Link Layer: Error detection and correction

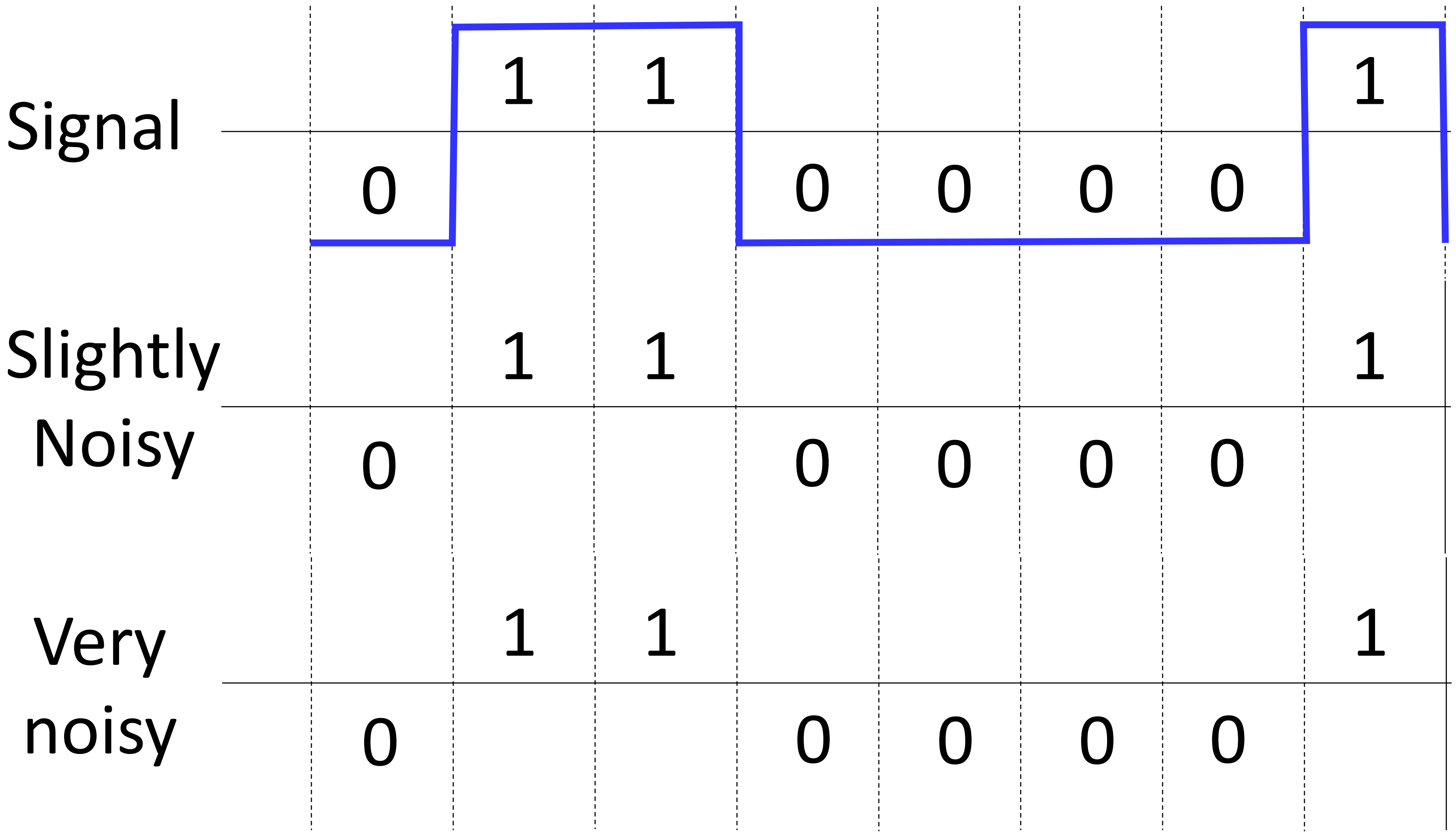
Topic

- Some bits will be received in error due to noise. What can we do?
  - Detect errors with codes
  - Correct errors with codes
  - Retransmit lost frames

Later

- Reliability is a concern that cuts across the layers

Problem – Noise may flip received bits



- Ideas?

Approach – Add Redundancy

- Error detection codes
  - Add check bits to the message bits to let some errors be detected
- Error correction codes
  - Add more check bits to let some errors be corrected

- Key issue is now to structure the code to detect many errors with few check bits and modest computation

- Ideas?

