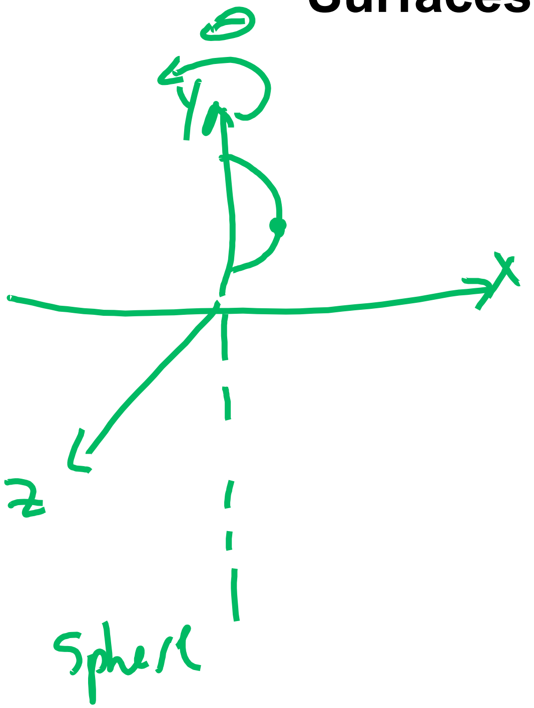
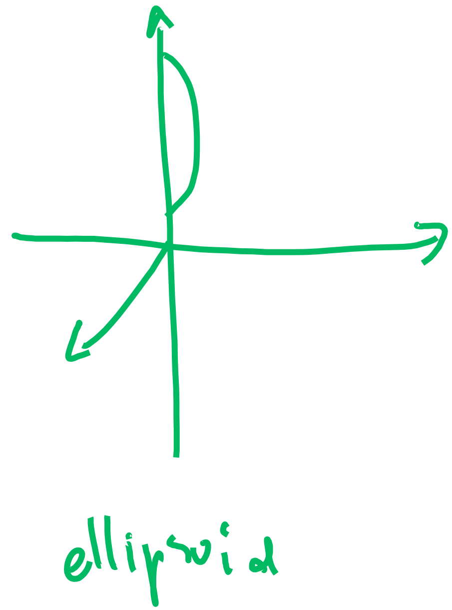
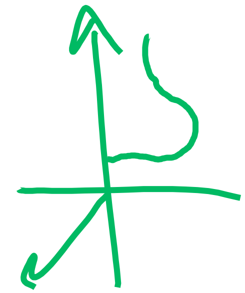
Surfaces of Revolution

