Lecture 10:
Projections

Reading

- Hearn and Baker, Sections 12.1-12.4

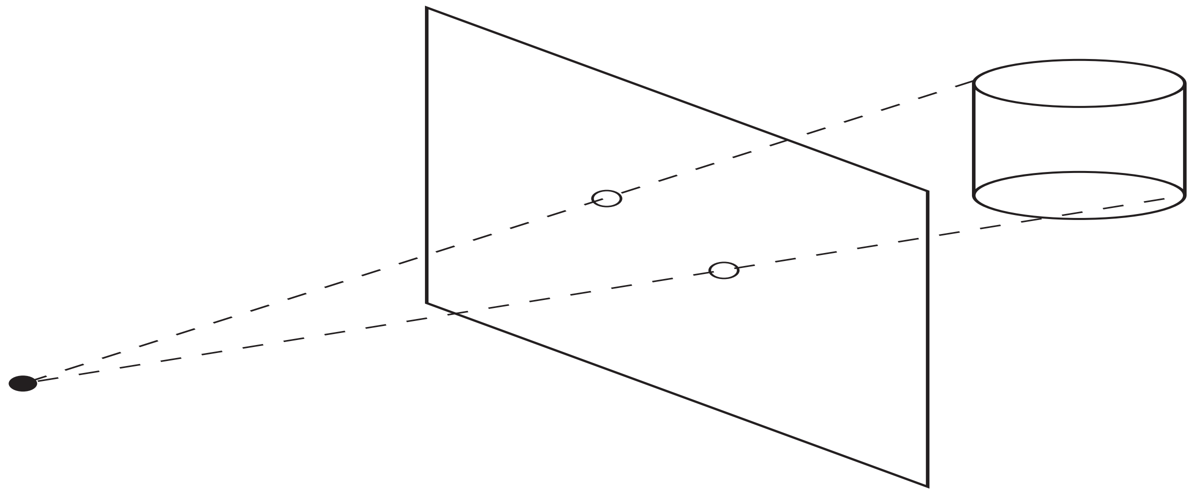
Optional

- Foley *et al.* Chapter 6

Projections

Projections transform points in *n*-space to *m*-space, where $m < n$.

In 3D, we map points from 3-space to the **projection plane (PP)** along **projectors** emanating from the **center of projection (COP)**.



There are two basic types of projections:

- **Perspective -** distance from COP to PP finite
- **Parallel -** distance from COP to PP infinite

Perspective vs. parallel projections

Perspective projections pros and cons:

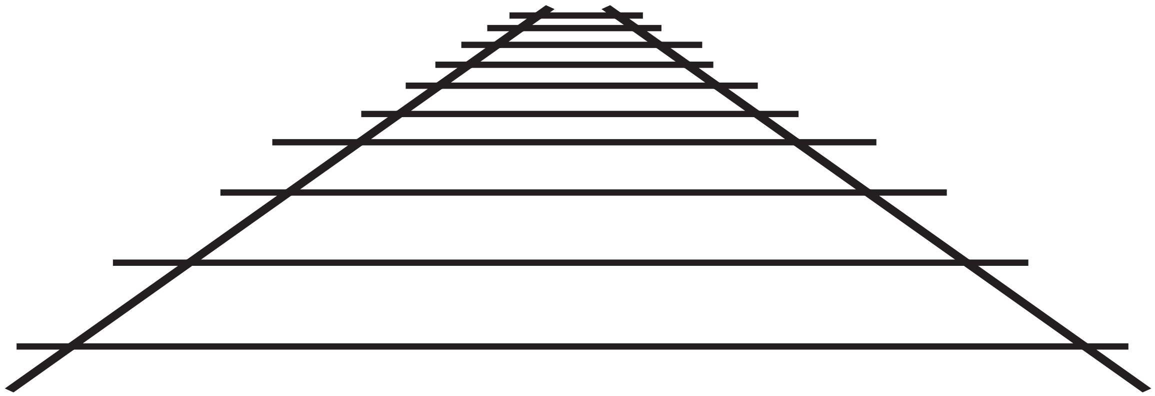
+ Size varies inversely with distance - looks realistic
– Distance and angles are not (in general) preserved
– Parallel lines do not (in general) remain parallel

Parallel projection pros and cons:

– Less realistic looking
+ Good for exact measurements
+ Parallel lines remain parallel
– Angles not (in general) preserved

Vanishing points

Under perspective projections, any set of parallel lines that are not parallel to the PP will converge to a **vanishing point**.