Motivating Example

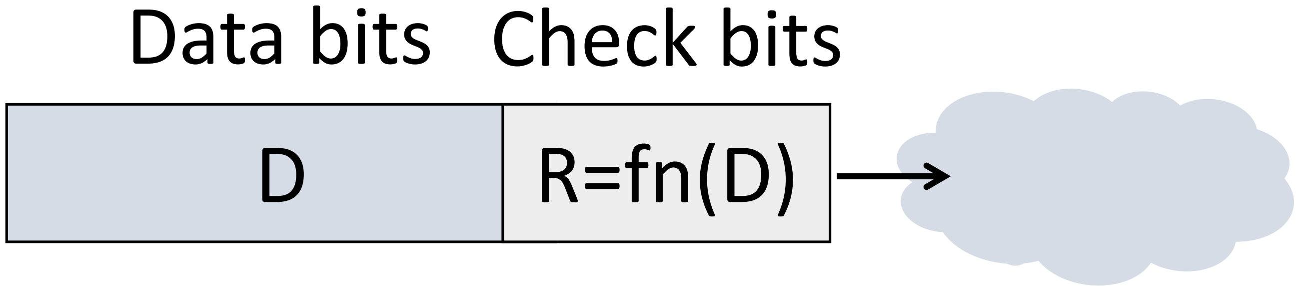
- A simple code to handle errors:
  - Send two copies! Error if different.
- How good is this code?
  - How many errors can it detect/correct?
  - How many errors will make it fail?

Motivating Example (2)

- We want to handle more errors with less overhead
  - Will look at better codes; they are applied mathematics
  - But, they can't handle all errors
  - And they focus on accidental errors (will look at secure hashes later)

Using Error Codes

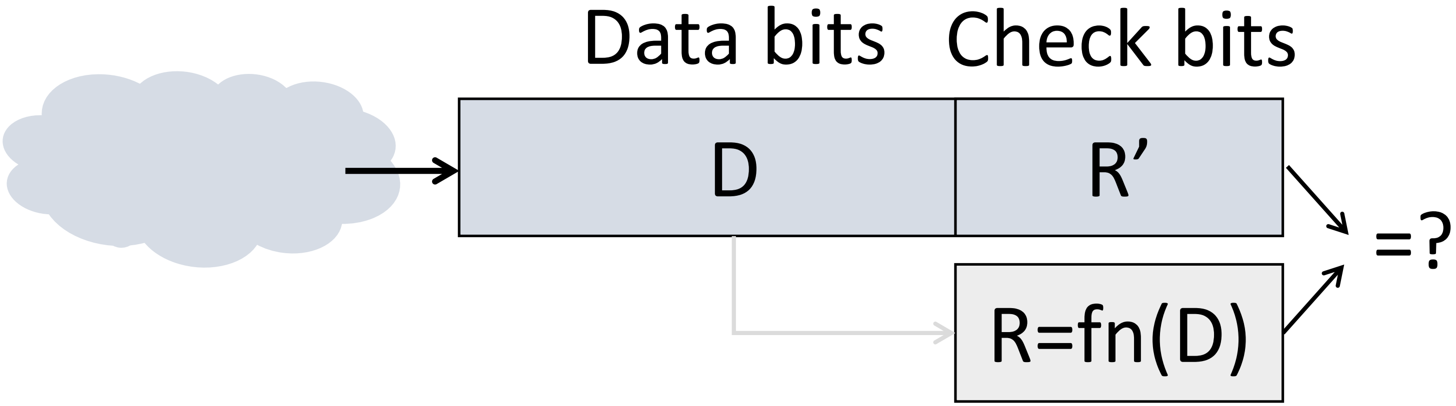
- Codeword consists of D data plus R check bits (=systematic block code)



- Sender:
  - Compute R check bits based on the D data bits; send the codeword of D+R bits

