Particle Systems

Zoran Popovic
CSE 457
Spring 2019

Reading

- Required:
  - Witkin, *Particle System Dynamics*, SIGGRAPH ’97 course notes on Physically Based Modeling.

- Optional
  - Witkin and Baraff, *Differential Equation Basics*, SIGGRAPH ’97 course notes on Physically Based Modeling.
  - Hocknew and Eastwood. *Computer simulation using particles*. Adam Hilger, New York, 1988.
  - Gavin Miller. “The motion dynamics of snakes and worms.” *Computer Graphics* 22:169-178, 1988.

What are particle systems?

A **particle system** is a collection of point masses that obeys some physical laws (e.g, gravity or spring behaviors).

Particle systems can be used to simulate all sorts of physical phenomena:

- Smoke
- Snow
- Fireworks
- Hair
- Cloth
- Snakes
- Fish

Overview

1. One lousy particle
2. Particle systems
3. Forces: gravity, springs
4. Implementation

Particle in a flow field

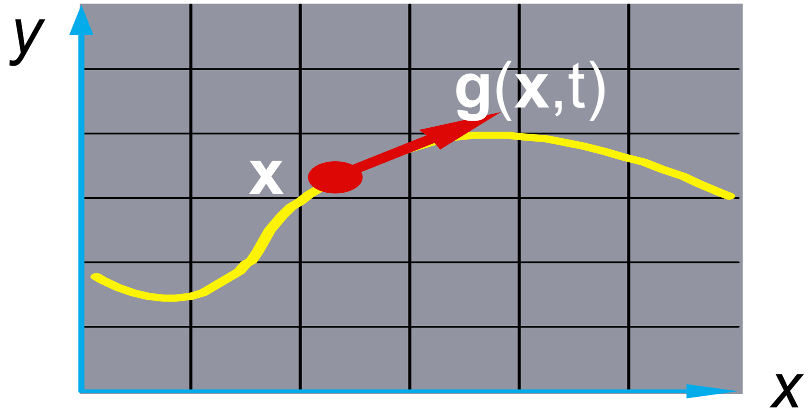
We begin with a single particle with:

- Position, $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

- Velocity, $\mathbf{v} \equiv \dot{\mathbf{x}} = \frac{d\mathbf{x}}{dt} = \begin{bmatrix} dx/dt \\ dy/dt \end{bmatrix}$

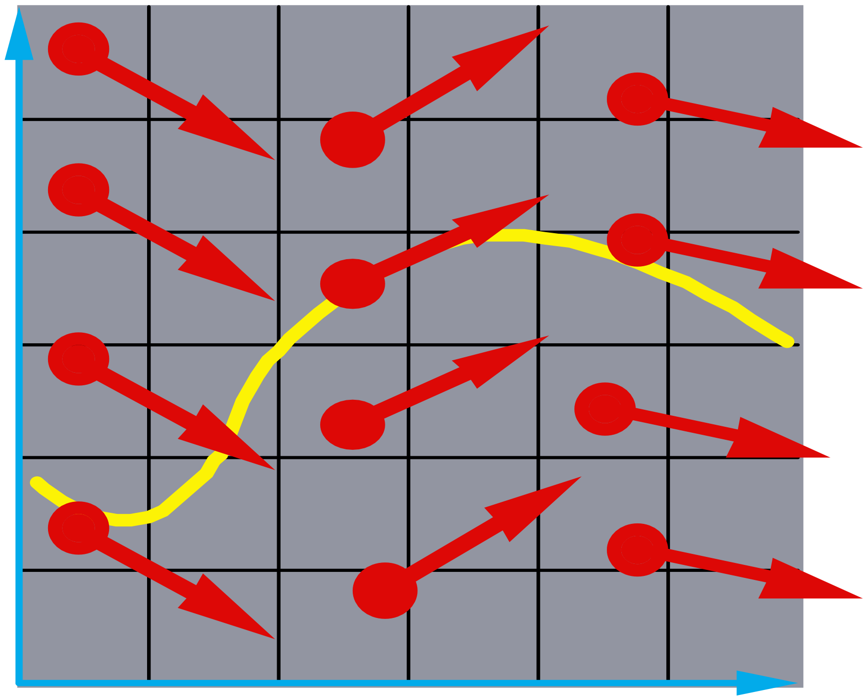
Suppose the velocity is dictated by some driving function **g**:

$\dot{\mathbf{x}} = \mathbf{g}(\mathbf{x}, t)$



Vector fields

At any moment in time, the function **g** defines a vector field over **x**:

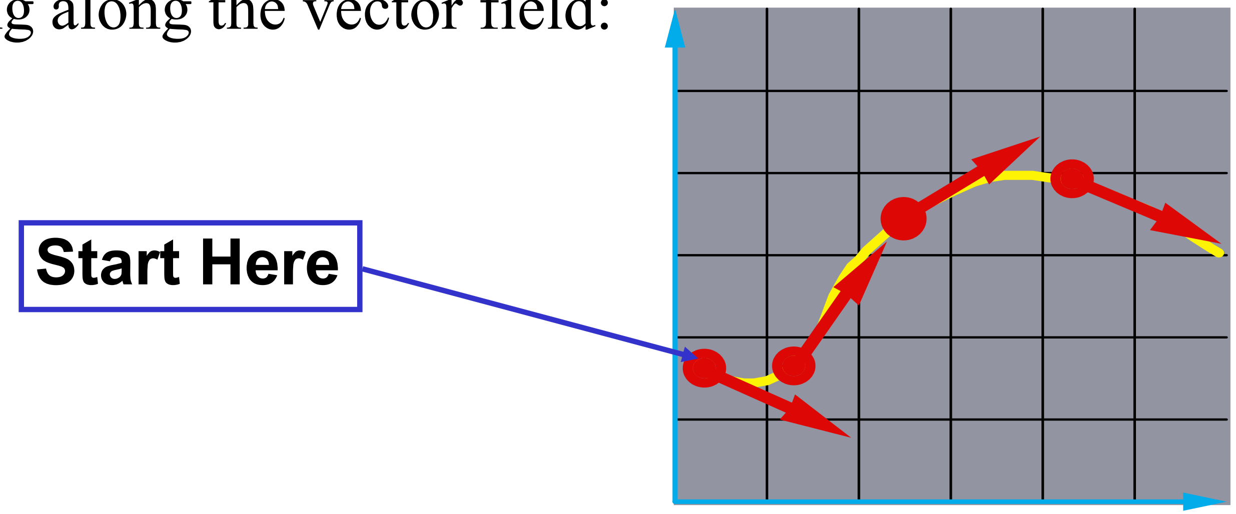


How does our particle move through the vector field?

