

# **Hierarchical Modeling**

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CSE 457  
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# Reading

Optional:

- ◆ Angel, sections 8.1 – 8.6, 8.8

Further reading:

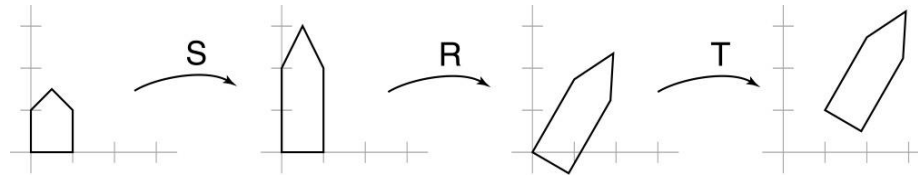
- ◆ *OpenGL Programming Guide*, chapter 3

# Symbols and instances

Most graphics APIs support a few geometric **primitives**:

- ◆ spheres
- ◆ cubes
- ◆ cylinders

These symbols are **instanced** using an **instance transformation**.



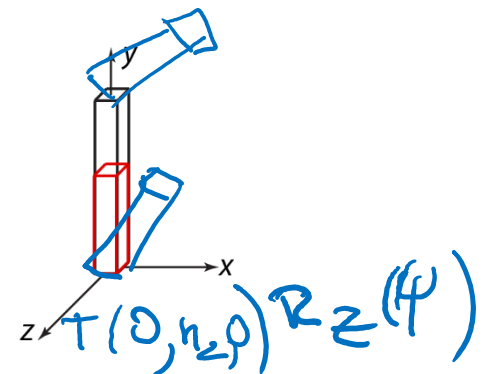
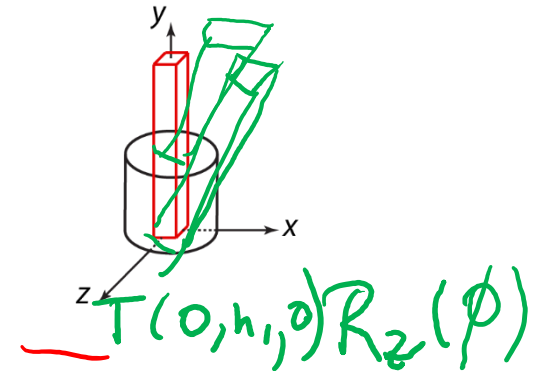
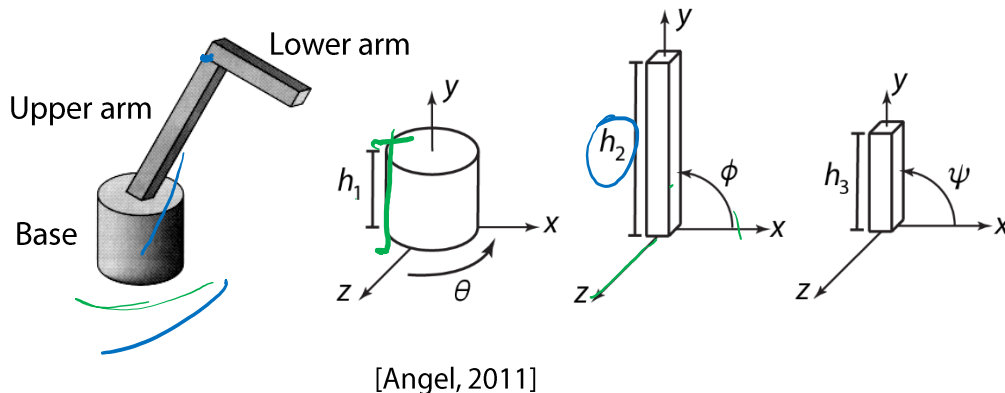
**Q:** What is the matrix for the instance transformation above?

$$M = T \cdot R \cdot S$$

### 3D Example: A robot arm

Let's build a robot arm out of a cylinder and two cuboids, with the following 3 degrees of freedom:

- ◆ Base rotates about its vertical axis by  $\theta$
- ◆ Upper arm rotates in its  $xy$ -plane by  $\phi$
- ◆ Lower arm rotates in its  $xy$ -plane by  $\psi$



(Note that the angles are set to zero in the figures on the right; i.e., the parts are shown in their "default" positions.)

Suppose we have transformations  $R_x(\cdot), R_y(\cdot), R_z(\cdot), T(\cdot, \cdot, \cdot)$ .