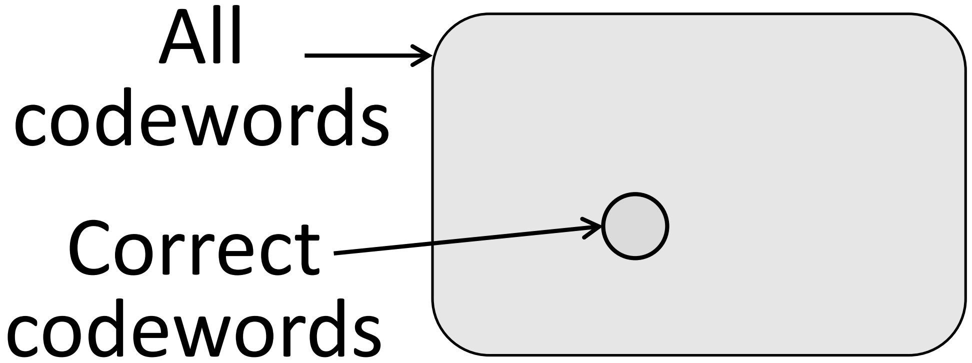
Using Error Codes (2)

- Receiver:
  - Receive D+R bits with unknown errors
  - Recompute R check bits based on the D data bits; error if R doesn't match R'



Intuition for Error Codes

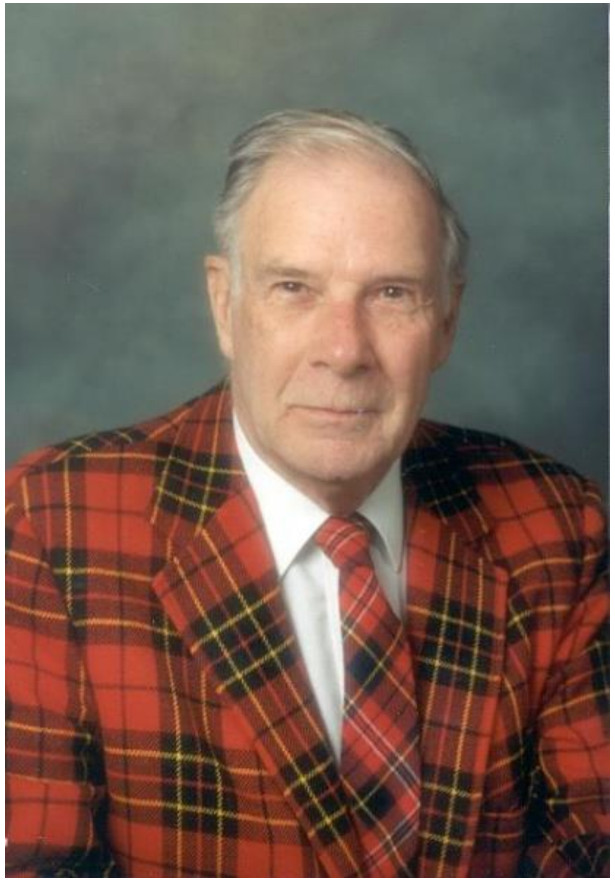
- For D data bits, R check bits:



- Randomly chosen codeword is unlikely to be correct; overhead is low

R.W. Hamming (1915-1998)

- Much early work on codes:
  - "Error Detecting and Error Correcting Codes", BSTJ, 1950
- See also:
  - "You and Your Research", 1986



Source: IEEE GHN, © 2009 IEEE

Hamming Distance

- Distance is the number of bit flips needed to change D_1 to D_2
- Hamming distance of a coding is the minimum distance between any pair of codewords (bit-strings) that cannot be detected

Hamming Distance (2)

- Error detection:
  - For a coding of distance d+1, up to d errors will always be detected
- Error correction:
  - For a coding of distance 2d+1, up to d errors can always be corrected by mapping to the closest valid codeword

Simple Error Detection – Parity Bit

- Take D data bits, add 1 check bit that is the sum of the D bits
  - Sum is modulo 2 or XOR

Parity Bit (2)

- How well does parity work?
  - What is the distance of the code?
  - How many errors will it detect/correct?
- What about larger errors?

Checksums

- Idea: sum up data in N-bit words
  - Widely used in, e.g., TCP/IP/UDP

1500 bytes	16 bits

- Stronger protection than parity

Internet Checksum

- Sum is defined in 1s complement arithmetic (must add back carries)
  - And it's the negative sum
- *"The checksum field is the 16 bit one's complement of the one's complement sum of all 16 bit words …"* – RFC 791

Internet Checksum (2)

Sending:

1. Arrange data in 16-bit words
2. Put zero in checksum position, add
3. Add any carryover back to get 16 bits
4. Negate (complement) to get sum

```
0001
f204
f4f5
f6f7
```

Internet Checksum (3)

Sending:

1. Arrange data in 16-bit words
2. Put zero in checksum position, add
3. Add any carryover back to get 16 bits
4. Negate (complement) to get sum

```
  0001
  f204
  f4f5
  f6f7
+(0000)
------
 2ddf1
   ↓
  ddf1
+    2
------
  ddf3
   ↓
  220c
```

Internet Checksum (4)

Receiving:

1. Arrange data in 16-bit words
2. Checksum will be non-zero, add
3. Add any carryover back to get 16 bits
4. Negate the result and check it is 0

```
  0001
  f204
  f4f5
  f6f7
+ 220c
------
```

Internet Checksum (5)

Receiving:

1. Arrange data in 16-bit words
2. Checksum will be non-zero, add
3. Add any carryover back to get 16 bits
4. Negate the result and check it is 0

```
   0001
   f204
   f4f5
   f6f7
 + 220c
 ------
  2fffd
    ↓
   fffd
 +    2
 ------
   ffff
    ↓
   0000
```

Internet Checksum (6)

- How well does the checksum work?
  - What is the distance of the code?
  - How many errors will it detect/correct?

- What about larger errors?

Cyclic Redundancy Check (CRC)

- Even stronger protection
  - Given n data bits, generate k check bits such that the n+k bits are evenly divisible by a generator C
- Example with numbers:
  - n = 302, k = one digit, C = 3

CRCs (2)

- The catch:
  - It's based on mathematics of finite fields, in which "numbers" represent polynomials
  - e.g, 10011010 is $x^7 + x^4 + x^3 + x^1$
- What this means:
  - We work with binary values and operate using modulo 2 arithmetic

CRCs (3)

- Send Procedure:

1. Extend the n data bits with k zeros
2. Divide by the generator value C
3. Keep remainder, ignore quotient
4. Adjust k check bits by remainder

- Receive Procedure:

1. Divide and check for zero remainder

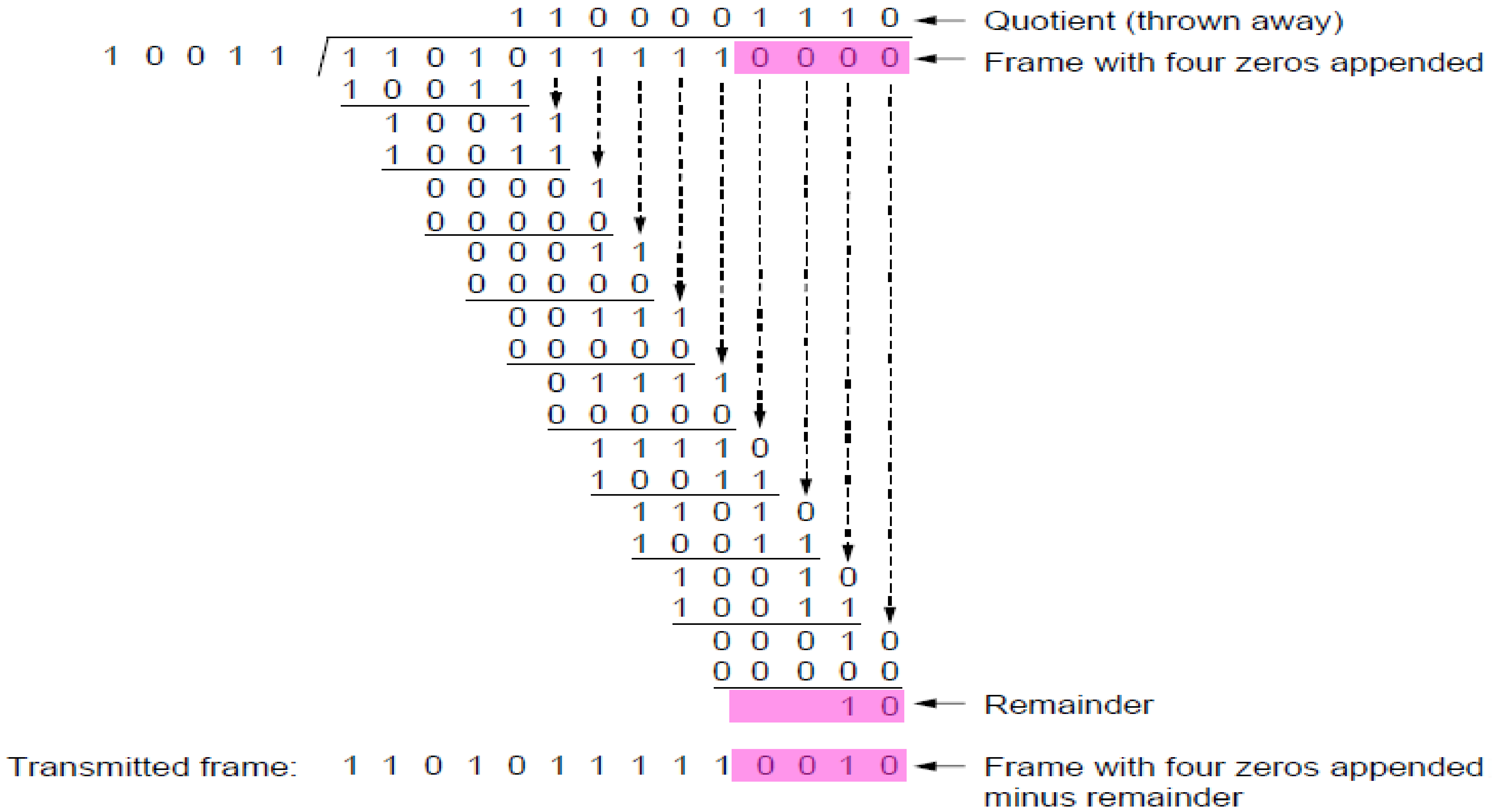
CRCs (4)

