Hierarchical Modeling

CSE 457

Reading

Required:

- Angel, sections 8.1 – 8.6, 8.8

Optional:

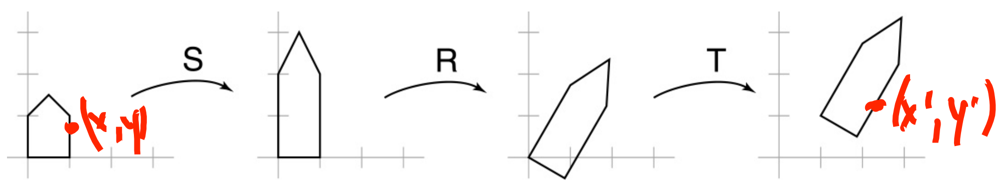
- *OpenGL Programming Guide*, chapter 3

Symbols and instances

Most graphics APIs support a few geometric **primitives**:

- spheres
- cubes
- cylinders

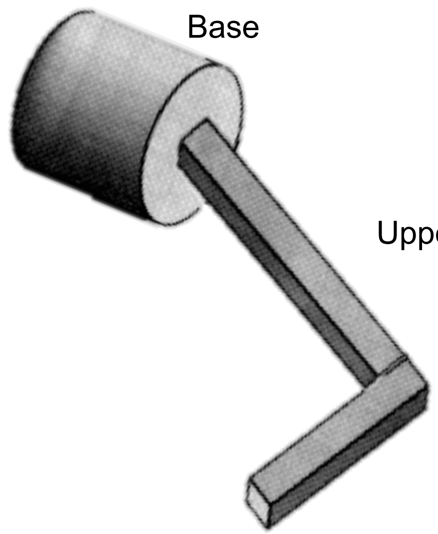
These symbols are **instanced** using an **instance transformation**.



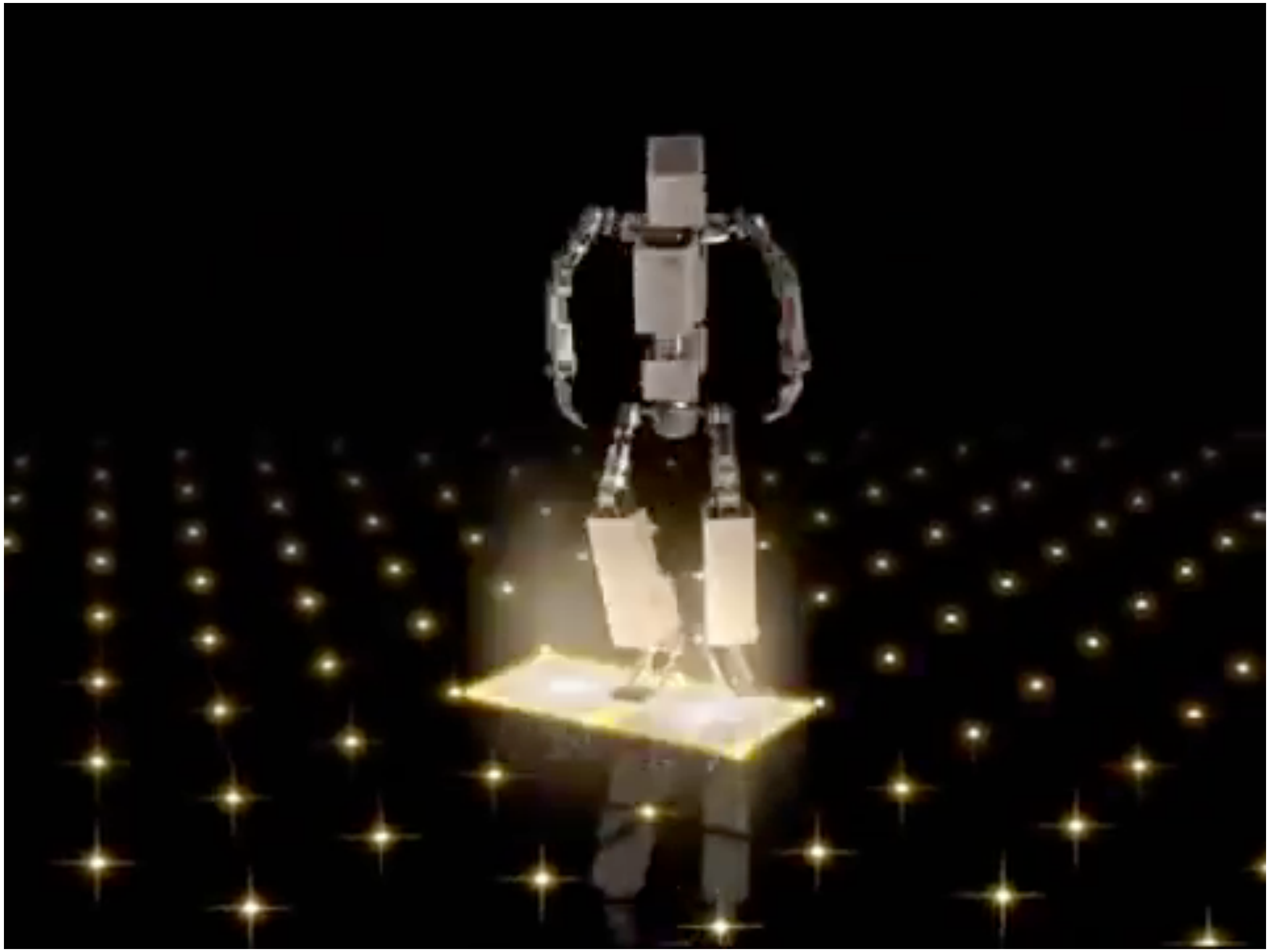
Q: What is the matrix for the instance transformation above?

