Subdivision surfaces

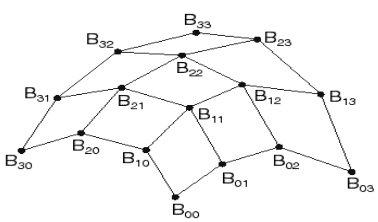
Reading

Stollnitz, DeRose, and Salesin. *Wavelets for Computer Graphics: Theory and Applications*, 1996, section 10.2.

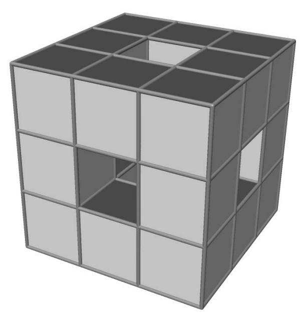
Recommended:

Warren and Weimer. *Subdivision Methods for Geometric Design*, 2002.

What's wrong with spline surfaces?



A “regular” mesh is required (all vertices valence 4)



What about meshes like this one?

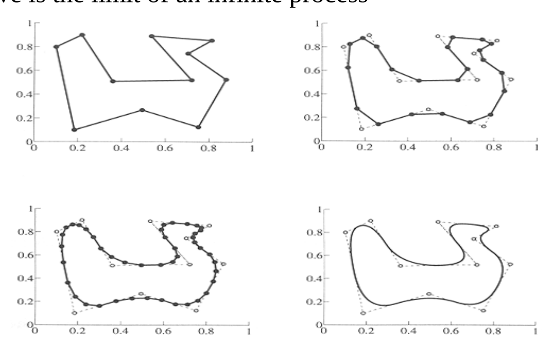
Subdivision curves

Idea:

- repeatedly refine the control polygon

$$P_0 \rightarrow P_1 \rightarrow P_2 \rightarrow \cdots$$
$$C = \lim_{i\to\infty} P_i$$

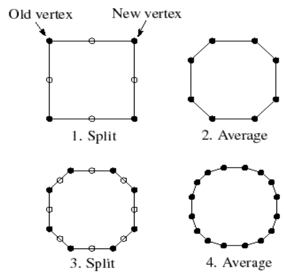
- curve is the limit of an infinite process



Chaikin's algorithm

Chakin introduced the following "corner-cutting" scheme in 1974:

- Start with a piecewise linear curve
- Insert new vertices at the midpoints (the **splitting step**)
- Average each vertex with the "next" neighbor (the **averaging step**)
- Go to the splitting step



Averaging masks

The limit curve is a quadratic B-spline!

Instead of averaging with the nearest neighbor, we can generalize by applying an **averaging mask** during the averaging step:

$$r = (\dots, r_{-1}, r_0, r_1, \dots)$$

In the case of Chaikin's algorithm:

$$r =$$

Lane-Riesenfeld algorithm (1980)

Use averaging masks from Pascal's triangle:

$$r = \frac{1}{2^n}\left(\binom{n}{0}, \binom{n}{1}, \dots, \binom{n}{n}\right)$$

Gives B-splines of degree n+1.

n=0:

n=1:

n=2:

General Subdivision Process

1. Initialize with a piecewise linear curve
2. **Splitting step**: add more vertices and edges
3. **Averaging step**: adjust the vertices by applying the *averaging mask*
4. Goto step 2.

Subdivide ad nauseum?

After each split-average step, we are closer to the limit surface.

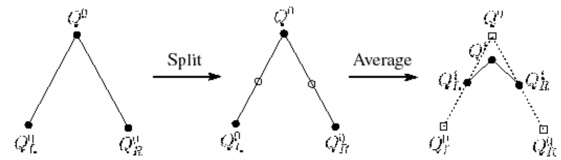
How many steps until we reach the final (limit) position?

Can we push a vertex to its limit position without infinite subdivision? Yes!

Local subdivision matrix

Consider the cubic B-spline subdivision mask: $\frac{1}{4}(1 \quad 2 \quad 1)$

Now consider what happens during splitting and averaging:



Applying the process recursively will at infinity converge to a specific point:

$$Q^{\infty} = S^{\infty} Q^{j} = U Q^{j}$$

It turns out that we can determine the closed form solution for the final position of that point by applying the *evaluation mask* **U**

Recipe for subdivision curves

The evaluation mask for the cubic B-spline is:

$$\frac{1}{6}(1 \quad 4 \quad 1)$$

Now we can cook up a simple procedure for creating subdivision curves:

1. Subdivide (split+average) the control polygon a few times. Use the *averaging mask*.
2. Push the resulting points to the limit positions using the *evaluation mask*.

