Computer Graphics
CSE 457 - Autumn 2015
Instructor: Ira Kemelmacher-Shlizerman

Homework 2

Assigned:
Friday, Nov 13, 2015

Due date:
Tuesday, Dec 1, 2015, in the beginning of the class.

Feel free to discuss but answer on your own.
Use your own paper to write the solutions.
Show derivations.
Write your name.

Good luck!

Problem 1. Blinn-Phong shading (16 Points)

The Blinn-Phong shading model for a scene illuminated by global ambient light and a single directional light can be summarized by the following equation:

$$I_{phong} = k_e + k_a I_a + k_d B\, I_L\, (\mathbf{N}\cdot\mathbf{L}) + k_s B\, I_L\, (\mathbf{N}\cdot\mathbf{H})_+^{n_s}$$

Imagine a scene with one white sphere illuminated by white global ambient light and a single white directional light. For sub-problems a) – f), describe – qualitatively, in words – the effect of each step on the shading of the object. At each incremental step, assume that all the preceding steps have been applied first. Assume that the directional light is oriented so that the viewer can see the shading over the surface, including diffuse and specular where appropriate.

a) (2 points) The directional light is off. How does the shading vary over the surface of the object?

b) (2 points) Now turn the directional light on. The specular reflection coefficient k_s of the material is zero, and the diffuse reflection coefficient k_d is non-zero. How does the shading vary over the surface of the object?

c) (2 points) Now translate the sphere straight toward the viewer. What happens to the shading over the object?

d) (2 points) Now increase the specular exponent n_s. What happens?

e) (2 points) Now increase the specular reflection coefficient k_s of the material to be greater than zero. What happens?

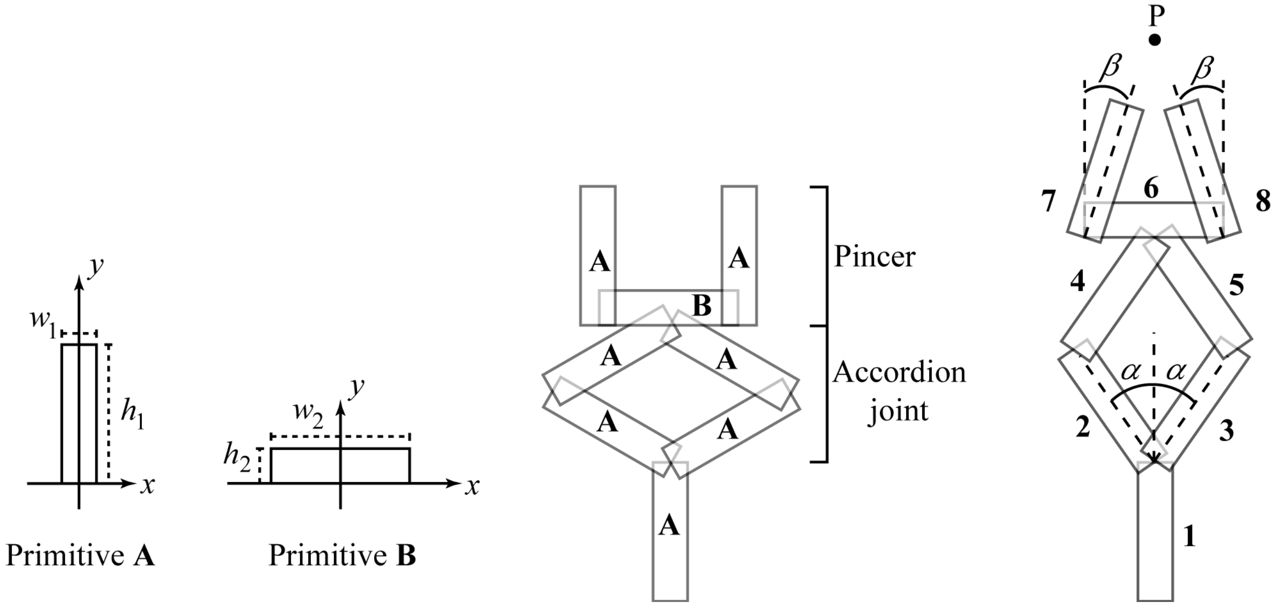
f) (2 points) Now decrease the specular exponent n_s. What happens?

g) (2 points) Suppose we assume that the viewing direction **V** is constant regardless of which pixel it passes through. What does this imply about the viewer?

h) (2 points) Assuming that **L** and **V** are constant everywhere, then with a little pre-computation, it is possible to shade faster (i.e., using fewer operations) using the Blinn-Phong model above, than it is to shade using the Phong model, which bases the specular component on $(\mathbf{V}\cdot\mathbf{R})_+^{n_s}$. Why would Blinn-Phong be faster than Phong in this situation? Explain.

