Surfaces of Revolution

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CSE 457
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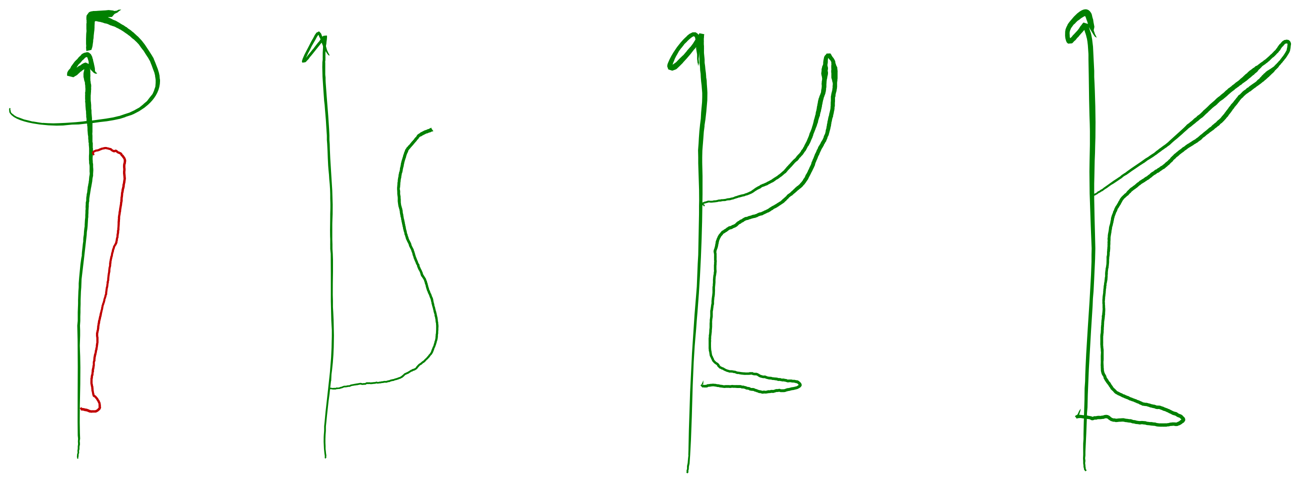
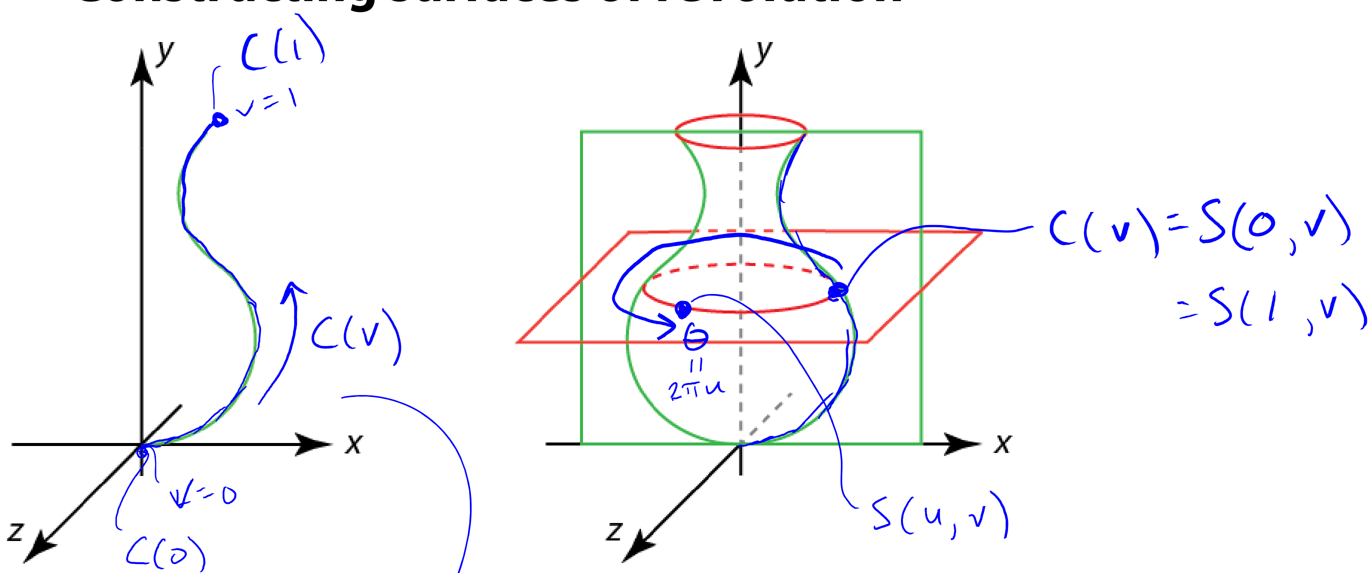
Surfaces of revolution



