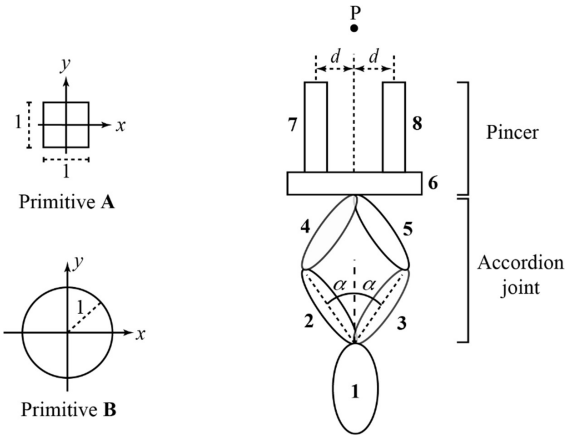
Problem 1: Hierarchical Modeling [100 points]

Suppose you want to model a pincer with accordion joint illustrated below. The model is comprised of 8 parts, using primitives **A** and **B**. The model is controlled by parameters α and d. The pincer operates by translating primitives **7** and **8** towards or away from each other according to d. The illustration on the right also shows a point P that the model is reaching towards.



Assume that α can take values in the range $[0, 90°]$. As part of your solution to this problem, you will need to define the instance transformations that should be applied to a given primitive on the left so that it is the same shape as a desired primitive on the right. Object **1** is 2 units long on the major axis and 1 unit long on the minor axis. Objects **2**, **3**, **4**, **5** are 1/2 units long on the minor axis and 2 units long on the major axis. Object **6** has length 3 and width 1/2. Objects **7** and **8** have width 1/2 and length 2.

The following transformations are available to you (use it as a shorthand for the transformation matrix it represents):

- $R(\theta)$ – rotate by θ degrees (counter clockwise)
- $T(a, b)$ – translate by $\begin{bmatrix} a \\ b \end{bmatrix}$
- $S(s_x, s_y)$ – scale the x-component by s_x and scale the y-component by s_y

(a) (20 points) Construct a tree to describe this hierarchical model using part **1** as the root. The child of each node can either be a part number (**1**...**8**) or a reference to the canonical geometry that will be drawn (**A** or **B**). A node can have one or more children. Two parts connected physically should be connected in the tree unless it is already connected to another part (i.e., if there are multiple parents, choose one).

(b) (40 points) Along each of the edges of the tree, write an expression for the transformations that are applied along that edge to connect parent to child. Write all transformations using the notation above; you do not need to write out the matrices, just the symbolic references to them and their arguments. Remember that the order of transformations is important! Show your work wherever the transformations are not "obvious." Assume that the center of part **1** sits on the origin in world coordinates.

(c) (20 points) Write out the full transformation expression for drawing the geometric primitive for part **7**. Again, use the symbolic matrix notation above that appears in your tree.

(d) (20 points) What values of α and d would have the model extend out and close the pincer just enough to precisely grasp the point $P = [0\ 6]^T$? Show your work.