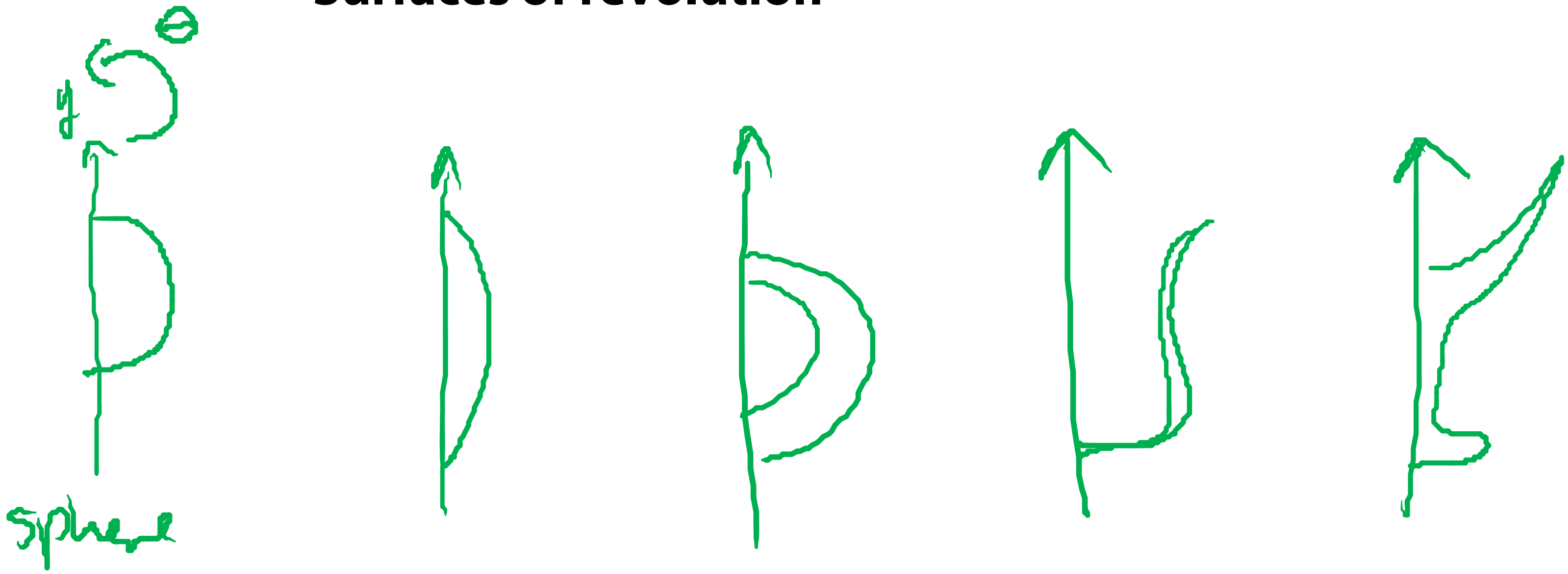
Surfaces of Revolution

CSE 457

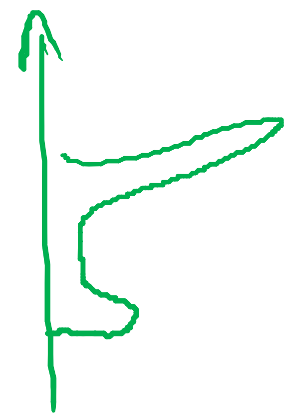
Winter 2015

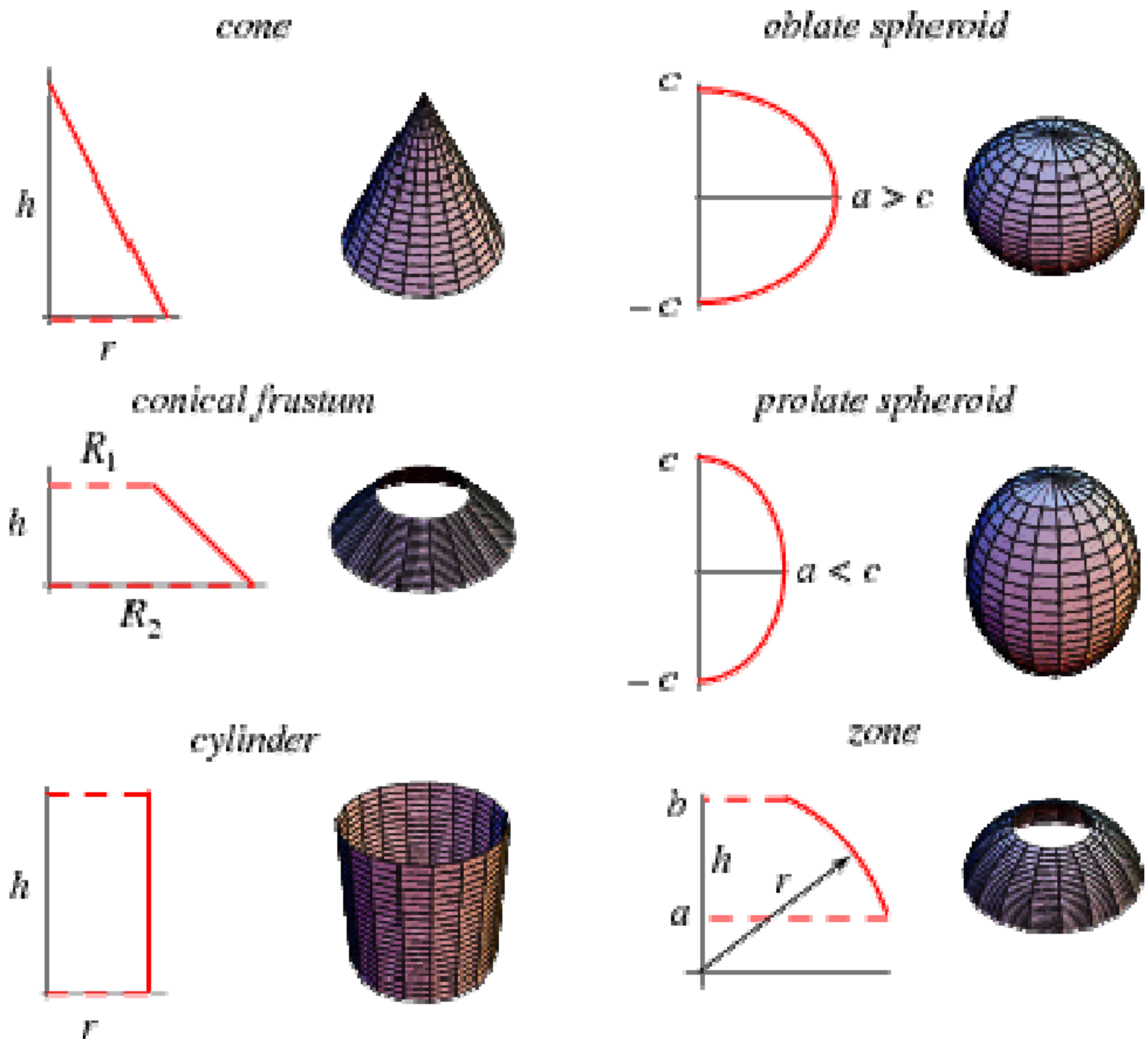