Idea: rotate a 2D **profile curve** around an axis.

What kinds of shapes can you model this way?

Constructing surfaces of revolution



Given: A curve $C(v)$ in the xy-plane:

$$C(v) = \begin{bmatrix} C_x(v) \\ C_y(v) \\ 0 \\ 1 \end{bmatrix}$$

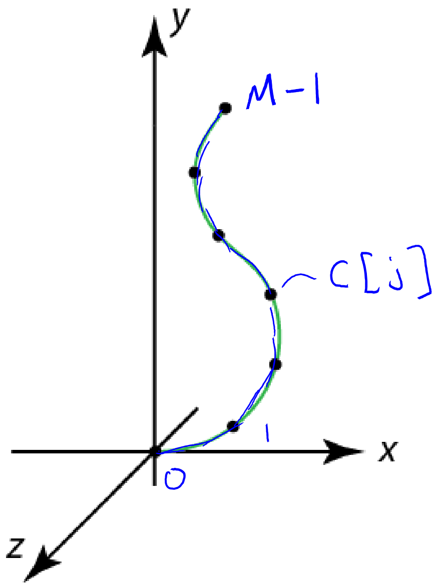
Let $R_y(\theta)$ be a rotation about the y- axis.

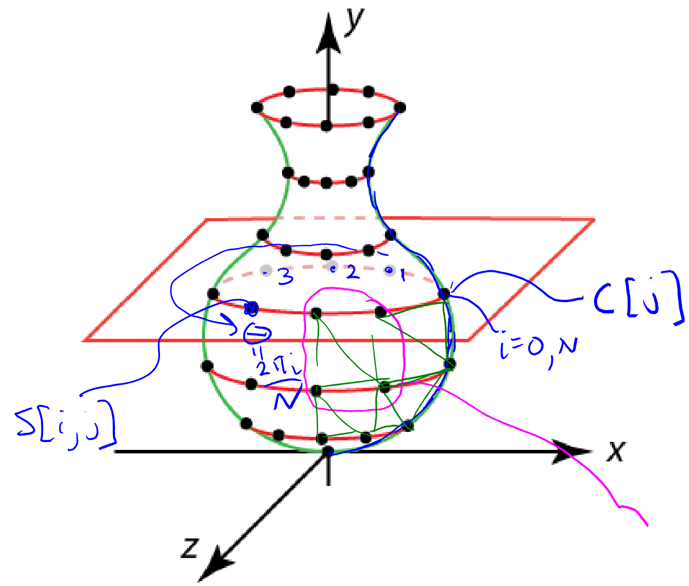
Find: A surface $S(u, v)$ which is $C(v)$ rotated $\theta = 2\pi u$
about the y-axis, where $u, v \in [0, 1]$.

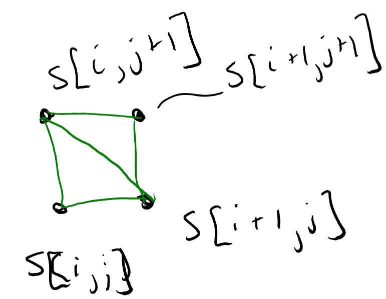
Solution: $S(u, v) = R_y(2\pi u)\, C(v)$

Constructing surfaces of revolution

We can sample in *u* and *v* to get a grid of points over the surface.