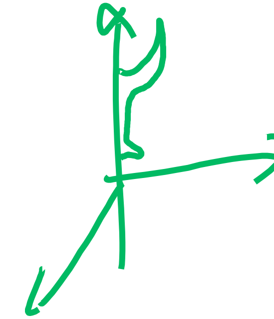
CSE 457

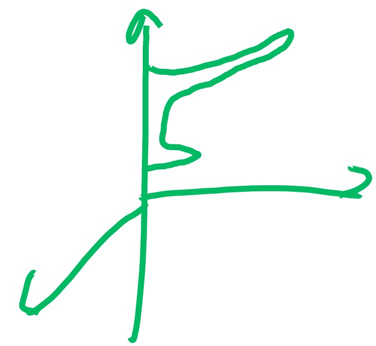
Surfaces of revolution







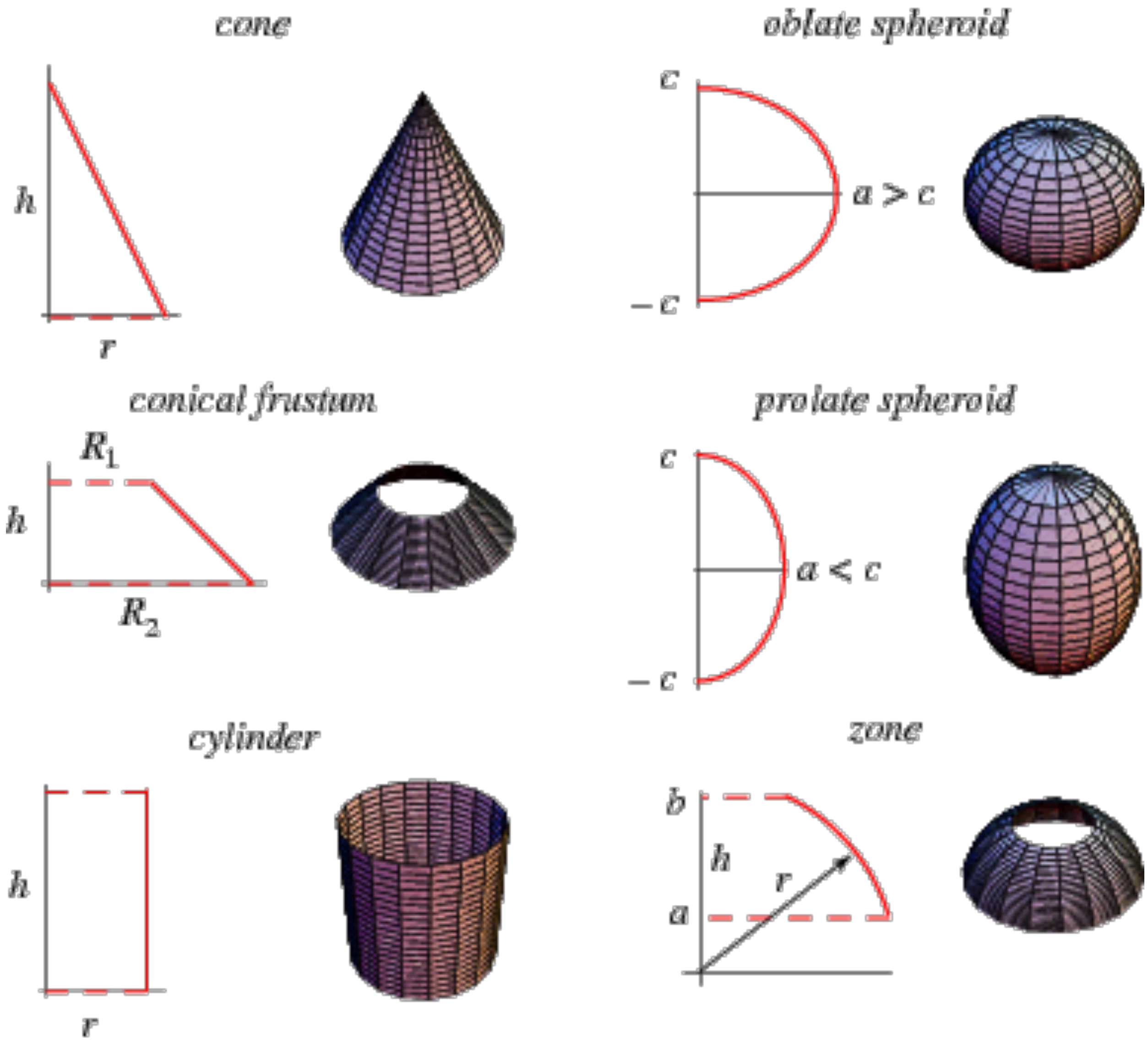