Diff eqs and integral curves

The equation [illegible] is actually a **first order differential equation**.

We can solve for **x** through time by starting at an initial point and stepping along the vector field:

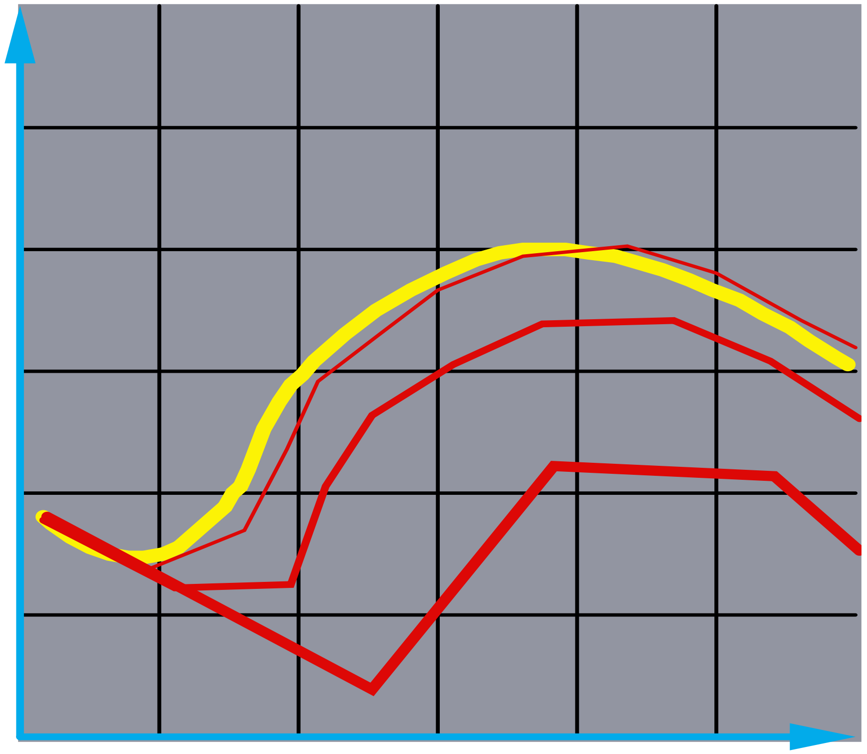


This is called an **intial value problem** and the solution is called an **integral curve**.

Euler's method

One simple approach is to choose a time step, Δt, and take linear steps along the flow:

$$\begin{aligned}\mathbf{x}(t+\Delta t) &= \mathbf{x}(t)+\Delta t\cdot\dot{\mathbf{x}}(t)\\ &= \mathbf{x}(t)+\Delta t\cdot\mathbf{g}(\mathbf{x},t)\end{aligned}$$



This approach is called **Euler's method** and looks like:

Properties:

- Simplest numerical method
- Bigger steps, bigger errors

Need to take pretty small steps, so not very efficient. Better (more complicated) methods exist, e.g., "Runge-Kutta."

Particle in a force field

- Now consider a particle in a force field **f**.
- In this case, the particle has:
  - Mass, *m*
  - Acceleration, $\mathbf{a} \equiv \ddot{} \quad \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{x}}{dt^2}$
- The particle obeys Newton's law: $\mathbf{f} = m\mathbf{a} = m\ddot{}$
- The force field **f** can in general depend on the position and velocity of the particle as well as time.
- Thus, with some rearrangement, we end up with:

$$\ddot{} \quad \frac{\mathbf{f}(\mathbf{x}, \dot{}}{m}$$

Second order equations

This equation: $\ddot{} \quad \frac{\mathbf{f}(\mathbf{x}\ \mathbf{v},t)}{m}$

is a **second order differential equation**.

Our solution method, though, worked on first order differential equations.

We can rewrite this as: $\begin{bmatrix} \dot{} & \\ \dot{} & \frac{\mathbf{f}(\mathbf{x}\ \mathbf{v}\ t)}{m} \end{bmatrix}$

where we have added a new variable **v** to get a pair
of **coupled first order equations**.

Phase space

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{v} \end{bmatrix}$$

Concatenate **x** and **v** to make a 6-vector: position in **phase space**.

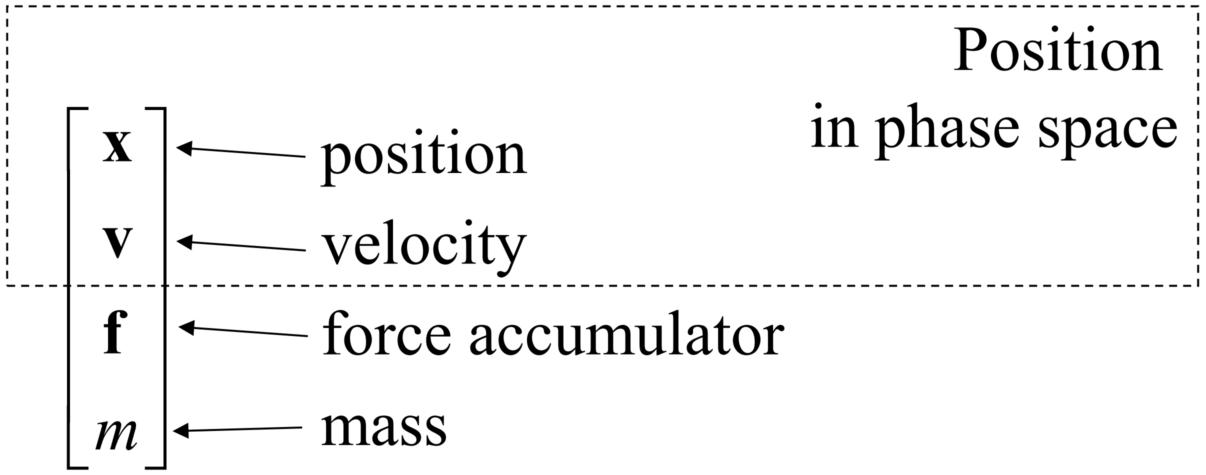
$$\begin{bmatrix} \dot{} \\ \dot{} \end{bmatrix}$$

Taking the time derivative: another 6-vector.

$$\begin{bmatrix} \dot{} \\ \dot{} \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

A vanilla 1st-order differential equation.

