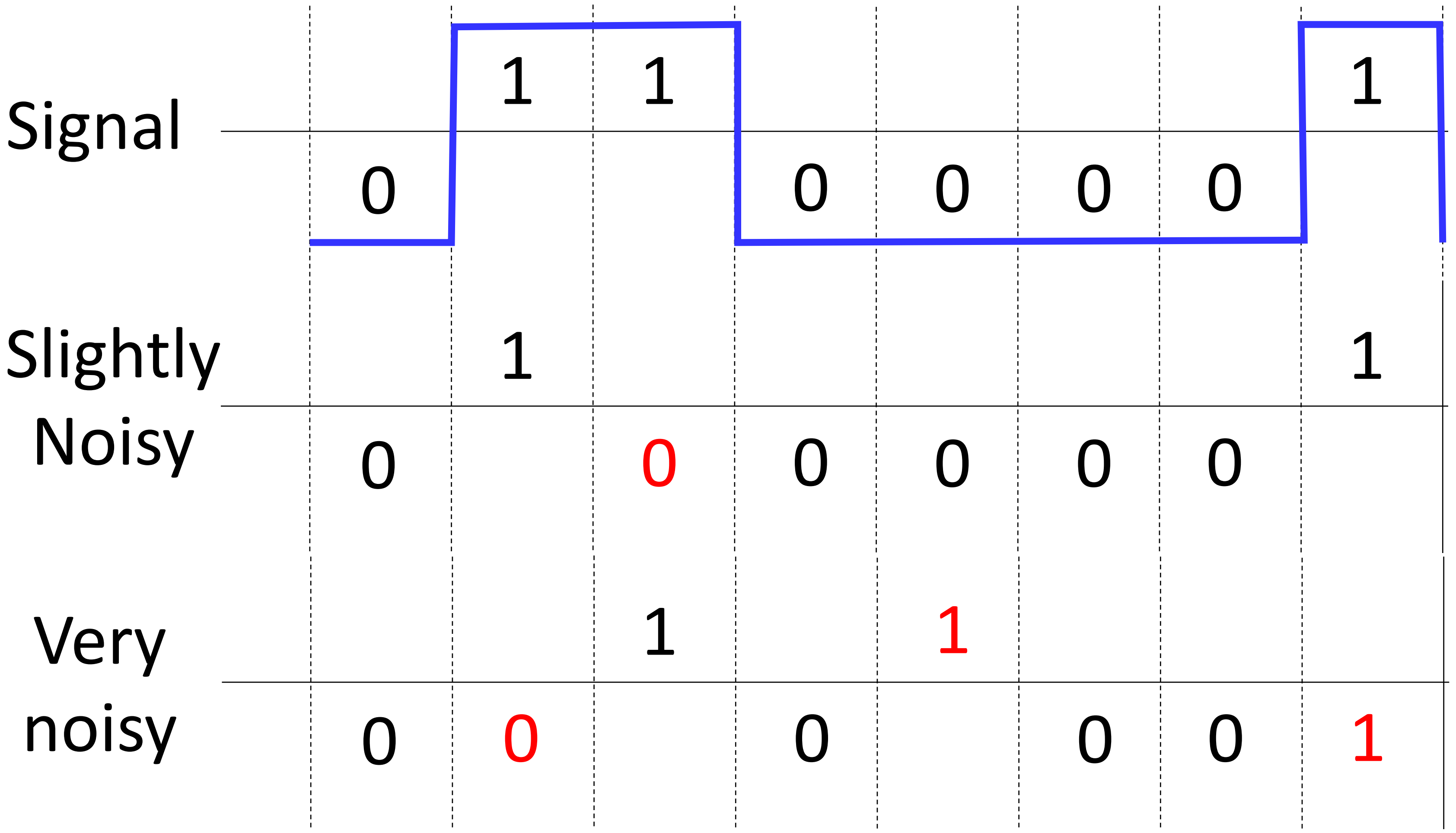
Link Layer: Error detection and correction

Problem: Noise may Flip Received Bits

- Link layers provides some protection
  - Detect errors with codes
  - Correct errors with codes
  - Retransmit lost frames Later
- Reliability concern cuts across the layers
  - E.g, TCP in the transport layer, DNS in the app layer

Problem: Noise may Flip Received Bits



Ideas?