Surfaces of revolution

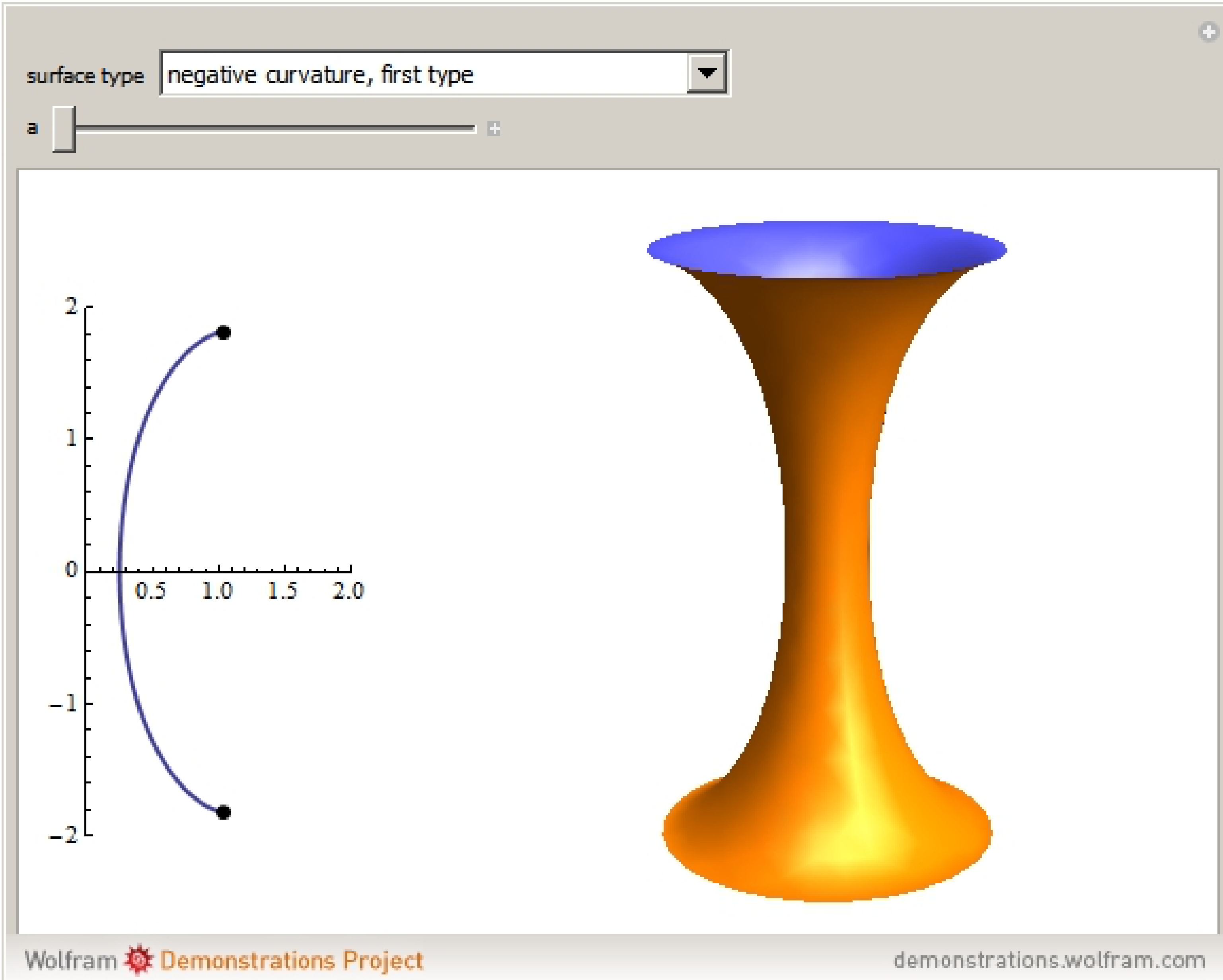


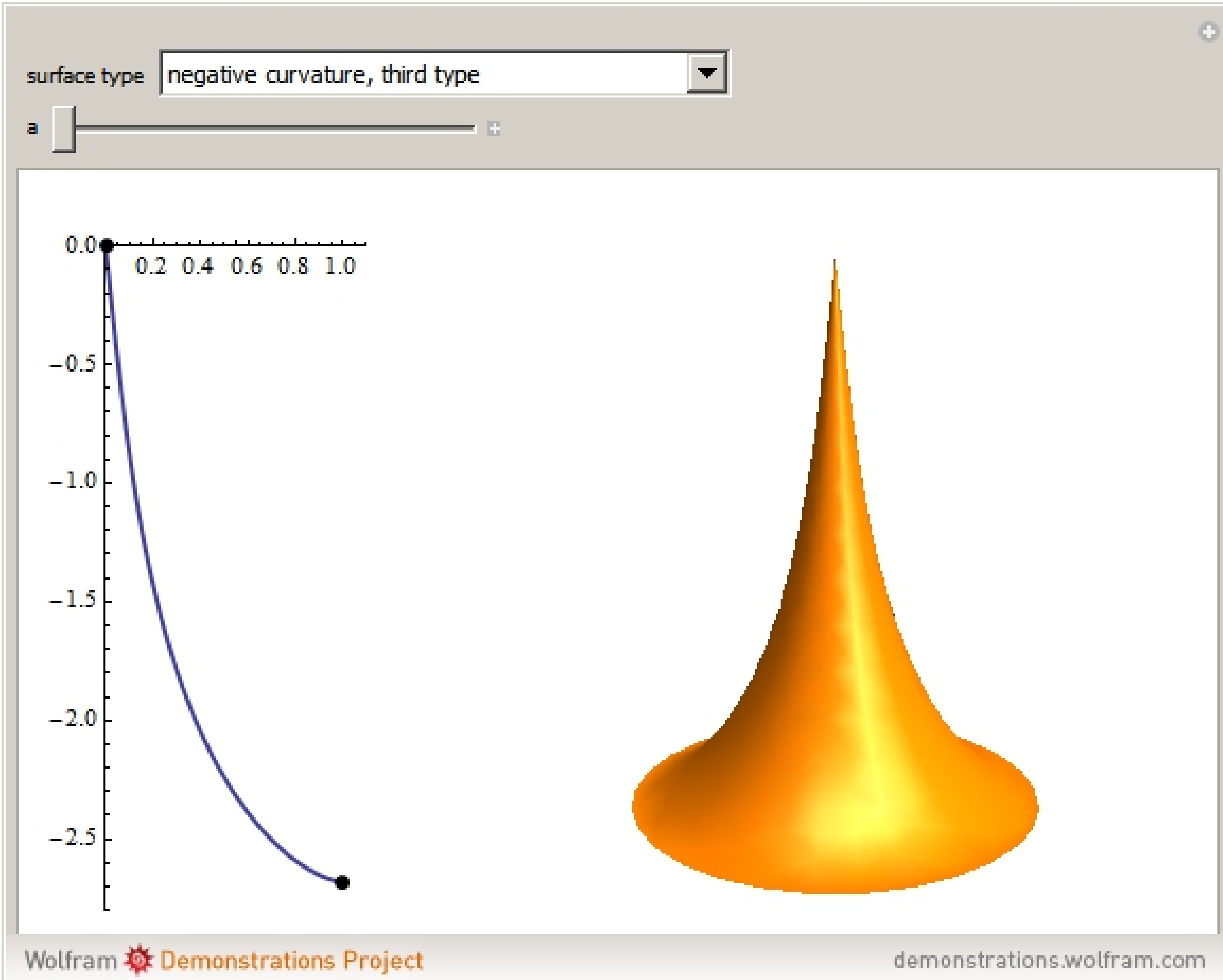
Idea: rotate a 2D **profile curve** around an axis.

What kinds of shapes can you model this way?




cone
h
r
oblate spheroid
c
a > c
− c
conical frustum
R_1
h
R_2
prolate spheroid
c
a < c
− c
cylinder
h
r
zone
b
h
r
a


surface type
negative curvature, first type
a
2
1
0
−1
−2
0.5
1.0
1.5
2.0
Wolfram Demonstrations Project
demonstrations.wolfram.com