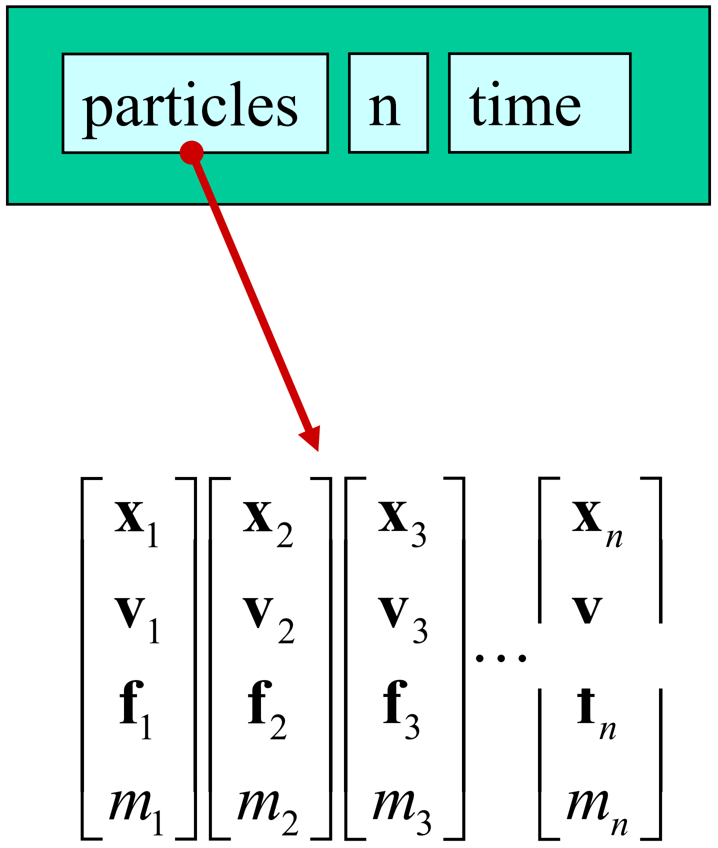
Particle structure



Solver interface

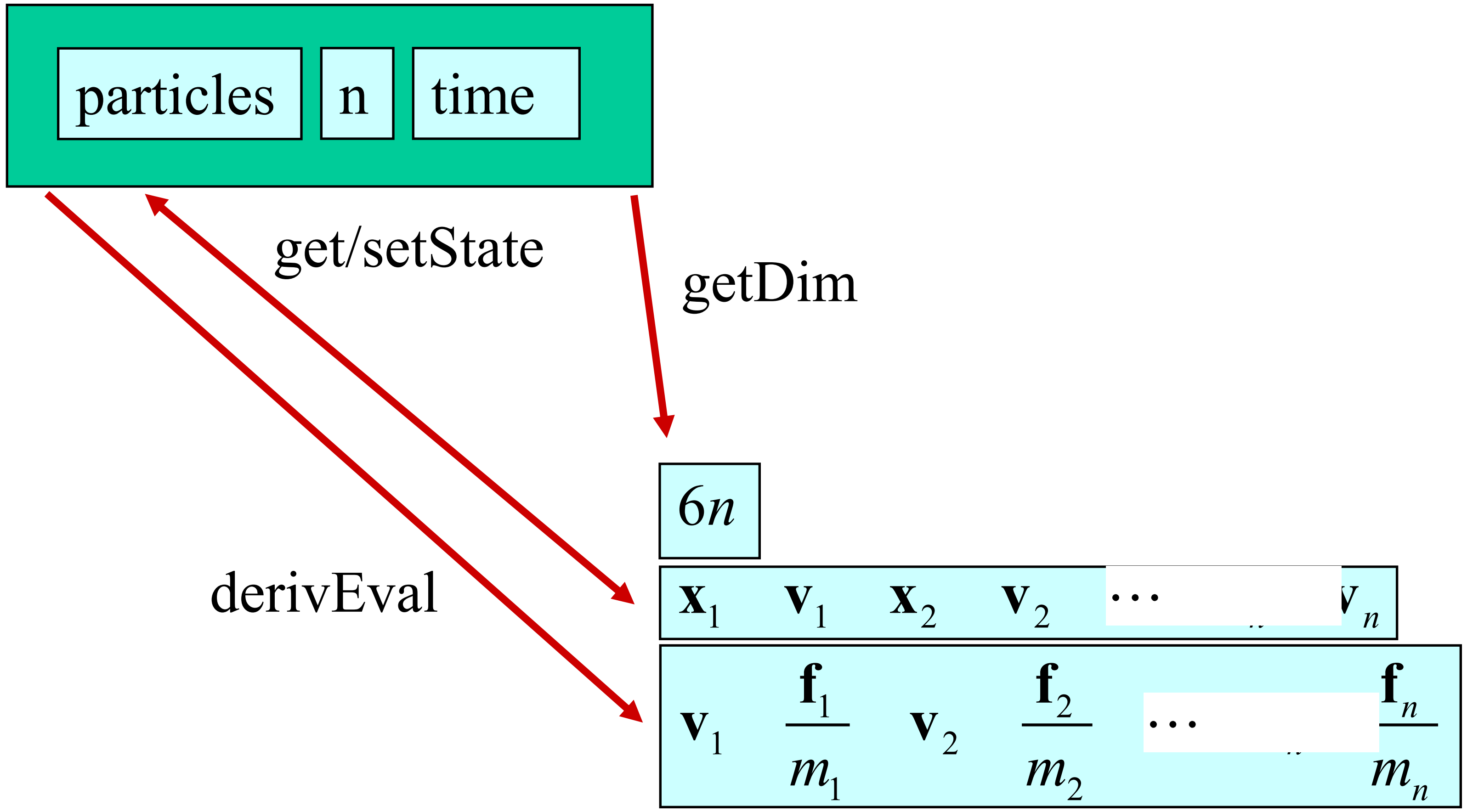
$$\begin{tikzcd}
 & \begin{bmatrix}6\end{bmatrix} \\
\begin{bmatrix}\mathbf{x}\\ \mathbf{v}\\ \mathbf{f}\\ m\end{bmatrix} \arrow[ur, "\text{getDim}"] \arrow[r, shift left, "\text{getState}"] \arrow[dr, "\text{derivEval}"'] & \begin{bmatrix}\mathbf{x}\\ \mathbf{v}\end{bmatrix} \arrow[l, shift left, "\text{setState}"] \\
 & \begin{bmatrix}\mathbf{v}\\ \mathbf{f}/m\end{bmatrix}
\end{tikzcd}$$

