2D Transformations

Introduction

- A *2D transform* is *a* "useful" *mapping of the 2D* pl*ane onto itself*.
  - H*omeomorphism*
  - P*reserves some amount of geometric information*
- M*any different functions are* possible u*nder this* de*finition*, bu*t we'll see that some in particular are* useful *for computer graphics*.

Applications

- S*ometimes*, it's easie*r to* specify *an object* usi*ng a transform than* directly.
- I*nstancing*
  - T*ransforms can* be *a more compact representation than* copies *of an object*.
  - A *transformed instance of an object preserves more information about the object than a* copy.
- Hie*rarchical modeling*
  - We *can group a* collectio*n of geometry under a transform node and manipulate it all at once*.

Linearity

- O*ne important* class *of transforms are the linear transforms*.
  - They c*orrespond to change-of-basis or change-of-coordinate-system transforms*.
- How c*an we represent linear transforms*?
- Which *transforms are linear*?

Linear Isn't Enough

- What important transform isn't linear?
  - Why *not*?
- How can we unify all of the transforms we've discussed?

Affine Geometry

- Treating the Euclidean plane as a vector space ignores the distinction between points and vectors.
- The affine plane is a set of points together with a set of vectors.
  - No distinguished origin
  - The difference between two points is a vector.
  - A vector added to a point yields a new point.
  - Vectors and points are not interchangeable.
- How can we represent the affine plane?
- How can we represent transforms in the affine plane?
- What geometric properties do affine transforms preserve?

Properties of Transforms

- Compact representation
- Fast implementation
- Easy to invert
- Easy to compose

Examples

- Rotation about a point other than the origin
- Reflection through a line other than an axis
- Viewport transform

Summary

- A small number of 2D transforms are especially useful in computer graphics: translation, rotation, scale, shear, reflection, and of course identity.
- Some of these transforms are linear and can be represented using a 2x2 matrix.
- Some are affine and can be represented using a 3x3 matrix acting on homogeneous coordinates.
- Affine transforms have certain properties which make them very practical for computer graphics.

Reading

The reading here covers 2D and 3D transformations.

- Angel, chapter 4
- Foley et al., chapter 5 and Appendix A
- Hearn & Baker, chapter 5 (2D), chapter 11 (3D)