Lecture 19:
Tensor Product Surfaces

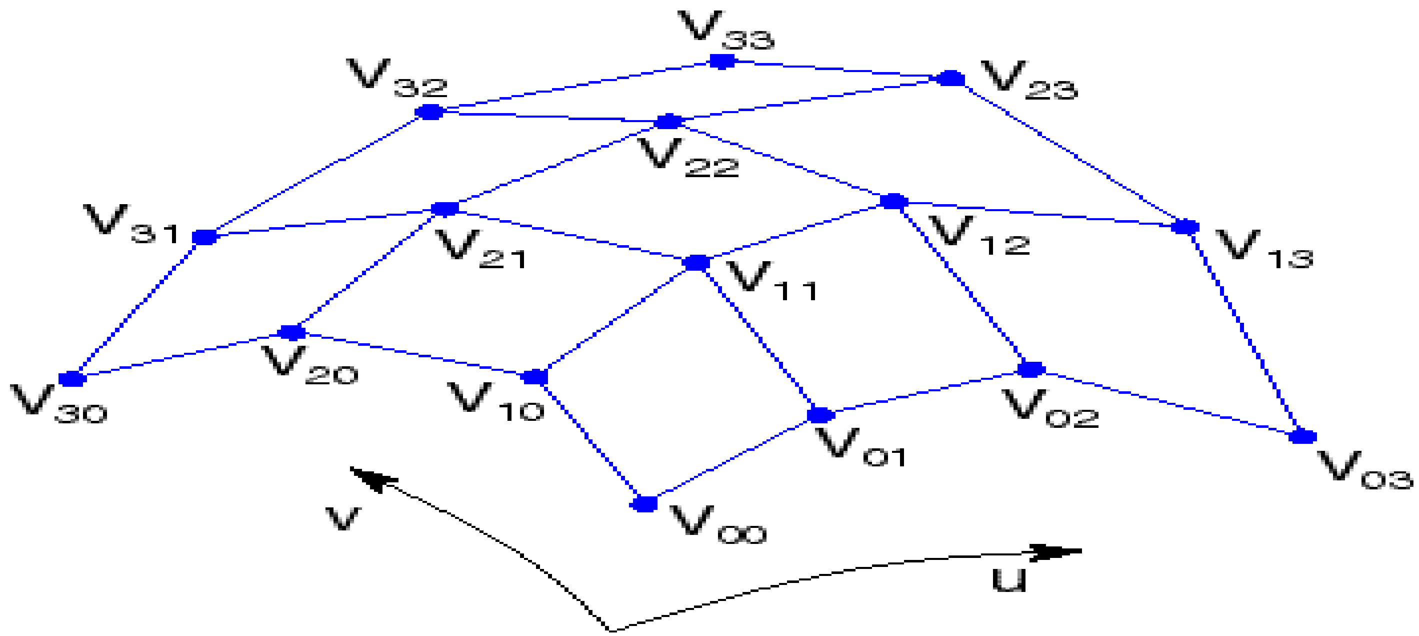
Reading

- Hearn and Baker, sections 10.8, 10.9, 10.14.

Recommended:

- Bartels, Beatty, and Barsky. *An Introduction to Splines for use in Computer Graphics and Geometric Modeling*, 1987.

Tensor product Bézier surfaces



Given a grid of control points V_{ij}, forming a **control net**, contruct a surface $S(u,v)$ by:

- treating rows of V as control points for curves $V_0(u),\ldots, V_n(u)$.
- treating $V_0(u),\ldots, V_n(u)$ as control points for a curve parameterized by v.

Building surfaces from curves

Let the geometry vector vary by a second paramter v:

$$S(u,v) = \mathbf{U} \cdot \mathbf{M} \cdot \begin{bmatrix} \mathbf{G}_1(v) \\ \mathbf{G}_2(v) \\ \mathbf{G}_3(v) \\ \mathbf{G}_4(v) \end{bmatrix}$$

$$\mathbf{G}_i(v) = \mathbf{V} \cdot \mathbf{M} \cdot \mathbf{g}_i$$

$$\mathbf{g}_i = \begin{bmatrix} \mathbf{g}_{i1} & \mathbf{g}_{i2} & \mathbf{g}_{i3} & \mathbf{g}_{i4} \end{bmatrix}^T$$

