Link Layer

Error Detection VS Error Correction

Question About Hamming Distance:

Hamming distance is the minimum change of bits to transform from one correct codeword to another.

Example: 1100 → 1111 HD = 2

Error Detection:

For a coding of HD = d + 1, up to d errors will always be detected.

Error Correction:

For a coding of HD = 2d + 1, up to d errors will always be corrected

by mapping to the closest valid codeword.

So why d + 1 and 2d + 1???

Lets take a look at the following 2-dimensional parity bit (even).

0 1 0 1 0 --> check bits for columns

1 1 0 0 0

0 0 0 0 0

1 0 0 1 0

HD = 4 detection: HD = $d_1 + 1 \Rightarrow d_1 = 3$ correction: HD = $2d_2 + 1 \Rightarrow d_2 = 1$

Error Detection: up to 3 bits of error will always be detected.

Error Correction: Up to 1 bit of error will always be corrected.

Conclusion

Error detection:

Suppose HD = d_1 + 1. Then, if there are HD errors, one valid codeword would just transform into another without being detected. However, if there are ≤ (HD - 1) errors, the transformed codeword would be invalid and thus we can detect the error. Hence, the number of error bits detectable is d_1 = (HD - 1).

Error correction:

Suppose HD = 2d_2 + 1. Then, if there are ≤ d_2 errors, we can correct the codeword by transforming it to the closest valid codeword. However, if there are d_2 + 1 errors, there would be two valid codewords that are of the same distances to the codeword. In this case, the error is uncorrectable. Hence, the number of error bits correctable is d2 = floor((HD - 1) / 2).

Detection vs Correction

a) When do we use error detection instead of error correction?

b) When do we use error correction instead of error detection?

a) When do we use error detection instead of error correction?

1. Errors are expected to be rare.
2. If errors do occur, the amount of errors is really large.
   In this case, retransmission is more effecient than correction.

b) When do we use error correction instead of error detection?

1. Errors are expected to occur from time to time, but there won't be many of them at each time.
2. Retransmission is time-consuming...

Internet Checksum

Suppose we are going to send a message in hex: 2188815eeee91

What is the **internet checksum** for this message?

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What is the internet checksum for this message?

ANS: E2F5

```
2188815eeee91 =>
   0002
   1888
   15ee
   ee91
+ (0000)
--------
  11d09
=>
   1d09
+     1
--------
   1d0a
=> e2f5
```

Hamming Codes

Suppose we want to send a message M of 11 bits and we add 4 check bits at the end.

```
k = 4 => n = 2^k-k-1 = 2^4-4-1 = 11
```

Following are the bits each check bit covers:

```
1 =>  1, 3, 5, 7, 9,11,13,15
2 =>  2, 3, 6, 7,10,11,14,15
4 =>  4, 5, 6, 7,12,13,14,15
8 =>  8, 9,10,11,12,13,14,15
```

Do you see a pattern of the bits being covered?

Hamming Codes

ANS: The corresponding bit of the check bit is always 1!

Take check bit 4 for example. 4 = 0b0100.

```
 4 = 0b0100
 5 = 0b0101
 6 = 0b0110
 7 = 0b0111
12 = 0b1100
13 = 0b1101
14 = 0b1110
15 = 0b1111
```

Hamming Codes

Now, the message M has been converted into M’ (15 bits) and sent to the receiver.

The receiver got M’ as follows:

```
0  0  0  0  1  0  1  0  1  0  0  1  0  0  0
```

Is there any error in message M’? (Hint: Try to calculate the syndrome.)

```
1 =>  1, 3, 5, 7, 9,11,13,15
2 =>  2, 3, 6, 7,10,11,14,15
4 =>  4, 5, 6, 7,12,13,14,15
8 =>  8, 9,10,11,12,13,14,15
```

Hamming Codes

The receiver got M' as follows:

0	0	0	0	1	0	1	0	1	0	0	1	0	0	0
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

p1=(0+0+1+1+1+0+0+0) mod 2=1
p2=(0+0+0+1+0+0+0+0) mod 2=1
p4=(0+1+0+1+1+0+0+0) mod 2=1
p8=(0+1+0+0+1+0+0+0) mod 2=0

=> syndrome=p8p4p2p1=0111

Hamming Codes

The receiver got M' as follows:

0	0	0	0	1	0	1	0	1	0	0	1	0	0	0
$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{5}$	$\overline{6}$	$\overline{7}$	$\overline{8}$	$\overline{9}$	$\overline{10}$	$\overline{11}$	$\overline{12}$	$\overline{13}$	$\overline{14}$	$\overline{15}$

The syndrome is 0111 and we know the mapping:

```
1 =>  1, 3, 5, 7, 9,11,13,15
2 =>  2, 3, 6, 7,10,11,14,15
4 =>  4, 5, 6, 7,12,13,14,15
8 =>  8, 9,10,11,12,13,14,15
```

So which bit is wrong??? What should be the correct value of message M?

Hamming Codes

The syndrome 0111 tells us that the error bit exists in the bits covered by check bit 1, 2, and 4, not 8.

Hence, the error bit is bit **7**!

0	0	0	0	1	0	**0**	0	1	0	0	1	0	0	0
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Message M = **01001001000**

What is difference between coding and modulation?

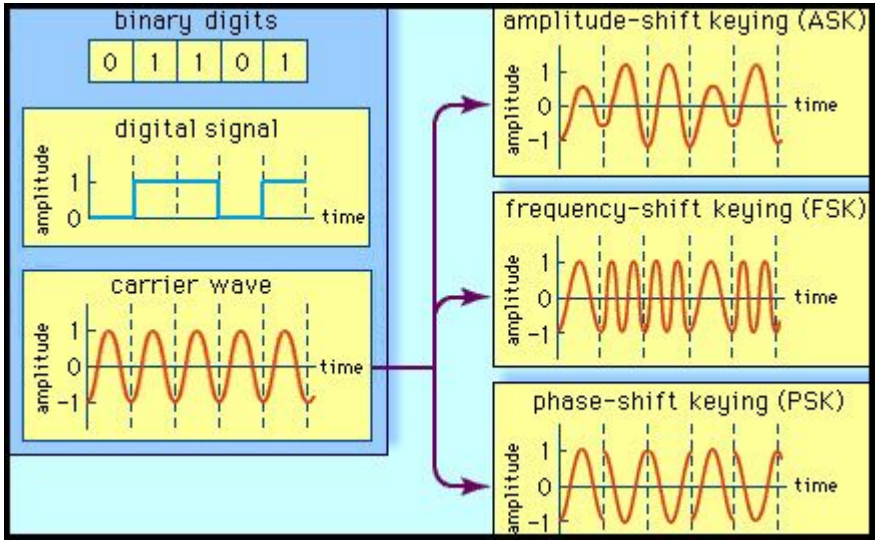
What is difference between coding and modulation?

Coding = determining the bit pattern that is sent
Modulation = changing the signal to transmit the bit pattern

What are 3 forms of modulation?

What are 3 forms of modulation?

1. Amplitude
2. Frequency
3. Phase



Coding

4B5B Coding Scheme

- Maps 4 bits of data onto 5 bits for transmission
- Clock Recovery

Data		4B5B code
(Hex)	(Binary)	
0	0000	11110
1	0001	01001
2	0010	10100
3	0011	10101
4	0100	01010
5	0101	01011
6	0110	01110
7	0111	01111

Data		4B5B code
(Hex)	(Binary)	
8	1000	10010
9	1001	10011
A	1010	10110
B	1011	10111
C	1100	11010
D	1101	11011
E	1110	11100
F	1111	11101