Suspect we sample:

- in v, to give $C[j]$ where $j \in [0..M\text{-}1]$
- in u, to give rotation angle $\theta[i] = 2\pi i/N$ where $i \in [0..N]$

We can now write the surface as:

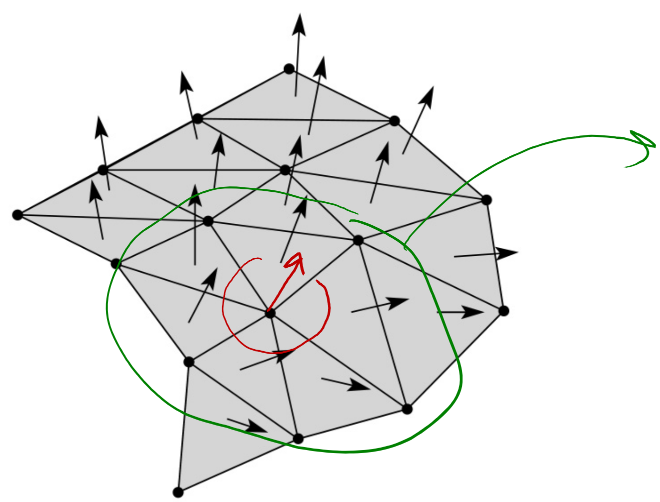
$$S[i,j] = R_y\left(\frac{2\pi i}{N}\right) C[j]$$

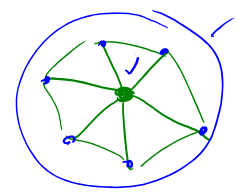
How would we turn this into a mesh of triangles?
How do we assign per-vertex normals?

Surface normals

Now that we describe the surface as a triangle mesh, we need to provide surface normals. As we'll see later, these normals are important for drawing and shading the surface (i.e., for "rendering").

One approach is to compute the normal to each triangle. How do we compute these normals?





1-ring neighborhood of v

For surfaces of revolution, we can get better-looking results by analytically computing the normal at each vertex…

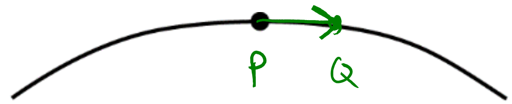
vertex normal =

compute normals of triangles in 1-ring neighborhood and average and normalize

neighbor vertices share an edge w/ v

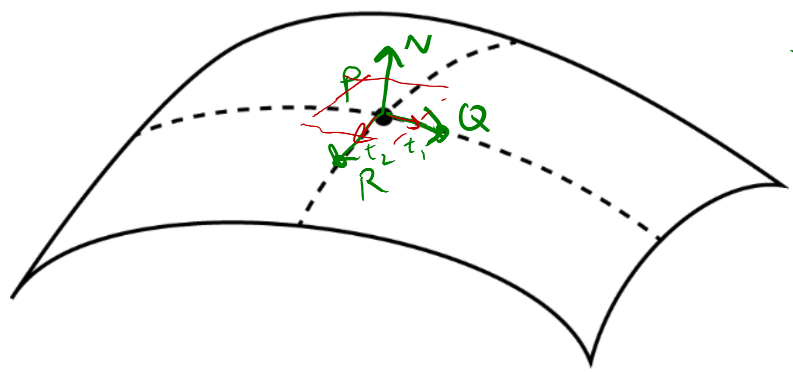
\# of neighbors ≡ valence (N)