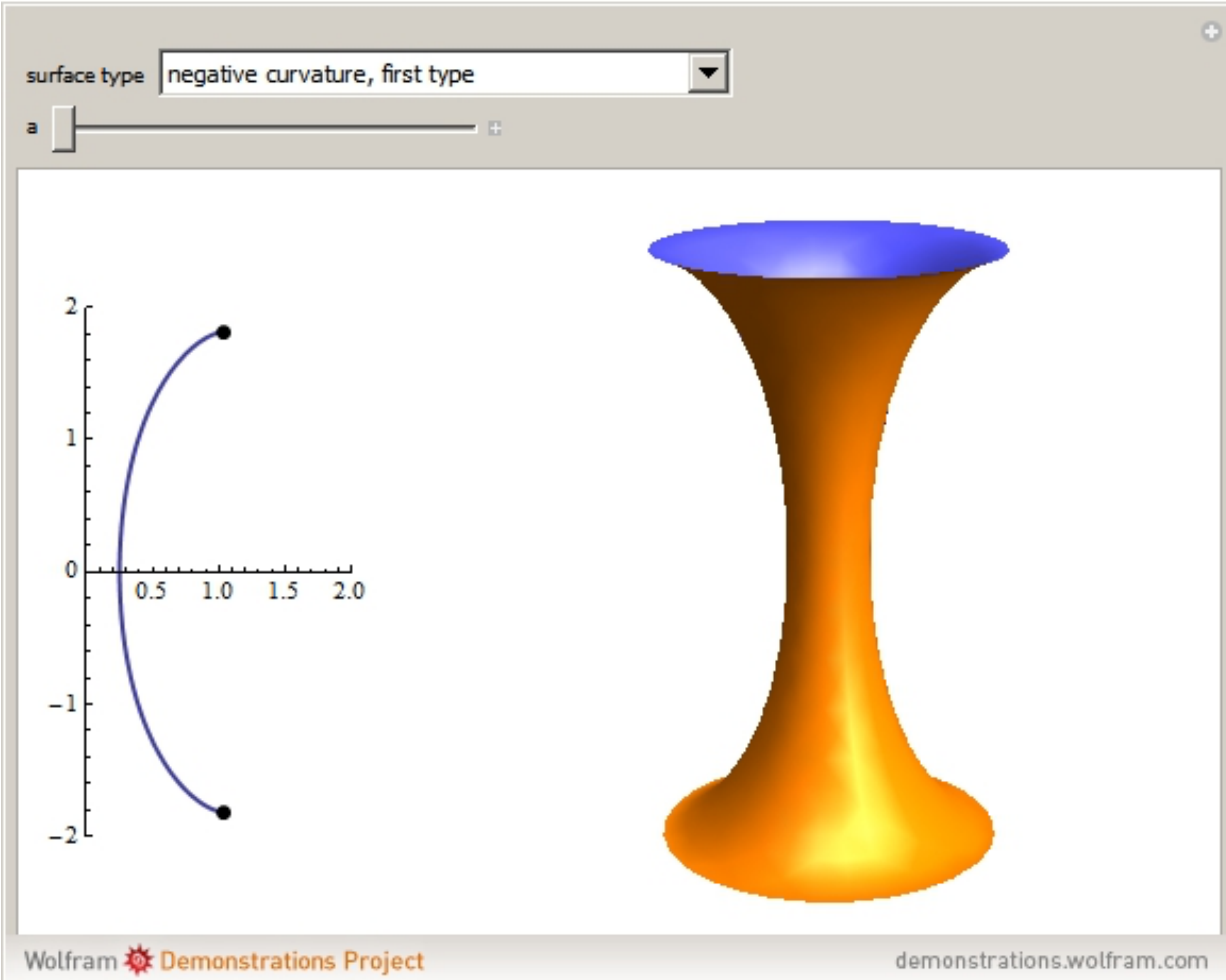


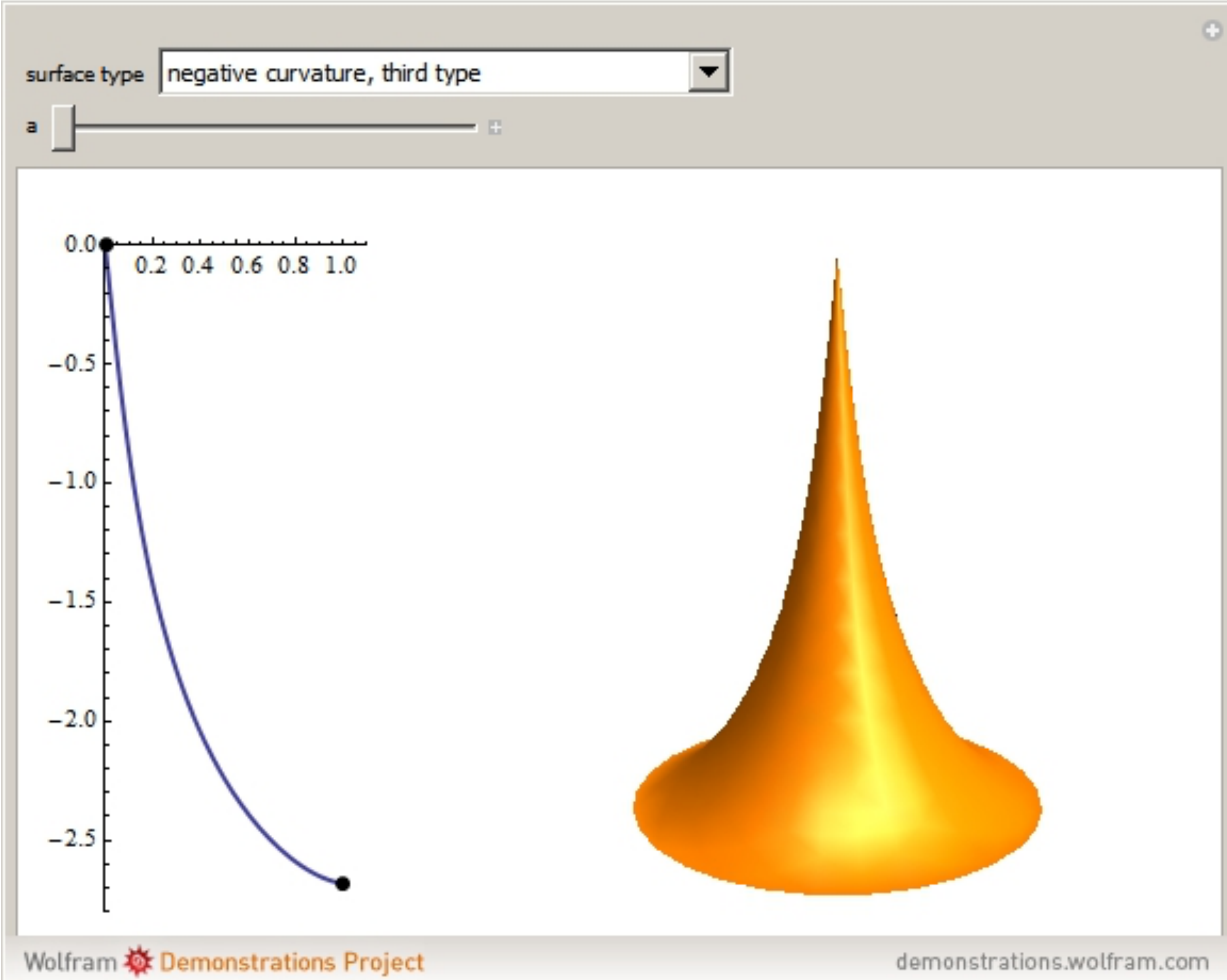
Idea: rotate a 2D **profile curve** around an axis.