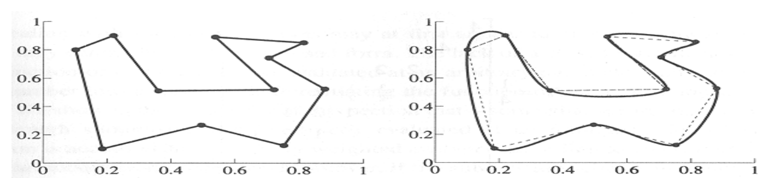
DLG interpolating scheme (1987)

Slight modification to algorithm:

- splitting step introduces midpoints
- averaging step *only changes midpoints*

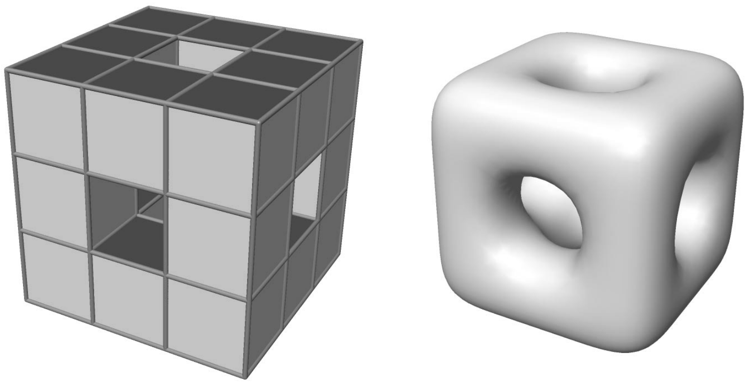
For DLG (Dyn-Levin-Gregory), use:

$$r = \frac{1}{16}(-2, 6, 10, 6, -2)$$



Since we are only changing the midpoints, the points after the averaging step do not move.

Building complex models



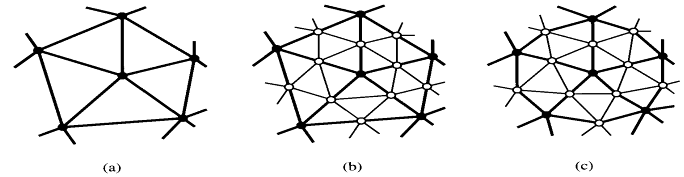
Subdivision surfaces

Chaikin's use of subdivision for curves inspired similar techniques for surfaces.

Iteratively refine a **control polyhedron** (or **control mesh**) to produce the limit surface

$$\sigma = \lim_{j \to \infty} M^{j}$$

using splitting and averaging steps.

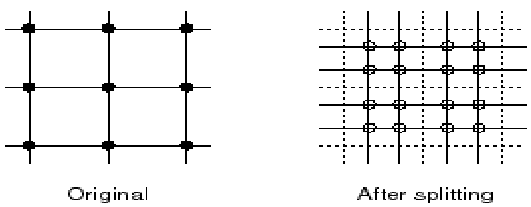


There are two types of splitting steps:

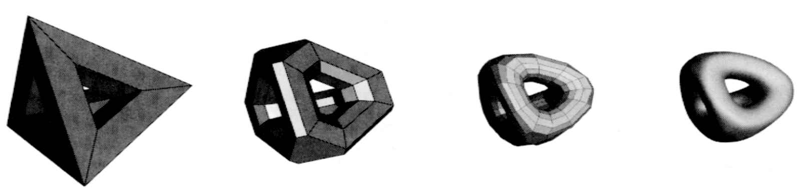
- **vertex schemes**
- **face schemes**

Vertex schemes

A vertex surrounded by *n* faces is split into *n* subvertices, one for each face:

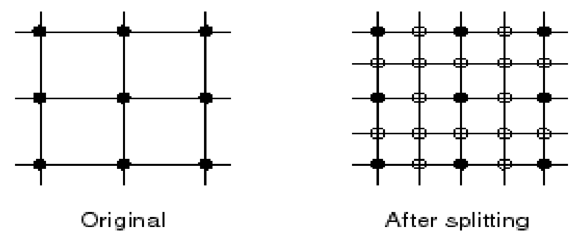


Doo-Sabin subdivision:

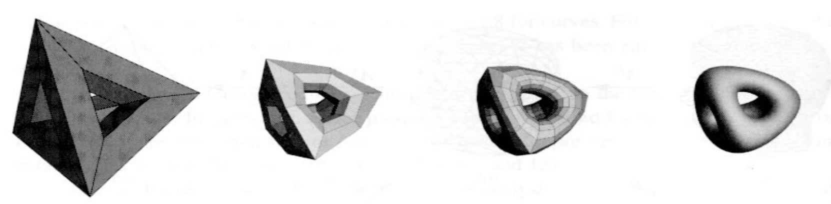


Face schemes

Each quadrilateral face is split into four subfaces:

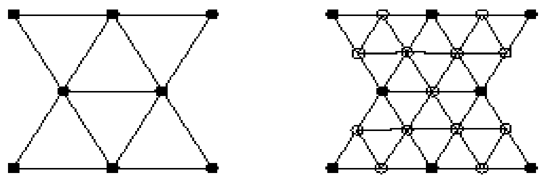


Catmull-Clark subdivision:



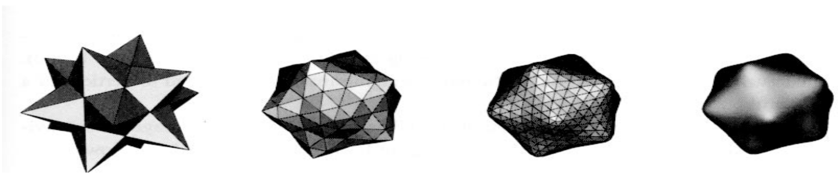
Face schemes, cont.

Each triangular face is split into four subfaces:



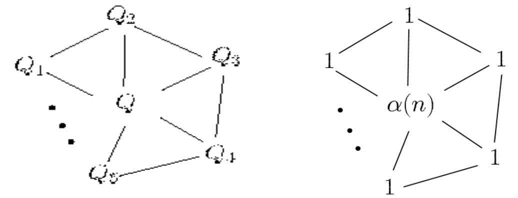
Original After splitting

Loop subdivision:



Averaging step

Once again we can use **masks** for the averaging step:



Vertex labeling Averaging mask

$$Q \leftarrow \frac{\alpha(n) + Q_1 + \cdots + Q_n}{\alpha(n) + n}$$

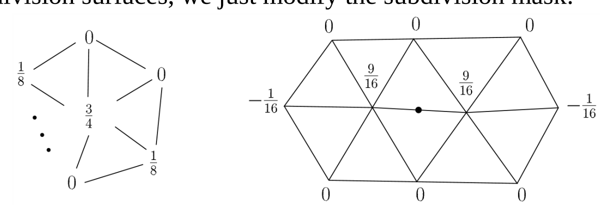
where

$$\alpha(n) = \frac{n(1-\beta(n))}{\beta(n)} \quad \beta(n) = \frac{5}{4} - \frac{(3 + 2\cos(2\pi / n))^2}{32}$$

(carefully chosen to ensure smoothness.)

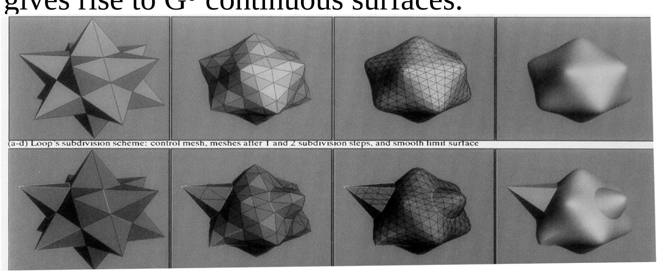
Adding creases without trim curves

Sometimes, particular feature such as a crease should be preserved. With NURBS surfaces, this required the use of trim curves.
For subdivision surfaces, we just modify the subdivision mask:



Loop crease/boundary edge Buttery crease/boundary edge

This gives rise to G^0 continuous surfaces.



Creases without trim curves, cont.

Here's an example using Catmull-Clark surfaces of the kind found in Geri's Game: