

Hierarchical Modeling

Zoran Popovic

CSE 457

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Reading

Optional:

- ◆ Angel, sections 8.1 – 8.6, 8.8

Further reading:

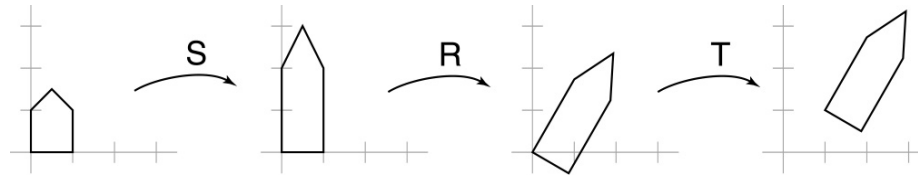
- ◆ *OpenGL Programming Guide*, chapter 3

Symbols and instances

Most graphics APIs support a few geometric **primitives**:

- ◆ spheres
- ◆ cubes
- ◆ cylinders

These symbols are **instanced** using an **instance transformation**.



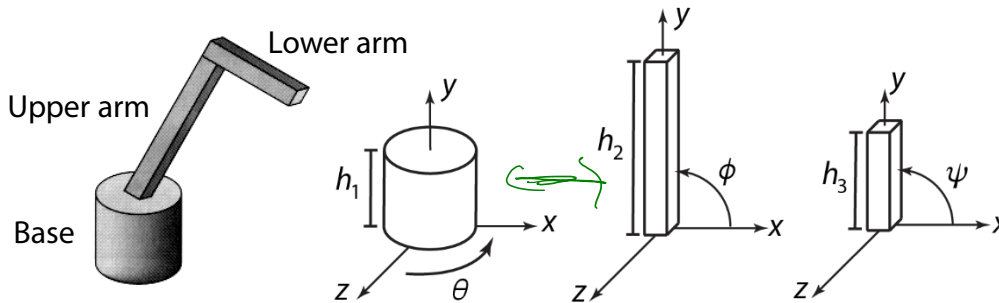
Q: What is the matrix for the instance transformation above?

$$TRS \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

3D Example: A robot arm

Let's build a robot arm out of a cylinder and two cuboids, with the following 3 degrees of freedom:

- ◆ Base rotates about its vertical axis by θ
- ◆ Upper arm rotates in its xy -plane by ϕ
- ◆ Lower arm rotates in its xy -plane by ψ



[Angel, 2011]

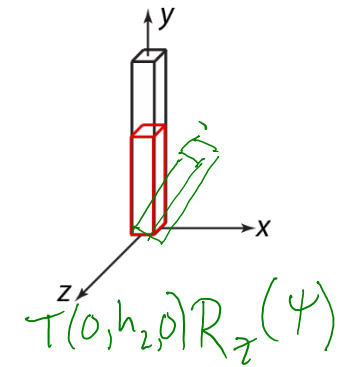
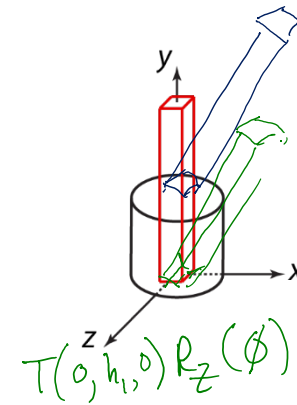
(Note that the angles are set to zero in the figures on the right; i.e., the parts are shown in their "default" positions.)

Suppose we have transformations $R_x(\cdot)$, $R_y(\cdot)$, $R_z(\cdot)$, $T(\cdot, \cdot, \cdot)$.

Q: What matrix do we use to transform the base?

Q: What matrix product for the upper arm?