Approach – Add Redundancy

- Error detection codes: Add check bits to the message bits to let some errors be detected
- Error correction codes: Add more check bits to let some errors be corrected

- Key issue: Structure the code such that
  - Need few check bits to detect/correct many errors
  - Modest computation

Motivating Example

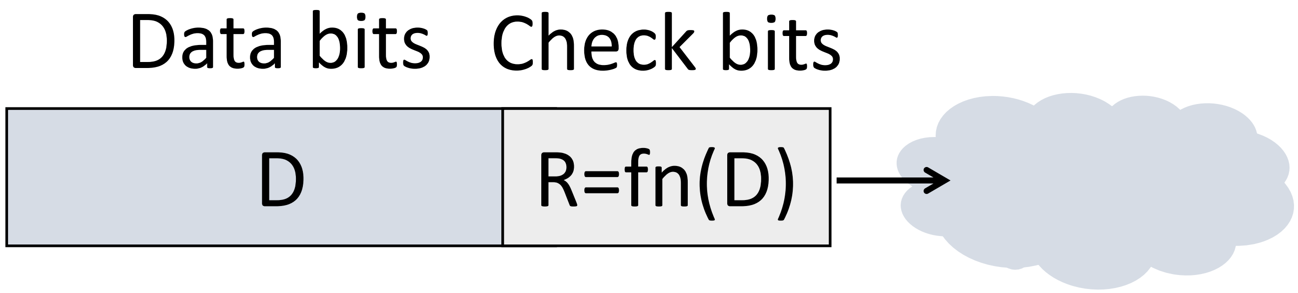
- A simple code to handle errors:
  - Send two copies! Error detected if different.
- How good is this code?
  - How many errors can it detect/correct?
  - How many errors will make it fail?

Want to Handle More Errors w/ Fewer Bits

- We’ll look at better codes (applied mathematics)
  - But, they can’t handle all errors
  - And they focus on accidental (random) errors

Using Error Codes

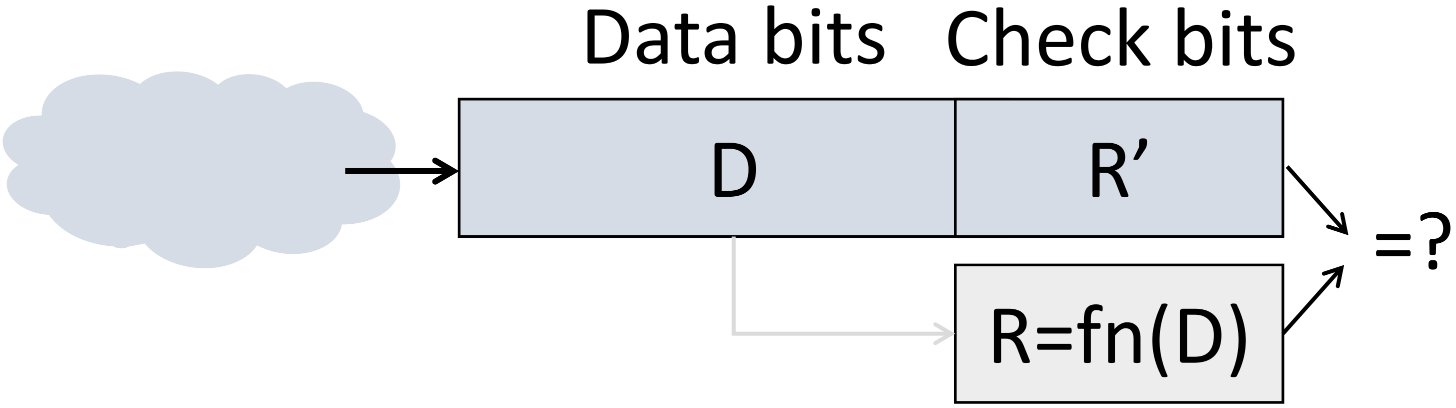
- Codeword consists of D data plus R check bits (=systematic block code)



- Sender:
  - Compute R check bits based on the D data bits; send the codeword of D+R bits

