Surfaces of Revolution

CSE 457
Surfaces of revolution

Idea: rotate a 2D profile curve around an axis.

What kinds of shapes can you model this way?
Constructing surfaces of revolution

Given: A curve $C(u)$ in the $xy$-plane:

Let $R_y(\theta)$ be a rotation about the $y$-axis.

Find: A surface $S(u,v)$ which is $C(u)$ rotated about the $y$-axis, where $u, v \in [0, 1]$.

Solution: $S(u,v) = R_y \left( 2\pi \frac{u}{N} \right) C(u)$
Constructing surfaces of revolution

We can sample in $u$ and $v$ to get a grid of points over the surface.

Suppose we sample:

- in $u$, to give $C[m]$ where $m \in [0..M-1]$
- in $v$, to give rotation angle $\theta[n] = \frac{2\pi n}{N}$ where $n \in [0..N-1]$

We can now write the surface as:

$$S[m,n] = R_y\left(\frac{2\pi n}{N}\right)C[m]$$

How would we turn this into a mesh of triangles?
Side note about Rotation matrices

\[ R = \begin{bmatrix} \begin{array}{ccc} 1 & & \\
\tilde{u} & 1 & \\
\tilde{v} & & 1 \end{array} \end{bmatrix} \quad R = R_x R_y R_z \]

\[ R^{-1} = ? \]

Orthogonal matrix

1. \( \| u \| = 1 \)
2. \( u^T v = 0 \)
3. \( \| v \| = 1 \)
4. \( u^T w = 0 \)
5. \( \| w \| = 1 \)
6. \( v^T w = 0 \)

\[ R^T = \begin{bmatrix} -u^T & -v^T & -w^T \end{bmatrix} \]

\[ R^T R = \begin{bmatrix} u^T & v^T & w^T \end{bmatrix} \begin{bmatrix} u & v & w \end{bmatrix} = \begin{bmatrix} u^T u & u^T v & u^T w \\
v^T u & v^T v & v^T w \\
w^T u & w^T v & w^T w \end{bmatrix} \]

\[ R^T R = I \quad \Rightarrow \quad R^T = R^{-1} \]
Surface normals

Now that we describe the surface as a triangle mesh, we need to provide surface normals. As we’ll see later, these normals are important for drawing and shading the surface (i.e., for “rendering”).

One approach is to compute the normal to each triangle. How do we compute these normals?

Later, we will see that we can get better-looking results by computing the normal at each vertex. How might we do this?
Tangent vectors and tangent planes

2D

\[ \mathbf{T} \approx \frac{\mathbf{Q} - \mathbf{P}}{||\mathbf{Q} - \mathbf{P}||} \]

3D

\[ \mathbf{T}_1 \approx \frac{\mathbf{Q} - \mathbf{P}}{||\mathbf{Q} - \mathbf{P}||} \]
\[ \mathbf{T}_2 \approx \frac{\mathbf{R} - \mathbf{P}}{||\mathbf{R} - \mathbf{P}||} \]

\[ \mathbf{N} = \frac{\mathbf{T}_1 \times \mathbf{T}_2}{|| \mathbf{T}_1 \times \mathbf{T}_2 ||} \]
Normals on a surface of revolution

\[ T_1 = s(m_{+1}, 0) - s(m_0) \]
\[ T_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \]

\[ N(m, 0) = \frac{T_1 \times T_2}{\|T_1\| \cdot \|T_2\|} \]
\[ N(m, n) = R_y \left( \frac{\pi n}{N} \right) N(m, 0) \]
Triangle meshes

How should we generally represent triangle meshes?
Represent triangles

Vertices = [A B C D ...]

Index Array = \[
\begin{pmatrix}
1 & 2 & 3 & 4 \\
3 & 4 & 5 & 6 \\
2 & 5 & 6 & \vdots
\end{pmatrix}
\]

Each vertex appears 6 times

Triangle strip

Arr: ABCDEFG ...