Geometry matrices

By transposing the geometry curve we get:

$$
\begin{aligned}
\mathbf{G}_i(v)^T &= \left(\mathbf{V}\cdot\mathbf{M}\cdot\mathbf{g}_i\right)^T \\
&= \mathbf{g}_i{}^T\cdot\mathbf{M}^T\cdot\mathbf{V}^T \\
&= \begin{bmatrix}\mathbf{g}_{i1} & \mathbf{g}_{i2} & \mathbf{g}_{i3} & \mathbf{g}_{i4}\end{bmatrix}\cdot\mathbf{M}^T\cdot\mathbf{V}^T
\end{aligned}
$$

Geometry matrices

Combining

$$\mathbf{G}_i(v) = \begin{bmatrix} \mathbf{g}_{i1} & \mathbf{g}_{i2} & \mathbf{g}_{i3} & \mathbf{g}_{i4} \end{bmatrix} \cdot \mathbf{M}^T \cdot \mathbf{V}^T$$

And

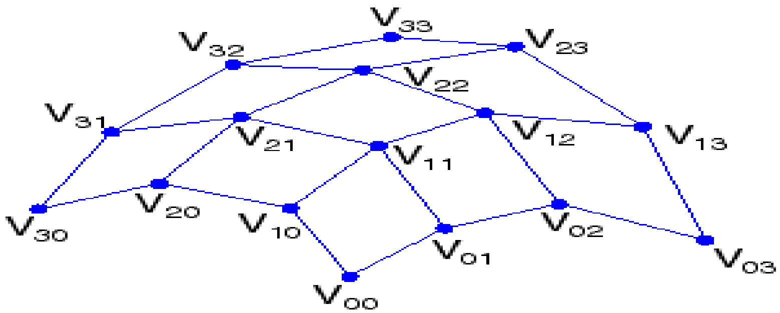
$$S(u,v) = \mathbf{U} \cdot \mathbf{M} \cdot \begin{bmatrix} \mathbf{G}_1(v) \\ \mathbf{G}_2(v) \\ \mathbf{G}_3(v) \\ \mathbf{G}_4(v) \end{bmatrix}^T$$

We get

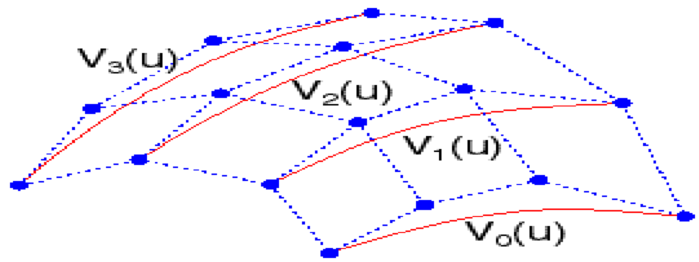
$$S(u,v) = \mathbf{U} \cdot \mathbf{M} \cdot \begin{bmatrix} \mathbf{g}_{11} & \mathbf{g}_{12} & \mathbf{g}_{13} & \mathbf{g}_{14} \\ \mathbf{g}_{21} & \mathbf{g}_{22} & \mathbf{g}_{23} & \mathbf{g}_{24} \\ \mathbf{g}_{31} & \mathbf{g}_{32} & \mathbf{g}_{33} & \mathbf{g}_{34} \\ \mathbf{g}_{41} & \mathbf{g}_{42} & \mathbf{g}_{43} & \mathbf{g}_{44} \end{bmatrix} \mathbf{M}^T \cdot \mathbf{V}^T$$

Tensor product surfaces, cont.

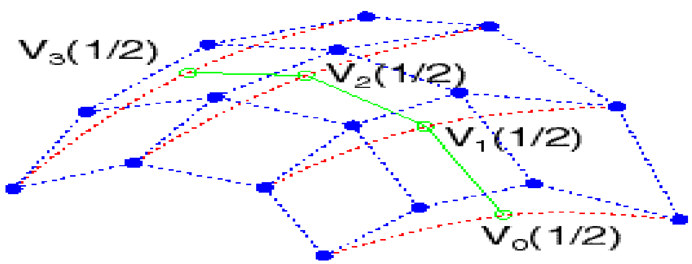
Let's walk through the steps:



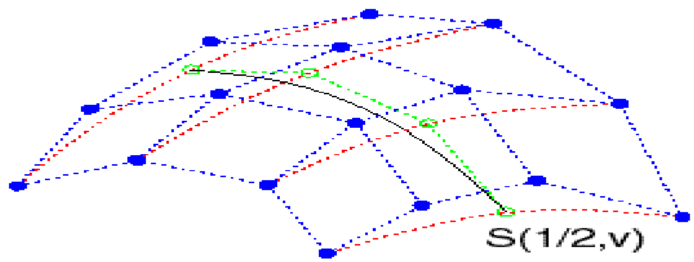
Control net



Control curves in u



Control polygon at u=1/2



Curve at $S(1/2,v)$

Which control points are interpolated by the surface?