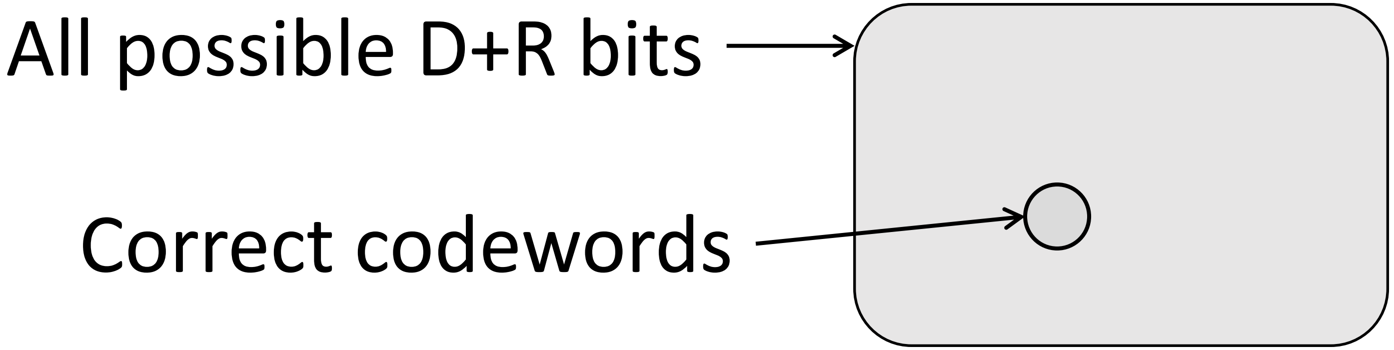
Using Error Codes (2)

- Receiver:
  - Receive D+R bits with unknown errors
  - Recompute R check bits based on the D data bits
  - Error detected if R doesn't match R'



Intuition for Error Codes

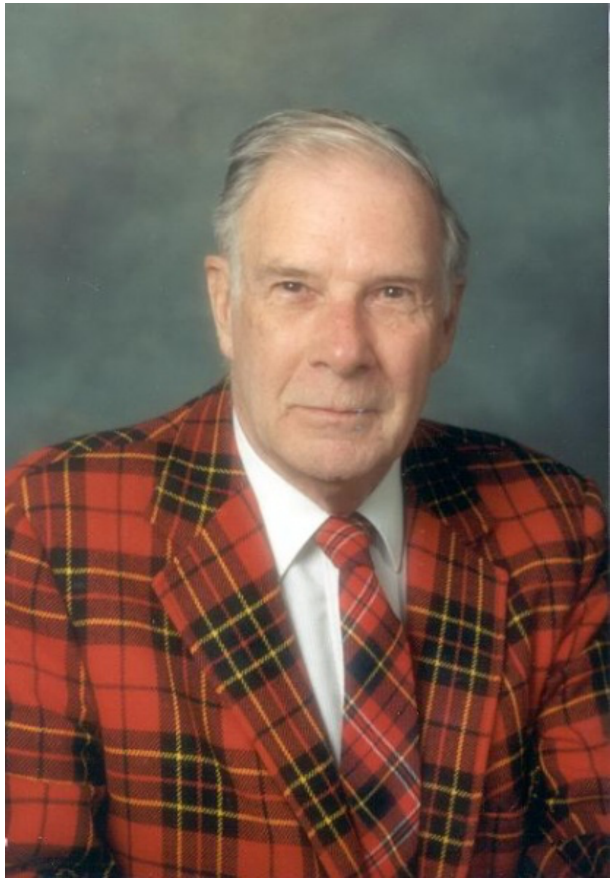
- For D data bits, R check bits:



- Randomly chosen D+R bits is unlikely to be correct
  - Low, controllable overhead

R.W. Hamming (1915-1998)

- Much early work on codes:
  - "Error Detecting and Error Correcting Codes", BSTJ, 1950
- See also:
  - "You and Your Research", 1986



Hamming Distance

- Distance is the number of bit flips needed to change D_1 to D_2

- Hamming distance of a coding is the minimum error distance between any pair of codewords (bit-strings) that cannot be detected

Hamming Distance (2)

- Error detection:
  - For a coding of distance d+1, up to d errors will always be detected
- Error correction:
  - For a coding of distance 2d+1, up to d errors can always be corrected by mapping to the closest valid codeword

Simple Error Detection – Parity Bit

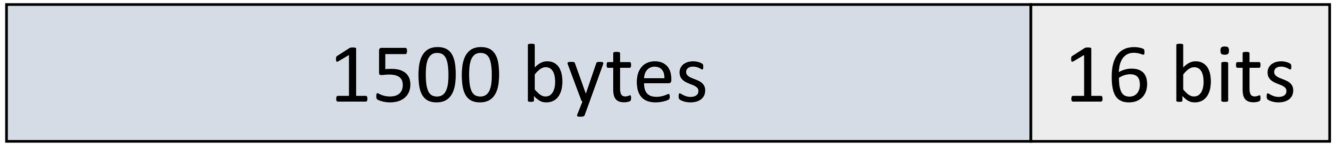
- Take D data bits, add 1 check bit
  - Check bit could be sum modulo 2 or XOR

Parity Bit (2)

- How well does parity work?
  - What is the distance of the code?
  - How many errors will it detect/correct?
- What about larger errors?