Particle systems



$$\begin{bmatrix}\mathbf{x}_1\\ \mathbf{v}_1\\ \mathbf{f}_1\\ m_1\end{bmatrix}\begin{bmatrix}\mathbf{x}_2\\ \mathbf{v}_2\\ \mathbf{f}_2\\ m_2\end{bmatrix}\begin{bmatrix}\mathbf{x}_3\\ \mathbf{v}_3\\ \mathbf{f}_3\\ m_3\end{bmatrix}\cdots\begin{bmatrix}\mathbf{x}_n\\ \mathbf{v}\\ \mathbf{t}_n\\ m_n\end{bmatrix}$$

Solver interface


particles
n
time
get/setState
getDim
derivEval
$6n$
$\mathbf{x}_1 \quad \mathbf{v}_1 \quad \mathbf{x}_2 \quad \mathbf{v}_2 \quad \cdots \quad \mathbf{v}_n$
$\mathbf{v}_1 \quad \frac{\mathbf{f}_1}{m_1} \quad \mathbf{v}_2 \quad \frac{\mathbf{f}_2}{m_2} \quad \cdots \quad \frac{\mathbf{f}_n}{m_n}$

Forces

- Constant (gravity)
- Position/time dependent (force fields)
- Velocity-dependent (drag)
- N-ary (springs)

Gravity

Force law:

$$\mathbf{f}_{grav} = m\mathbf{G}$$

```
p->f += p->m * F->G
```

Viscous drag

Force law:

$$\mathbf{f}_{drag} = -k_{drag}\mathbf{v}$$

```
p->f -= F->k * p->v
```

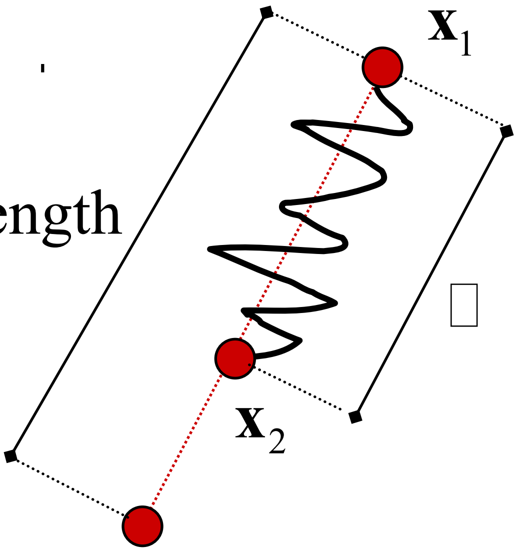
Damped spring

Force law:

$$\mathbf{f}_1 = -\left[k_s(|\square \qquad k_d \left(\frac{\square \quad \square}{|\square} \right) \right] \frac{\square}{\square}$$