Data bits:
1101011111

$$10011 \overline{)\,1\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ 1\ 1\ }$$

Check bits:
$C(x)=x^4+x^1+1$
C = 10011
k = 4

CRCs (5)


1 1 0 0 0 0 1 1 1 0
Quotient (thrown away)
1 0 0 1 1
1 1 0 1 0 1 1 1 1 1 0 0 0 0
Frame with four zeros appended
1 0 0 1 1
1 0 0 1 1
1 0 0 1 1
0 0 0 0 1
0 0 0 0 0
0 0 0 1 1
0 0 0 0 0
0 0 1 1 1
0 0 0 0 0
0 1 1 1 1
0 0 0 0 0
1 1 1 1 0
1 0 0 1 1
1 1 0 1 0
1 0 0 1 1
1 0 0 1 0
1 0 0 1 1
0 0 0 1 0
0 0 0 0 0
1 0
Remainder
Transmitted frame:
1 1 0 1 0 1 1 1 1 1 0 0 1 0
Frame with four zeros appended
minus remainder

CRCs (6)

- Protection depend on generator
  - Standard CRC-32 is 10000010 01100000 10001110 110110111

- Properties:
  - HD=4, detects up to triple bit errors
  - Also odd number of errors
  - And bursts of up to k bits in error
  - Not vulnerable to systematic errors like checksums

Why Error Correction is Hard

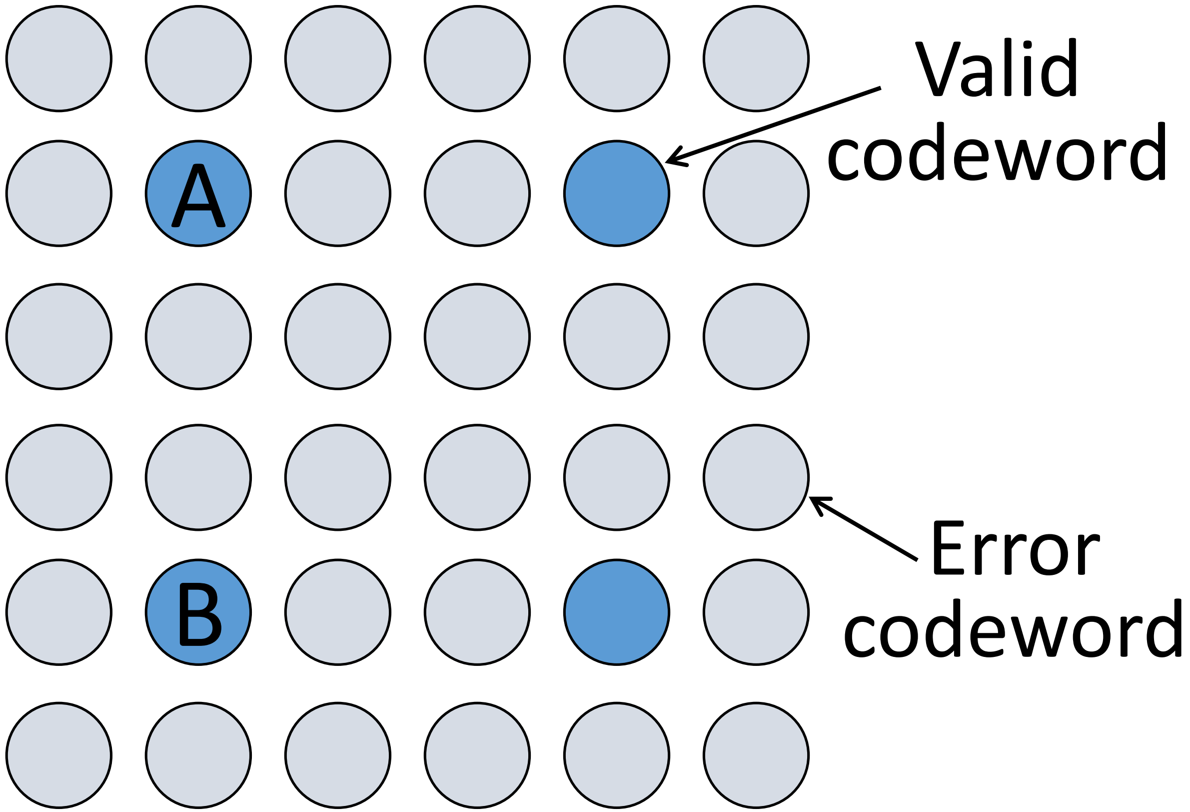
- If we had reliable check bits we could use them to narrow down the position of the error
  - Then correction would be easy
- But error could be in the check bits as well as the data bits!
  - Data might even be correct