Q: What matrix do we use to transform the base?

Q: What matrix product for the upper arm?

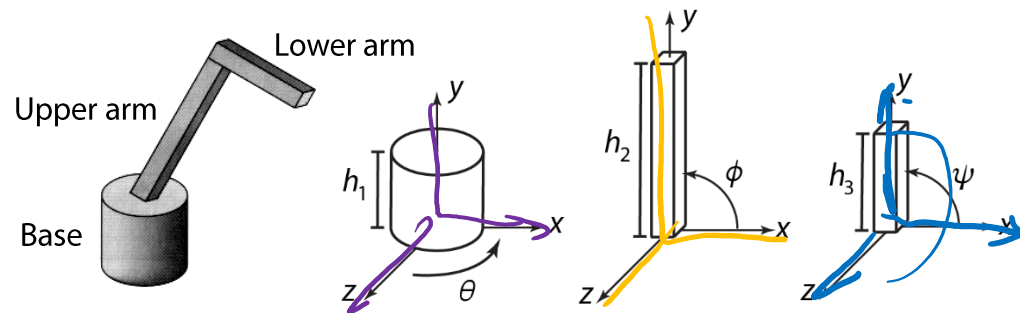
Q: What matrix product for the lower arm?

Handwritten notes and equations:

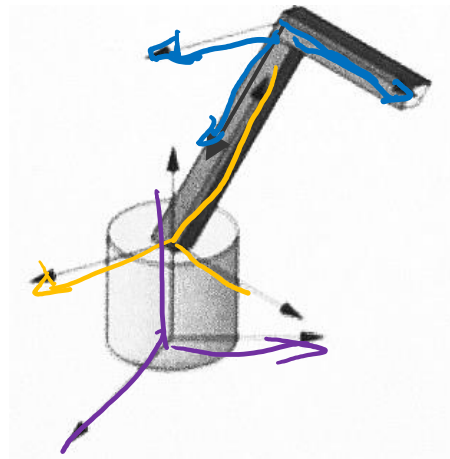
- $R_y(\cdot)$  (crossed out)
- $T(0, h_1, 0)R_z(\phi)$
- $T(0, h_2, 0)R_z(\psi)$
- A green bracket underlines the two transformation equations above, with the label  $U A$  written below it.
- A blue bracket underlines the entire set of equations, with the label  $2 A$  written below it.

## 3D Example: A robot arm

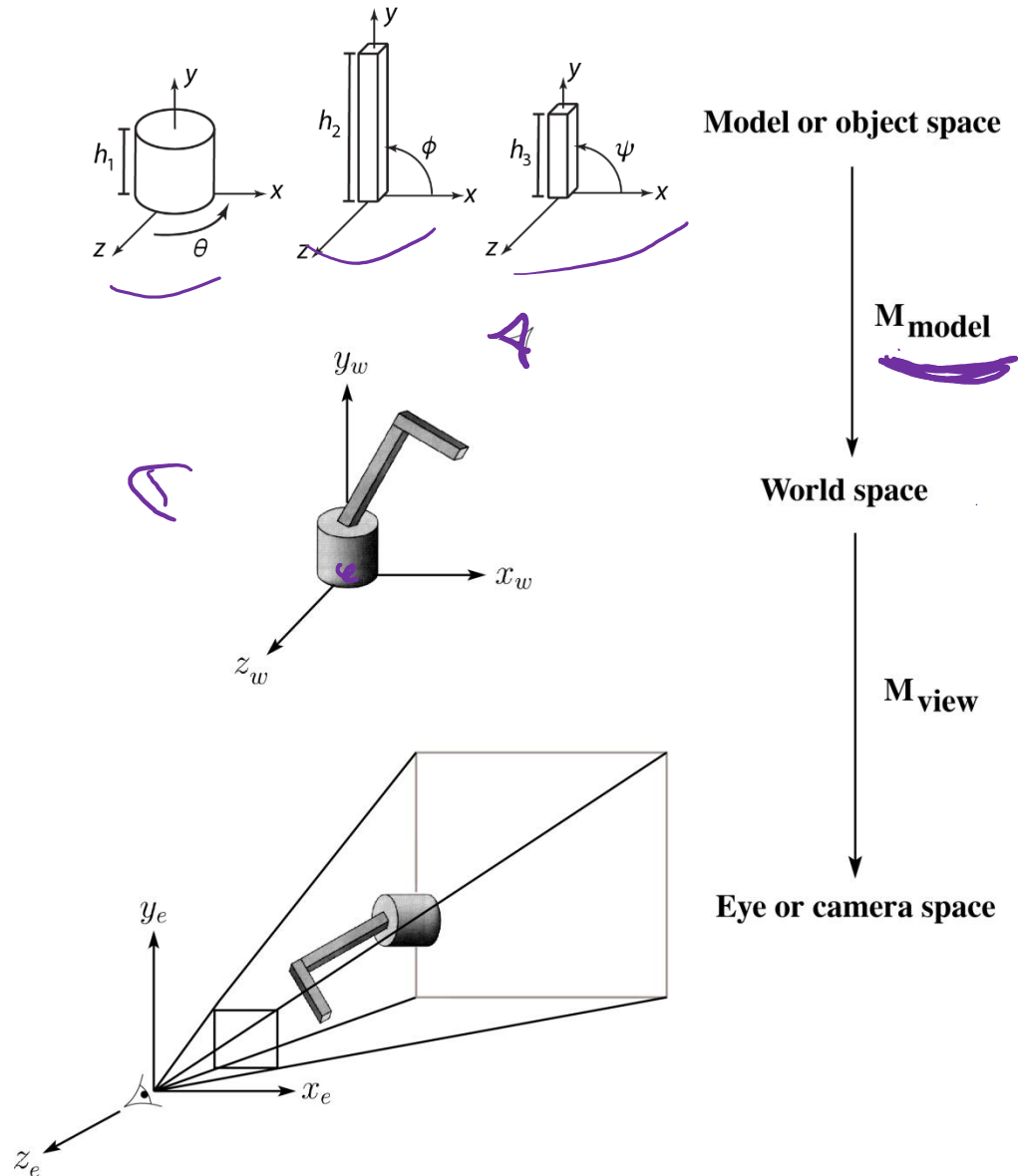
An alternative interpretation is that we are taking the original coordinate frames...



