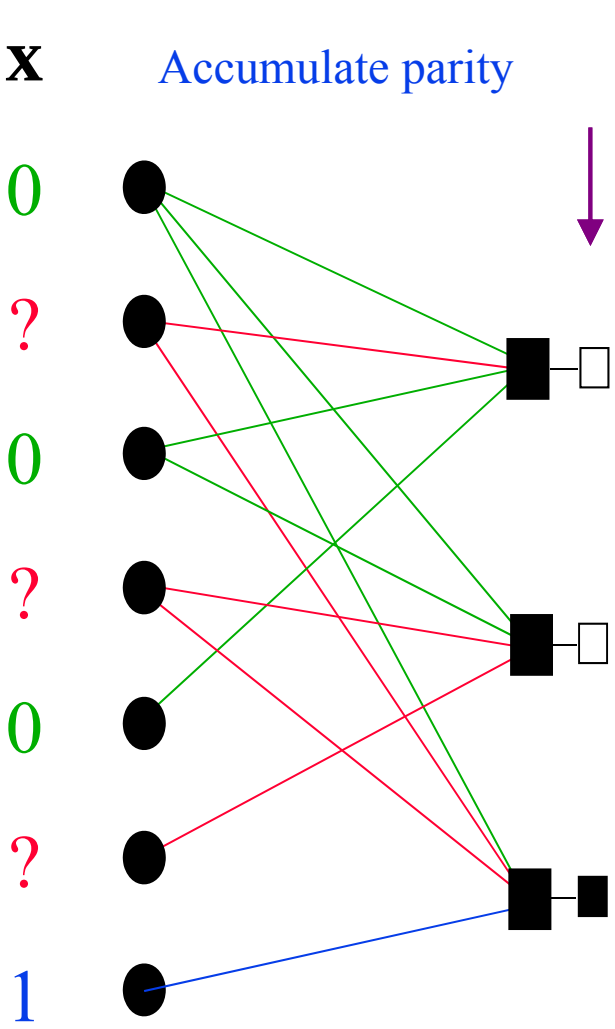


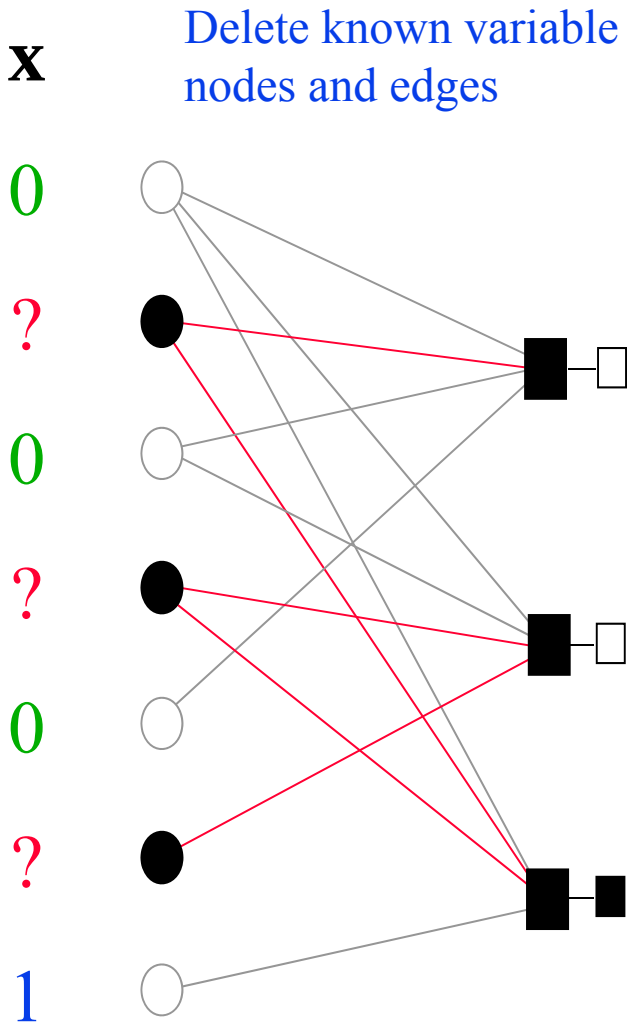
Peeling Decoder – Initialization





Peeling Decoder - Initialization

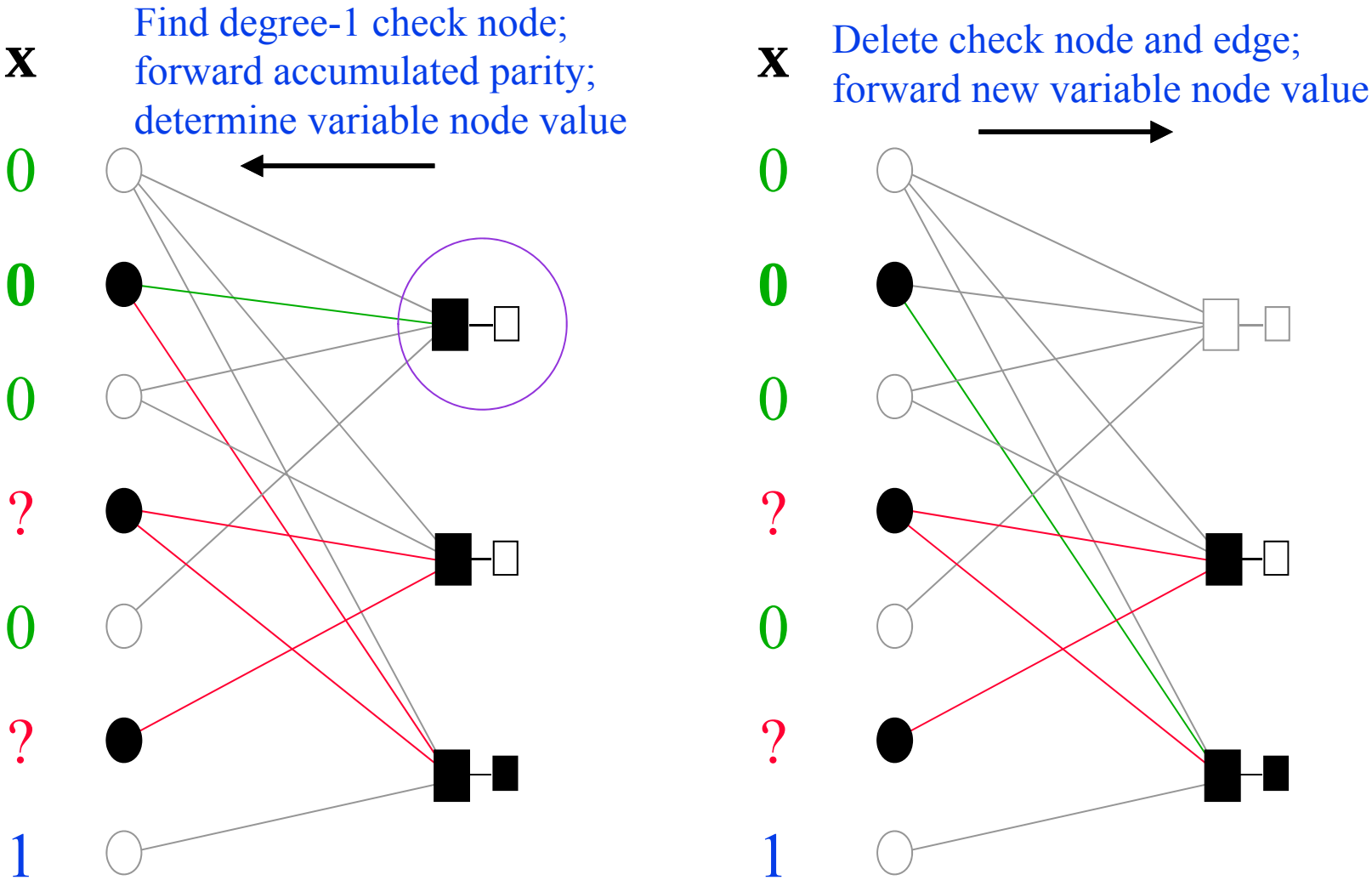




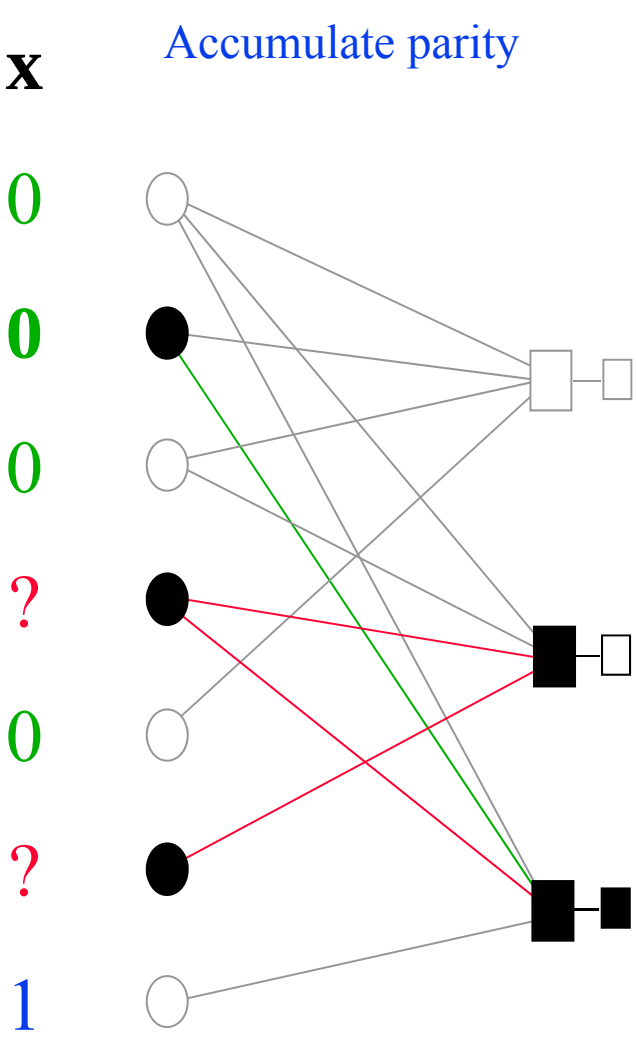