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = T R S \begin{pmatrix} x \\ y \end{pmatrix}$$

3D Example: A robot arm



Have to be constrained via a hierarchical model



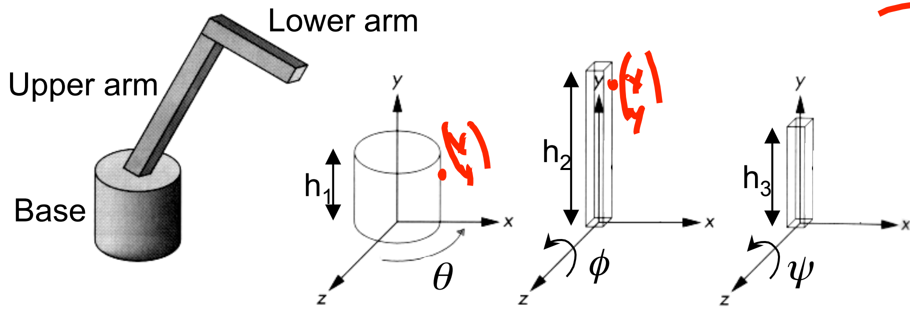
“Number One” Playgroup – Duran Duboi

3D Example: A robot arm

Consider this robot arm with 3 degrees of freedom:

- Base rotates about its vertical axis by θ
- Upper arm rotates in its *xy*-plane by ϕ
- Lower arm rotates in its *xy*-plane by ψ

$R(\theta)$
$R(\phi)$
$R(\psi)$
$T(\cdot)$



[Angel, 2011]

(Note that the angles are set to zero in the figure;
i.e., the parts are shown in their "default" positions.)

Q: What matrix do we use to transform the base?

$R(\theta)\begin{pmatrix}x\\y\end{pmatrix}$

Q: What matrix for the upper arm?

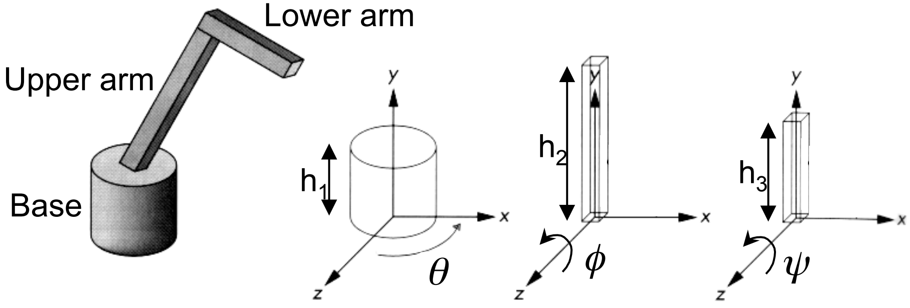
$R(\theta)\,T(h_1)\,R(\phi)\begin{pmatrix}x\\y\end{pmatrix}$

Q: What matrix for the lower arm?