What kinds of shapes can you model this way?

symmetric


cone
h
r
oblate spheroid
c
a > c
− c
conical frustum
R1
h
R2
prolate spheroid
c
a < c
− c
cylinder
h
r
zone
b
h
r
a


surface type
negative curvature, first type
a
2
1
0
0.5
1.0
1.5
2.0
−1
−2
Wolfram Demonstrations Project
demonstrations.wolfram.com


surface type
negative curvature, third type
a
0.0
0.2
0.4
0.6
0.8
1.0
−0.5
−1.0
−1.5
−2.0
−2.5
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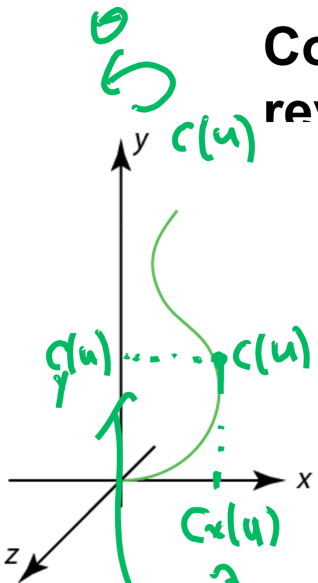
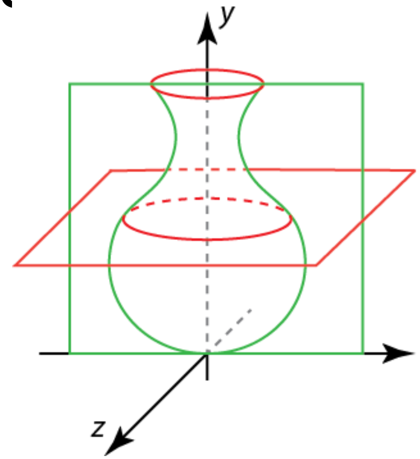
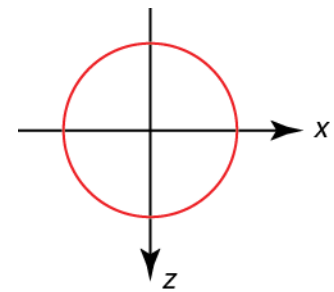
Constructing surfaces of revolution







Given: A curve $C(u)$ in the xy-plane:

$$C(u) = \begin{bmatrix} c_x(u) \\ c_y(u) \\ 0 \\ 1 \end{bmatrix}$$

→ on xy plane

→ homogenous coord.