$$\mathbf{f}_2 = -\mathbf{f}_1$$

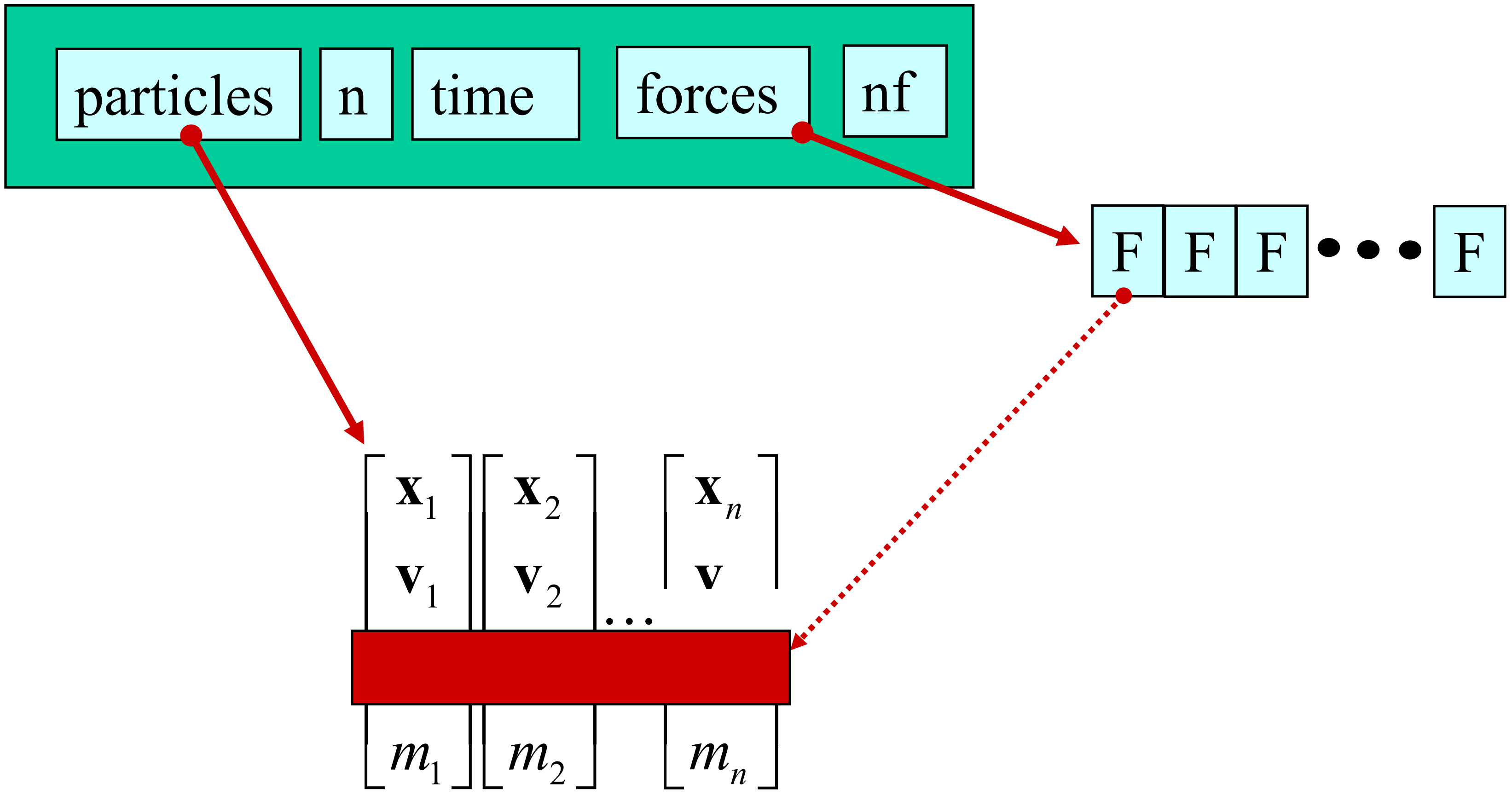
$\mathbf{r}$ = rest length



$\square \qquad \mathbf{x}_2$

$\square \qquad \mathbf{v}_2$

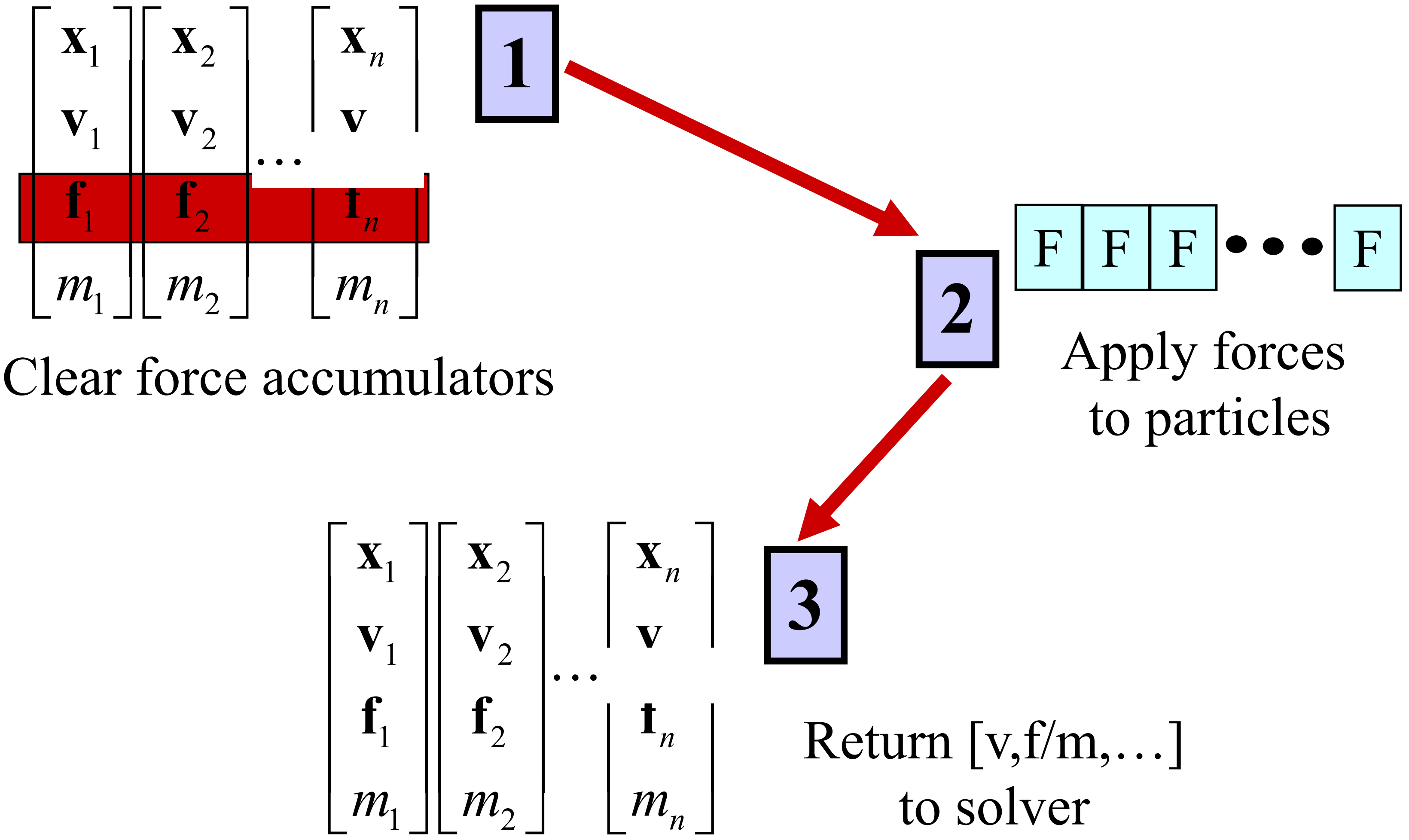
Particle systems with forces


particles
n
time
forces
nf
F
F
F
F
$\mathbf{x}_1$ $\mathbf{v}_1$ m_1
$\mathbf{x}_2$ $\mathbf{v}_2$ m_2
$\mathbf{x}_n$ $\mathbf{v}$ m_n

derivEval loop

1. Clear forces
   - Loop over particles, zero force accumulators
2. Calculate forces
   - Sum all forces into accumulators
3. Gather
   - Loop over particles, copying v and f/m into destination array