...and translating and rotating them into place:



# From parts to model to viewer



# Robot arm implementation

The robot arm can be displayed by keeping a global matrix and computing it at each step:

```
Matrix M, M_model, M_view;
```

```
main()
```

```
{  
    . . .  
    M_view = compute_view_transform();  
    robot_arm();  
    . . .  
}
```

```
robot_arm()
```

```
{  
    M_model = R_y(theta);  
    M = M_view*M_model;  
    base();  
    M_model = R_y(theta)*T(0,h1,0)*R_z(phi);  
    M = M_view*M_model;  
    upper_arm();  
    M_model = R_y(theta)*T(0,h1,0)*R_z(phi)*T(0,h2,0)*R_z(psi);  
    M = M_view*M_model;  
    lower_arm();  
}
```

Do the matrix computations seem wasteful?

## Robot arm implementation, better

Instead of recalculating the global matrix each time, we can just update it *in place* by concatenating matrices on the right:

```
Matrix M_modelview;

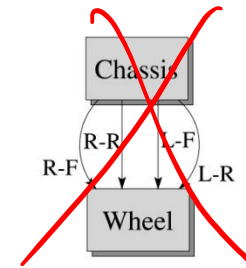
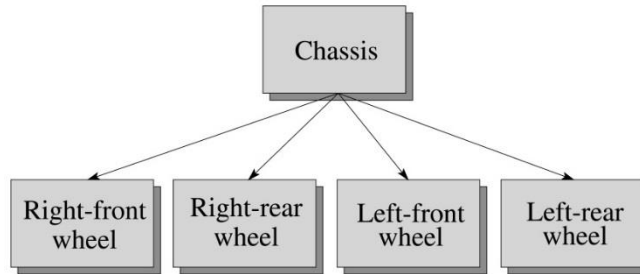
main()
{
    . . .
    M_modelview = compute_view_transform();
    robot_arm();
    . . .
}

robot_arm()
{
    M_modelview *= R_y(theta);
    base();
    M_modelview *= T(0,h1,0)*R_z(phi);
    upper_arm();
    M_modelview *= T(0,h2,0)*R_z(psi);
    lower_arm();
}
```