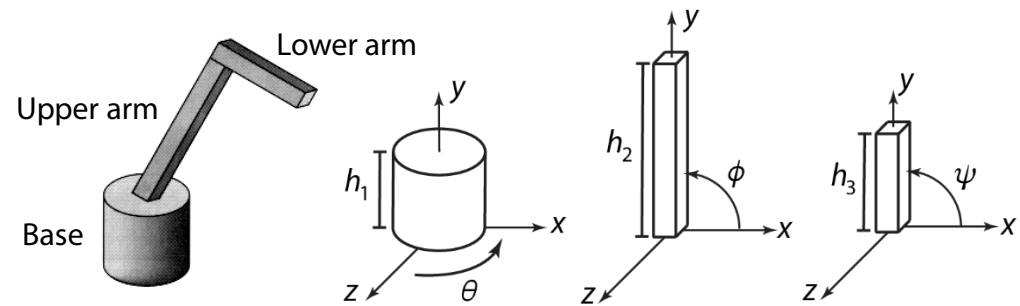
Q: What matrix product for the lower arm?



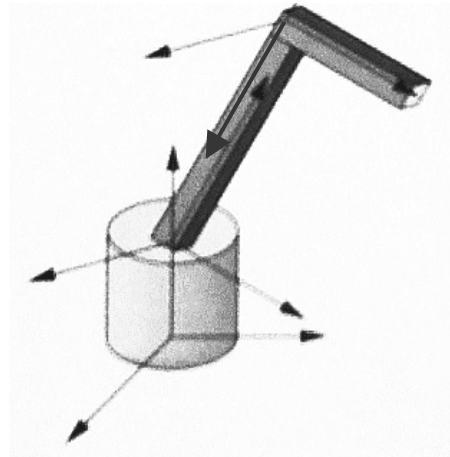
$$\underbrace{R_y(\theta)}_{\text{base}} \underbrace{T(0, h_1, 0)R_z(\phi)}_{\text{UA}} \underbrace{T(0, h_2, 0)R_z(\psi)}_{\text{LA}}$$

3D Example: A robot arm

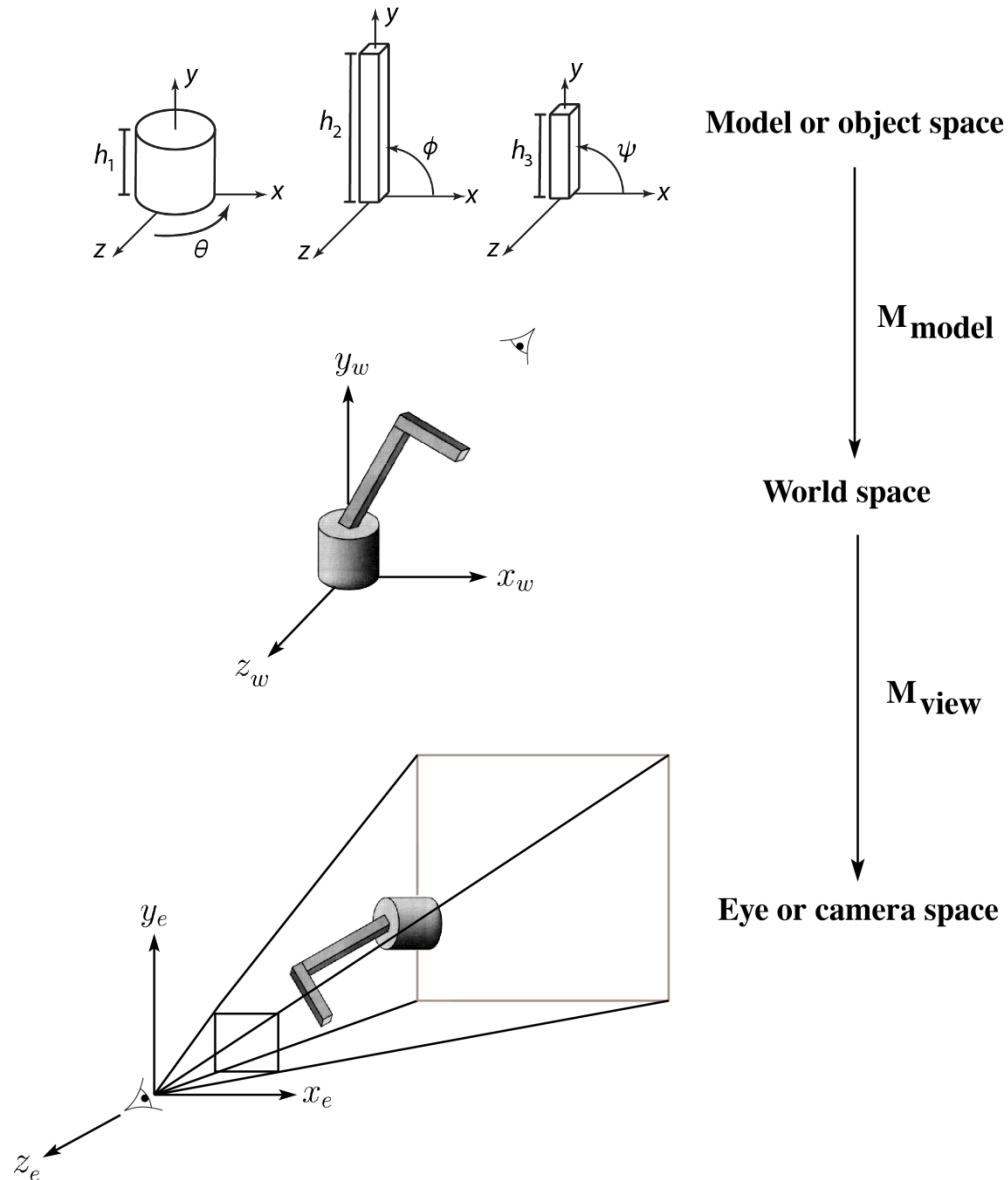
An alternative interpretation is that we are taking the original coordinate frames...



...and translating and rotating them into place:



From parts to model to viewer



Robot arm implementation

The robot arm can be displayed by keeping a global matrix and computing it at each step:

```
Matrix M, M_model, M_view;
```

```
main()
```

```
{
```

```
    . . .
```

```
    M_view = compute_view_transform();
```

```
    robot_arm();
```

```
    . . .
```

```
}
```

```
robot_arm()
```

```
{
```

```
    M_model = R_y(theta);
```

```
    M = M_view*M_model;
```

```
    base();
```

```
    M_model = R_y(theta)*T(0,h1,0)*R_z(phi);
```

```
    M = M_view*M_model;
```

```
    upper_arm();
```

```
    M_model = R_y(theta)*T(0,h1,0)*R_z(phi)*T(0,h2,0)*R_z(psi);
```

```
    M = M_view*M_model;
```

```
    lower_arm();
```

```
}
```

Do the matrix computations seem wasteful?

Robot arm implementation, better

Instead of recalculating the global matrix each time, we can just update it *in place* by concatenating matrices on the right:

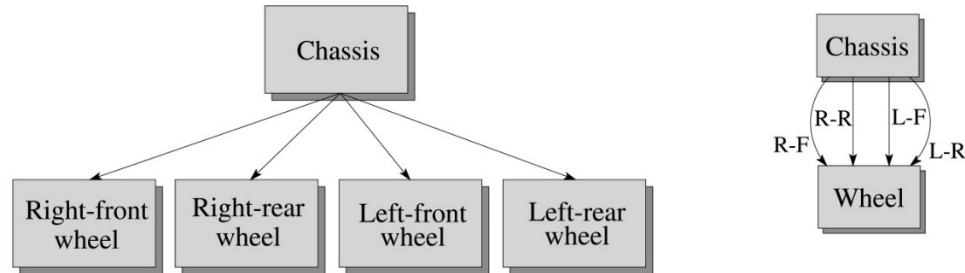
```
Matrix M_modelview;

main()
{
    . . .
    M_modelview = compute_view_transform();
    robot_arm();
    . . .
}

robot_arm()
{
    M_modelview *= R_y(theta);
    base();
    M_modelview *= T(0,h1,0)*R_z(phi);
    upper_arm();
    M_modelview *= T(0,h2,0)*R_z(psi);
    lower_arm();
}
```


