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Homework 2: Image Processing, Image Compositing, and Geometric Transformations

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Directions

Please provide short written answers to the questions in the space provided. If you require extra space, you may staple additional pages to the back of your assignment. Feels free to talk over the problems with classmates but please answer the questions on your own.

Name:____

Problem 1 True or False. Please justify your answers

(a) T/F: Impulse noise contains random occurrences of black and white pixels.

(b) T/F: Mean filters can be used for smoothing an image as well as reducing noise.

(c) T/F: When using a mean filter, increasing the size of the filter sharpens the image.

(d) T/F: A median filter does a better job of throwing out impulse and salt and pepper noise than a mean filter.

(e) T/F: A median filter is a type of convolution filter.

(f) T/F: At any location on an image, the gradient points in the direction perpendicular to the maximum change in intensity.

(g) T/F: The lower the frequency of a sinusoid, the more samples that must be taken to gain an accurate representation of the wave.

Problem 2 Image Processing

(a) Convolution filtering can modify images in a variety of ways. Describe the expected effect of filtering an image using the following convolution kernel. Justify your answer.

$$\begin{matrix} \mathbf{-1} & \mathbf{-3} & \mathbf{-1} \\ \mathbf{-3} & \mathbf{17} & \mathbf{-3} \\ \mathbf{-1} & \mathbf{-3} & \mathbf{-1} \end{matrix}$$

(b) An image I(i,j) is given as an array of grayscale values, each between 0 and 1. If we filter an image I(i,j) with the convolution kernel given above, we get an image $I'(i,j)$. What are the minimum and maximum values the filtered image $I'(i,j)$ can take on at a particular pixel?

(c) Why might the following convolution kernel be a good choice for smoothing images taken from interlaced video?

$$\begin{matrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{8} & \mathbf{16} & \mathbf{8} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{matrix}$$

Problem 3 Compositing

A common technique used in special effects work is one in which a foreground image (such as that of an actor) is extracted from a background of a constant *backing color* - usually blue. The foreground image, along with its **matte**, or alpha channel, can then be composited over a different background. Thus, an actor can be filmed in front of a blue screen and then made to appear as if he or she is in some other environment. This technique is called “blue screen matting,” and the computation of the foreground’s alpha channel is called “pulling a matte”.

While the blue screen matting problem is very difficult in general, there are a number of situations in which a **priori** knowledge of the foreground image colors can make the problem relatively simple. So, let’s try pulling some mattes for some of these special cases.

In all of the following notation, you should assume premultiplied alphas for all RGB components; that is, (r, b, g, α) indicates a pixel that is α covered by the color $(r/\alpha, g/\alpha, b/\alpha)$**:**

- Let $c_k = (r_k, g_k, b_k, \alpha_k)$ be the backing color which for the rest of this problem we’ll assume to be 100% blue, or (0,0,1,1).
- Let $c_f = (r_f, g_f, b_f, \alpha_f)$ be the color of a given pixel in the foreground image, once it is extracted from the backing color.
- Finally, let $c_o = (r_o, g_o, b_o, \alpha_o)$ be the color of that same pixel as it is originally seen by the camera-that is, c_o is the color of the foreground c_f over the backing color c_k. We can assume that every pixel in this image is opaque-i.e., that α_o=1.

In general, c_k and c_o are the “knowns,” and c_f is the unknown that we need to solve for. “Pulling a matte” amounts to computing α_f. Once we know α_f, it is easy to solve for the other color components of the foreground image (as you’ll show in part(b) below).

Problem 3 (continued)

(a) Write an expression for c_o in terms of c_f and c_k.

(b) Express r_f, g_f, b_f, as functions of α_f and (some or all) of the components of $c_{o.}$

(c) Assume that the foreground image has no blue in it whatsoever—i.e., that $b_f = 0$. What is α_f, expressed as (some or all) of the components of $c_{o.}$

(d) Assume that the foreground is a perfectly grey spaceship-i.e. that $r_f = g_f = b_f$. What is α_f, expressed as a function of (some or all of) the components of c_o?

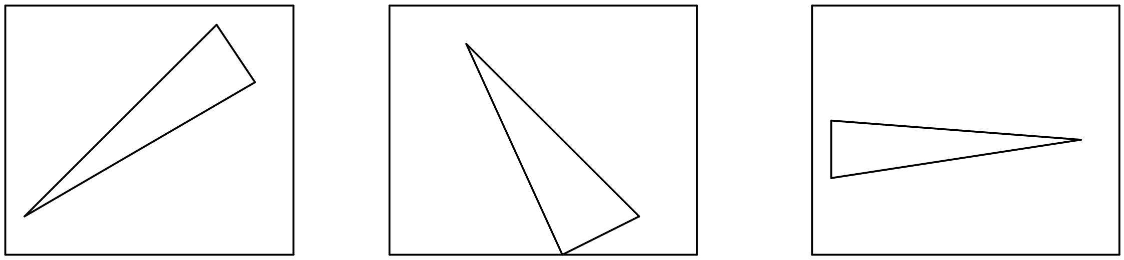
(e) Flesh of all races tends to have an RGB color expressible as [d,.5d,.5d], with d being a lightening factor depending on the individual. Assuming a (flesh colored) human actor as the foreground element, what is $\alpha_{f,}$ expressed as a function of (some or all of) the components of c_o?

Problem 3(continued)

(f) There is a problem with this type of matte extraction technique. Blue objects that appear in the foreground image will disappear, allowing the background matte to show through. Suppose that all objects were also shot against a black background. Write an expression for the foreground color r_f, g_f, b_f, and α_f (as described in part b) using a black background, or (0,0,0,1).