Intuition for Error Correcting Code

- Suppose we construct a code with a Hamming distance of at least 3
  - Need ≥3 bit errors to change one valid codeword into another
  - Single bit errors will be closest to a unique valid codeword
- If we assume errors are only 1 bit, we can correct them by mapping an error to the closest valid codeword
  - Works for d errors if HD ≥ 2d + 1

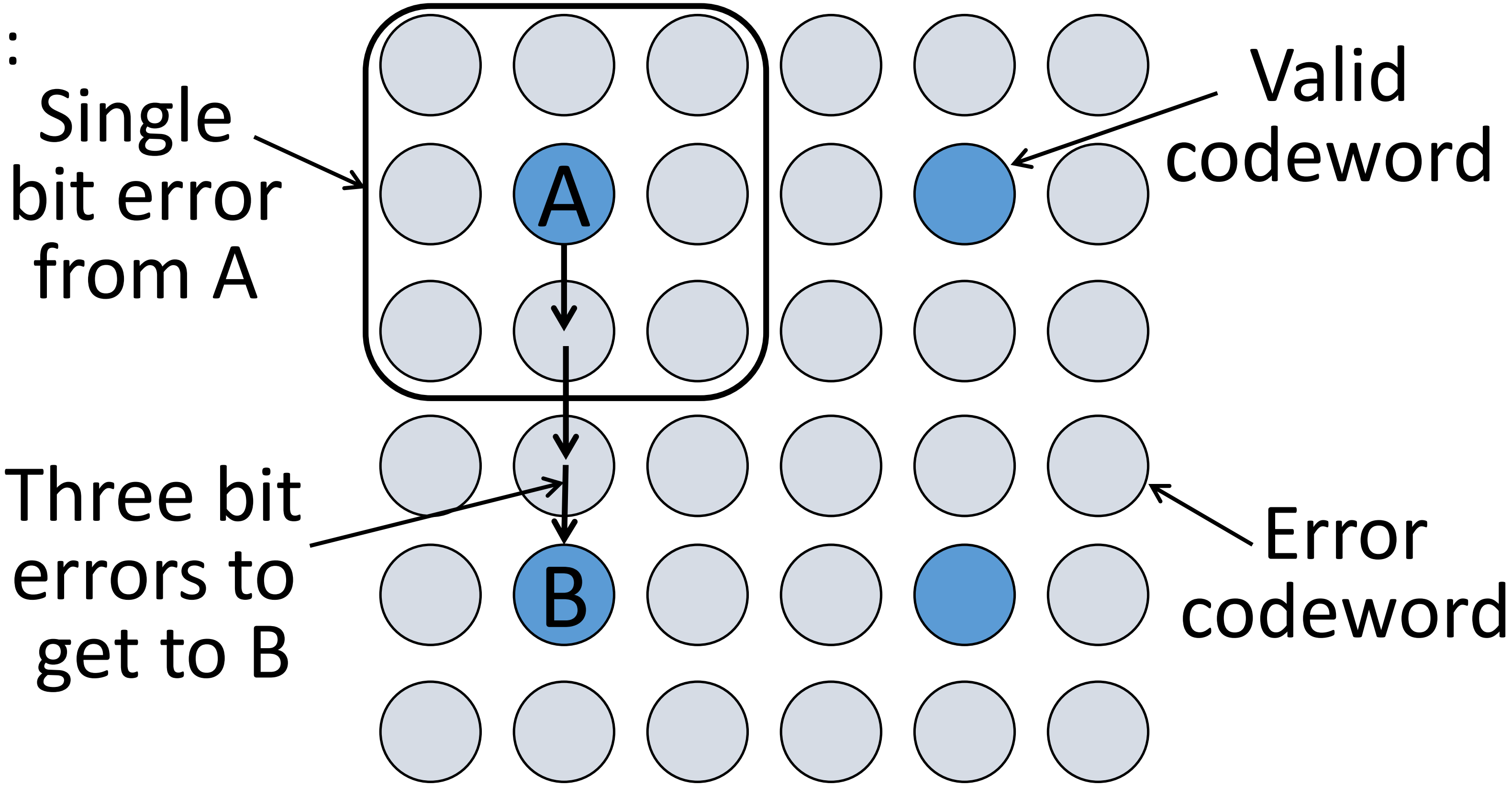
Intuition (2)