# Hierarchical modeling

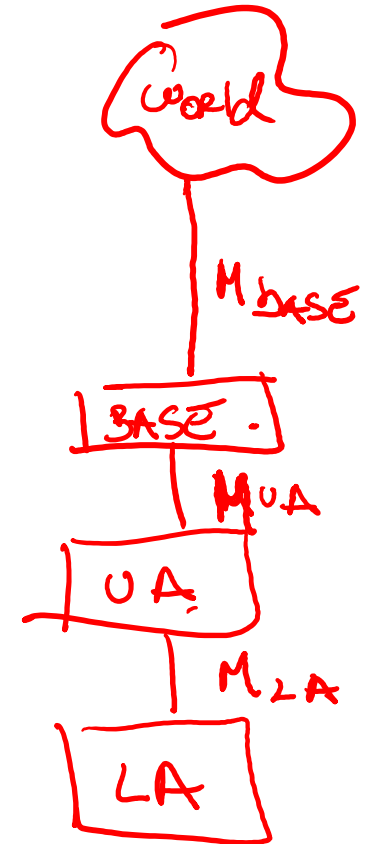
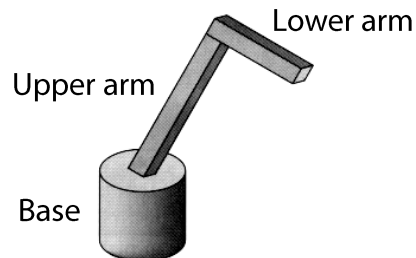
Hierarchical models can be composed of instances using trees or DAGs:



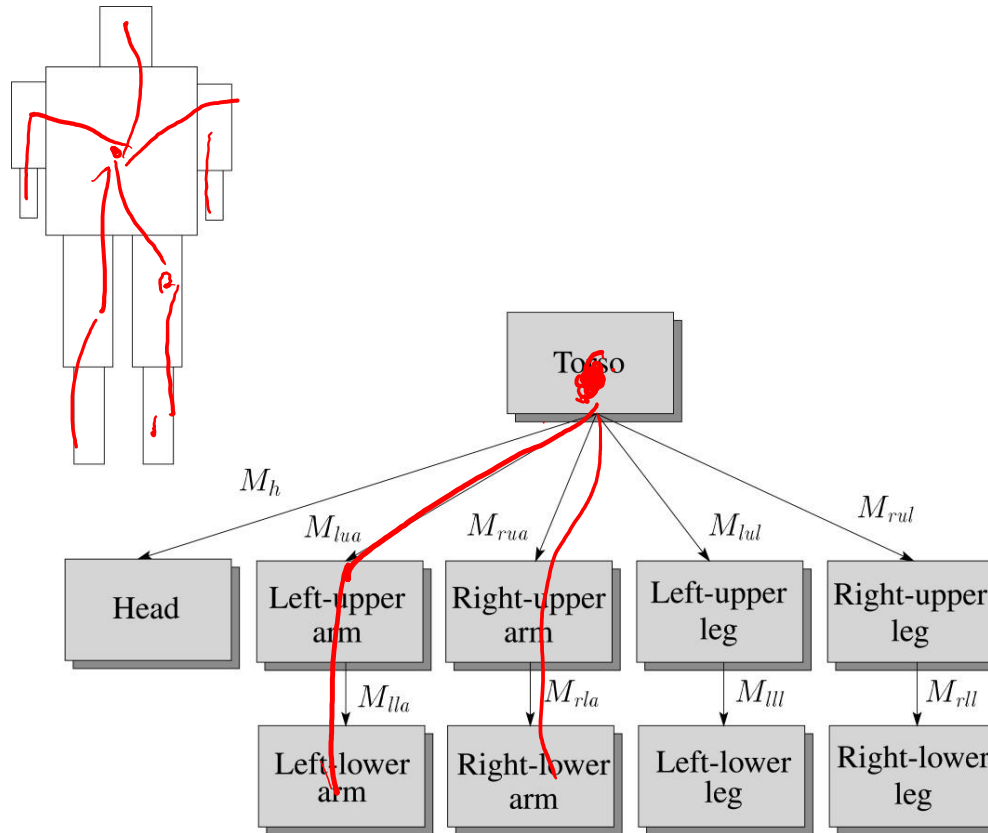
- ◆ edges contain geometric transformations
- ◆ nodes contain geometry (and possibly drawing attributes)

We will use trees for hierarchical models.

How might we draw the tree for the robot arm?