Vanishing points of lines parallel to a principal axis *x*, *y*, or *z* are called **principal vanishing points**.

How many of these can there be?

Types of perspective drawing

Perspective drawings are often classified by the number of principal vanishing points.

- One-point perspective — simplest to draw
- Two-point perspective — gives better impression of depth
- Three-point perspective — most difficult to draw

All three types are equally simple with computer graphics.

Parallel projections

For parallel projections, we specify a **direction of projection (DOP)** instead of a COP.

There are two types of parallel projections:

- **Orthographic projection** — DOP perpendicular to PP
- **Oblique projection** — DOP not perpendicular to PP

There are two especially useful kinds of oblique projections:

- **Cavalier projection**
  - DOP makes 45° angle with PP
  - Does not foreshorten lines perpendicular to PP
- **Cabinet projection**
  - DOP makes 63.4° angle with PP
  - Foreshortens lines perpendicular to PP by one-half

Orthographic Projections

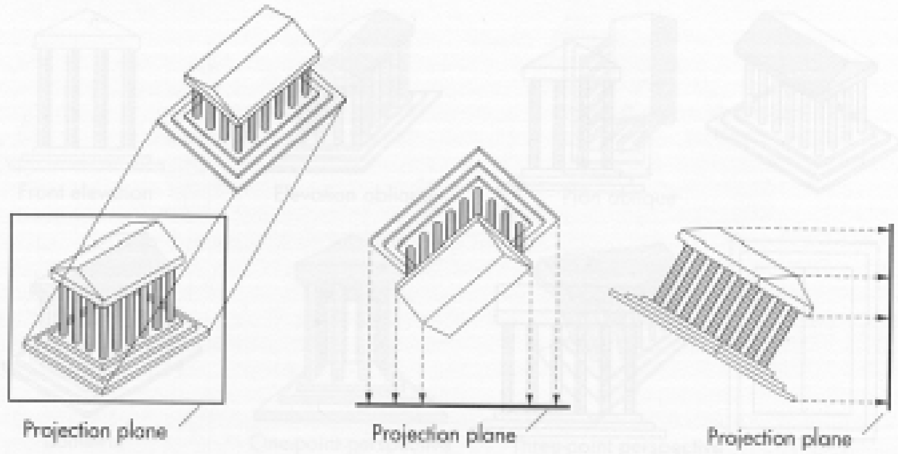


Figure 5.6 Axonometric projections. (a) Construction of trimteric-view projections. (b) Top view. (c) Side view.