surface type
negative curvature, third type
a
0.0
0.2 0.4 0.6 0.8 1.0
−0.5
−1.0
−1.5
−2.0
−2.5
Wolfram Demonstrations Project
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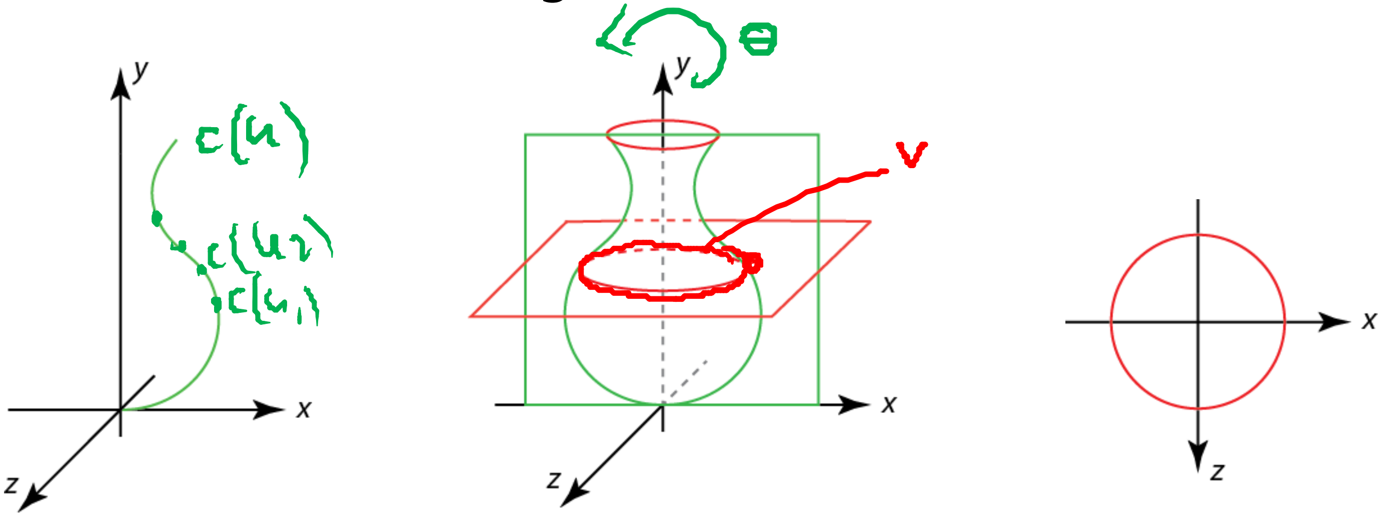
Constructing surfaces of revolution



Given: A curve $C(u)$ in the xy-plane:

profile curve

$$C(u) = \begin{bmatrix} c_x(u) \\ c_y(u) \\ 0 \\ 1 \end{bmatrix}$$

$$\theta = 2\pi \frac{n}{N}$$

Let $R_y(\theta)$ be a rotation about the y-axis.

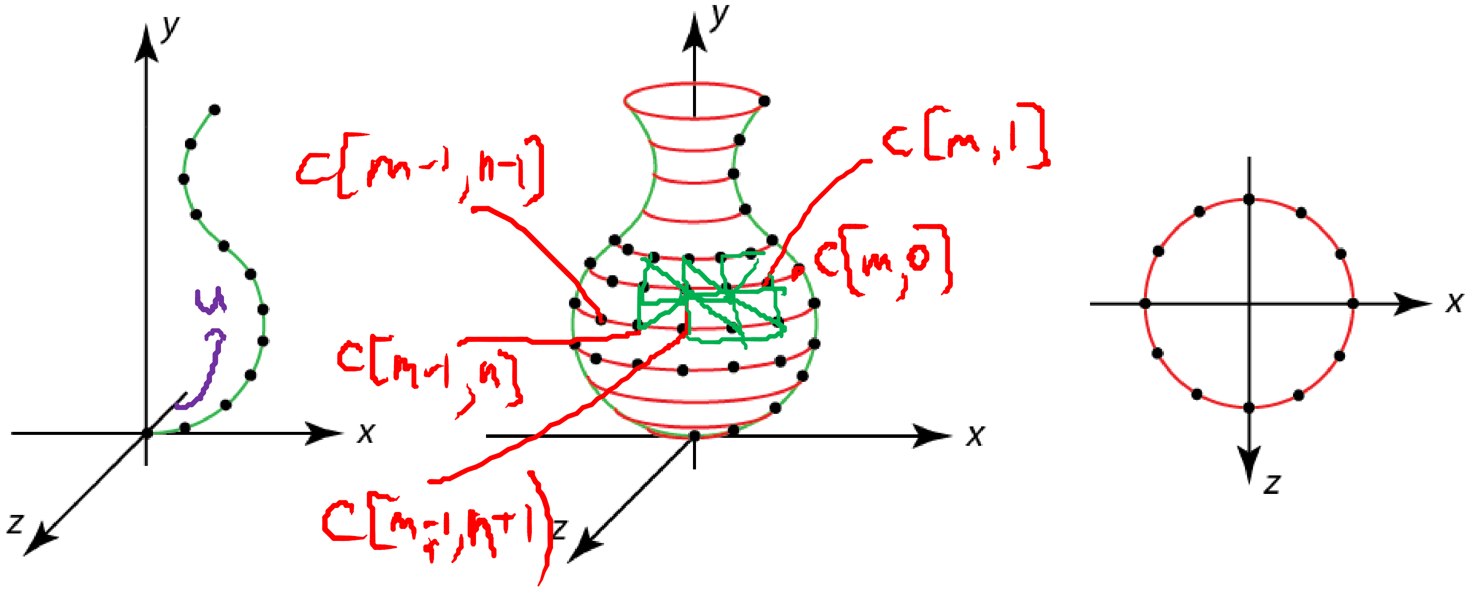
Find: A surface $S(u,v)$ which is $C(u)$ rotated about the y-axis, where $u, v \in [0, 1]$.

