CSE 457 : Homework # 2 Zoran Popović

> Assigned 10/18Due 11/1

The number of points assigned to each problem corresponds to the number of minutes I expect it will take you to complete the problem if you already know the material. Feel free to talk over the problems with classmates, but please answer the questions on your own.

Name: \_\_\_\_\_

# 1 5 points

Give three two-dimensional transformation operations that can be performed on points using a  $2 \times 2$  matrix. Which important operation can not be expressed in a  $2 \times 2$  matrix? How do we solve this problem?

# 2 15 points

Please give the matrix (or matrices) for reflecting across the arbitrary line given by the following equation:

y = ax + b

# 3 30 points

The objective of this question is to construct a hierarchy model of a human hand using geometric transformations on a sphere primitive.



#### 3.1 Model Primitives

You will need to transform a sphere (which starts off with a radius of 1 unit) into the appropriate shapes for an upper arm, lower arm, and hand (i.e. there will be three transformed spheres in this model). The length of the upper arm will be u units longs, the lower arm will be l units long, and the hand

will be h units long. The radial thickness of the upper arm, lower arm, and hand is t units. Construct three hierarchies using a unit sphere with scaling and translation transformations that will draw the upper arm, lower arm, and hand. Explicitly use the parameters u, l, h, and t in your hierarchies. You will need to translate the upper arm so that the shoulder joint is at the upper arm object space origin, the lower arm so that the elbow joint is at lower arm object space origin, and the hand so that the wrist joint is at the hand object space origin.

#### 3.2 Model Hierarchy

The shoulder, elbow, and wrist have a rotation matrix associated with them that represents the relationship between the upper arm and body, the lower arm and upper arm, and the hand and lower arm respectively (appropriately named as  $R_s$ ,  $R_e$ ,  $R_w$ ). Using the hierarchy primitives defined in the previous section (upper arm, lower arm, and hand) as well as additional rotation and translation transformations, draw a hierarchy tree showing how you would connect the model hierarchy primitives in the form of an arm.

### 3.3 Hierarchies in OpenGL

To draw this arm in OpenGL, the arm model primitives need to be drawn with the proper transformations applied. We need to iterate over the tree while maintaining the proper transformation matrix stack. Show (in pseudocode) how you would combine the information and structure of the hierarchy with the minimum number of glPushMatrix() and glPopMatrix() function calls to properly draw the object.

### 4 20 points

Pretend we have a model for a windshield wiper (with the origin of the object space defined at the axis around which the windshield wiper rotates). Windshield wipers rotate through an angle *theta* that ranges from 0 degrees to 90 degrees. Some cars have two identical windshield wipers that are mirror images of each other; we can use a single model with different transformations to draw both windshield wipers. These two windshield wipers are each d units of distance away from the center of the windshield. Show how to do this task in pseudocode using the following operations (you may use glRotate() only once; do not worry about intersections between the two windshield wipers):

- 1. glScale()
- 2. glTranslate()
- 3. glRotate()
- 4. drawWindshieldWiper()

This is a visual representation of the model in object space. Note the angle theta around which the windshield wiper rotates. The windshield wiper rotates around the origin of object space.



This is what the combined model should look like in world space. Note the distance d from the center point to the point around which the wiper rotates.



# 5 20 points

Under perspective projections, any set of parallel lines that are not parallel to the projection plane will converge to a "vanishing point". Vanishing points of lines parallel to a principal axis x, y, or z are called "principal vanishing points".

1. How many different vanishing points can a perspective drawing have?

2. How many different **principal** vanishing points can a perspective drawing have?

### 6 10 points

### 6.1 Perspective Projections

1. true or false - Size varies inversely with distance.

- 2. true or false Distance and angles are preserved.
- 3. true or false Parallel lines do not remain parallel.
- 4. true or false Perspective projections (i.e. non-linear spaces) make z-buffers more imprecise.

#### 6.2 Parallel Projections

- 1. true or false Parallel projections are more realistic looking than perspective projections.
- 2. true or false Parallel projections are good for exact measurements.
- 3. true or false Parallel lines do not remain parallel.
- 4. true or false Angles (in general) are not preserved.
- 5. true or false Lengths vary with distance to the eye.

## 7 Extra Credit - 20 points

In two dimensions, rotations of an angle  $\theta$  can be specified as a series of shear transformation matrices. Give these matrices, or if it can't be done, prove it.