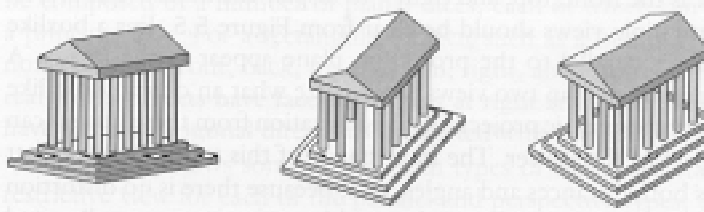
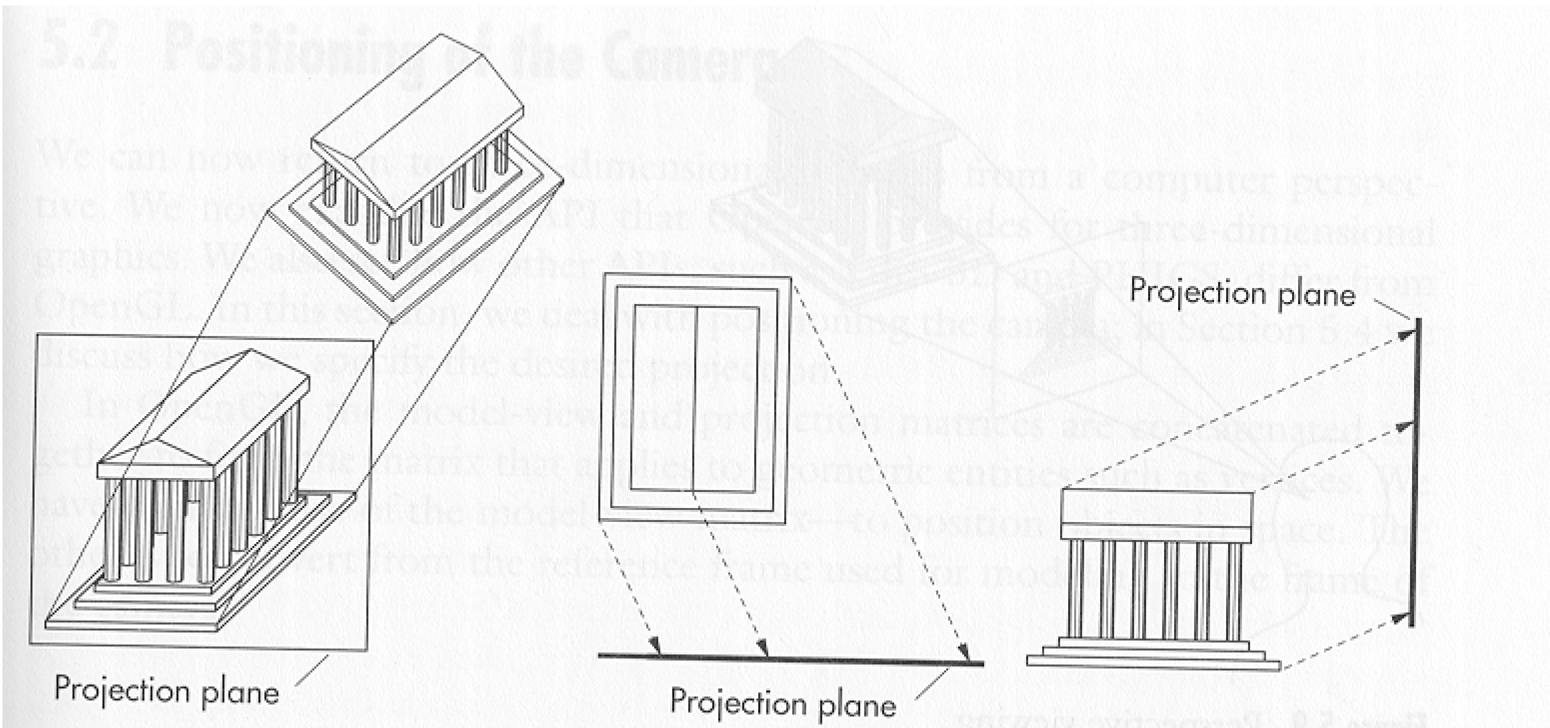
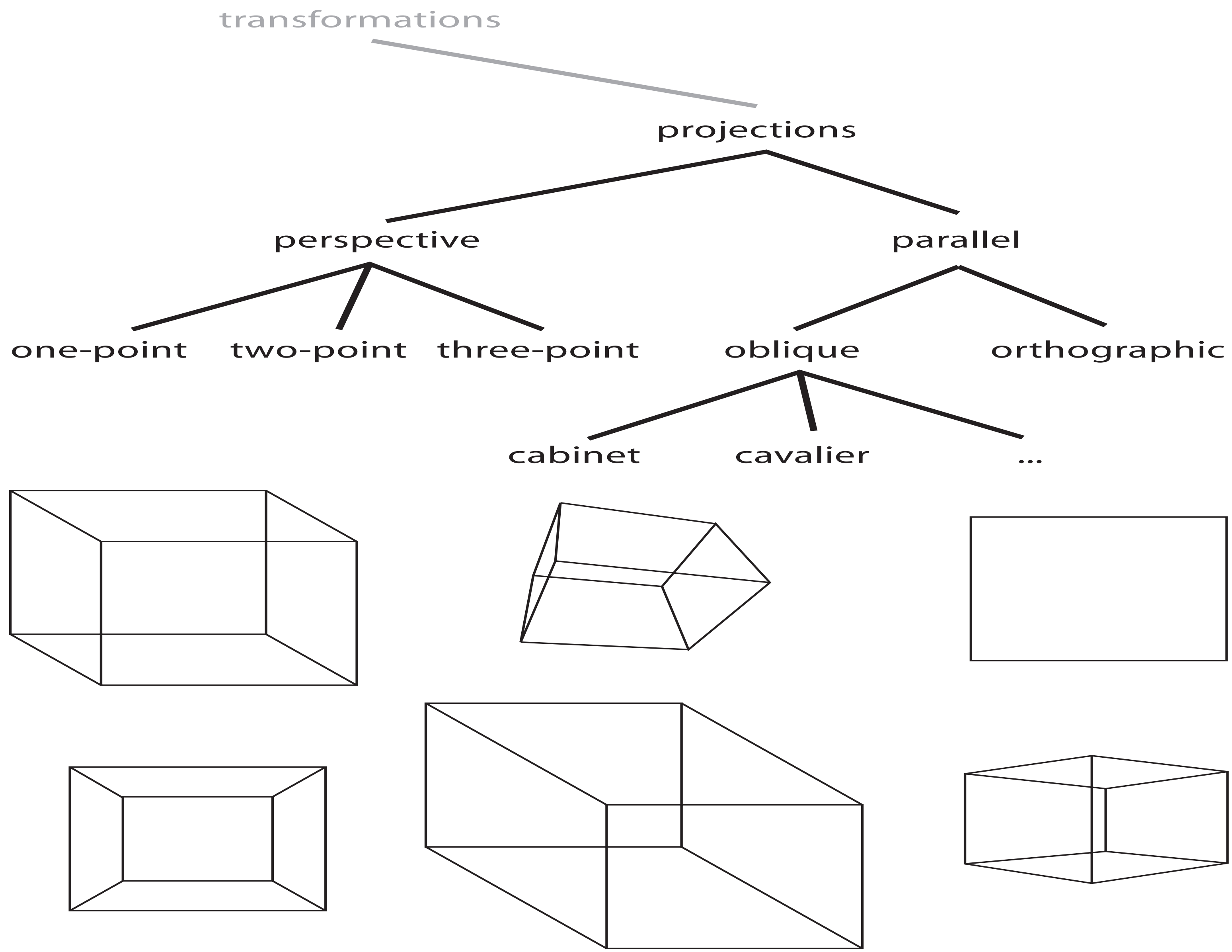