Solution: $S(u,v) = R_y(\theta) \cdot C(u)$

$= R_y(2\pi \frac{n}{N}) \cdot C(u)$

Constructing surfaces of revolution

We can sample in *u* and *v* to get a grid of points over the surface.



Suppose we sample:

- in *u*, to give $C[m]$ where $m \in [0..M\text{-}1]$
- in *v*, to give rotation angle $\theta[n] = 2\pi n/N$ where $n \in [0..N\text{-}1]$

We can now write the surface as:

$$S[m,n] = R_y\left(\frac{2\pi n}{N}\right) C[m]$$

How would we turn this into a mesh of triangles?

3D $\boxed{R = \begin{bmatrix} \vec{u} & \vec{v} & \vec{w} \end{bmatrix}}$

$R = R_x R_y R_z$

cos

sin

orthonormal matrix

1) $\|u\|=1$, $\|v\|=1$, $\|w\|=1$ unit vectors

2) $u^T v = 0$ $v^T w = 0$ .....

$$R^T = \begin{bmatrix} — u^T — \\ — v^T — \\ — w^T — \end{bmatrix}$$

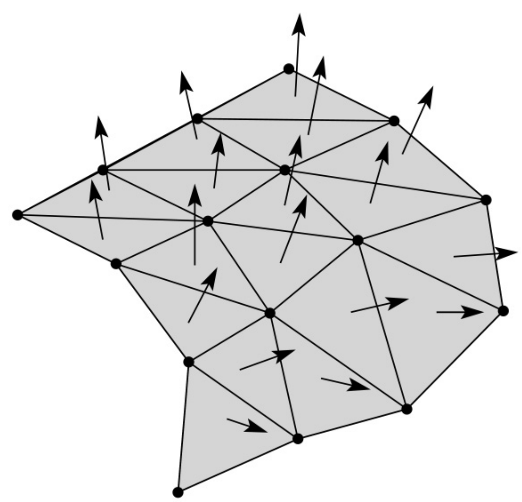
$$R^T R = \begin{bmatrix} u^T \\ v^T \\ w^T \end{bmatrix} \begin{bmatrix} u & v & w \end{bmatrix} = \begin{bmatrix} u^T u & u^T v & u^T w \\ \cdots & v^T v & \cdots \\ \cdots & \cdots & w^T w \end{bmatrix}$$

$$\Rightarrow R^T R = I \Rightarrow \boxed{R^{-1} = R^T} \quad = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Surface normals

Now that we describe the surface as a triangle mesh, we need to provide surface normals. As we'll see later, these normals are important for drawing and shading the surface (i.e., for "rendering").

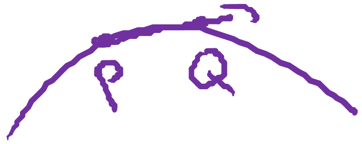
One approach is to compute the normal to each triangle. How do we compute these normals?