Hierarchical modeling

Hierarchical models can be composed of instances using trees or DAGs:

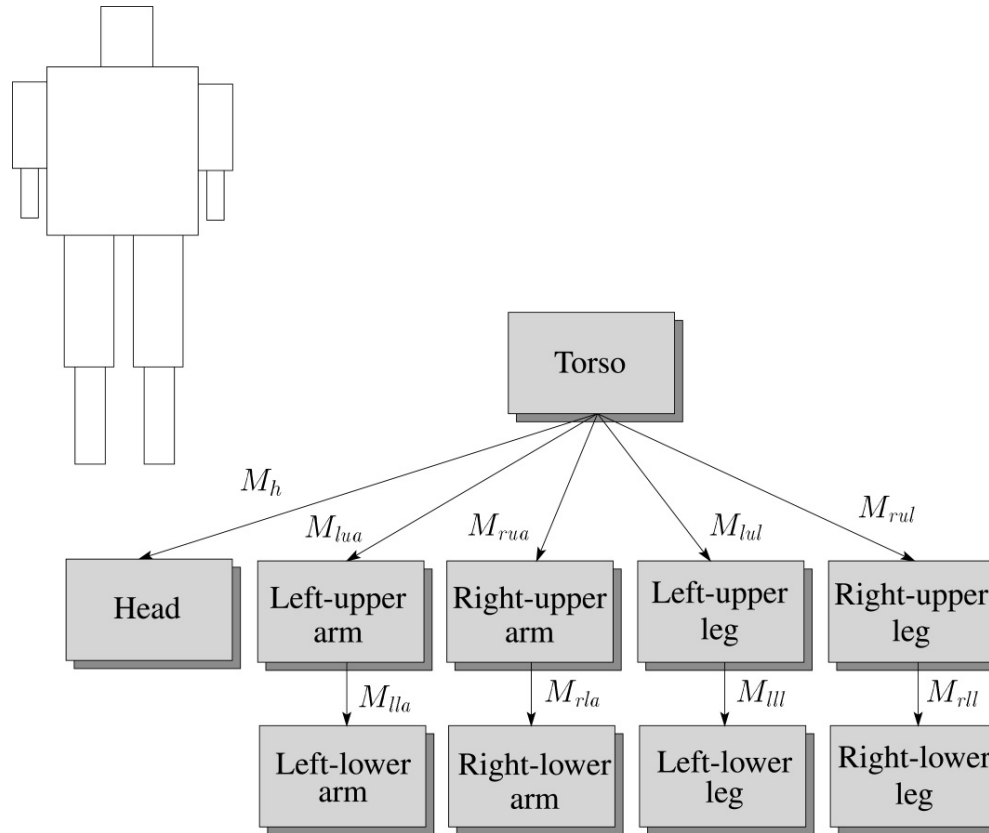


- ◆ edges contain geometric transformations
- ◆ nodes contain geometry (and possibly drawing attributes)

We will use trees for hierarchical models.

How might we draw the tree for the robot arm?

A complex example: human figure



Q: What's the most sensible way to traverse this tree?

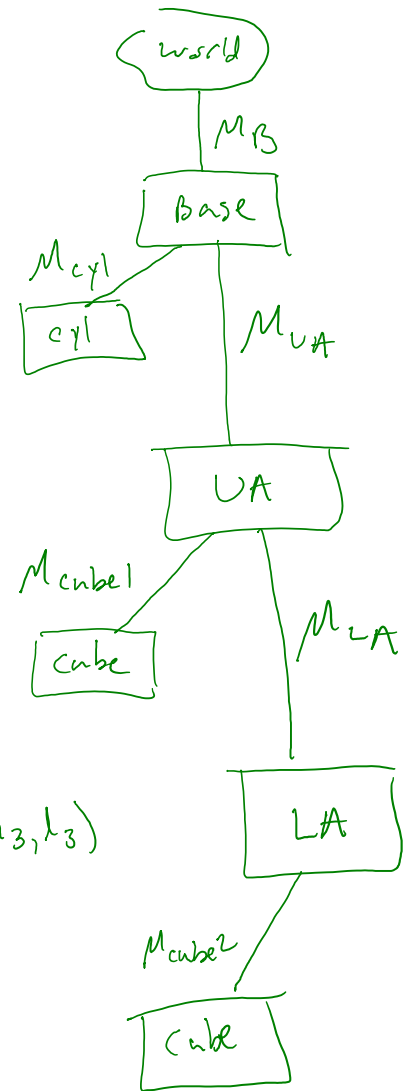
Using canonical primitives

Consider building the robot arm again, but this time the building blocks are canonical primitives like a unit cylinder and a unit cube.

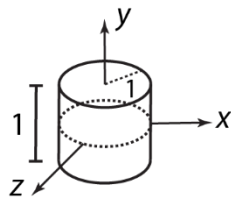
$$S(-, -, -)$$

What additional transformations are needed?

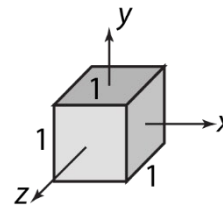
What does the hierarchy look like now?



Canonical primitives



Unit cylinder



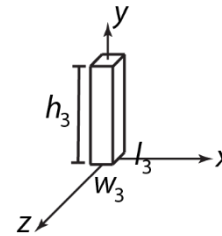
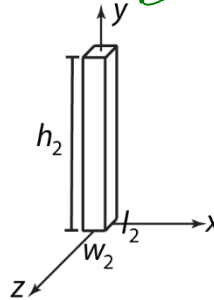
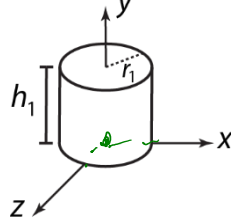
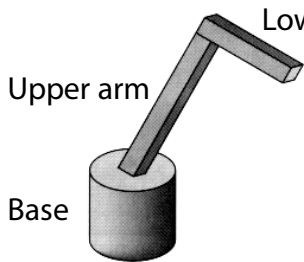
Unit cube

$$M_{cyl} = T(0, \frac{h}{2}, 0) S(r, h, r)$$

$$S(r, h, r)$$

$$M_{cube1} = T(0, \frac{h_2}{2}, 0) S(w_2, h_2, l_2)$$

$$M_{cube2} = T(0, \frac{h_3}{2}, 0) S(w_3, h_3, l_3)$$



Animation

The above examples are called **articulated models**:

- ◆ rigid parts
- ◆ connected by joints

They can be animated by specifying the joint angles (or other display parameters) as functions of time.

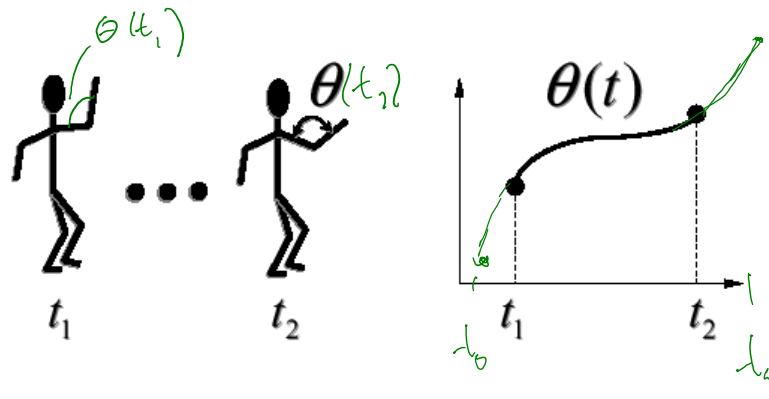
Key-frame animation

The most common method for character animation in production is **key-frame animation**.

- ◆ Each joint specified at various **key frames** (not necessarily the same as other joints)
- ◆ System does interpolation or **in-betweening**

Doing this well requires:

- ◆ A way of smoothly interpolating key frames: **splines**
- ◆ A good interactive system
- ◆ A lot of skill on the part of the animator

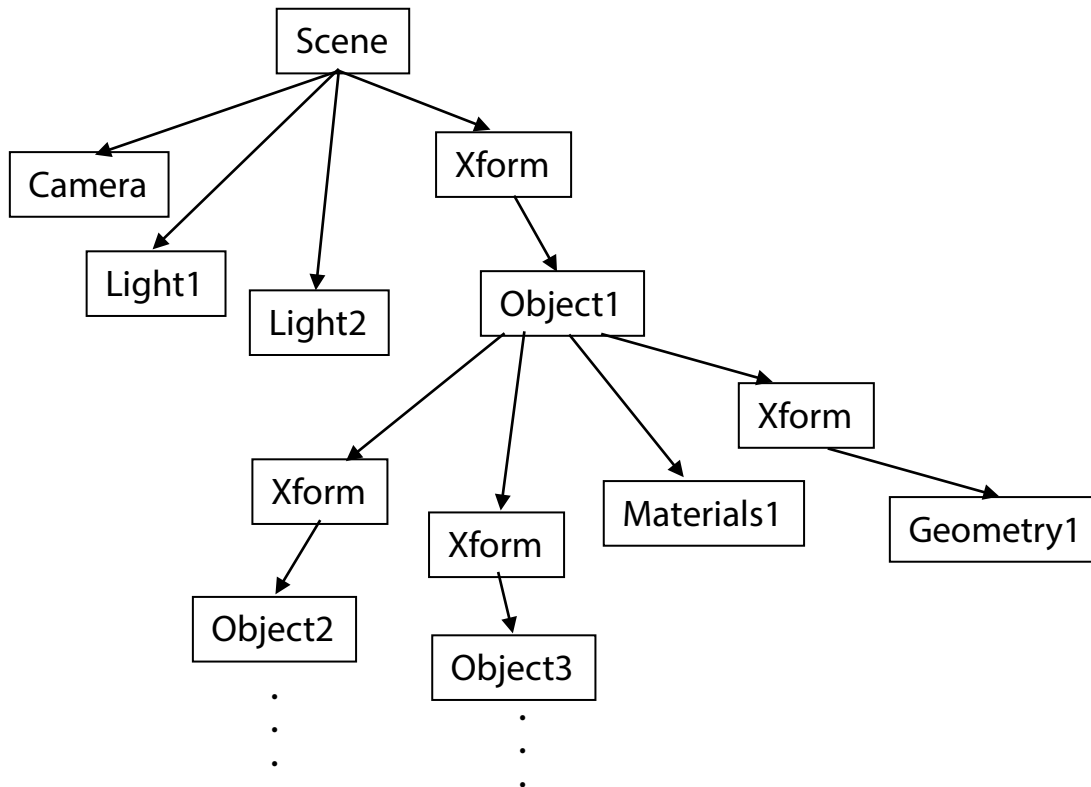


Scene graphs

The idea of hierarchical modeling can be extended to an entire scene, encompassing:

- ◆ many different objects
- ◆ lights
- ◆ camera position

This is called a **scene tree** or **scene graph**.



Summary

Here's what you should take home from this lecture:

- ◆ All the **boldfaced terms**.
- ◆ How primitives can be instantiated and composed to create hierarchical models using geometric transforms.
- ◆ How the notion of a model tree or DAG can be extended to entire scenes.
- ◆ How OpenGL transformations can be used in hierarchical modeling.
- ◆ How keyframe animation works.