Figure 5.7 Axonometric views.

Oblique Projections



Projection taxonomy

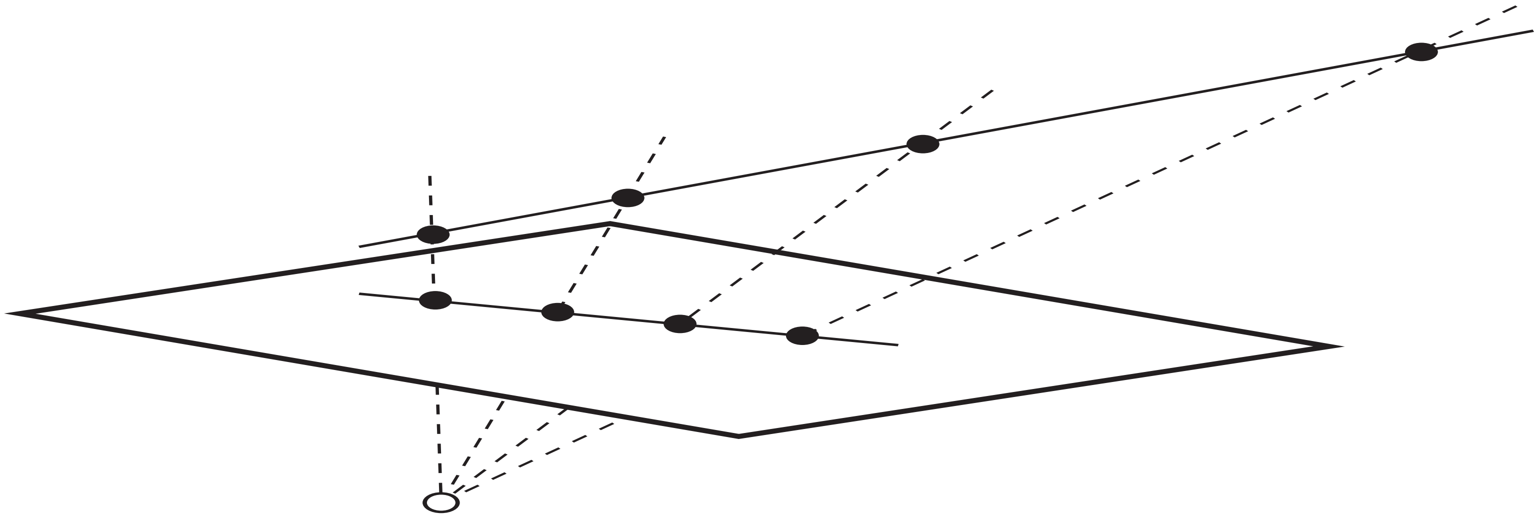


Properties of projections

The perspective projection is an example of a **projective transformation**.

Here are some properties of projective transformations:

- Lines map to lines
- Parallel lines *don't* necessarily remain parallel
- Ratios are *not* preserved

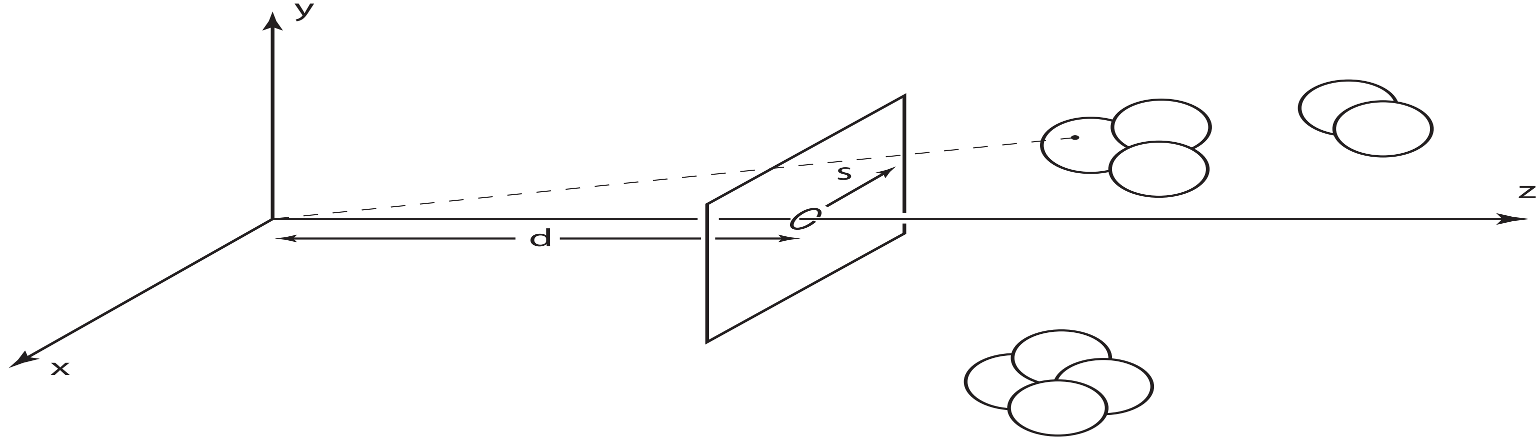


Coordinate systems for CG

The real computer graphics guru uses lots of different coordinate systems:

- **Model space** — for describing the objections (aka “object space”, “world space”)
- **World space** — for assembling collections of objects (aka “object space”, “problem space”, “application space”)
- **Eye space** — a canonical space for viewing (aka “camera space”)
- **Screen space** — the result of perspective transformation (aka “normalized device coordinate space”, “normalized projection space”)
- **Image space** — a 2D space that uses device coordinates (aka “window space”, “screen space”, “normalized device coordinate space”, “raster space”)