derivEval Loop



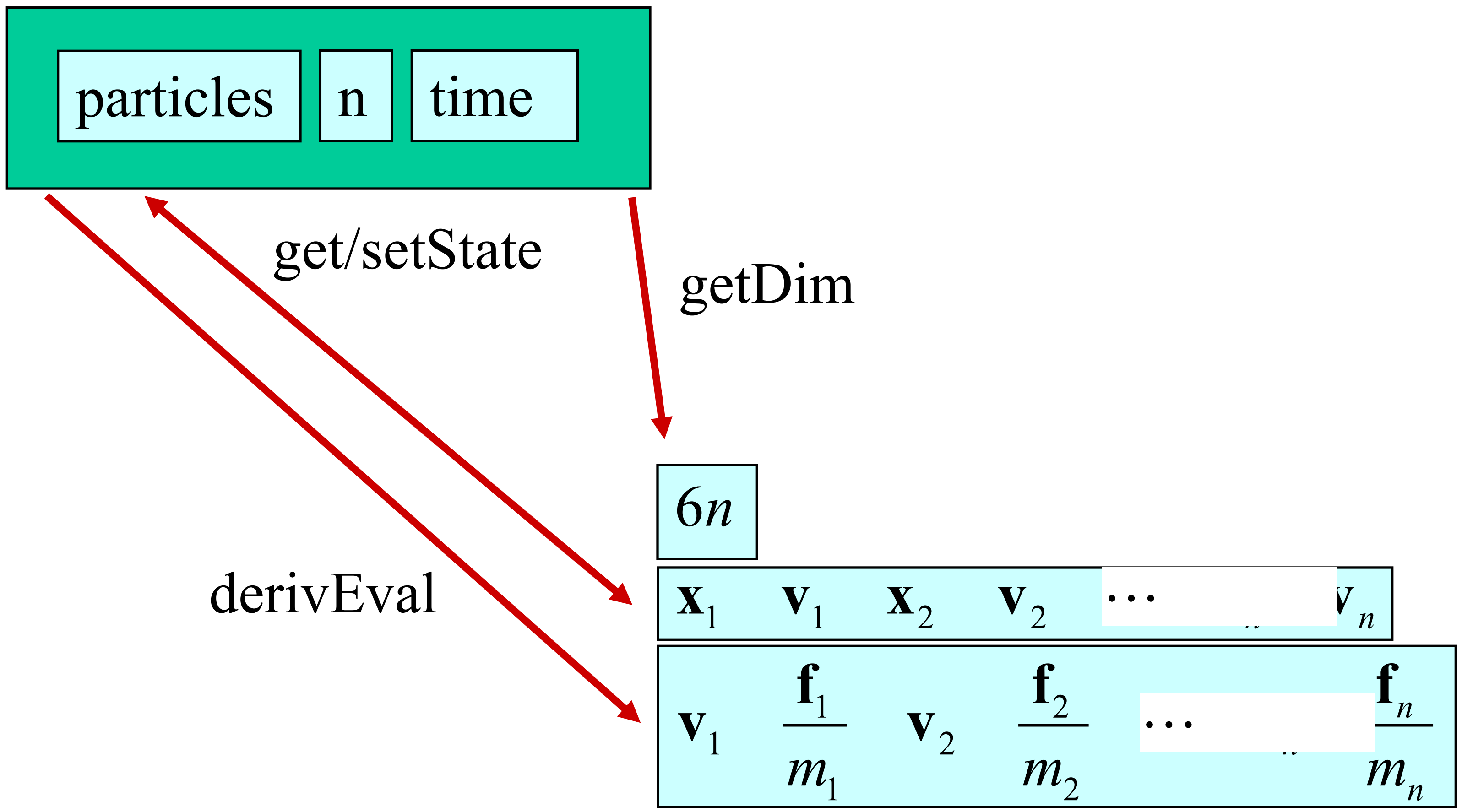
$\mathbf{x}_1$ $\mathbf{v}_1$ $\mathbf{f}_1$ m_1
$\mathbf{x}_2$ $\mathbf{v}_2$ $\mathbf{f}_2$ m_2
…
$\mathbf{x}_n$ $\mathbf{v}$ $\mathbf{f}_n$ m_n
1
2
F F F • • • F
3
$\mathbf{x}_1$ $\mathbf{v}_1$ $\mathbf{f}_1$ m_1
$\mathbf{x}_2$ $\mathbf{v}_2$ $\mathbf{f}_2$ m_2
…
$\mathbf{x}_n$ $\mathbf{v}$ $\mathbf{f}_n$ m_n

Clear force accumulators

Apply forces to particles

Return [v,f/m,…] to solver

Solver interface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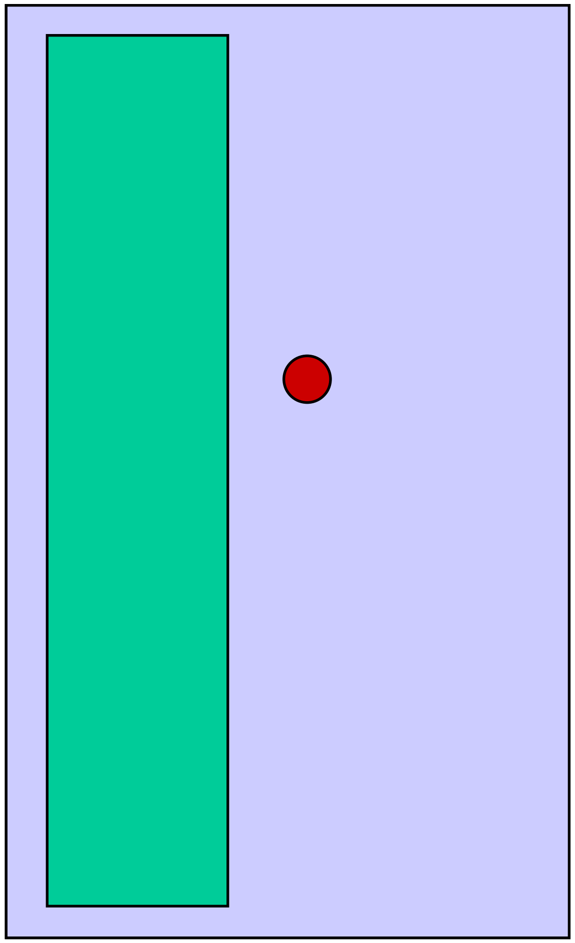
$\mathbf{v}_1 \quad \frac{\mathbf{f}_1}{m_1} \quad \mathbf{v}_2 \quad \frac{\mathbf{f}_2}{m_2} \quad \cdots \quad \frac{\mathbf{f}_n}{m_n}$

Differential equation solver

[illegible]