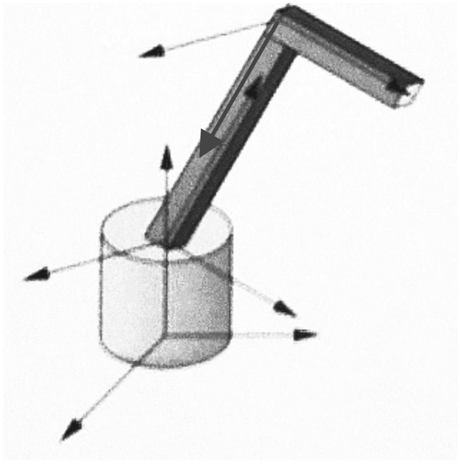
$R(\theta)\,T(h_1)\,R(\phi)\,T(h_2)\,R(\psi)\begin{pmatrix}x\\y\end{pmatrix}$

3D Example: A robot arm

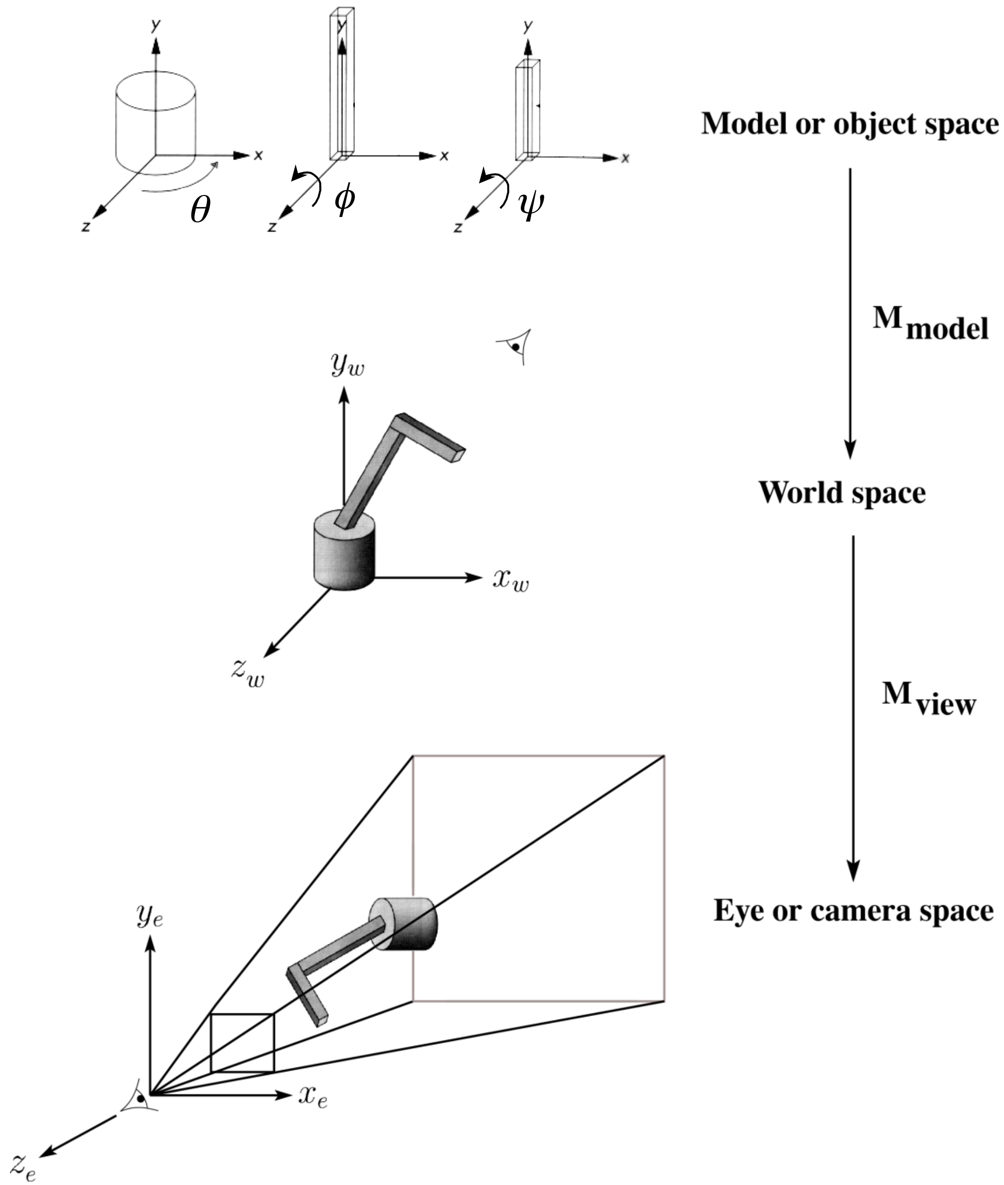
An alternative interpretation is that we are taking the original coordinate frames…