# A complex example: human figure



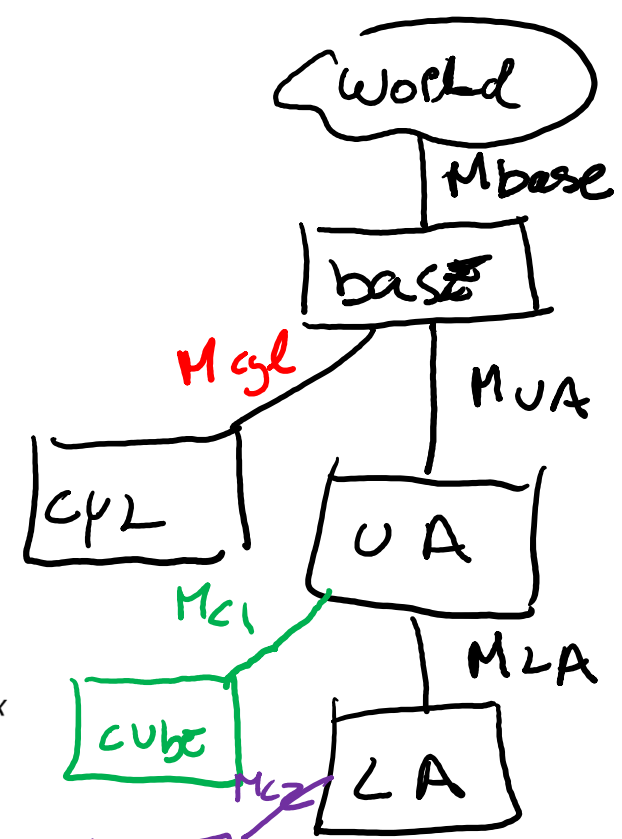
**Q:** What's the most sensible way to traverse this tree?

*depth first w/ stack*

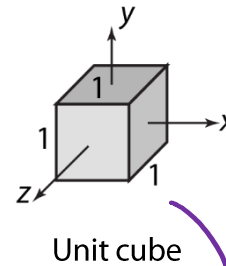
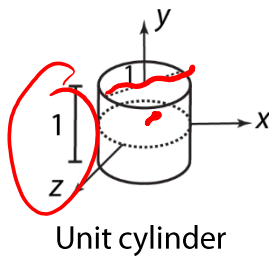
# Using canonical primitives

Consider building the robot arm again, but this time the building blocks are canonical primitives like a unit cylinder and a unit cube. We can use transformations like  $T(t_x, t_y, t_z)$ ,  $S(s_x, s_y, s_z)$ ,  $R_y(\theta)$ , etc.

What additional transformations are needed?  
 What does the hierarchy look like now?



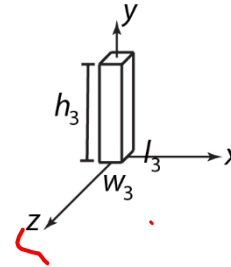
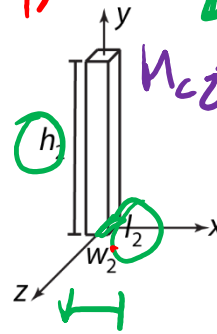
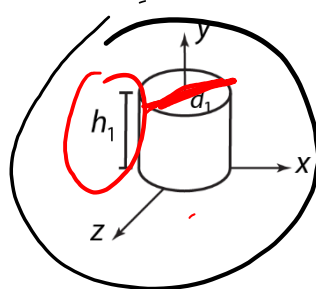
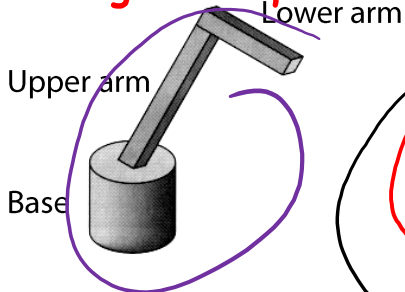
Canonical primitives



$$M_{cube1} = T(0, h_2/2, 0) \cdot S(w_2, h_2/2)$$

$$M_{cyl} = T(0, h_2/2, 0) \cdot S(d_1, h_2, d_1)$$

$$M_{c2} = T(0, h_3/2, 0) \cdot S(w_3, h_3, l_3)$$



# Animation

The above examples are called **articulated models**:

- ◆ rigid parts
- ◆ connected by joints

They can be animated by specifying the joint angles (or other display parameters) as functions of time.