Decoding with the Tanner Graph: an a-Peeling Decoder

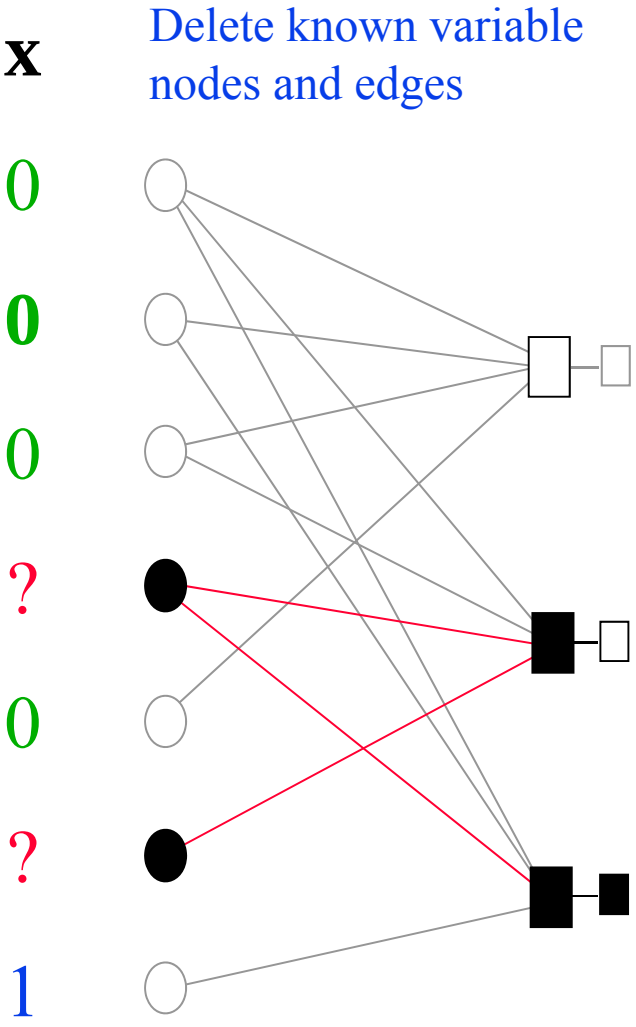
- Decoding step:
  - Select, if possible, a check node with one edge remaining; forward its parity, thereby determining the connected variable node
  - Delete the check node and its outgoing edge
  - Follow procedure in the initialization process at the known variable node
- Termination
  - If remaining graph is empty, the codeword is determined
  - If decoding step gets stuck, declare decoding failure