Later, we will see that we can get better-looking results by computing the normal at each vertex. How might we do this?

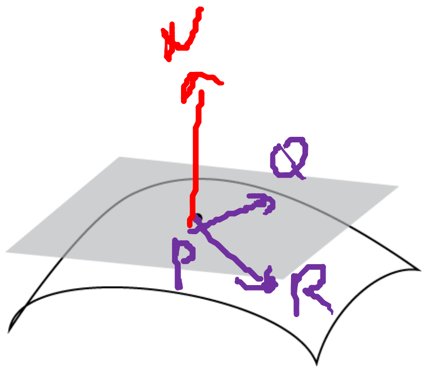
Tangent vectors and tangent planes

2D



$$T \simeq \frac{Q-P}{\|Q-P\|}$$

3D



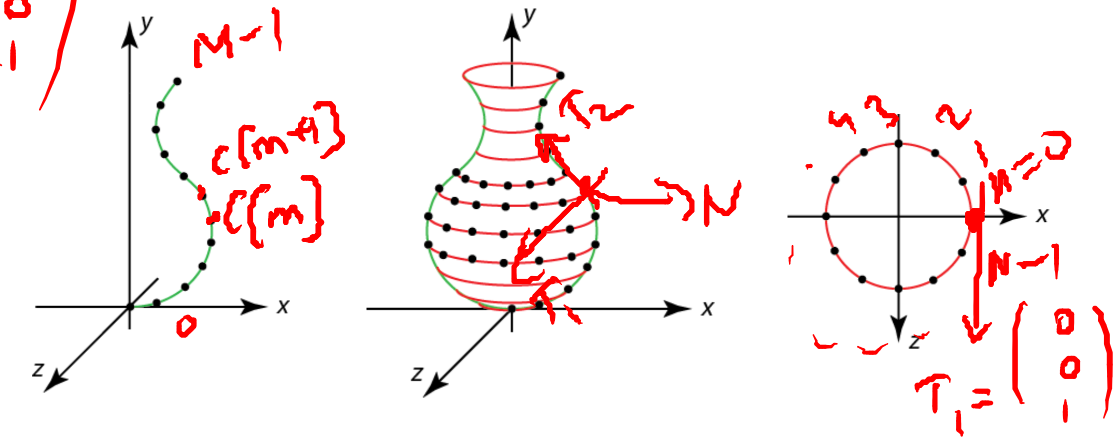
$$T_1 \simeq \frac{Q-P}{\|Q-P\|}$$

$$T_2 \simeq \frac{R-P}{\|R-P\|}$$

$$N = \frac{T_2 \times T_1}{\|T_2 \times T_1\|}$$

Normals on a surface of revolution

$$C[m] = \begin{pmatrix} x \\ y \\ 0 \\ 1 \end{pmatrix}$$



$$T_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad T_2 = C[m+1] - C[m]$$