Problem 2: Hierarchical modeling (24 points)

Suppose you want to model the pincer with accordion joint illustrated below. The model is comprised of 8 parts, using primitives **A** and **B**. The model is shown in two poses below, with the controlling parameters of the model illustrated on the far right. The illustration on the right also shows a point P that the model is reaching toward, as described in sub-problem **c**).



Assume that α and β can take values in the range [0, 90°]. Also assume that all parts use primitive **A**, except for part **6**, which uses primitive **B**. The model on the left shows the primitives used, the model on the right shows the enumeration (naming) of the parts.

The following transformations are available to you:

- R(θ) – rotate by θ degrees (counter clockwise)
- T(a, b) – translate by $\begin{bmatrix} a \\ b \end{bmatrix}$

a) (16 points) Construct a tree to describe this hierarchical model using part **1** as the root. Along each of the edges of the tree, write expressions for the transformations that are applied along that edge, using the notation given above (you do not need to write out the matrices). Remember that the order of transformations is important! Show your work wherever the transformations are not "obvious." Your tree should contain a bunch of boxes (or circles) each containing one part number (1…8); these boxes should be connected by line segments, each labeled with a corresponding transformation that connects child to parent. The tree must have one or more branches in it. If two parts are connected physically, then they should be connected in the tree, as long as you don't form a cycle by connecting them.

b) (2 points) Write out the full transformation expression for part **7**.

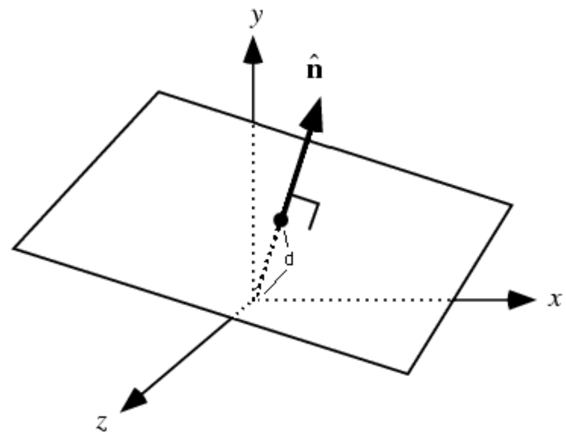
c) (6 points) Suppose the primitives are infinitesimally thin, $w_1 = h_2 = 0$, and have lengths $h_1 = 10$ and $w_2 = 12$. Assume that part **1** sits right on the origin in world coordinates. What would the α

and β parameters have to be so that the model extends out and closes the pincer just enough to precisely grasp the point $P = \begin{bmatrix} 0 & 28 \end{bmatrix}^T$, in world coordinates. Show your work.

Problem 3: 3D Affine Transformations (16 points)

The equation $\hat{\mathbf{n}} \bullet \vec{\mathbf{x}} = d$ describes the plane pictured below which has unit length normal $\hat{\mathbf{n}}$ pointing away from the origin and is a distance d from the origin (in the direction of the normal vector). Any point $\vec{\mathbf{x}} = \begin{bmatrix} x & y & z \end{bmatrix}$ on the plane must satisfy the plane equation $\hat{\mathbf{n}} \bullet \vec{\mathbf{x}} = d$.

$$M_{xy} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



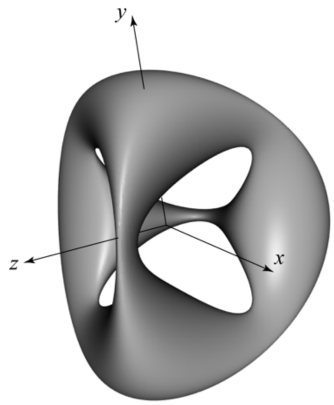
Now consider a plane with normal lying in the y-z plane. The normal will have the form (0, sinθ, cosθ) for some θ. The equation for the plane is then $y\sin\theta + z\cos\theta = d$. Write out the product of 4x4 matrices that would perform a reflection across this plane. One of these matrices will be a reflection matrix; you must use the matrix M_{xy} above, which performs a reflection across the x-y plane. You must write out the elements of the matrices and the product order in which they would be applied, but you do not need to multiply them out. Justify your answer with words and/or drawings.