Peeling Decoder – Step 1



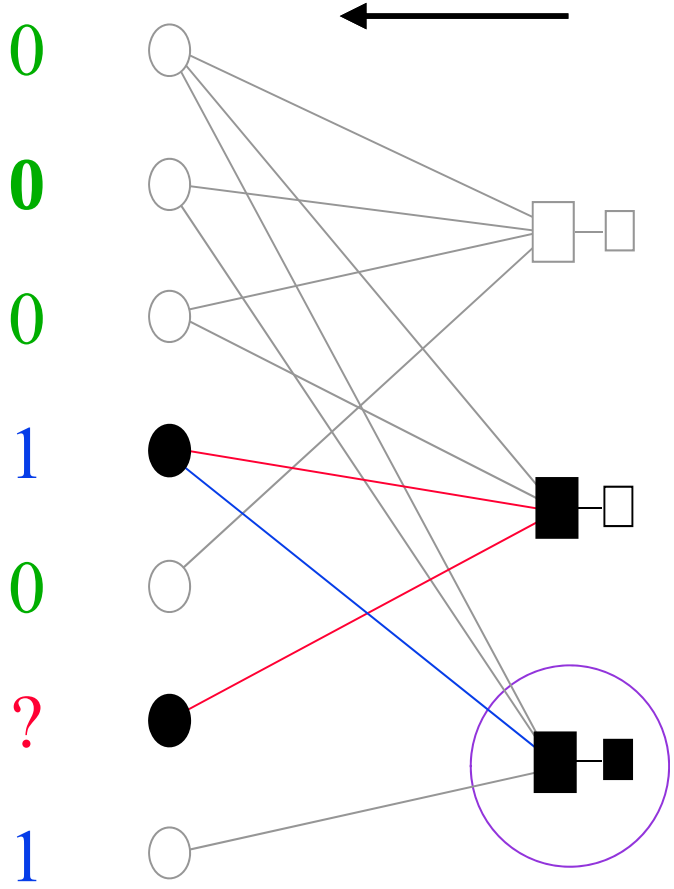
Peeling Decoder – Step 1



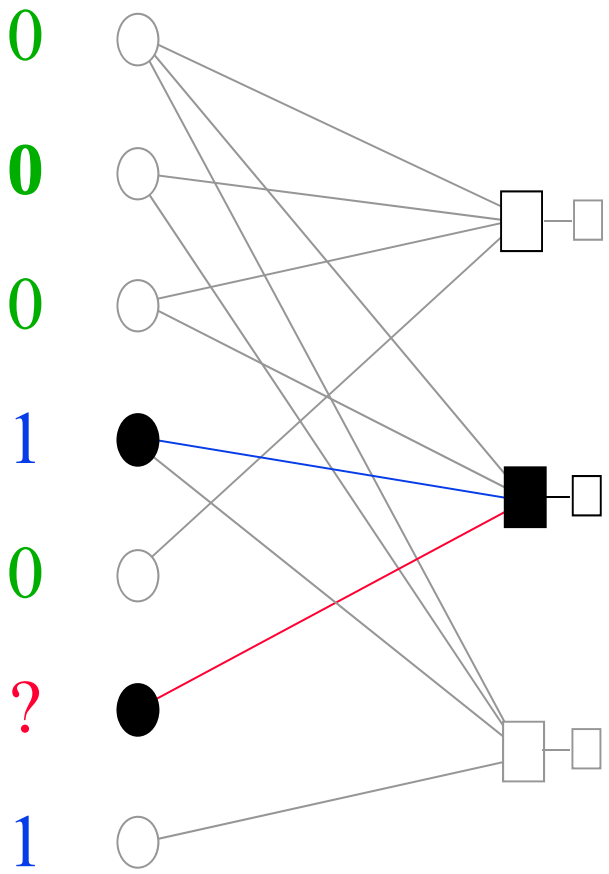


Peeling Decoder – Step 2

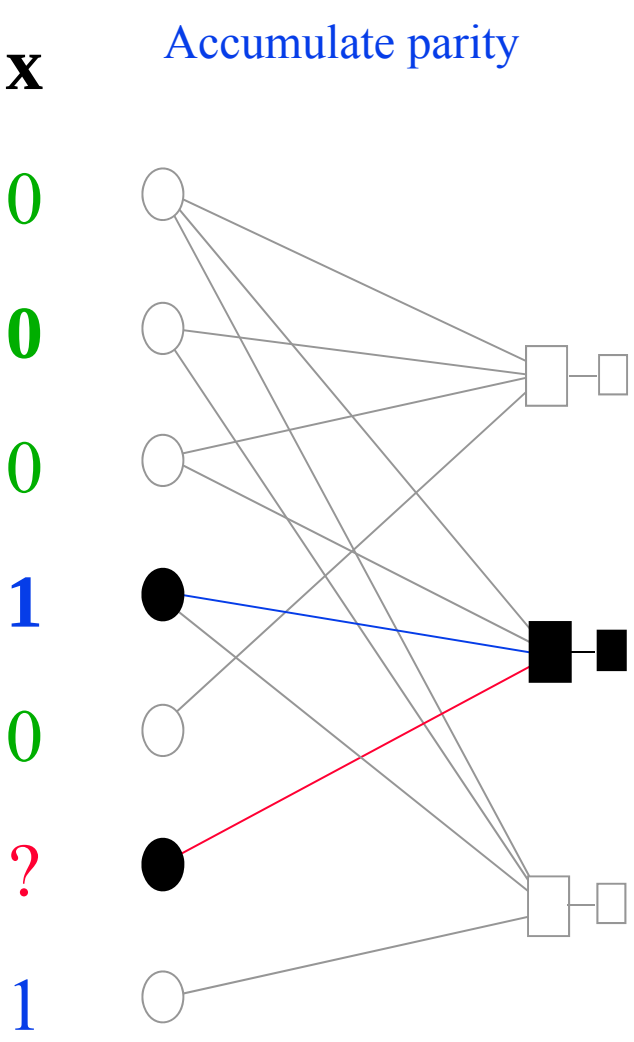
Find degree-1 check node;
forward accumulated parity;
determine variable node value

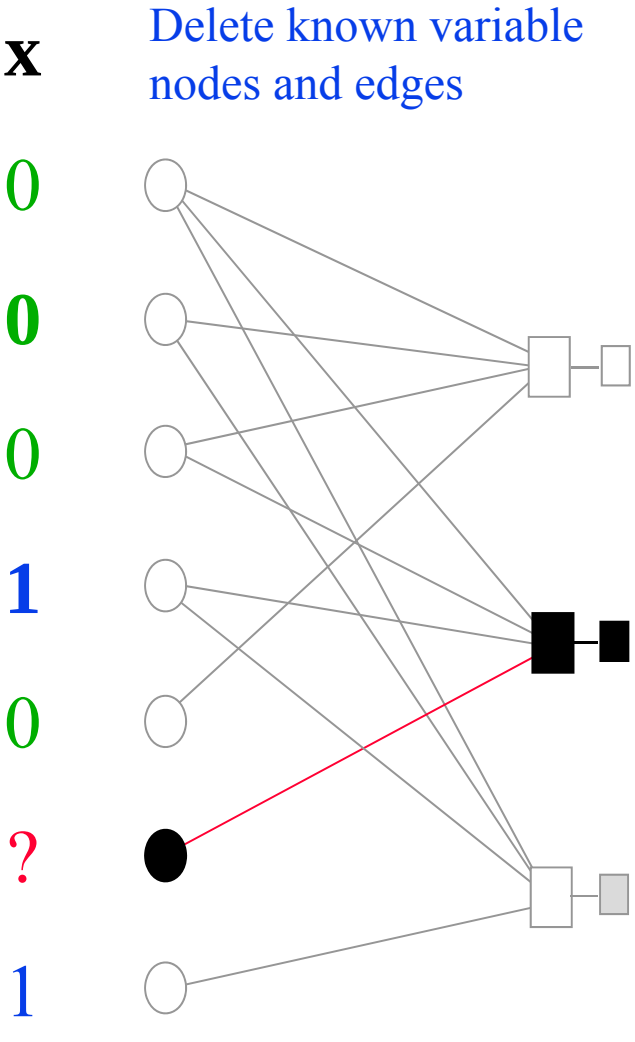


Delete check node and edge;
forward new variable node value



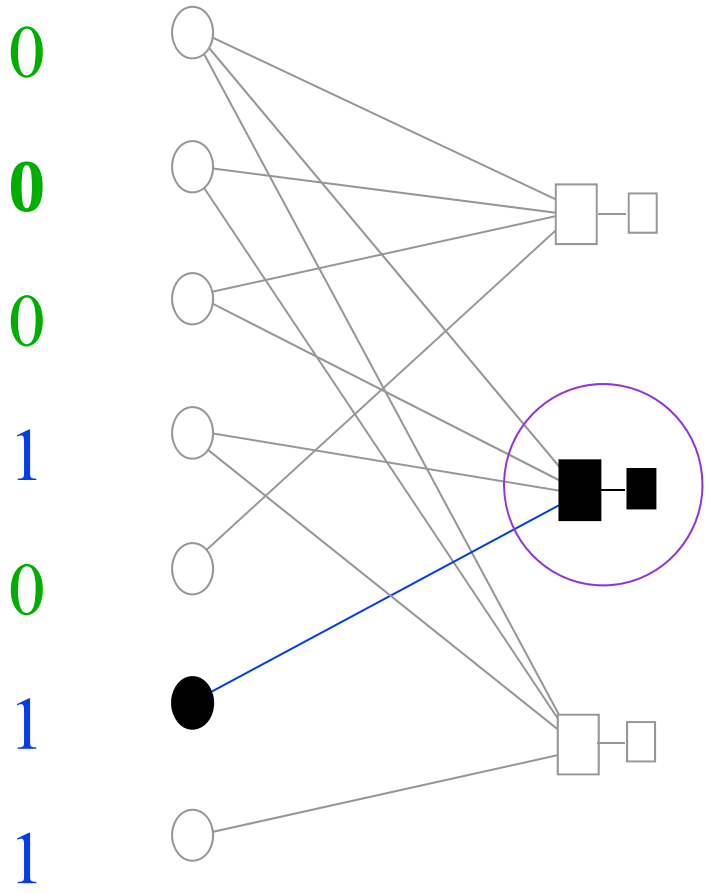
Peeling Decoder – Step 2



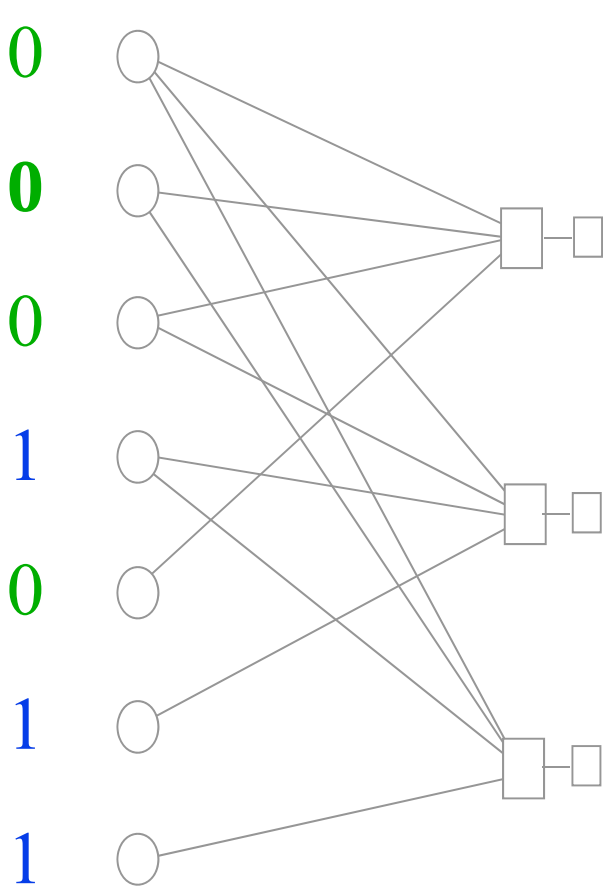


Peeling Decoder – Step 3

Find degree-1 check node;
forward accumulated parity;
determine variable node value



Delete check node and edge;
decoding complete



Message-Passing Decoding

- The local decoding procedure can be described in terms of an iterative, "message-passing" algorithm in which all variable nodes and all check nodes in parallel iteratively pass messages along their adjacent edges.
- The values of the code bits are updated accordingly.
- The algorithm continues until all erasures are filled in, or until the completion of a specified number of iterations.