Let $R_y(\theta)$ be a rotation about the y-axis. $S[u,v] = R_y(\theta)\, C(u)$

Find: A surface $S(u,v)$ which is $C(u)$ rotated about the y-axis, where $u, v \in [0, 1]$.

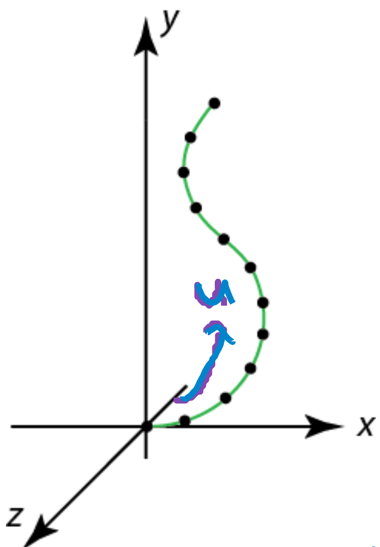
$\theta = 2\pi \cdot \frac{n}{N}$

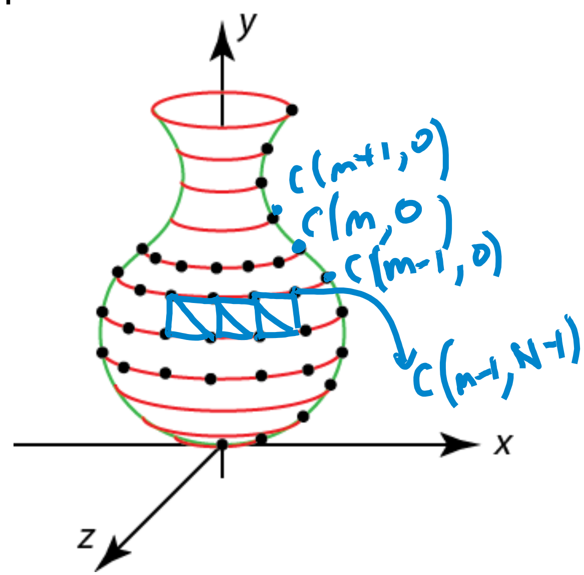
Solution: $S(u,v) = R_y\left(2\pi \frac{n}{N}\right) C(u)$

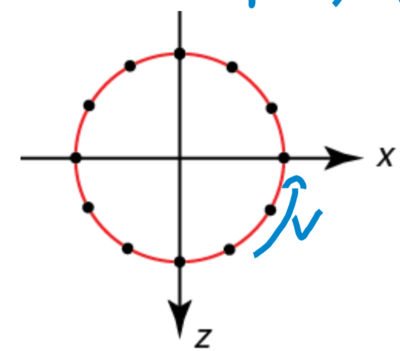
Constructing surfaces of revolution

We can sample in *u* and *v* to get a grid of points over the surface

point on the circle = N
$n = 0, \dots, N-1$







#points on the curve = M
$m = 0, \dots, M-1$

Suppose we sample:

- in *u*, to give $C[m]$ where $m \in [0..M-1]$
- in *v*, to give rotation angle $\theta[n] = 2\pi n/N$ where $n \in [0..N-1]$

We can now write the surface as:

$$S[m,n] = R_y\left(2\pi \frac{n}{N}\right) C[m]$$

How would we turn this into a mesh of triangles?

Side note about Rotation matrices

$R = \begin{bmatrix} | & | & | \\ \vec{u} & \vec{v} & \vec{w} \\ | & | & | \end{bmatrix}$

3×3

$R = \underbrace{R_x}_{\substack{\cos \\ \sin \ldots}} R_y R_z$

$R^{-1} = ?$

orthonormal matrix

(1) $\|u\| = 1$

$\|v\| = 1$

$\|w\| = 1$

(2) $u^T v = 0$

$u^T w = 0$

$v^T w = 0$

$R^T = \begin{pmatrix} — u^T — \\ — v^T — \\ — w^T — \end{pmatrix}$

$R^T R = \begin{pmatrix} u^T \\ v^T \\ w^T \end{pmatrix} (u \; v \; w) = \begin{pmatrix} u^T u & u^T v & u^T w \\ v^T u & v^T v & \\ & & w^T w \end{pmatrix}$

