Topic

- Some bits may be received in error due to noise. How do we detect this?
  - Parity »
  - Checksums »
  - CRCs »
- Detection will let us fix the error, for example, by retransmission (later).

Simple Error Detection – Parity Bit

- Take D data bits, add 1 check bit that is the sum of the D bits
  - Sum is modulo 2 or XOR

Parity Bit (2)

- How well does parity work?
  - What is the distance of the code?
  - How many errors will it detect/correct?
- What about larger errors?

Checksums

- Idea: sum up data in N-bit words
  - Widely used in, e.g., TCP/IP/UDP

1500 bytes	16 bits

- Stronger protection than parity

Internet Checksum

- Sum is defined in 1s complement arithmetic (must add back carries)
  - And it’s the negative sum
- *“The checksum field is the 16 bit one's complement of the one's complement sum of all 16 bit words …”* – RFC 791

Internet Checksum (2)

Sending:

1. Arrange data in 16-bit words
2. Put zero in checksum position, add
3. Add any carryover back to get 16 bits
4. Negate (complement) to get sum

```
0001
f203
f4f5
f6f7
```

Internet Checksum (3)

Sending:

1. Arrange data in 16-bit words
2. Put zero in checksum position, add
3. Add any carryover back to get 16 bits
4. Negate (complement) to get sum

```
   0001
   f203
   f4f5
   f6f7
+(0000)
------
  2ddf0
    ↓
   ddf0
+     2
------
   ddf2
    ↓
   220d
```

Internet Checksum (4)

Receiving:

1. Arrange data in 16-bit words
2. Checksum will be non-zero, add

```
  0001
  f203
  f4f5
  f6f7
+ 220d
------
```

3. Add any carryover back to get 16 bits

4. Negate the result and check it is 0

Internet Checksum (5)

Receiving:

1. Arrange data in 16-bit words
2. Checksum will be non-zero, add
3. Add any carryover back to get 16 bits
4. Negate the result and check it is 0

```
  0001
  f203
  f4f5
  f6f7
+ 220d
------
 2fffd
   ↓
  fffd
+    2
------
  ffff
   ↓
  0000
```

Internet Checksum (6)

- How well does the checksum work?
  - What is the distance of the code?
  - How many errors will it detect/correct?

- What about larger errors?

Cyclic Redundancy Check (CRC)

- Even stronger protection
  - Given n data bits, generate k check bits such that the n+k bits are evenly divisible by a generator C
- Example with numbers:
  - n = 302, k = one digit, C = 3

CRCs (2)

- The catch:
  - It's based on mathematics of finite fields, in which "numbers" represent polynomials
  - e.g, 10011010 is $x^7 + x^4 + x^3 + x^1$
- What this means:
  - We work with binary values and operate using modulo 2 arithmetic

CRCs (3)

- Send Procedure:

1. Extend the n data bits with k zeros
2. Divide by the generator value C
3. Keep remainder, ignore quotient
4. Adjust k check bits by remainder

- Receive Procedure:

1. Divide and check for zero remainder

CRCs (4)

Data bits:
1101011111

Check bits:
$C(x)=x^4+x^1+1$
C = 10011
k = 4

$$1\,0\,0\,1\,1 \,\overline{)\,1\;1\;0\;1\;0\;1\;1\;1\;1\;1\qquad}$$

CRCs (5)

```
                      1 1 0 0 0 0 1 1 1 0        ← Quotient (thrown away)
1 0 0 1 1 / 1 1 0 1 0 1 1 1 1 1 0 0 0 0          ← Frame with four zeros appended
            1 0 0 1 1
              1 0 0 1 1
              1 0 0 1 1
                0 0 0 0 1
                0 0 0 0 0
                  0 0 0 1 1
                  0 0 0 0 0
                    0 0 1 1 1
                    0 0 0 0 0
                      0 1 1 1 1
                      0 0 0 0 0
                        1 1 1 1 0
                        1 0 0 1 1
                          1 1 0 1 0
                          1 0 0 1 1
                            1 0 0 1 0
                            1 0 0 1 1
                              0 0 0 1 0
                              0 0 0 0 0
                                    1 0      ← Remainder

Transmitted frame: 1 1 0 1 0 1 1 1 1 1 0 0 1 0   ← Frame with four zeros appended
                                                   minus remainder
```