Checksums

- Idea: sum up data in N-bit words
  - Widely used in, e.g., TCP/IP/UDP



- Stronger protection than parity

Internet Checksum

- Sum is defined in 1s complement arithmetic (must add back carries)
  - And it's the negative sum
- *"The checksum field is the 16 bit one's complement of the one's complement sum of all 16 bit words …"* – RFC 791

Internet Checksum (2)

Sending:

1. Arrange data in 16-bit words
2. Put zero in checksum position, add
3. Add any carryover back to get 16 bits
4. Negate (complement) to get sum

```
0001
f204
f4f5
f6f7
```

Internet Checksum (3)

Sending:

1. Arrange data in 16-bit words
2. Put zero in checksum position, add
3. Add any carryover back to get 16 bits
4. Negate (complement) to get sum

```
  0001
  f204
  f4f5
  f6f7
+(0000)
------
 2ddf1

  ddf1
+    2
------
  ddf3

  220c
```

Internet Checksum (4)

Receiving:

1. Arrange data in 16-bit words
2. Checksum will be non-zero, add
3. Add any carryover back to get 16 bits
4. Negate the result and check it is 0

```
  0001
  f204
  f4f5
  f6f7
+ 220c
------
```

Internet Checksum (5)

Receiving:

1. Arrange data in 16-bit words
2. Checksum will be non-zero, add
3. Add any carryover back to get 16 bits
4. Negate the result and check it is 0

```
  0001
  f204
  f4f5
  f6f7
+ 220c
------
 2fffd

  fffd
+    2
------
  ffff

  0000
```

Internet Checksum (6)

- How well does the checksum work?
  - What is the distance of the code?
  - How many errors will it detect/correct?

Why Error Correction is Hard

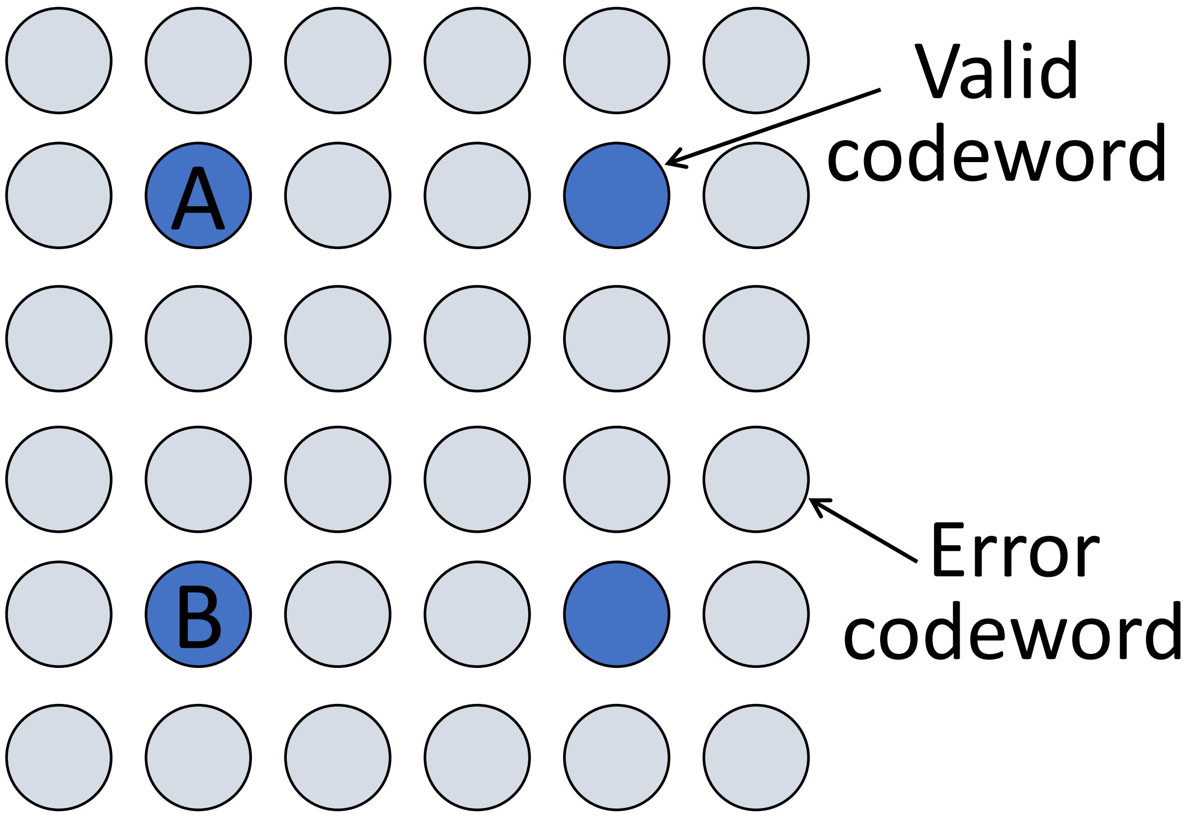
- If we had reliable check bits we could use them to narrow down the position of the error
  - Then correction would be easy
- But error could be in the check bits as well as the data bits!
  - Data might even be correct