…and translating and rotating them into place:



From parts to model to viewer



Robot arm implementation

The robot arm can be displayed by keeping a global matrix and computing it at each step:

```
Matrix M_model;
Matrix M_view;

main()
{
    . . .
    M_view = compute_view_transform();
    robot_arm();
    . . .
}

robot_arm()
{
    M_model = M_view*R_y(theta);
    base();
    M_model = M_View*R_y(theta)*T(0,h1,0)*R_z(phi);
    upper_arm();
    M_model = M_view*R_y(theta)*T(0,h1,0)
                    *R_z(phi)*T(0,h2,0)*R_z(psi);
    lower_arm();
}
```

Do the matrix computations seem wasteful?

Robot arm implementation, better

Instead of recalculating the global matrix each time, we can just update it *in place* by concatenating matrices on the right:

```
Matrix M_modelview;

main()
{
    . . .
    M_modelview = compute_view_transform();
    robot_arm();
    . . .
}

robot_arm()
{
    M_modelview *= R_y(theta);
    base();
    M_modelview *= T(0,h1,0)*R_z(phi);
    upper_arm();
    M_modelview *= T(0,h2,0)*R_z(psi);
    lower_arm();
}
```

Robot arm implementation, OpenGL

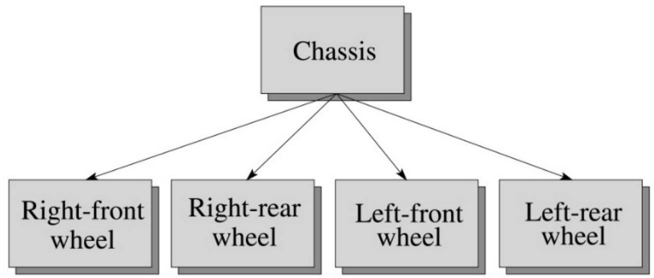
OpenGL maintains a global state matrix called the **model-view matrix**, which is updated by concatenating matrices on the ***right***.

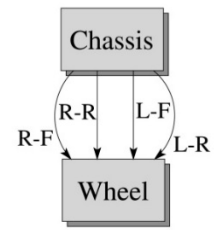
```
main()
{
    . . .
    glMatrixMode( GL_MODELVIEW );
    Matrix M = compute_view_xform();
    glLoadMatrixf( M );
    robot_arm();
    . . .
}

robot_arm()
{
    glRotatef( theta, 0.0, 1.0, 0.0 );
    base();
    glTranslatef( 0.0, h1, 0.0 );
    glRotatef( phi, 0.0, 0.0, 1.0 );
    lower_arm();
    glTranslatef( 0.0, h2, 0.0 );
    glRotatef( psi, 0.0, 0.0, 1.0 );
    upper_arm();
}
```

Hierarchical modeling

Hierarchical models can be composed of instances using trees or DAGs:

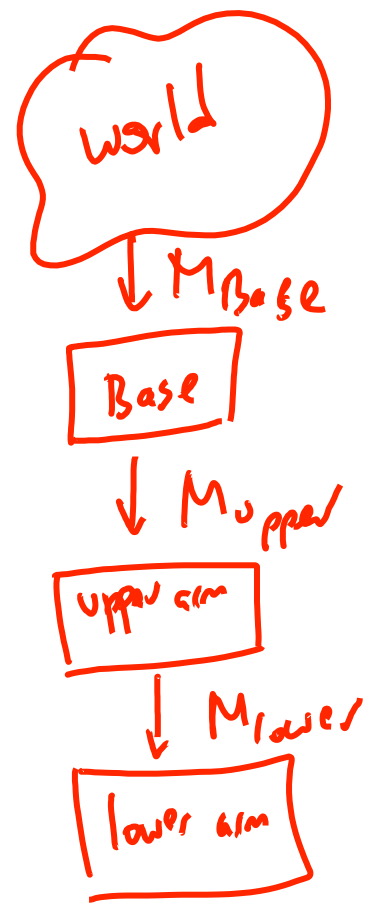




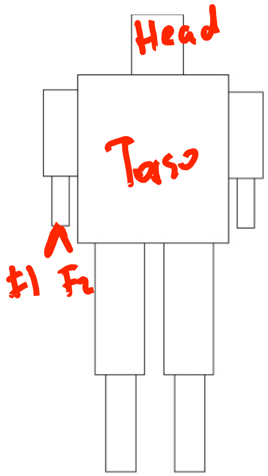
- edges contain geometric transformations
- nodes contain geometry (and possibly drawing attributes)