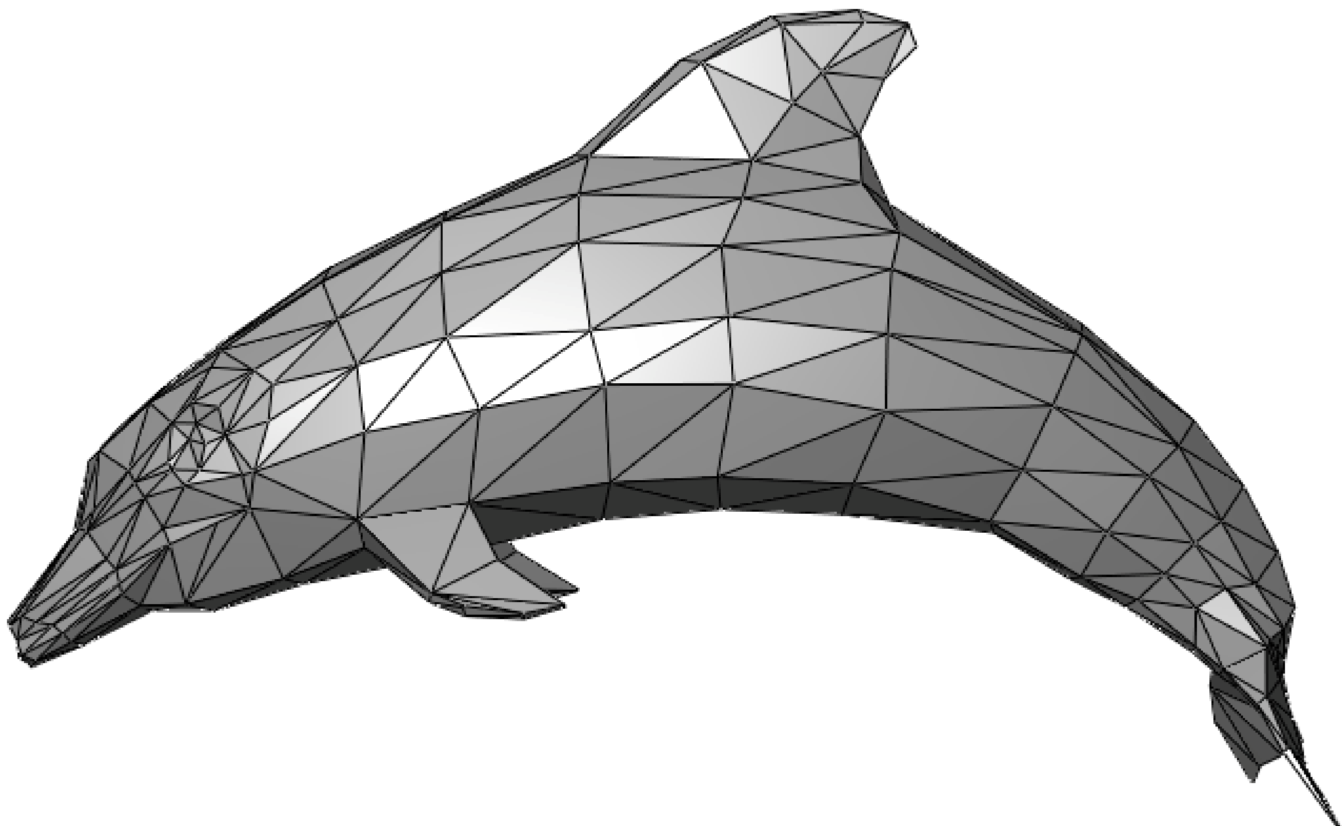
normal

$$\vec{N}_{curve} = \frac{T_1 \times T_2}{\|\cdot\|} \qquad \boxed{\vec{N}[n,m] = R_y\left(\frac{2\pi n}{N}\right)\vec{N}[m]}$$

points on a circle

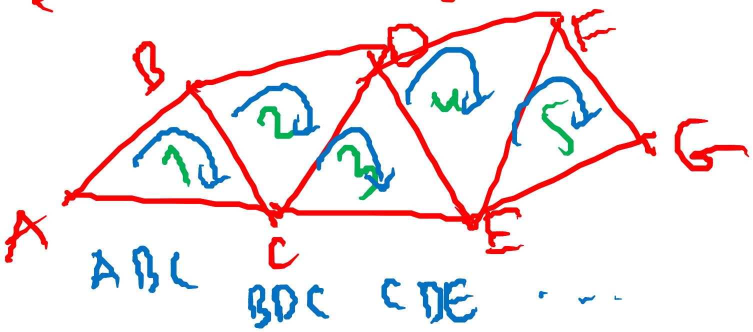
Triangle meshes

How should we generally represent triangle meshes?



Representation

(1) Triangle strip



$\underbrace{ABC}_{1}$, $\underbrace{BCD}_{2}$, $\underbrace{CDE}_{3}$

ABCDEFG ....

GL_TRIANGLE_STRIP

Glvertex3f

:

(2) index array

$$V = [\underset{1}{A}\ \underset{2}{D}\ \underset{3}{C}\ D\ \cdots\ ]$$

$$I = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ & \vdots & \end{bmatrix}$$

Gldrawelement

VBO