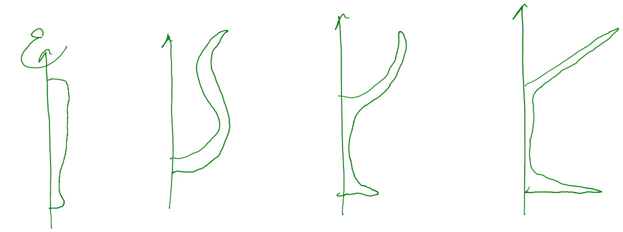
Surfaces of Revolution

Brian Curless
CSE 457
Autumn 2017

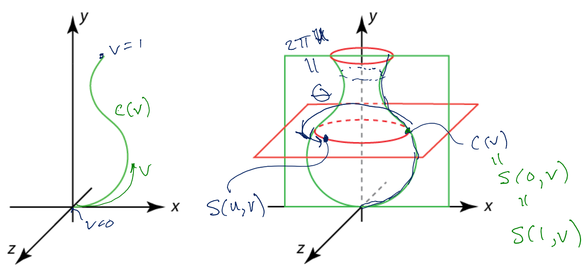
Surfaces of revolution



Idea: rotate a 2D **profile curve** around an axis.

What kinds of shapes can you model this way?

Constructing surfaces of revolution



Given: A curve $C(v)$ in the xy-plane:

$$C(v) = \begin{bmatrix} C_x(v) \\ C_y(v) \\ 0 \\ 1 \end{bmatrix}$$

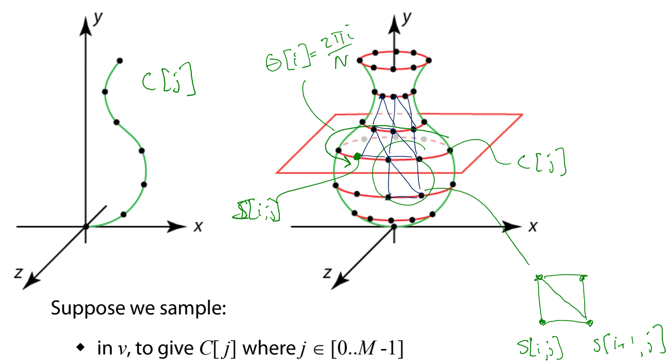
Let $R_y(\theta)$ be a rotation about the y-axis.

Find: A surface $S(u,v)$ which is $C(v)$ rotated about the y-axis, where $u,v \in [0, 1]$.

Solution: $R(\theta)C(v)$ $\quad$ $R(2\pi u)C(v)$

Constructing surfaces of revolution

We can sample in u and v to get a grid of points over the surface.



Suppose we sample:

- in v, to give $C[j]$ where $j \in [0..M-1]$
- in u, to give rotation angle $\theta[i] = 2\pi i/N$ where $i \in [0..N]$

We can now write the surface as:

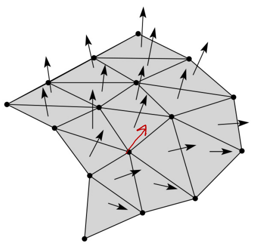
$$S[i;j] = R\left(\frac{2\pi i}{N}\right)C[j]$$

How would we turn this into a mesh of triangles?
How do we assign per-vertex normals?

Surface normals

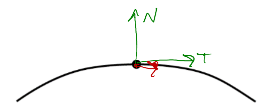
Now that we describe the surface as a triangle mesh, we need to provide surface normals. As we'll see later, these normals are important for drawing and shading the surface (i.e., for "rendering").

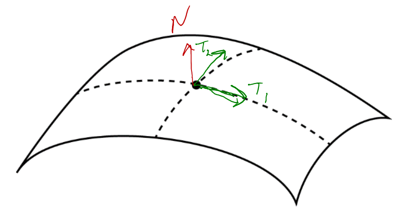
One approach is to compute the normal to each triangle. How do we compute these normals?



For surfaces of revolution, we can get better-looking results by analytically computing the normal at each vertex…

Tangent vectors, tangent planes, and normals

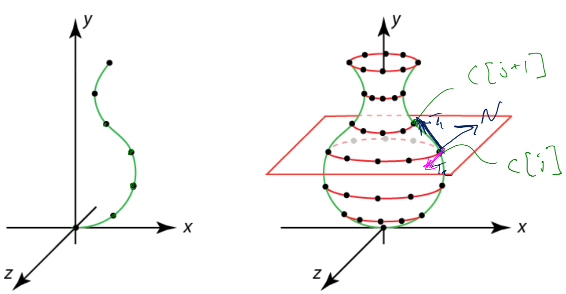




$N \sim T_1 \times T_2$

$\hat{N} = \frac{T_1 \times T_2}{\|T_1 \times T_2\|}$

Normals on a surface of revolution



We can compute tangents to the curve points in the *xy*-plane:

$\mathbf{T}_1[0, j] \approx C[j+1] - C[j]$

$\mathbf{T}_2[0, j] = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

to get the normal in that plane:

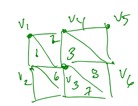
$\hat{\mathbf{N}}[0, j] = T_1 \times T_2 / \|T_1 \times T_2\|$

and then rotate it around:

$N[i,j] = R\left(\frac{2\pi i}{N}\right) \hat{N}[0,j]$

Triangle meshes

How should we generally represent triangle meshes?

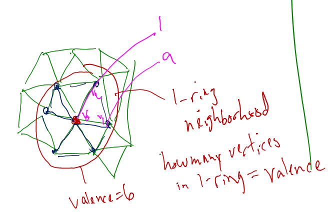


V_1, N_1
V_2, N_2
V_3, N_3

V_1, N_1
V_3, N_3
V_4, N_4

Vertices
V_1, N_1
V_2, N_2
V_3, N_3
V_4, N_4

Tri indices
1, 2, 3
1, 3, 4



$V_0' = \frac{1 \cdot V_0 + a \cdot V_1 + a \cdot V_2 \cdots}{1 + aN} = \frac{V_0 + \sum_{i=1}^{N} a V_i}{1 + aN}$

Then, re-compute Normal as average of face (Δ) normals, then normalizes

Summary

What to take away from this lecture:

- All the names in boldface.
- How to compute a surface of revolution given a profile curve.
- How to represent a surface of revolution as a triangle mesh.
- How to compute per-vertex normals for a surface of revolution.