Intuition for Error Correcting Code

- Suppose we construct a code with a Hamming distance of at least 3
  - Need ≥3 bit errors to change one valid codeword into another
  - Single bit errors will be closest to a unique valid codeword
- If we assume errors are only 1 bit, we can correct mapping an error to the closest valid codeword
  - Works for d errors if HD ≥ 2d + 1

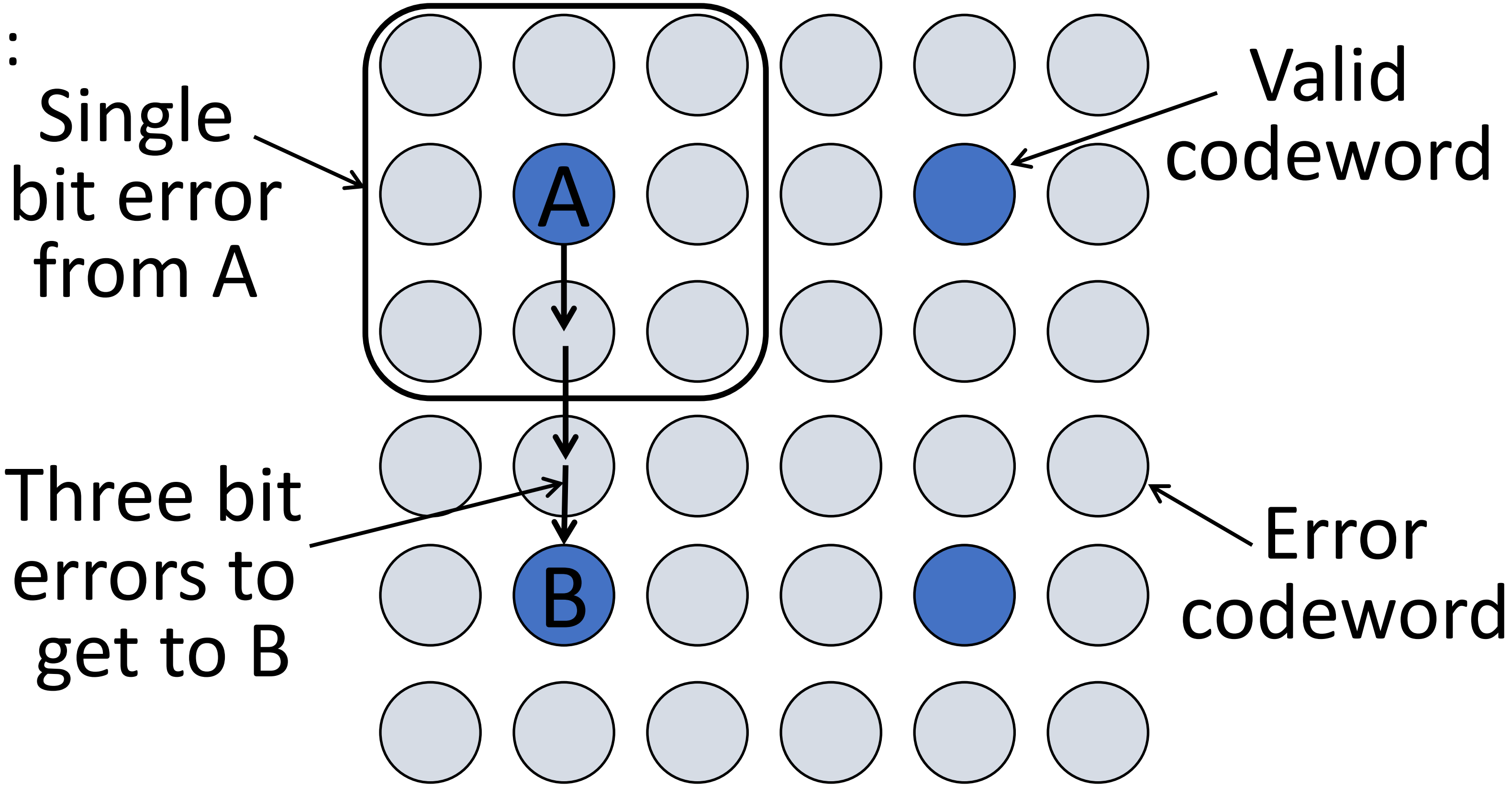
Intuition (2)