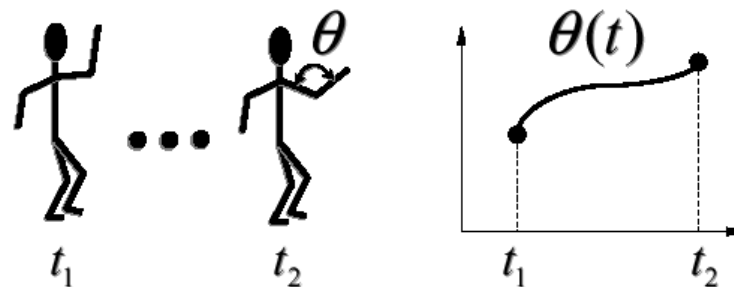
# Key-frame animation

The most common method for character animation in production is **key-frame animation**.

- ◆ Each joint specified at various **key frames** (not necessarily the same as other joints)
- ◆ System does interpolation or **in-betweening**

Doing this well requires:

- ◆ A way of smoothly interpolating key frames: **splines**
- ◆ A good interactive system
- ◆ A lot of skill on the part of the animator

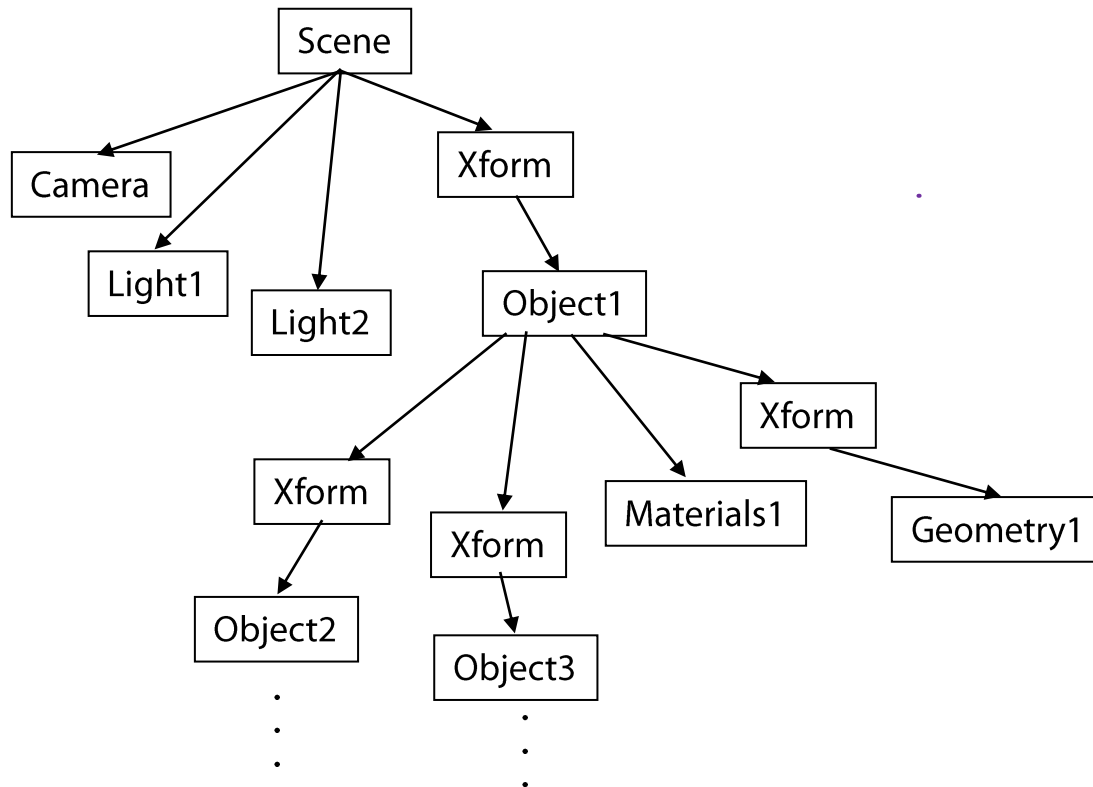


# Scene graphs

The idea of hierarchical modeling can be extended to an entire scene, encompassing:

- ◆ many different objects
- ◆ lights
- ◆ camera position

This is called a **scene tree** or **scene graph**.



# Summary

Here's what you should take home from this lecture:

- ◆ All the **boldfaced terms**.
- ◆ How primitives can be instantiated and composed to create hierarchical models using geometric transforms.
- ◆ How the notion of a model tree or DAG can be extended to entire scenes.
- ◆ How OpenGL transformations can be used in hierarchical modeling.
- ◆ How keyframe animation works.