Matrix form

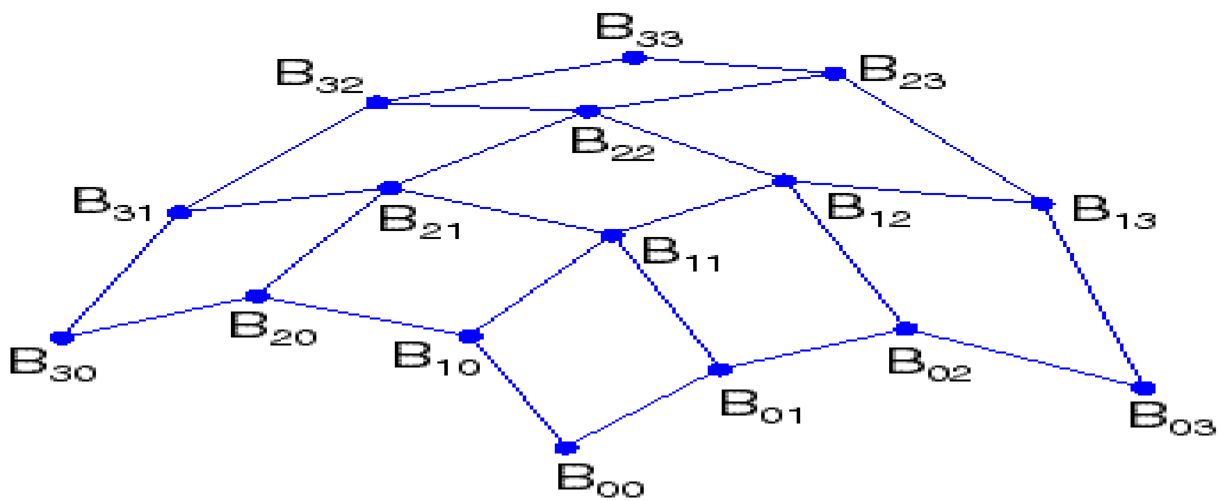
Tensor product surfaces can be written out explicitly:

$$S(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{n} V_{ij} B_i^n(u) B_j^n(v)$$

$$= \begin{bmatrix} v^3 & v^2 & v & 1 \end{bmatrix} M_{Bézier} \; V \; M_{Bézier}^T \begin{bmatrix} u^3 \\ u^2 \\ u \\ 1 \end{bmatrix}$$

