Lecture 20:
Surface Modeling

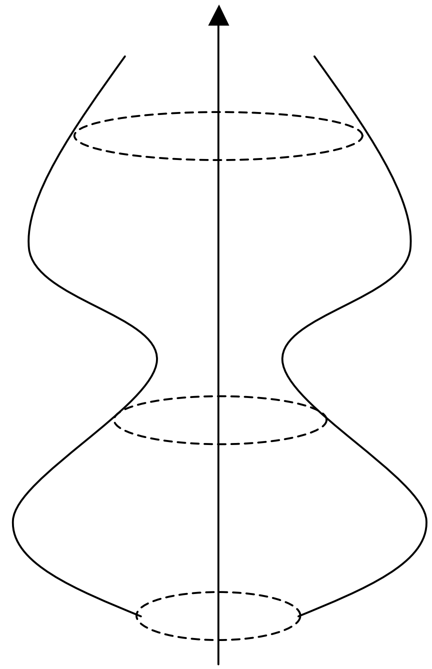
Reading

Hearn and Baker, sections 10.8, 10.9, 10.14.

Optional:

- Bartels, Beatty, and Barsky. *An Introduction to Splines for use in Computer Graphics and Geometric Modeling*, 1987.
- Stollnitz, DeRose, and Salesin. *Wavelets for Computer Graphics: Theory and Applications*, 1996, section 10.2.

Surfaces of revolution



Idea: rotate a 2D **profile curve** around an axis.

What kinds of shapes can you model this way?

Variations

Several variations are possible:

- Scale $C(u)$ as it moves, possibly using length of $T(v)$ as a scale factor.
- Morph $C(u)$ into some other curve $C'(u)$ as it moves along $T(v)$.
- …

Constructing surfaces of revolution

Given: A curve $C(u)$ in the yz-plane:

$$C(u) = \begin{bmatrix} 0 \\ c_y(u) \\ c_z(u) \\ 1 \end{bmatrix}$$

Let $R_x(\theta)$ be a rotation about the x-axis.

Find: A surface $S(u,v)$ which is $C(u)$ rotated about the z-axis.

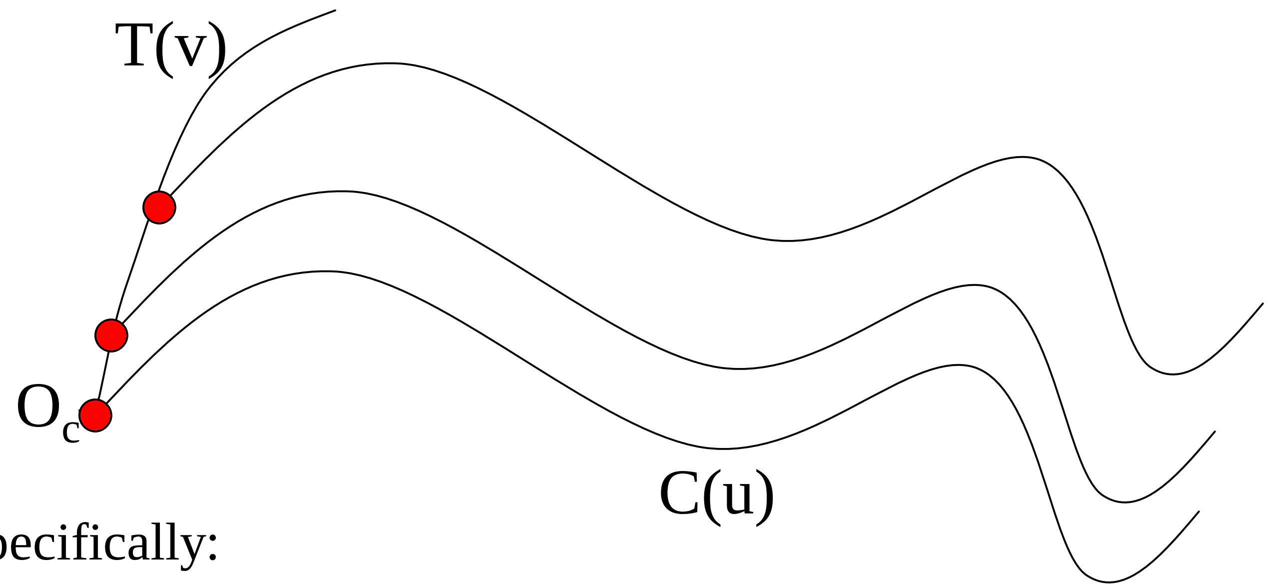
$$S(u,v) = \mathbf{R_x}(v) \cdot C(u)$$

General sweep surfaces

The **surface of revolution** is a special case of a **swept surface**.

Idea: Trace out surface $S(u,v)$ by moving a **profile curve** $C(u)$ along a **trajectory curve** $T(v)$.

$$S(u,v) = \mathbf{T}(T(v)) \cdot C(u)$$



More specifically:

- Suppose that $C(u)$ lies in an (x_c,y_c) coordinate system with origin O_c.
- For every point along $T(v)$, lay $C(u)$ so that O_c coincides with $T(v)$.

Orientation

The big issue:

- How to orient $C(u)$ as it moves along $T(v)$?

Here are two options:

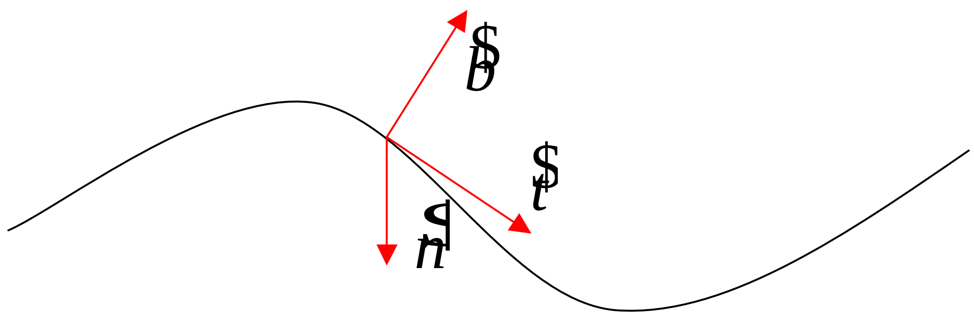
1. **Fixed** (or **static**): Just translate O_c along $T(v)$.

2. Moving. Use the **Frenet frame** of $T(v)$.

- Allows smoothly varying orientation.
- Permits surfaces of revolution, for example.

Frenet frames

Motivation: Given a curve $T(v)$, we want to attach a smoothly varying coordinate system.



To get a 3D coordinate system, we need 3 independent direction vectors.

$$\hat{t}(v) = normalize(T'(v))$$

$$\hat{b}(v) = normalize(T'(v) \times T''(v))$$

$$\hat{n}(v) = \hat{b}(v) \times \hat{t}(v)$$

As we move along $T(v)$, the Frenet frame (t,b,n) varies smoothly.

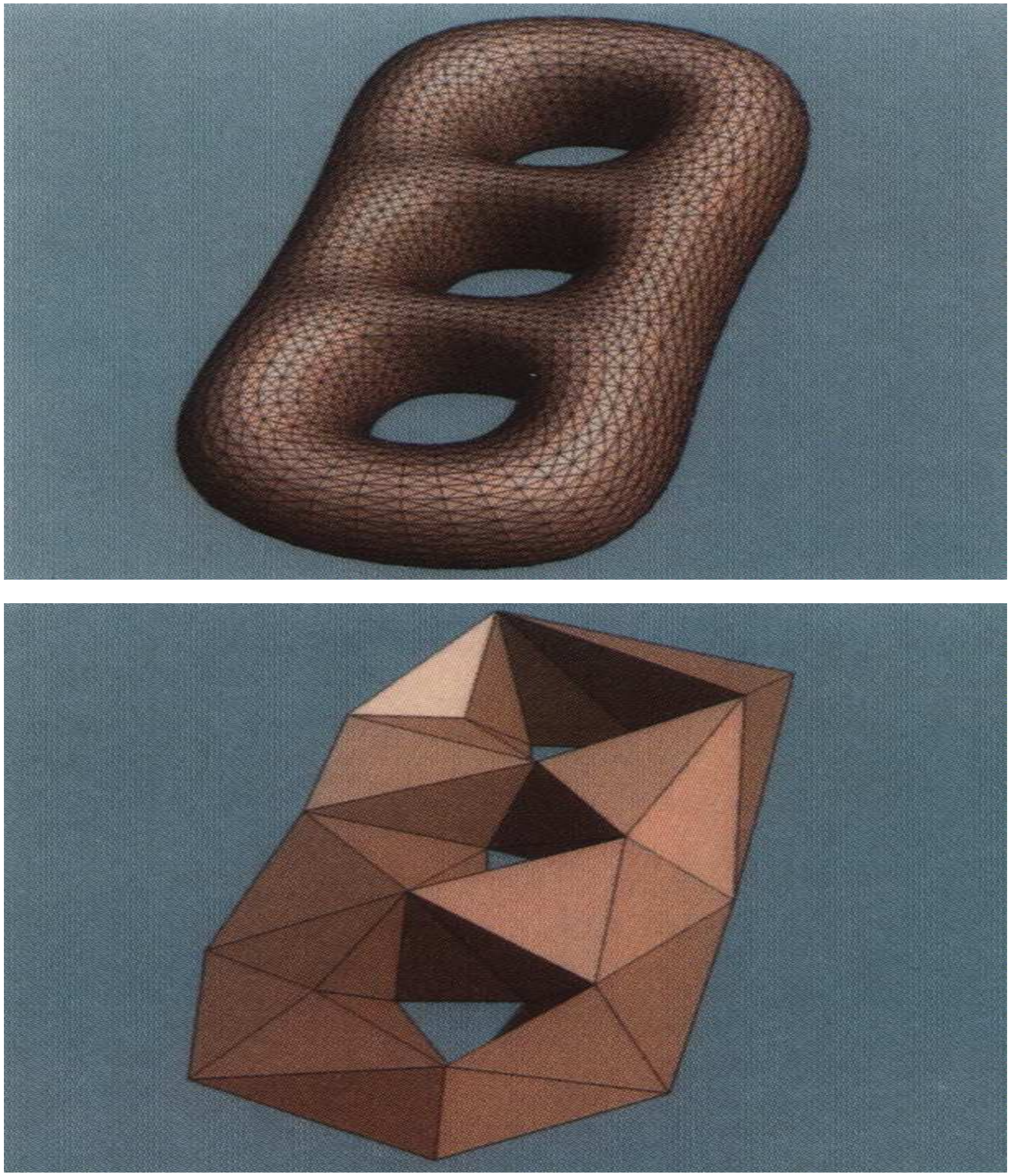
Frenet swept surfaces

Orient the profile curve $C(u)$ using the Frenet frame of the trajectory $T(v)$:

- Put $C(u)$ in the **normal plane** nb.
- Place O_c on $T(v)$.
- Align x_c for $C(u)$ with $-n$.
- Align y_c for $C(u)$ with b.

If $T(v)$ is a circle, you get a surface of revolution exactly?

Building complex models



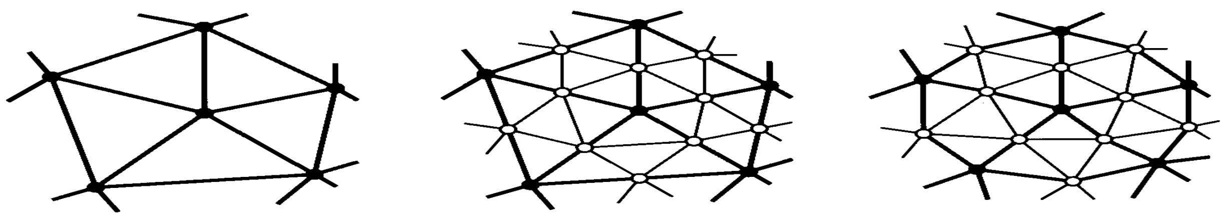
Subdivision surfaces

Chaikin's use of subdivision for curves inspired similar techniques for subdivision.

Iteratively refine a **control polyhedron** (or **control mesh**) to produce the limit surface

$$\sigma = \lim_{j \to \infty} M^j$$

using splitting and averaging steps.



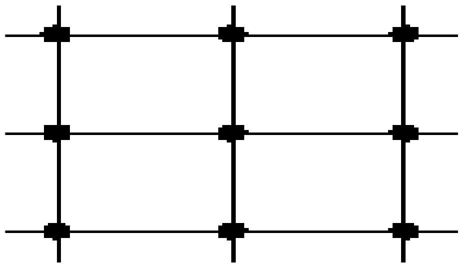
(a) (b) (c)

There are two types of splitting steps:

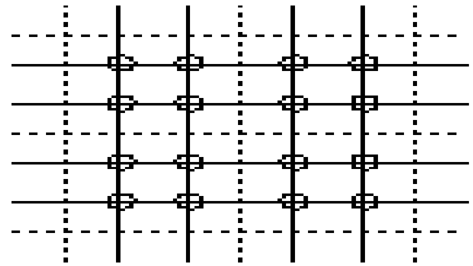
- **vertex schemes**
- **face schemes**

Vertex schemes

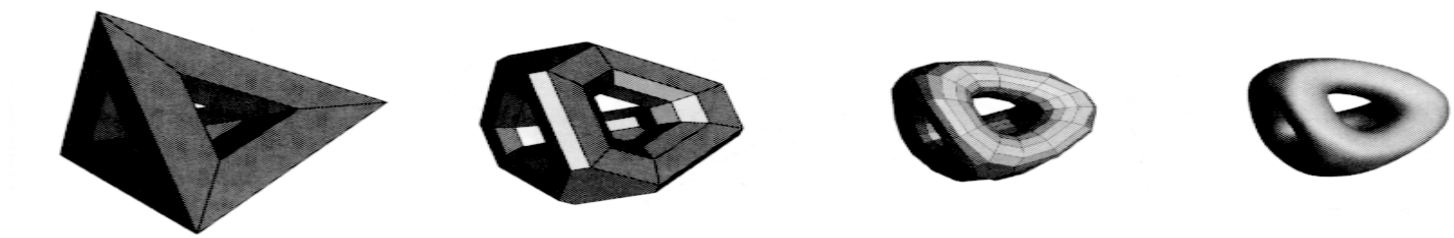
A vertex surrounded by *n* faces is split into *n* subvertices, one for each face:



Original



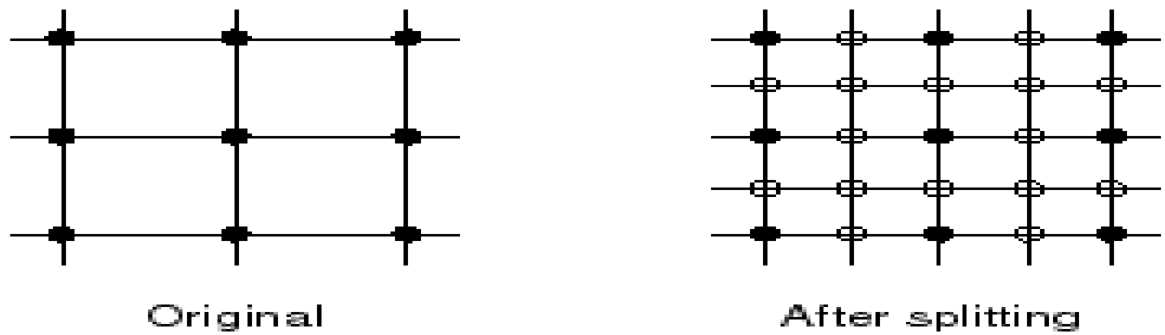
After splitting



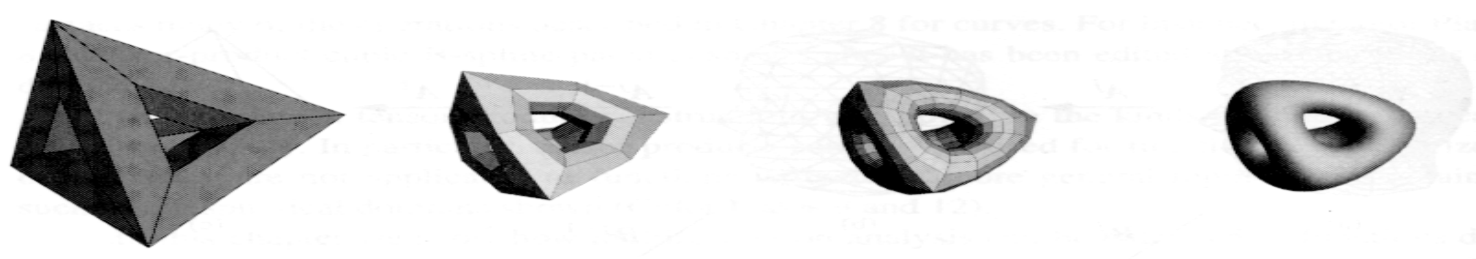
Doo-Sabin subdivision:

Face schemes

Each quadrilateral face is split into four subfaces:

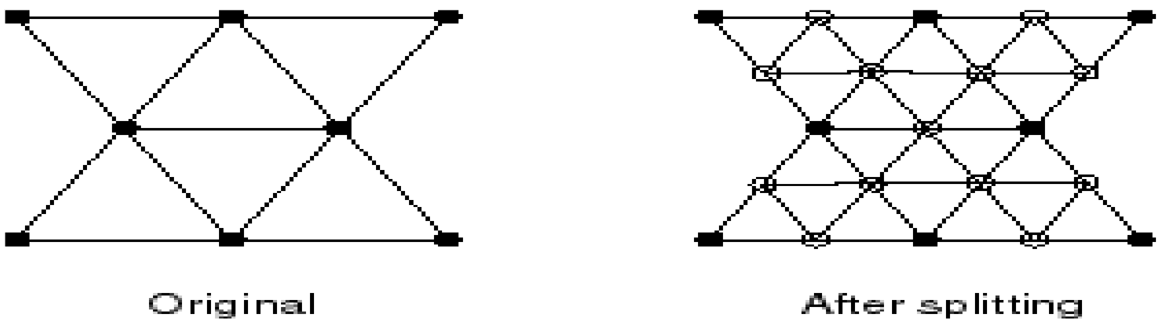


Catmull-Clark subdivision:

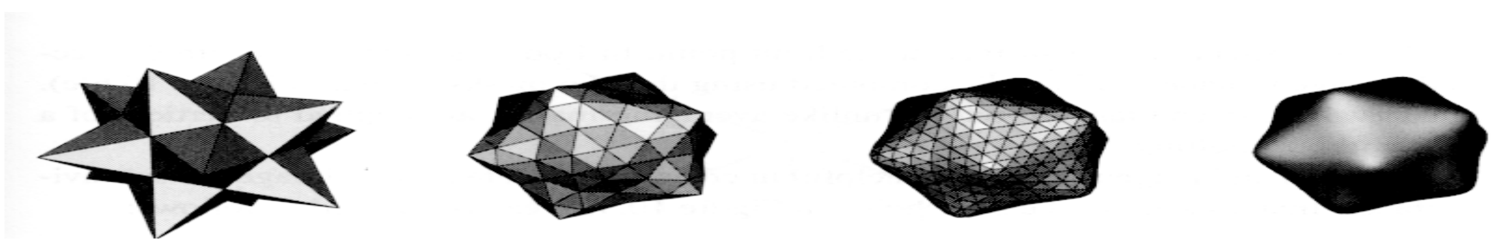


Face schemes, cont.

Each triangular face is split into four subfaces:



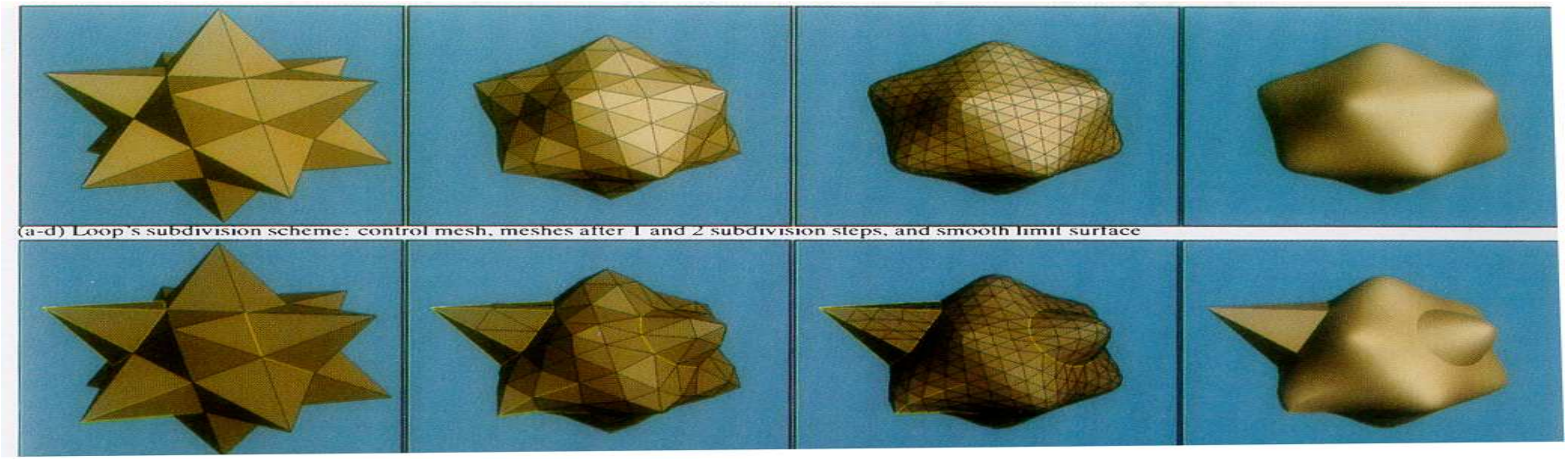
Loop subdivision:



Adding creases without trim curves

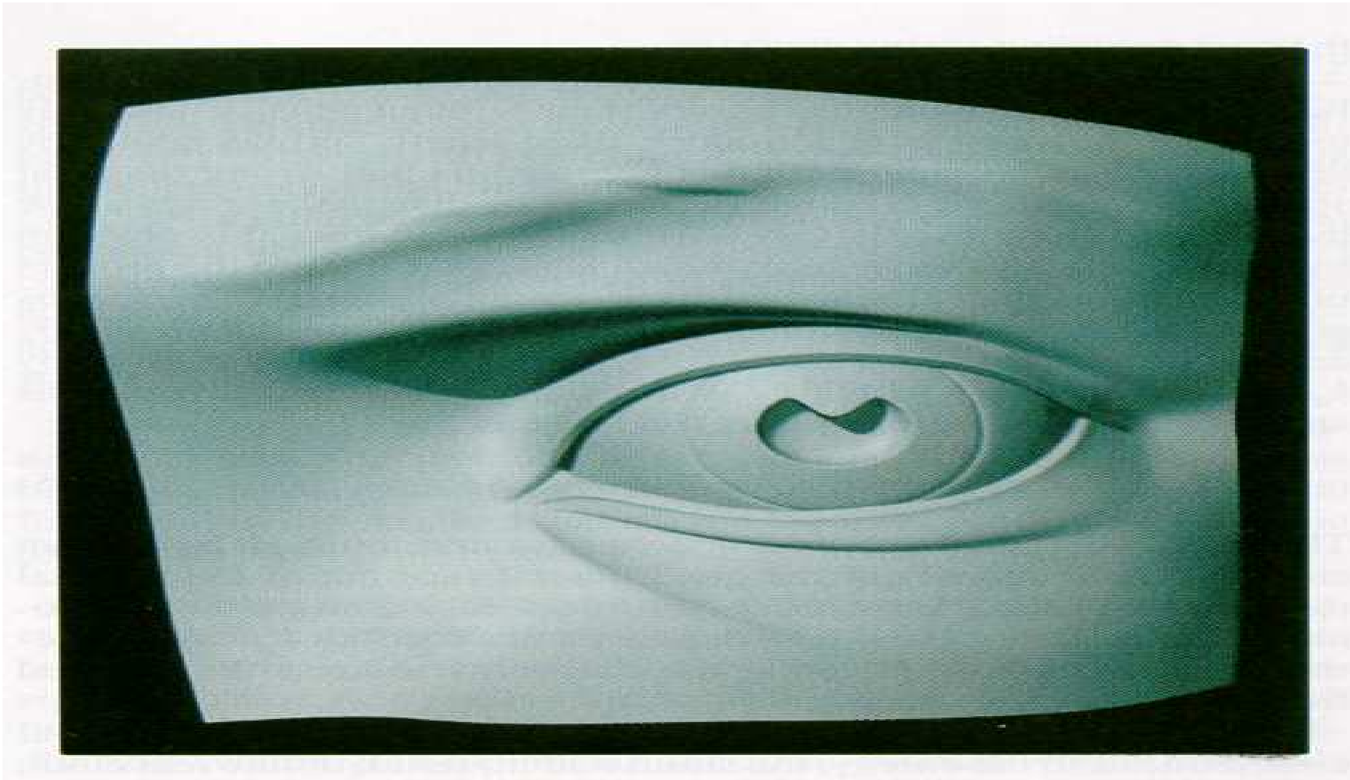
In some cases, we want a particular feature such as a crease to be preserved. With NURBS surfaces, this required the use of trim curves.

For subdivision surfaces, we can just modify the subdivision mask.



Creases with trim curves, cont.

Here's an example using Catmull-Clark surfaces of the kind found in Geri's Game:



Summary

What to take home:

- Surfaces of revolution
- How to construct swept surfaces from a profile and trajectory curve:
  - with a fixed frame
  - with a Frenet frame
- Subdivision surfaces