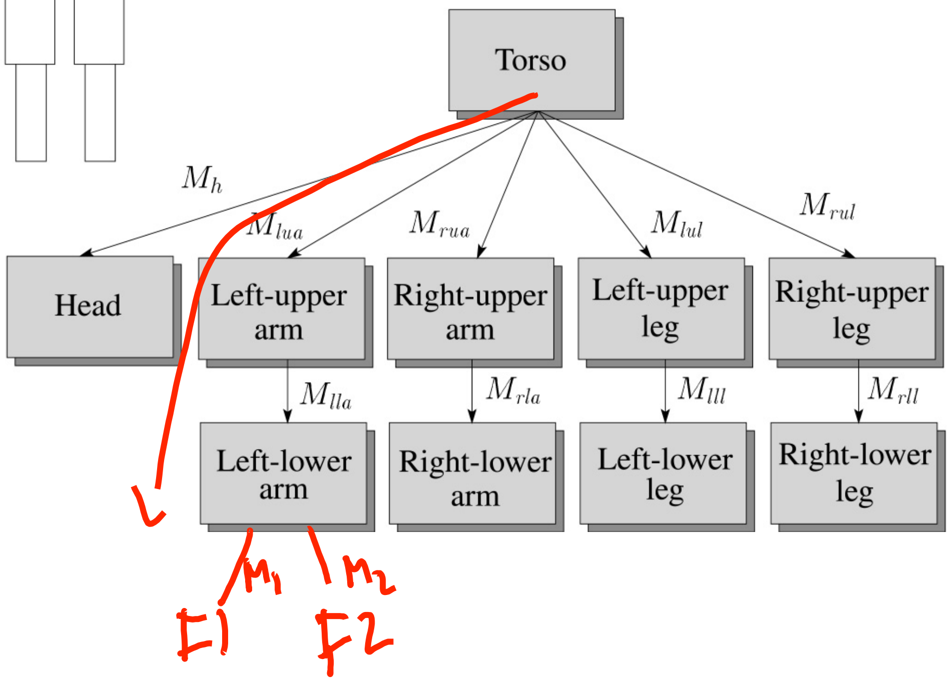
How might we draw the tree for the robot arm?

concat. on the right



A complex example: human figure

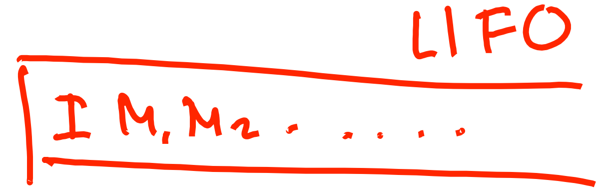




Q: What's the most sensible way to traverse this tree?

depth first

Implementing hierarchies:



A matrix stack that you can push/pop (LIFO).

Recursive algorithm that descends the model tree:

- Load identity matrix
- For each node:
  - Push a new matrix onto stack
  - Concatenate transformations onto current
  - Recursively descend the tree
  - Pop matrix out of stack
- For each leaf node:
  - Draw using the current transformation matrix

Human figure implementation, OpenGL

```
figure()
{
    torso();
    glPushMatrix();
        glTranslate( ... );
        glRotate( ... );
        head();
    glPopMatrix();
    glPushMatrix();
        glTranslate( ... );
        glRotate( ... );
        left_upper_arm();
        glPushMatrix();
            glTranslate( ... );
            glRotate( ... );
            left_lower_arm();
        glPopMatrix();
     glPopMatrix();
     . . .
}
```

$I T_1 R_1$

T_1 R_1

T R

Fingers

gl Push

Animation

The above examples are called **articulated models**:

- rigid parts
- connected by joints

They can be animated by specifying the joint angles (or other display parameters) as functions of time.