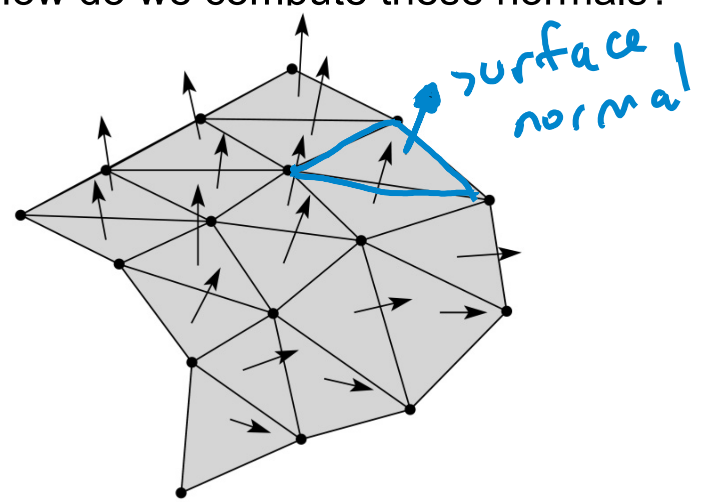
$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$R^T R = I \Rightarrow R^T = R^{-1}$

Surface normals

Now that we describe the surface as a triangle mesh, we need to provide surface normals. As we'll see later, these normals are important for drawing and shading the surface (i.e., for "rendering").

One approach is to compute the normal to each triangle. How do we compute these normals?

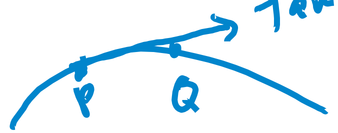


Later, we will see that we can get better-looking results by computing the normal at each vertex. How might we do this?