Tangent vectors, tangent planes, and normals



$t \approx Q - P$ ←

$$\hat{t} = \lim_{Q \to P} \frac{Q-P}{\|Q-P\|}$$

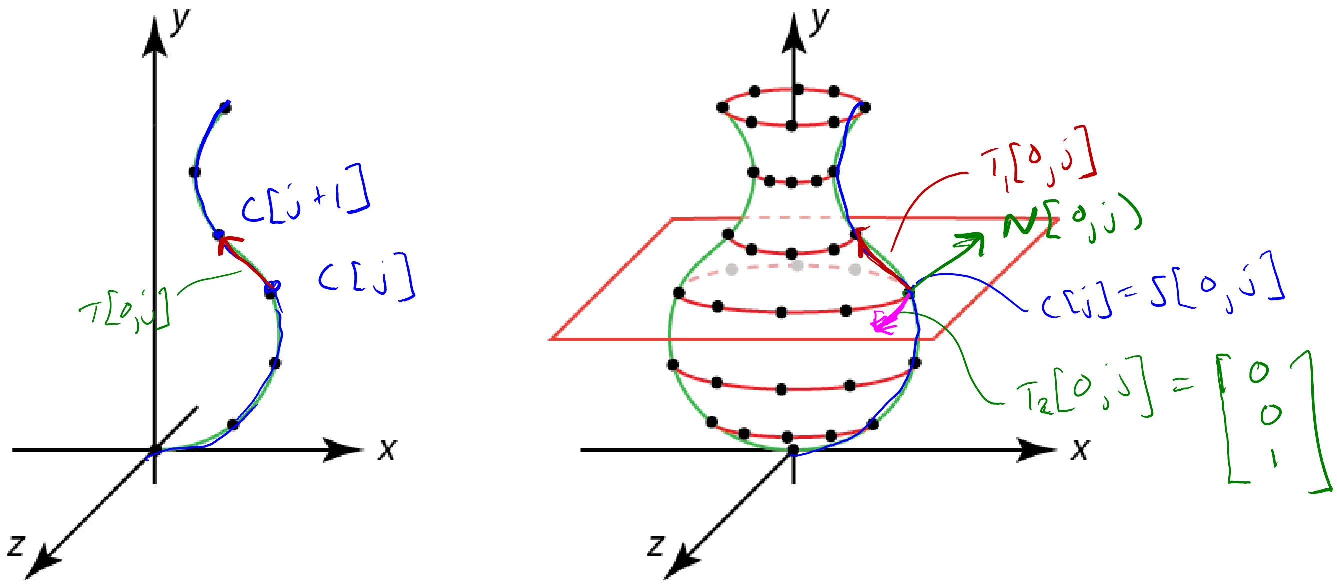


$t_1 \approx Q - P$

$t_2 \approx R - P$

$N \approx t_2 \times t_1$

Normals on a surface of revolution



We can compute tangents to the curve points in the x-y plane:

$$\mathbf{T}_1[0,j] \approx C[j+1] - C[j]$$

$$\mathbf{T}_2[0,j] = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \left(\text{as affine vector } \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right)$$

to get the normal in that plane:

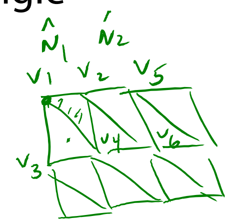
$$\mathbf{N}[0,j] = T_1[0,j] \times T_2[0,j] \qquad \hat{N}[0,j] = \frac{N[0,j]}{\| N[0,j] \|}$$

and then rotate it around:

$$\hat{N}[i,j] = R_y\left(\frac{2\pi i}{N}\right) \hat{N}[0,j]$$

Triangle meshes

How should we generally represent triangle meshes?



$V_1, \hat{N}_1$

$V_4, \hat{N}_4$

$V_2, \hat{N}_2$

Draw Δ

$V_1, \hat{N}_1$

$V_3, \hat{N}_3$

$V_4, \hat{N}_4$

Draw Δ

Vertex list

$V_1, \hat{N}_1$

$V_2, \hat{N}_2$

$V_3, \hat{N}_3$

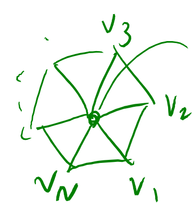
$\vdots$

Δ list

1, 4, 2

1, 3, 4

Do this



v + 1-ring neighbourhood

$$\bar{v} = v + \frac{a}{N} v_1 + \frac{a}{N} v_2 \dots$$

$$\bar{v} = \frac{v + \frac{a}{N} \sum v_i}{1 + a}$$

$$\sum \text{weights} = 1 + \frac{a}{N} + \frac{a}{N} + \dots$$
$$= 1 + a$$

$a \in [-1/2, 1/2]$

Summary

What to take away from this lecture:

- All the names in boldface.
- How to compute a surface of revolution given a profile curve.
- How to represent a surface of revolution as a triangle mesh.
- How to compute per-vertex normals for a surface of revolution.