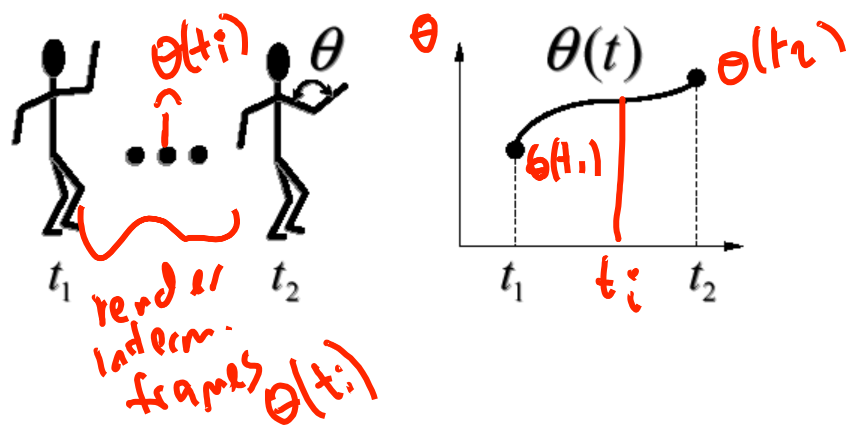
Key-frame animation

The most common method for character animation in production is **key-frame animation**.

- Each joint specified at various **key frames** (not necessarily the same as other joints)
- System does interpolation or **in-betweening**

Doing this well requires:

- A way of smoothly interpolating key frames: **splines**
- A good interactive system
- A lot of skill on the part of the animator

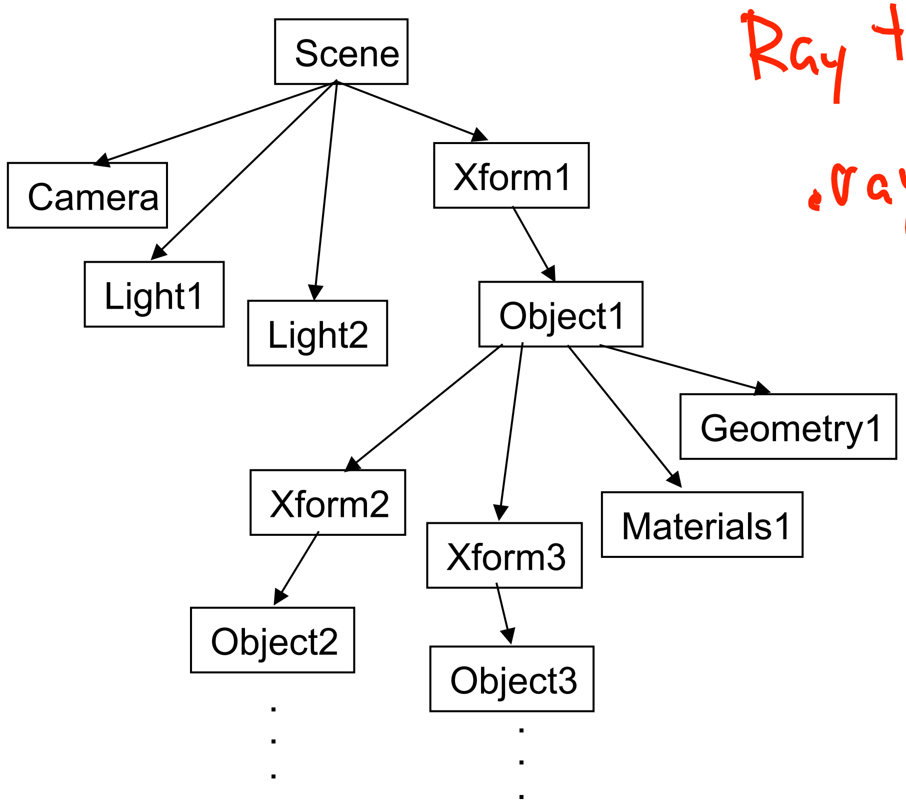


Scene graphs

The idea of hierarchical modeling can be extended to an entire scene, encompassing:

- many different objects
- lights
- camera position

This is called a **scene tree** or **scene graph**.



Ray tracing
.ray files

Summary

Here's what you should take home from this lecture:

- All the **boldfaced terms**.
- How primitives can be instanced and composed to create hierarchical models using geometric transforms.
- How the notion of a model tree or DAG can be extended to entire scenes.
- How OpenGL transformations can be used in hierarchical modeling.
- How keyframe animation works.