Compute surface normals

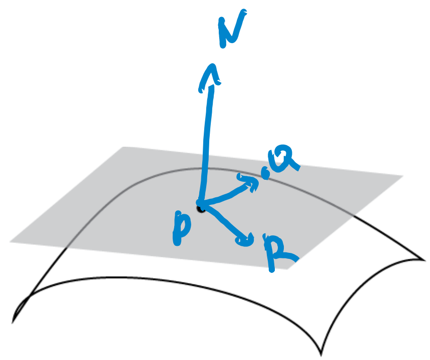
Tangent vectors and tangent planes

2D



$$T \approx \frac{Q-P}{\|Q-P\|}$$

3D

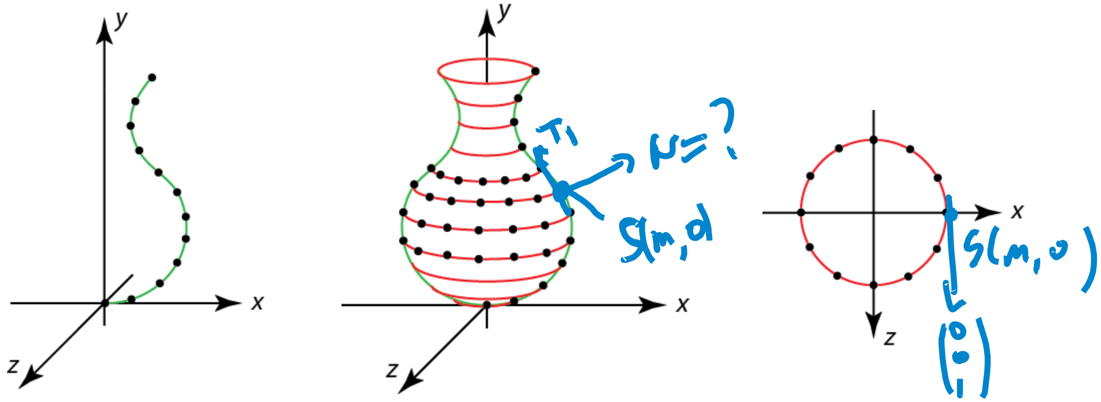


$$T_1 \approx \frac{Q-P}{\|Q-P\|}$$

$$T_2 \approx \frac{R-P}{\|R-P\|}$$

$$\boxed{N = \frac{T_1 \times T_2}{\|\cdot\|}}$$

Normals on a surface of revolution



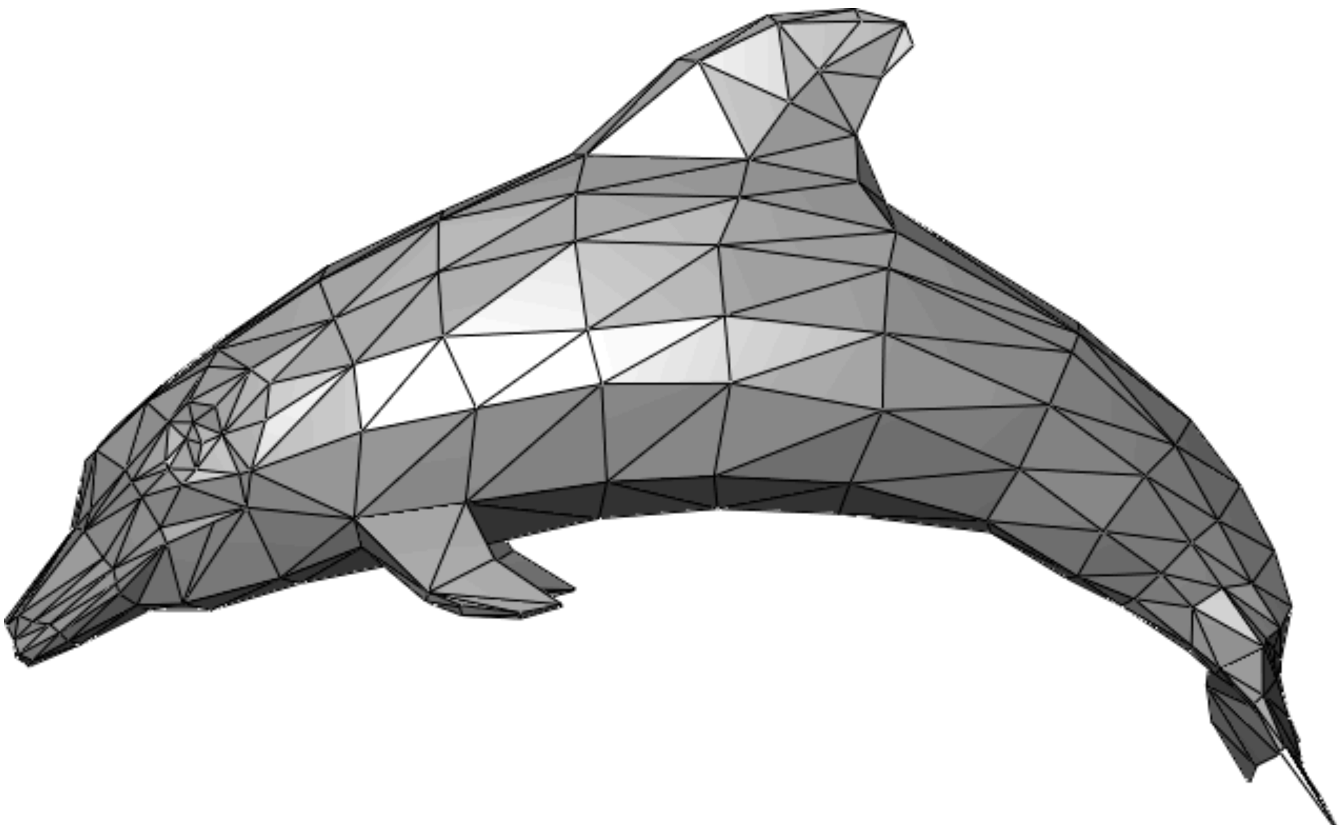
$$T_1 = S(m+1,0) - S(m,0)$$

$$T_2 = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$N(m,0) = \frac{T_1 \times T_2}{\| \cdot \|} \qquad N(m,n) = R_y\left(\frac{2\pi n}{N}\right) N(m,0)$$

Triangle meshes

How should we generally represent triangle meshes?



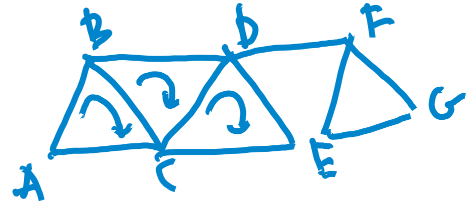
Represent triangles

① $\text{Vertices} = \begin{array}{ccccc} 1 & 2 & 3 & 4 & \cdots \\ \end{array}$
$\left[\begin{array}{ccccc} A & B & C & D & \cdots \end{array}\right]$

$\text{Index Array} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 2 & 5 & 6 \\ & \vdots & \end{pmatrix} \begin{array}{l} \text{triangle \#1} \\ \#2 \\ \#3 \\ \vdots \end{array}$

each vertex appears 6 times

② Triangle strip



Arr ABCDEFG.....