- Visualization of code:



Intuition (3)

- Visualization of code:



Hamming Code

- Gives a method for constructing a code with a distance of 3
  - Uses $n = 2^k - k - 1$, e.g., n=4, k=3
  - Put check bits in positions p that are powers of 2, starting with position 1
  - N-th check bit is parity of bit positions with n-th LSBit is same as p's
- Plus an easy way to correct [soon]

Hamming Code (2)

Cheat sheet

1: 0001

2: 0010

3: 0011

4: 0100

5: 0101

6: 0110

7: 0111

- Example: data=0101, 3 check bits
  - 7 bit code, check bit positions 1, 2, 4
  - Check 1 covers positions 1, 3, 5, 7 (LSB is 1)
  - Check 2 covers positions 2, 3, 6, 7 (2nd LSB is 1)
  - Check 4 covers positions 4, 5, 6, 7 (3rd LSB is 1)

0	1	0	0	1	0	1
1	2	3	4	5	6	7

$p_1 = 0+1+1 = 0$, $p_2 = 0+0+1 = 1$, $p_4 = 1+0+1 = 0$

Hamming Code (3)

- To decode:
  - Recompute check bits (with parity sum including the check bit)
  - Arrange as a binary number
  - Value (syndrome) tells error position
  - Value of zero means no error
  - Otherwise, flip bit to correct

Hamming Code (5)

- Example, continued

$$\longrightarrow \underline{0}\ \underline{1}\ 0\ \underline{0}\ 1\ 0\ 1$$

1 2 3 4 5 6 7

p_1=

p_2=

p_4=

Syndrome =

Data =

Hamming Code (6)

- Example, continued

$$\longrightarrow \underline{0}\ \underline{1}\ 0\ \underline{0}\ 1\ 0\ 1$$

1 2 3 4 5 6 7

p_1= 0+0+1+1 = 0, p_2= 1+0+0+1 = 0,
p_4= 0+1+0+1 = 0

Syndrome = 000, no error
Data = 0 1 0 1

Hamming Code (7)

- Example, continued

$$\longrightarrow \underline{0}\ \ \underline{1}\ \ 0\ \ \underline{0}\ \ 1\ \ 1\ \ 1$$

1 2 3 4 5 6 7

p_1=

p_2=

p_4=

Syndrome =

Data =

Hamming Code (8)

- Example, continued

$$\longrightarrow \underset{1}{\underline{0}}\ \ \underset{2}{\underline{1}}\ \ \underset{3}{0}\ \ \underset{4}{\underline{0}}\ \ \underset{5}{1}\ \ \underset{6}{1}\ \ \underset{7}{1}$$

$p_1 = 0+0+1+1 = 0$, $p_2 = 1+0+1+1 = 1$,
$p_4 = 0+1+1+1 = 1$

Syndrome = 1 1 0, flip position 6
Data = 0 1 0 1 (correct after flip!)