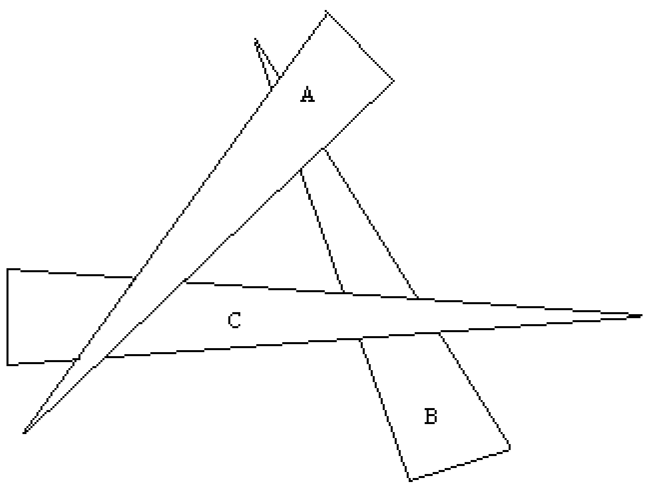
(g) Using both the images in (f), could we extract blue objects so that they would appear opaque in the final composite? If not, explain why not. Otherwise justify your answer by computing α_f.

Problem 4 Compositing
Suppose you were given three different elements that you wish to composite together into a single image:



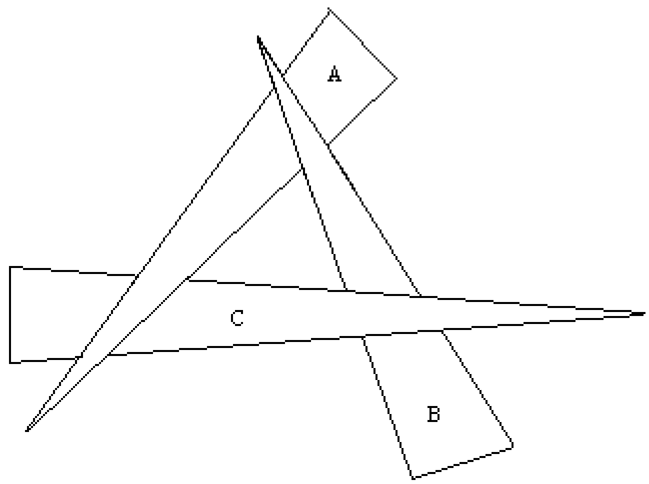
A **B** **C**

(a) Gives a concise compositing expression that yields the following figure.



Problem 4 (continued)

(b) Let's assume for the moment that we are not concerned about fractional coverage, i.e., $\alpha = 0$ or $\alpha = 1$ everywhere. Give a concise compositing expression that yields the following figure. More credit will be given for shorter expressions.



(c) Give another concise expression for the figure in part (b).
(Bonus points will be given if you don't use the same set of operators)

Problem 4 (continued)

(d) Now suppose that we take fractional alpha values into account. For the input images A and B, will the compositing expressions **(B over A)** and **(A out B) over B** produce the same output images? **Justify your answer.**

Problem 5 Geometric Transformations

As discussed in class, any two-dimensional affine transformation can be represented as a 3X3 matrix. Here are some useful matrices:

$$A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad B(a,b)=\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \qquad C(a,b)=\begin{bmatrix} \cos a & \sin a & 0 \\ -\sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D(a,b)=\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad E=\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad F(a)=\begin{bmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(a) Which of the matrices above implements each of the following transformations?

Rotation about the origin

Reflection through the line y=x

Translation

Shearing

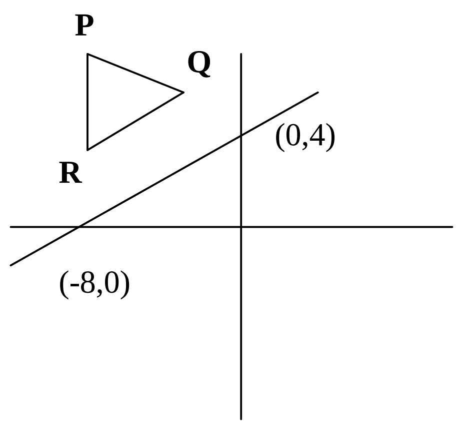
Differential (Non-Uniform) Scaling

Reflection through the x-axis

Problem 5 (continued)

(b) Consider the line L and the triangle PQR shown below. The equation of the line L is

$$\mathbf{Y = \tfrac{1}{2}X + 4}$$



Describe a 3x3 transformation matrix that reflects triangle PQR through the line L. (Hint: You should express your answer as the product of matrices shown on the previous page. All matrices should be filled out with exact numerical values to receive full credit).

Problem 6 Projections

(a) Complete the following table, summarizing properties of these two types of projections:

PROPERTY	PERSPECTIVE	PARALLEL
Parallel lines remain parallel[Y/N]:	________	________
Angles are preserved [Y/N]:	________	________
Lengths vary with distance to eye[Y/N]:	________	________

Under perspective projections, any set of parallel lines that are not parallel to the projection place will converge to a "vanishing point." Vanishing points of lines parallel to a principal axis x, y, or z are called "principal vanishing points".

a. How many different vanishing points can a perspective drawing have?

c. How many different principal vanishing points can a perspective drawing have?

d. How many vanishing points are in each of the following drawings?