CRCs (6)

- Protection depend on generator
  - Standard CRC-32 is 10000010 01100000 10001110 110110111
- Properties:
  - HD=4, detects up to triple bit errors
  - Also odd number of errors
  - And bursts of up to k bits in error
  - Not vulnerable to systematic errors like checksums

Error Detection in Practice

- CRCs are widely used on links
  - Ethernet, 802.11, ADSL, Cable …
- Checksum used in Internet
  - IP, TCP, UDP … but it is weak
- Parity
  - Is little used

Topic

- Some bits may be received in error due to noise. How do we fix them?
  - Hamming code »
  - Other codes »
- And why should we use detection when we can use correction?

Why Error Correction is Hard

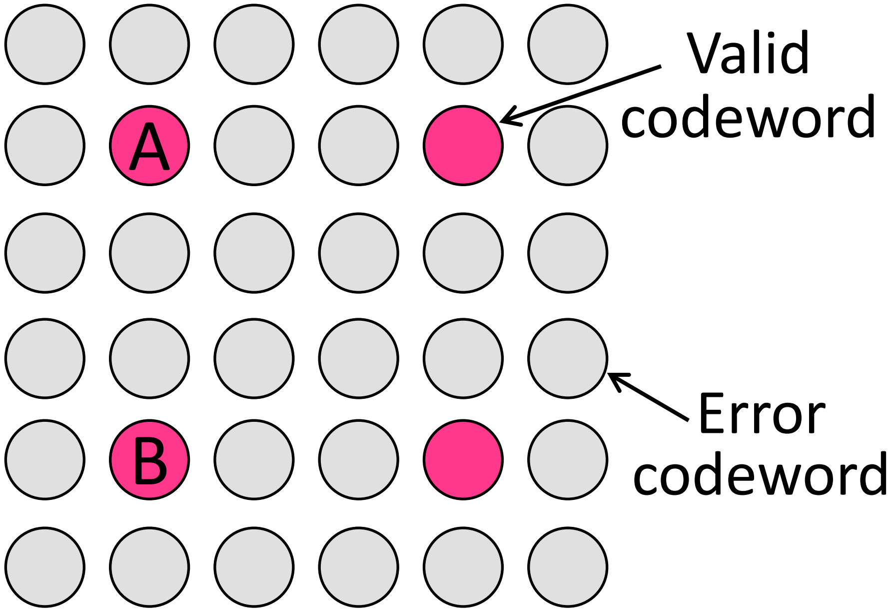
- If we had reliable check bits we could use them to narrow down the position of the error
  - Then correction would be easy
- But error could be in the check bits as well as the data bits!
  - Data might even be correct

Intuition for Error Correcting Code

- Suppose we construct a code with a Hamming distance of at least 3
  - Need ≥3 bit errors to change one valid codeword into another
  - Single bit errors will be closest to a unique valid codeword
- If we assume errors are only 1 bit, we can correct them by mapping an error to the closest valid codeword
  - Works for d errors if HD ≥ 2d + 1