Hamming Code (3)

- Example: bad message 0100111
  - 7 bit code, check bit positions 1, 2, 4
  - Check 1 covers positions 1, 3, 5, 7
  - Check 2 covers positions 2, 3, 6, 7
  - Check 4 covers positions 4, 5, 6, 7

$$\begin{array}{ccccccc} 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \longrightarrow$$

$p_1 = 0+0+1+1 = 0$, $p_2 = 1+0+1+1 = 1$, $p_4 = 0+1+1+1 = 1$

Hamming Code (3)

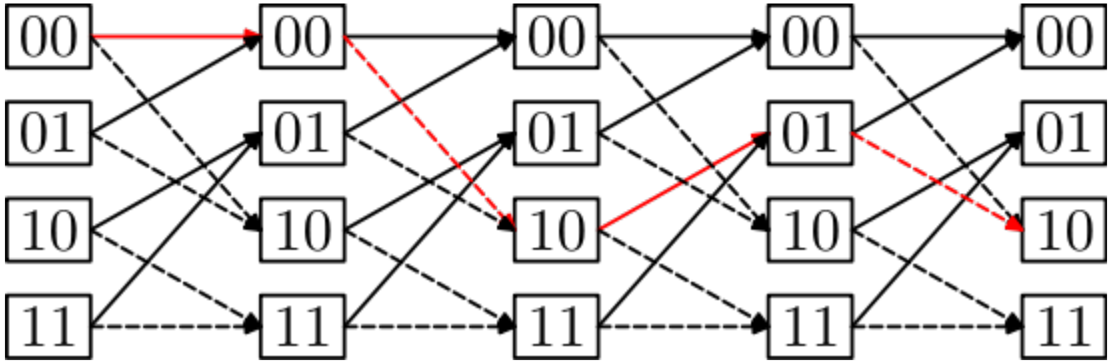
- Example: bad message 0100111
  - 7 bit code, check bit positions 1, 2, 4
  - Check 1 covers positions 1, 3, 5, 7
  - Check 2 covers positions 2, 3, 6, 7
  - Check 4 covers positions 4, 5, 6, 7

$$\underset{1}{0}\ \underset{2}{1}\ \underset{3}{0}\ \underset{4}{0}\ \underset{5}{1}\ \underset{6}{1}\ \underset{7}{1} \longrightarrow$$

$p_1 = 0+0+1+1 = 0$, $p_2 = 1+0+1+1 = 1$, $p_4 = 0+1+1+1 = 1$

Other Error Correction Codes

- Real codes are more involved than Hamming
- E.g., Convolutional codes (§3.2.3)
  - Take a stream of data and output a mix of the input bits
  - Makes each output bit less fragile
  - Decode using Viterbi algorithm (uses bit confidence values)



Detection vs. Correction

- Which is better will depend on the pattern of errors. For example:
  - 1000 bit messages with a bit error rate (BER) of 1 in 10000

- Which has less overhead?

Detection vs. Correction

- Which is better will depend on the pattern of errors. For example:
  - 1000 bit messages with a bit error rate (BER) of 1 in 10000

- Which has less overhead?
  - It still depends! We need to know more about the errors

Detection vs. Correction (2)

Assume bit errors are random

- Messages have 0 or maybe 1 error (1/10 of the time)

Error correction:

- Need ~10 check bits per message
- Overhead:
  - 10 bits per message

Error detection:

- Need ~1 check bits per message plus 1000 bit retransmission
- Overhead:
  - 101 bits per message

Detection vs. Correction (3)

Assume errors come in bursts of 100

- Only 1 or 2 messages in 1000 have significant (multi-bit) errors

Error correction:

- Need >>100 check bits per message
- Overhead:
  - >> 100 bpm

Error detection:

- Need 32 check bits per message plus 1000 bit resend 2/1000 of the time
- Overhead:
  - 34 bits per message

Detection vs. Correction (4)

- Error correction:
  - Needed when errors are expected
  - Or when no time for retransmission

- Error detection:
  - More efficient when errors are not expected
  - And when errors are large when they do occur

Error Correction in Practice

- Heavily used in physical layer
  - Used for demanding links like 802.11, DVB, WiMAX, power-line, …
  - Convolutional codes widely used in practice
- Error detection (w/ retransmission) is used in the link layer and above for residual errors
- Correction also used in the application layer
  - Called Forward Error Correction (FEC)
  - Normally with an erasure error model
  - E.g., Reed-Solomon (CDs, DVDs, etc.)

Error Correction in Practice (2)

- Everywhere! It is a key issue
  - Different layers contribute differently