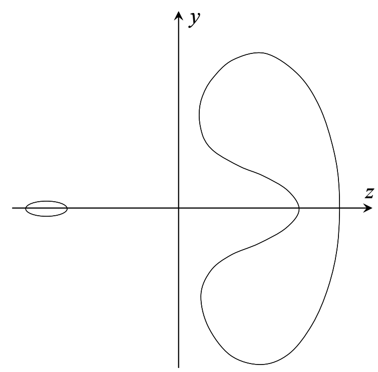
Problem 4. Ray intersection with implicit surfaces (23 points)

There are many ways to represent a surface. One way is to define a function of the form $f(x,y,z)=0$. Such a function is called an *implicit surface* representation. For example, the equation $f(x,y,z)=x^2+y^2+z^2-r^2=0$ defines a sphere of radius *r*. Suppose we wanted to ray trace a "quartic chair," described by the equation:

$$(x^2+y^2+z^2-ak^2)^2-b\left[(z-k)^2-2x^2\right]\left[(z+k)^2-2y^2\right]=0$$

On the left is a picture of a quartic chair, and on the right is a slice through the *y-z* plane.





For this problem, we will assume a = 0.95, b = 0.8, and k = 5.

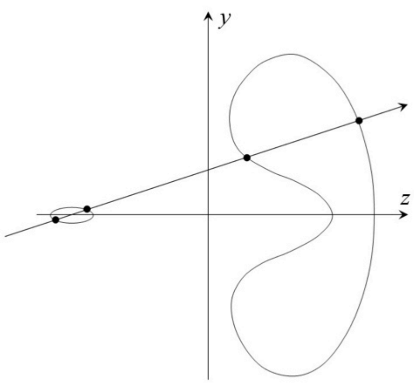
In the next problem steps, you will be asked to solve for and/or discuss ray intersections with this primitive. Performing the ray intersections will amount to solving for the roots of a polynomial, much as it did for sphere intersection. For your answers, you need to keep a few things in mind:

- You will find as many roots as the order (largest exponent) of the polynomial.
- You may find a mixture of real and complex roots. When we say complex here, we mean a number that has a non-zero imaginary component.
- All complex roots occur in complex conjugate pairs. If $A + iB$ is a root, then so is $A - iB$.
- Sometimes a real root will appear more than once, i.e., has multiplicity > 1. Consider the case of sphere intersection, which we solve by computing the roots of a quadratic equation. A ray that intersects the sphere will usually have two distinct roots (each has multiplicity = 1) where the ray enters and leaves the sphere. If we were to take such a ray and translate it away from the center of the sphere, those roots get closer and closer together, until they merge into one root. They merge when the ray is tangent to the sphere. The result is one distinct real root with multiplicity = 2.

a) (8 points) Consider the ray $P+t\mathbf{d}$, where $P=\begin{pmatrix}0 & 0 & 0\end{pmatrix}$ and $\mathbf{d}=\begin{pmatrix}0 & 0 & 1\end{pmatrix}$. Solve for all values of *t* where the ray intersects the quartic chair (including negative values of *t*). Which value of *t* represents the intersection we care about for ray tracing? In the process of solving for t, you will be computing the roots of a polynomial. How many distinct real roots do you find? How many of them have multiplicity > 1? How many complex roots do you find?

Problem 4 (cont'd)

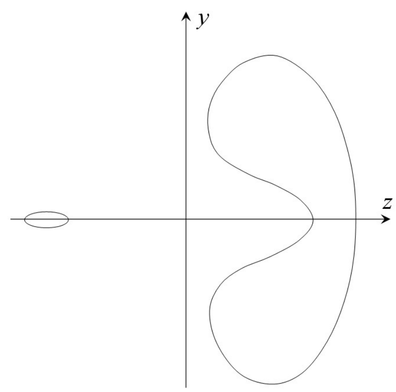
b) (15 points) What are all the possible combinations of roots, not counting the one in part (a)? For each combination, describe the 4 roots as in part (a), draw a ray in the *y-z* plane that gives rise to that combination, and place a dot at each intersection point. There are five diagrams below that have not been filled in. You may not need all five; on the other hand, if you can actually think of more distinct cases than spaces provided, then we might just give extra credit. The first one has already been filled in. (Note: not all conceivable combinations can be achieved on this particular implicit surface. For example, there is no ray that will give a root with multiplicity 4.) ***Please write on this page and include it with your homework solution. You do not need to justify your answers.***