A typical eye space

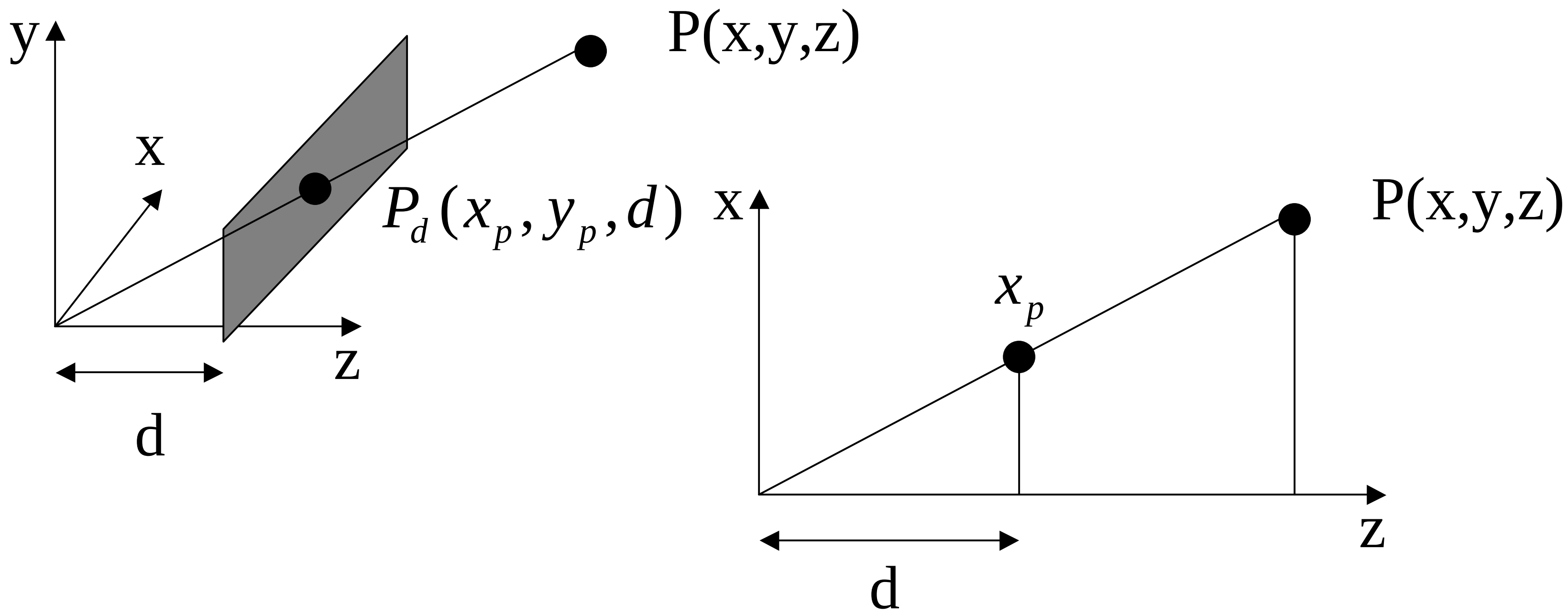


- **Eye**
  - Acts as the COP
  - Placed at the origin
  - Looks down the *z*-axis
- **Screen**
  - Lies in the PP
  - Perpendicular to *z*-axis
  - At distance *d* from the eye
  - Centered on *z*-axis, with radius *s*

Q: Which objects are visible?

Eye space ➔ screen space

Q: How do we perform the perspective projection from eye space into screen space?



Using similar triangles gives:

Eye space ➔ screen space, cont.

We can write this transformation in matrix form:

$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & 1/d & 0 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix} = MP = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

$$\begin{bmatrix} X/W \\ Y/W \\ Z/W \\ W/W \end{bmatrix} = \begin{bmatrix} \dfrac{x}{z/d} \\ \dfrac{y}{z/d} \\ d \\ 1 \end{bmatrix}$$

General perspective projection

Now, at last, we can see what the "last row" does.

In general, the matrix

$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ p & q & r & s \end{bmatrix}$$

performs a perspective projection into the plane $px + qy + rz + s = 1$.

Q: Suppose we have a cube C whose edges are aligned with the principal axes. Which matrices give drawings of C with

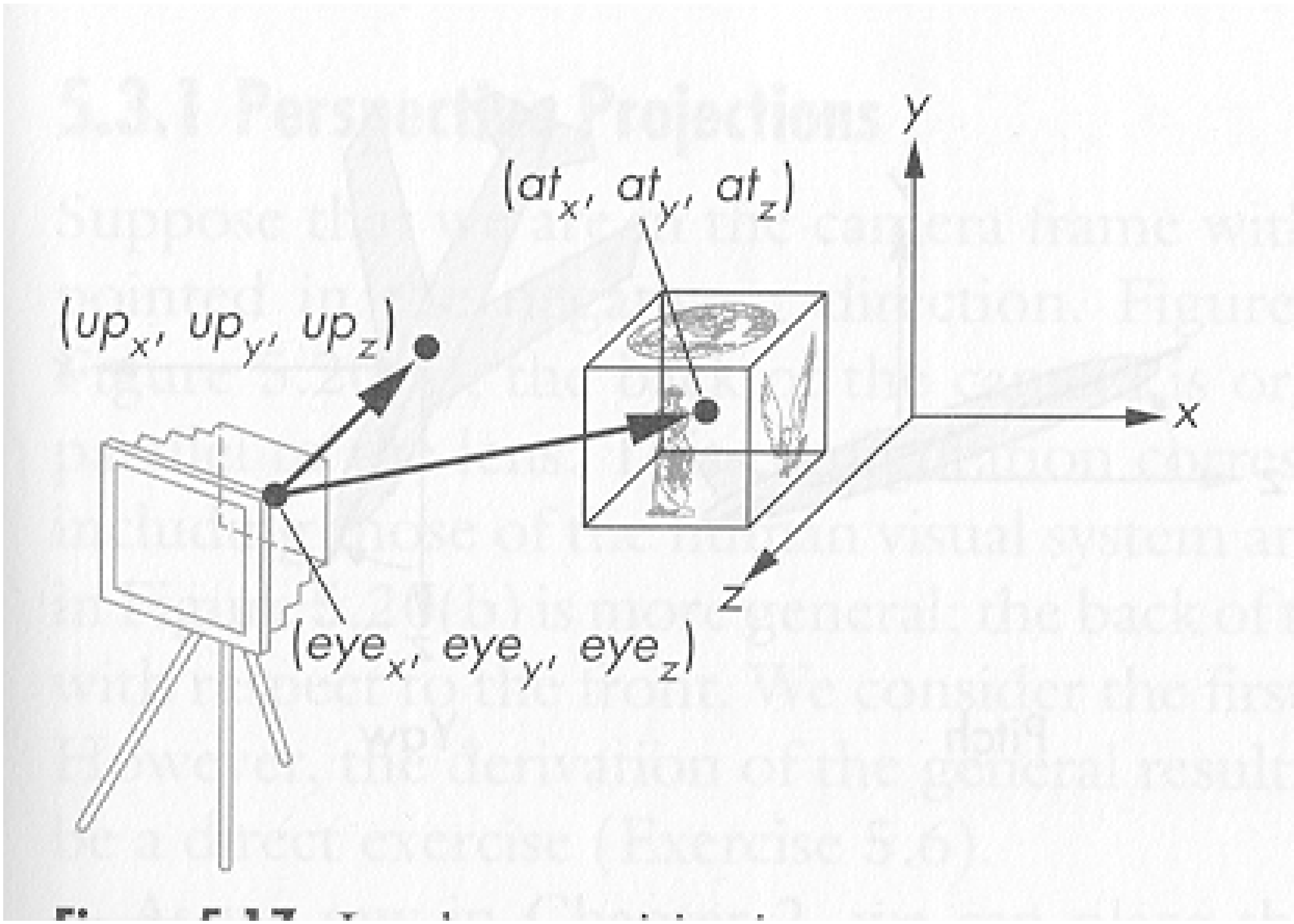
- one-point perspective?
- two-point perspective?
- three-point perspective?

Orthographic Projection

$$d = \infty$$

$$M_{ort} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

World Space Camera



Perspective depth

Q: What did our perspective projection do to z?

Often, it's useful to have a z around — e.g., for hidden surface calculations.

Hither and yon planes

In order to preserve depth, we set up two planes:

- The **hither** plane $z_e = N$
- The **yon** plane $z_e = F$

We'll map:

Exercise: Derive the matrix to do this projection.

Summary

Here's what you should take home from this lecture:

- What homogeneous coordinates are and how they work.
- Mathematical properties of affine vs. projective transformations.
- The classification of different types of projections.
- The concepts of vanishing points and one-, two-, and three-point perspective.
- An appreciation for the various coordinate systems used in computer graphics.
- How the perspective transformation works.