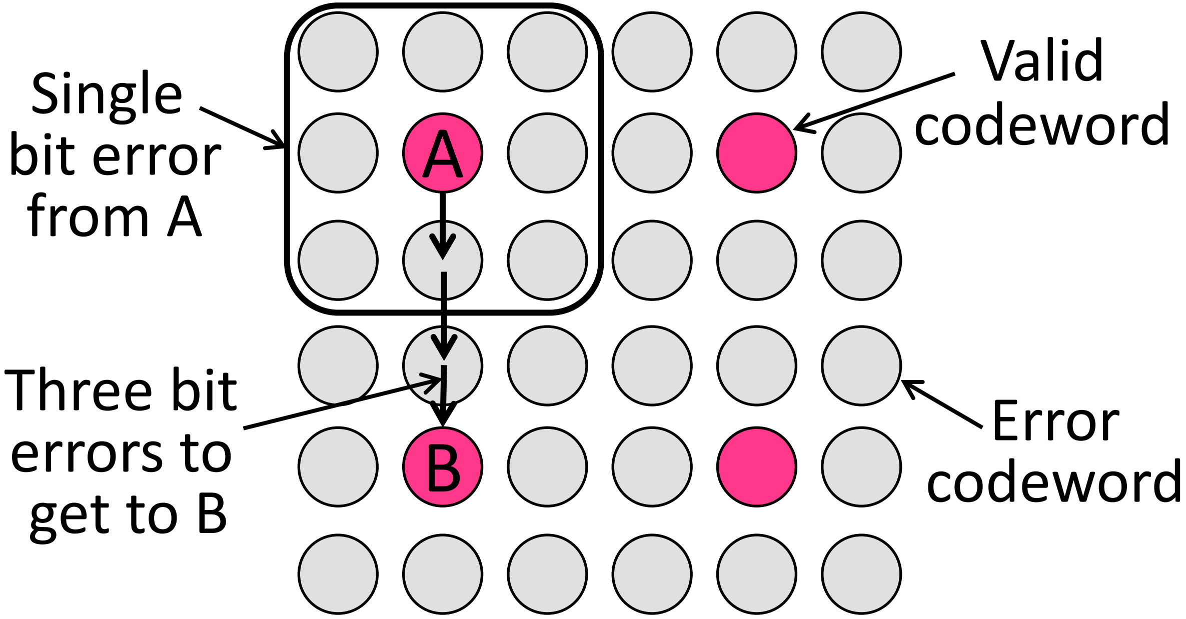
Intuition (2)

- Visualization of code:



Intuition (3)

- Visualization of code:



Hamming Code

- Gives a method for constructing a code with a distance of 3
  - Uses $n = 2^k - k - 1$, e.g., n=4, k=3
  - Put check bits in positions p that are powers of 2, starting with position 1
  - Check bit in position p is parity of positions with a p term in their values
- Plus an easy way to correct [soon]

Hamming Code (2)

- Example: data=0101, 3 check bits
  - 7 bit code, check bit positions 1, 2, 4
  - Check 1 covers positions 1, 3, 5, 7
  - Check 2 covers positions 2, 3, 6, 7
  - Check 4 covers positions 4, 5, 6, 7

_ _ _ _ _ _ _
1 2 3 4 5 6 7

Hamming Code (3)

- Example: data=0101, 3 check bits
  - 7 bit code, check bit positions 1, 2, 4
  - Check 1 covers positions 1, 3, 5, 7
  - Check 2 covers positions 2, 3, 6, 7
  - Check 4 covers positions 4, 5, 6, 7

$$\begin{array}{ccccccc} \underline{0} & \underline{1} & 0 & \underline{0} & 1 & 0 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \longrightarrow$$

$p_1 = 0+1+1 = 0$, $p_2 = 0+0+1 = 1$, $p_4 = 1+0+1 = 0$

Hamming Code (4)

- To decode:
  - Recompute check bits (with parity sum including the check bit)
  - Arrange as a binary number
  - Value (syndrome) tells error position
  - Value of zero means no error
  - Otherwise, flip bit to correct

Hamming Code (5)

- Example, continued

$$\longrightarrow \underline{0}\ \underline{1}\ 0\ \underline{0}\ 1\ 0\ 1$$

1 2 3 4 5 6 7

p_1=

p_2=

p_4=

Syndrome =

Data =

Hamming Code (6)

- Example, continued

$$\longrightarrow \underline{0}\ \ \underline{1}\ \ 0\ \ \underline{0}\ \ 1\ \ 0\ \ 1$$

1 2 3 4 5 6 7

$p_1 = 0+0+1+1 = 0$, $p_2 = 1+0+0+1 = 0$,
$p_4 = 0+1+0+1 = 0$

Syndrome = 000, no error
Data = 0 1 0 1

Hamming Code (7)

- Example, continued

$\longrightarrow$ $\underline{0}$ $\underline{1}$ 0 $\underline{0}$ 1 1 1

1 2 3 4 5 6 7

p_1=

p_2=

p_4=

Syndrome =

Data =

Hamming Code (8)