- Visualization of code:



Intuition (3)

- Visualization of code:



Hamming Code

- Gives a method for constructing a code with a distance of 3
  - Uses $n = 2^k - k - 1$, e.g., n=4, k=3
  - Put check bits in positions p that are powers of 2, starting with position 1
  - Check bit in position p is parity of positions with a p term in their values
- Plus an easy way to correct [soon]

Hamming Code (2)

- Example: data=0101, 3 check bits
  - 7 bit code, check bit positions 1, 2, 4
  - Check 1 covers positions 1, 3, 5, 7
  - Check 2 covers positions 2, 3, 6, 7
  - Check 4 covers positions 4, 5, 6, 7

_ _ _ _ _ _ _
1 2 3 4 5 6 7

Hamming Code (3)

- Example: data=0101, 3 check bits
  - 7 bit code, check bit positions 1, 2, 4
  - Check 1 covers positions 1, 3, 5, 7
  - Check 2 covers positions 2, 3, 6, 7
  - Check 4 covers positions 4, 5, 6, 7

$$\begin{array}{ccccccc} \underline{0} & \underline{1} & 0 & \underline{0} & 1 & 0 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \longrightarrow$$

$p_1 = 0+1+1 = 0$, $p_2 = 0+0+1 = 1$, $p_4 = 1+0+1 = 0$

Hamming Code (4)

- To decode:
  - Recompute check bits (with parity sum including the check bit)
  - Arrange as a binary number
  - Value (syndrome) tells error position
  - Value of zero means no error
  - Otherwise, flip bit to correct

Hamming Code (5)

- Example, continued

$\longrightarrow$ $\underline{0}\ \underline{1}\ 0\ \underline{0}\ 1\ 0\ 1$

1 2 3 4 5 6 7

$p_1=$ $p_2=$

$p_4=$

Syndrome =

Data =

Hamming Code (6)

- Example, continued

$$\longrightarrow \underline{0}\ \ \underline{1}\ \ 0\ \ \underline{0}\ \ 1\ \ 0\ \ 1$$

1 2 3 4 5 6 7

$p_1 = 0+0+1+1 = 0$, $p_2 = 1+0+0+1 = 0$,
$p_4 = 0+1+0+1 = 0$

Syndrome = 000, no error

Data = 0 1 0 1

Hamming Code (7)

- Example, continued

$\longrightarrow$ $\underline{0}$ $\underline{1}$ 0 $\underline{0}$ 1 1 1

1 2 3 4 5 6 7

p_1=　　　　　　　p_2=

p_4=

Syndrome =

Data =

Hamming Code (8)