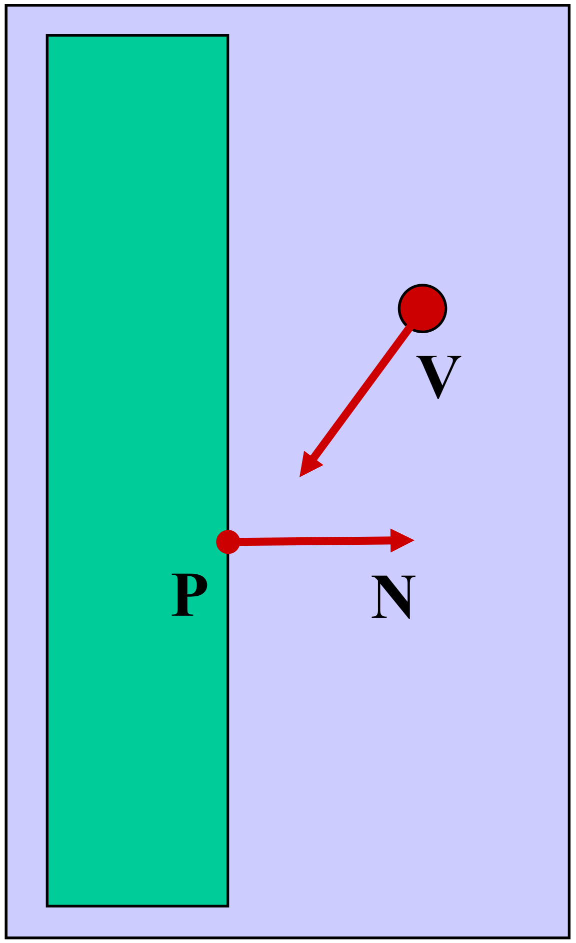
Euler method: $$\begin{bmatrix} \mathbf{x}_1^{\,i+1} \\ \mathbf{v}_1^{\,i+1} \\ \vdots \\ \mathbf{x}_n^{\,i+1} \\ \mathbf{v}_n^{\,i+1} \end{bmatrix} \quad \begin{bmatrix} \mathbf{x}_1^{\,i} \\ \mathbf{v}_1^{\,i} \\ \vdots \\ \mathbf{x}_n^{\,i} \\ \mathbf{v}_n^{\,i} \end{bmatrix} \qquad \begin{bmatrix} \mathbf{v}_1^{\,i} \\ \mathbf{f}_1^{\,i} / m_1 \\ \vdots \\ \mathbf{v}_n^{\,i} \\ \mathbf{f}_n^{\,i} / m_n \end{bmatrix}$$

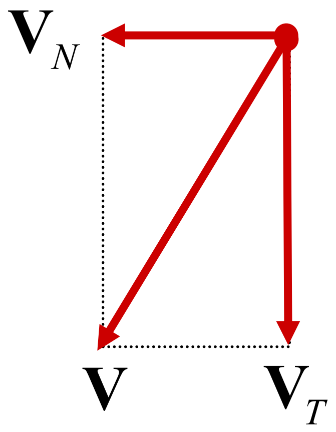
Bouncing off the walls



- Add-on for a particle simulator
- For now, just simple point-plane collisions

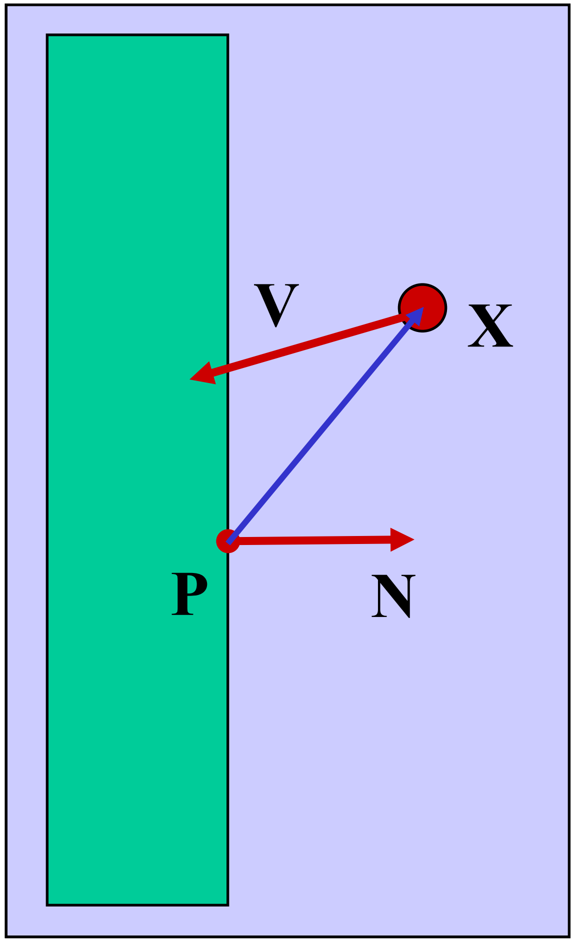
Normal and tangential components





$$\mathbf{V}_N = (\mathbf{N}\cdot\mathbf{V})\mathbf{N}$$
$$\mathbf{V}_T = \mathbf{V} - \mathbf{V}_N$$