- Example, continued

$$\longrightarrow \underline{0}\ \ \underline{1}\ \ 0\ \ \underline{0}\ \ 1\ \ 1\ \ 1$$

1 2 3 4 5 6 7

p_1= 0+0+1+1 = 0, p_2= 1+0+1+1 = 1,
p_4= 0+1+1+1 = 1

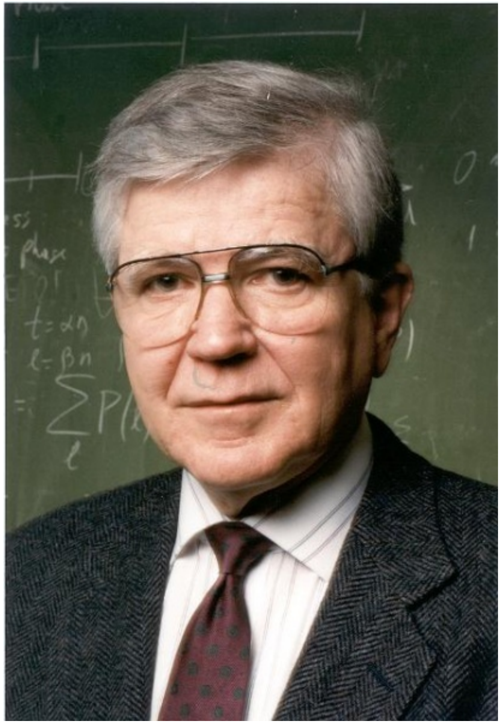
Syndrome = 1 1 0, flip position 6
Data = 0 1 0 1 (correct after flip!)

Other Error Correction Codes

- Codes used in practice are much more involved than Hamming
- Convolutional codes (§3.2.3)
  - Take a stream of data and output a mix of the recent input bits
  - Makes each output bit less fragile
  - Decode using Viterbi algorithm (which can use bit confidence values)

Other Codes (2) – LDPC

- Low Density Parity Check (§3.2.3)
  - LDPC based on sparse matrices
  - Decoded iteratively using a belief propagation algorithm
  - State of the art today
- Invented by Robert Gallager in 1963 as part of his PhD thesis
  - Promptly forgotten until 1996 …



Source: IEEE GHN, © 2009 IEEE

Detection vs. Correction

- Which is better will depend on the pattern of errors. For example:
  - 1000 bit messages with a <u>bit error rate</u> (<u>BER</u>) of 1 in 10000

- Which has less overhead?

Detection vs. Correction

- Which is better will depend on the pattern of errors. For example:
  - 1000 bit messages with a bit error rate (BER) of 1 in 10000

- Which has less overhead?
  - It still depends! We need to know more about the errors

Detection vs. Correction (2)

1. Assume bit errors are random
   - Messages have 0 or maybe 1 error

- Error correction:
  - Need ~10 check bits per message
  - Overhead:

- Error detection:
  - Need ~1 check bits per message plus 1000 bit retransmission 1/10 of the time
  - Overhead:

Detection vs. Correction (3)

2. Assume errors come in bursts of 100
   - Only 1 or 2 messages in 1000 have errors

- Error correction:
  - Need >>100 check bits per message
  - Overhead:

- Error detection:
  - Need 32? check bits per message plus 1000 bit resend 2/1000 of the time
  - Overhead:

Detection vs. Correction (4)

- Error correction:
  - Needed when errors are expected
  - Or when no time for retransmission
- Error detection:
  - More efficient when errors are not expected
  - And when errors are large when they do occur

Error Correction in Practice

- Heavily used in physical layer
  - LDPC is the future, used for demanding links like 802.11, DVB, WiMAX, LTE, power-line, …
  - Convolutional codes widely used in practice
- Error detection (w/ retransmission) is used in the link layer and above for residual errors
- Correction also used in the application layer
  - Called Forward Error Correction (FEC)
  - Normally with an erasure error model
  - E.g., Reed-Solomon (CDs, DVDs, etc.)