- Example, continued

$\longrightarrow \underline{0}\ \underline{1}\ 0\ \underline{0}\ 1\ 1\ 1$

1 2 3 4 5 6 7

$p_1 = 0+0+1+1 = 0$, $p_2 = 1+0+1+1 = 1$,
$p_4 = 0+1+1+1 = 1$

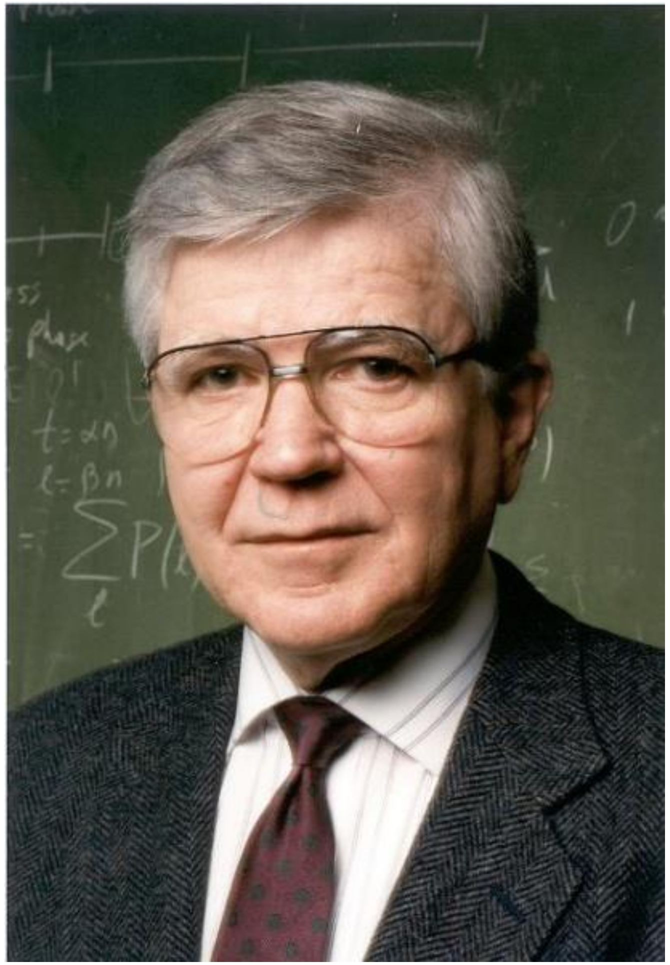
Syndrome = 1 1 0, flip position 6
Data = 0 1 0 1 (correct after flip!)

Other Error Correction Codes

- Codes used in practice are more involved than Hamming
- Convolutional codes (§3.2.3)
  - Take a stream of data and output a mix of the input bits
  - Makes each output bit less fragile
  - Decode using Viterbi algorithm (which can use bit confidence values)

Other Codes (2) – LDPC

- Low Density Parity Check (§3.2.3)
  - LDPC based on sparse matrices
  - Decoded iteratively using a belief propagation algorithm
  - State of the art today
- Invented by Robert Gallager in 1963 as part of his PhD thesis
  - Promptly forgotten until 1996 …



Source: IEEE GHN, © 2009 IEEE

Detection vs. Correction

- Which is better will depend on the pattern of errors. For example:
  - 1000 bit messages with a <u>bit error rate</u> (<u>BER</u>) of 1 in 10000
- Which has less overhead?

Detection vs. Correction

- Which is better will depend on the pattern of errors. For example:
  - 1000 bit messages with a <u>bit error rate</u> (<u>BER</u>) of 1 in 10000
- Which has less overhead?
  - It still depends! We need to know more about the errors

Detection vs. Correction (2)

Assume bit errors are random

- Messages have 0 or maybe 1 error (1/10 of the time)

Error correction:

- Need ~10 check bits per message
- Overhead:

Error detection:

- Need ~1 check bits per message plus 1000 bit retransmission
- Overhead:

Detection vs. Correction (3)

Assume errors come in bursts of 100

- Only 1 or 2 messages in 1000 have significant (multi-bit) errors

Error correction:

- Need >>100 check bits per message
- Overhead:

Error detection:

- Need 32 check bits per message plus 1000 bit resend 2/1000 of the time
- Overhead:

Detection vs. Correction (4)

- Error correction:
  - Needed when errors are expected
  - Or when no time for retransmission
- Error detection:
  - More efficient when errors are not expected
  - And when errors are large when they do occur

Error Correction in Practice

- Heavily used in physical layer
  - LDPC is the future, used for demanding links like 802.11, DVB, WiMAX, LTE, power-line, …
  - Convolutional codes widely used in practice

- Error detection (w/ retransmission) is used in the link layer and above for residual errors

- Correction also used in the application layer
  - Called Forward Error Correction (FEC)
  - Normally with an erasure error model
  - E.g., Reed-Solomon (CDs, DVDs, etc.)