# of distinct real roots: **4**

# of real roots w/ multiplicity > 1: **0**

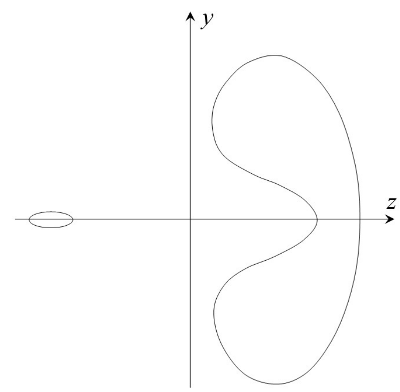
# of complex roots: **0**



# of distinct real roots:

# of real roots w/ multiplicity > 1:

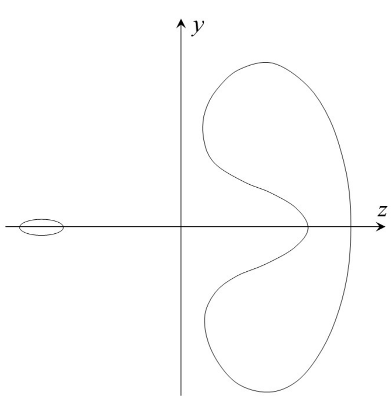
# of complex roots:



# of distinct real roots:

# of real roots w/ multiplicity > 1:

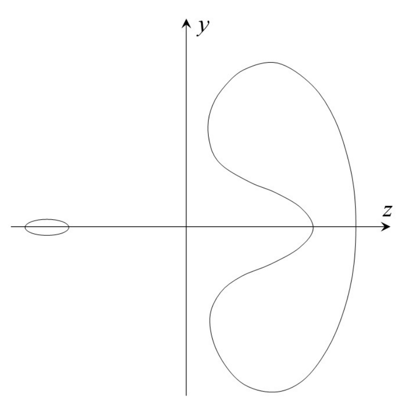
# of complex roots:



# of distinct real roots:

# of real roots w/ multiplicity > 1:

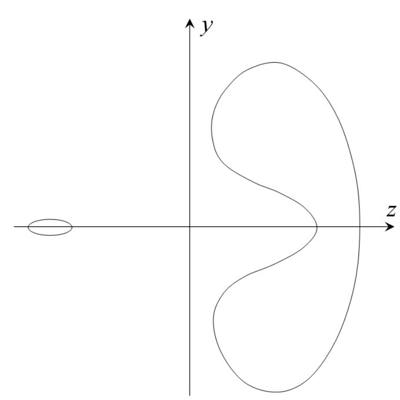
# of complex roots:



# of distinct real roots:

# of real roots w/ multiplicity > 1:

# of complex roots:



# of distinct real roots:

# of real roots w/ multiplicity > 1:

# of complex roots:

Problem 5. Bezier splines (21 points)

Consider a Bezier curve segment defined by three control points V_0, V_1, and V_2.

a) (3 points) What is the polynomial form of this curve, when written out in the form $Q(u) = A_n u^n + A_{n-1} u^{n-1} + \ldots + A_0$, where n is determined by the number of control points. The coefficients $A_0, \ldots, A_n$ should be substituted in the polynomial equation with expressions that depend on the control points V_0, V_1, and V_2. You may start with recursive subdivision or with the summation over Bernstein polynomials provided in lecture. Either way, show your work.

b) (3 points) What is the first derivative of $Q(u)$ evaluated at $u = 0$ and at $u = 1$ (i.e., what are $Q'(0)$ and $Q'(1)$)? Show your work.

c) (3 points) What is the second derivative of $Q(u)$ evaluated at $u = 0$ and at $u = 1$ (i.e., what are $Q''(0)$ and $Q''(1)$)? Show your work.

d) (5 points) To create a spline curve, we can stitch together consecutive Bezier curves. In this problem, we can add control points W_0, W_1, and W_2. What constraints must be placed on W_0, W_1, and/or W_2 so that, when combined with V_0, V_1, and V_2, the resulting spline curve is C^1 continuous at the joint between the Bezier segments? Write out equations for W_0, W_1, and/or W_2 in terms of V_0, V_1, and/or V_2. (It may be that not all of the W control points are constrained, in which case you would have fewer than three equations.) Show your work. Draw a copy of the control polygon below (shown at the bottom of the page) and place all constrained vertices exactly, and unconstrained vertices wherever you like, and then sketch the spline curve.

e) (4 points) Suppose we wanted to make the spline curve C^2 continuous at the joint between the Bezier segments. Now what constraints must be placed on W_0, W_1, and W_2? Write out equations for W_0, W_1, and/or W_2 in terms of V_0, V_1, and/or V_2. (It may be that not all of the W control points are constrained, in which case you would have fewer than three equations.) Show your work. Draw a copy of the control polygon below (shown at the bottom of the page) and place all constrained vertices exactly, and unconstrained vertices wherever you like, and then sketch the spline curve.

f) (3 points) Is it possible to achieve C^3 continuity with this spline? Explain.