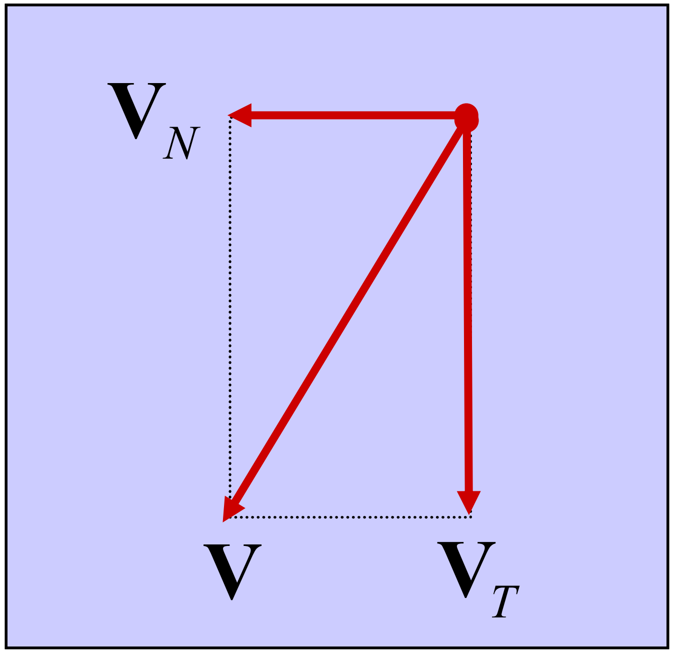
Collision Detection



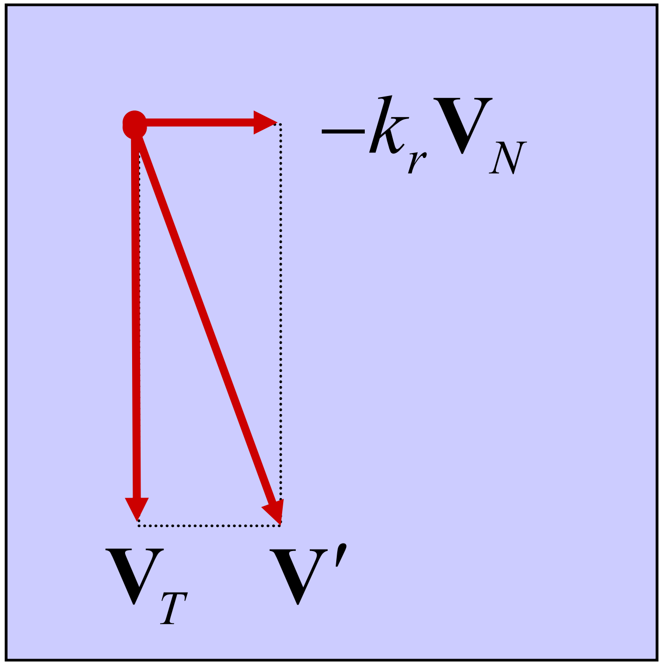
$(\mathbf{X}-\mathbf{P})\cdot\mathbf{N} < \varepsilon$ Within ε of the wall

$\mathbf{N}\cdot\mathbf{V} < 0$ Heading in

Collision Response



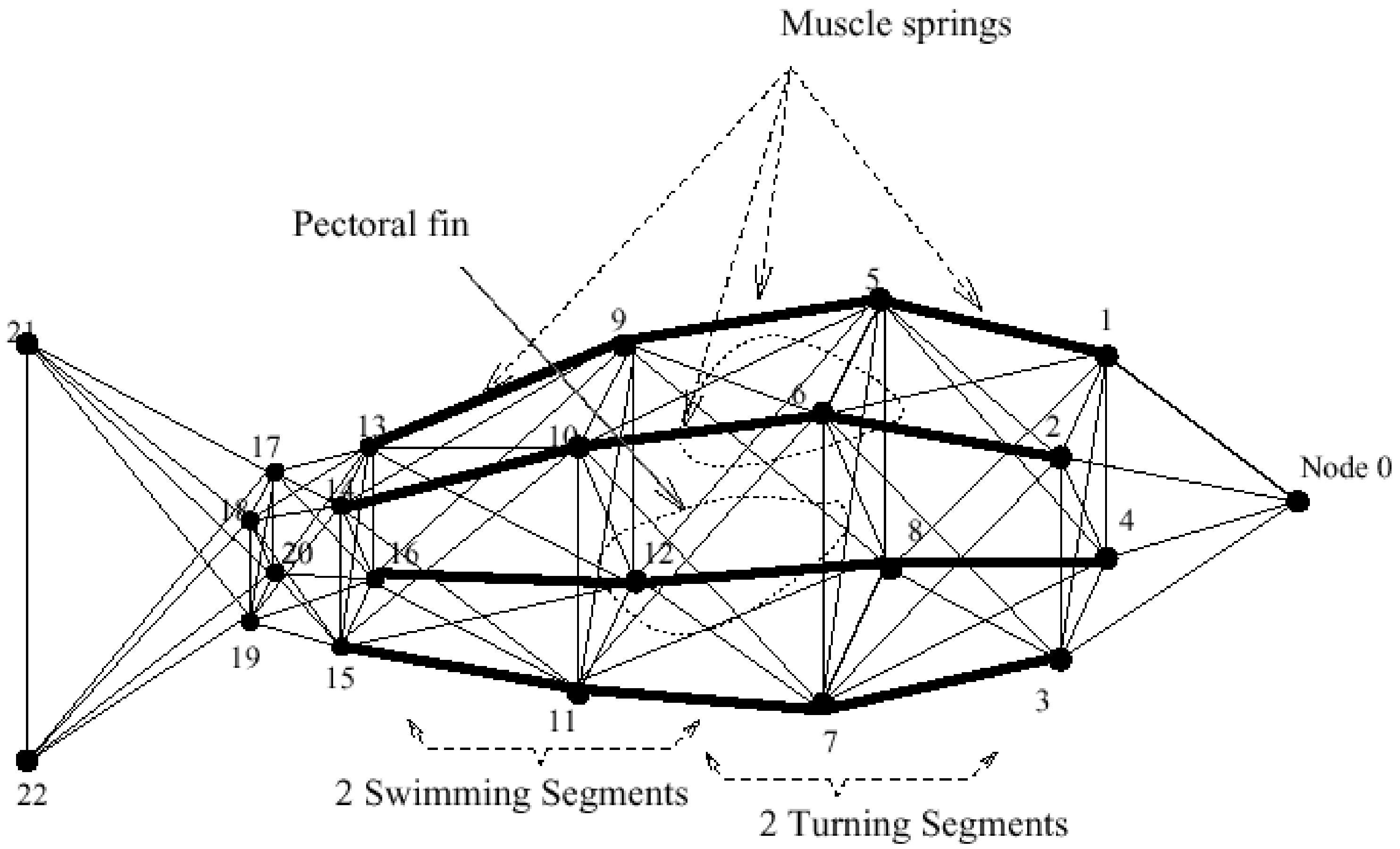
before



after

$$\mathbf{V}' = \mathbf{V}_T - k_r \mathbf{V}_N$$

Artificial Fish



Related Research

- Determining dynamic parameters for cloth simulation

Summary

What you should take away from this lecture:

- The meanings of all the **boldfaced** terms
- Euler method for solving differential equations
- Combining particles into a particle system
- Physics of a particle system
- Various forces acting on a particle
- Simple collision detection