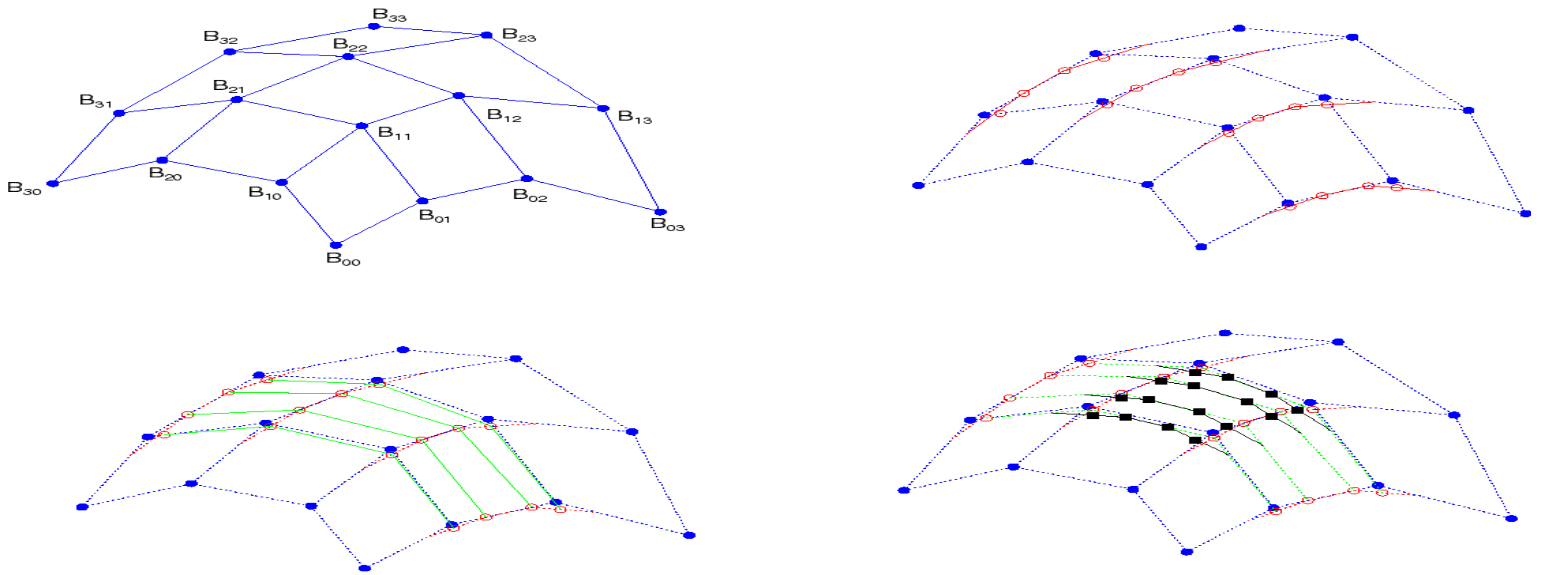
Tensor product B-spline surfaces

As with spline curves, we can piece together a sequence of Bézier surfaces to make a spline surface. If we enforce C2 continuity and local control, we get B-spline curves:



- treat rows of B as control points to generate Bézier control points in u.
- treat Bézier control points in u as B-spline control points in v.
- treat B-spline control points in v to generate Bézier control points in u.

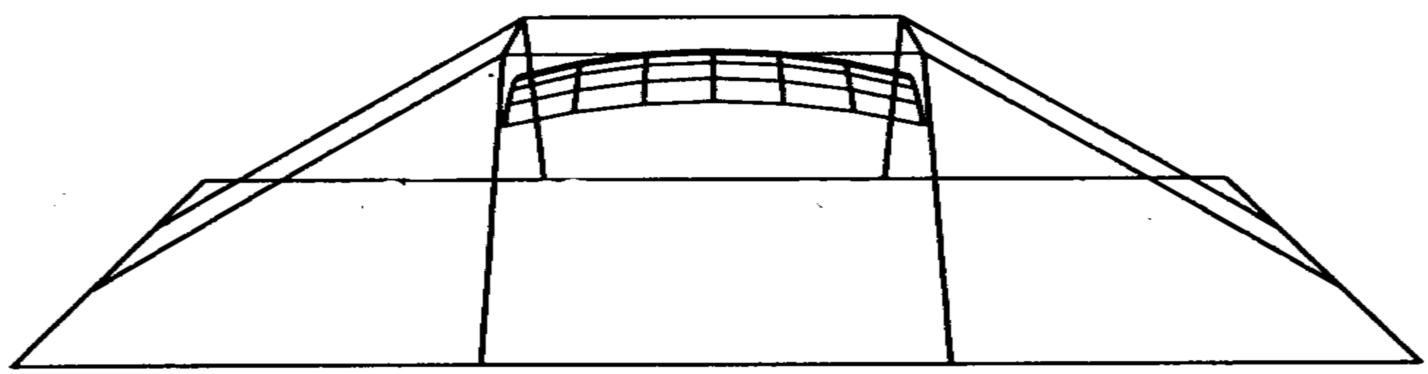
Tensor product B-splines, cont.



Which B-spline control points are interpolated by the surface?

Tensor product B-splines, cont.

Another example:

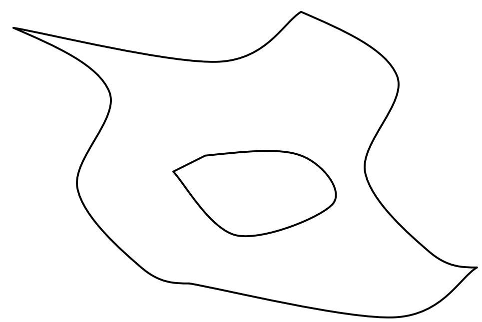


Trimmed NURBS surfaces

Uniform B-spline surfaces are a special case of NURBS surfaces.

Sometimes, we want to have control over which parts of a NURBS surface get drawn.

For example:



We can do this by **trimming** the *u*-*v* domain.

- Define a closed curve in the *u*-*v* domain (a **trim curve**)
- Do not draw the surface points inside of this curve.

It's really hard to maintain continuity in these regions, especially while animating.

Summary

What to take home:

- How to construct tensor product Bézier surfaces
- How to construct tensor product B-spline surfaces