Hidden Surface Algorithms

Reading

Reading:

- Angel 5.6, 9.10.3

Optional reading:

- Foley, van Dam, Feiner, Hughes, Chapter 15
- I. E. Sutherland, R. F. Sproull, and R. A. Schumacker, A characterization of ten hidden surface algorithms, *ACM Computing Surveys* 6(1): 1-55, March 1974.

Introduction

In the previous lecture, we figured out how to transform the geometry so that the relative sizes will be correct if we drop the *z* component.

But, how do we decide which geometry actually gets drawn to a pixel?

Known as the **hidden surface elimination problem** or the **visible surface determination problem**.

There are <u>dozens</u> of hidden surface algorithms.

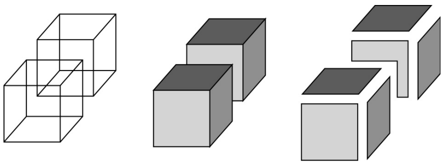
They can be characterized in at lease three ways:

- Object-precision vs. image-precision (a.k.a., object-space vs. image-space)
- Object order vs. image order
- Sort first vs. sort last

Object-precision algorithms

<u>Basic idea:</u>

- Operate on the geometric primitives themselves. (We'll use "object" and "primitive" interchangeably.)
- Objects typically intersected against each other
- Tests performed to high precision
- Finished list of visible objects can be drawn at any resolution



<u>Complexity:</u>

- For n objects, can take $O(n^2)$ time to compute visibility.
- For an *mxm* display, have to fill in colors for m^2 pixels.
- Overall complexity can be $O(k_{obj} n^2 + k_{disp} m^2)$.

<u>Implementation:</u>

- Difficult to implement
- Can get numerical problems

Image-precision algorithm

Basic idea:

- Find the closest point as seen through each pixel
- Calculations performed at display resolution
- Does not require high precision

Complexity:

- Naïve approach checks all n objects at every pixel. Then, $O(n\ m^2)$.
- Better approaches check only the objects that *could* be visible at each pixel. Let's say, on average, *d* objects project to each pixel (a.k.a., depth complexity). Then, $O(d\ m^2)$.

Implementation:

- Very simple to implement.
  - Used a lot in practice.

Object order vs. image order

Object order:

- Consider each object only once, draw its pixels, and move on to the next object.
- Might draw to the same pixel multiple times.

Image order:

- Consider each pixel only once, find nearest object, and move on to the next pixel.
- Might compute relationships between objects multiple times.

Sort first vs. sort last

Sort first:

- Find some depth-based ordering of the objects relative to the camera, then draw back to front.
- Build an ordered data structure to avoid duplicating work.

Sort last:

- Determine depth observed at each pixel and draw the color corresponding to the closest depth
- Can be done by considering all depths together or by "lazily" keeping track of depths as they arrive.

Three hidden surface algorithms

- Z-buffer
- Ray casting
- Binary space partitioning (BSP) trees

Z-buffer

The **Z-buffer** or **depth buffer** algorithm [Catmull, 1974] is probably the simplest and most widely used.

Here is pseudocode for the Z-buffer hidden surface algorithm:

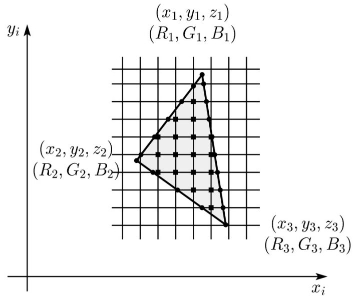
```
for each pixel (i,j) do
    Z-buffer [i,j] ← FAR
    Framebuffer[i,j] ← <background color>
end for
for each polygon A do
    for each pixel in A do
        Compute depth z and shade s of A at (i,j)
        if z > Z-buffer [i,j] then
            Z-buffer [i,j] ← z
            Framebuffer[i,j] ← s
        end if
    end for
end for
```

Q: What should FAR be set to?

Rasterization

The process of filling in the pixels inside of a polygon is called **rasterization**.

During rasterization, the *z* value and shade *s* can be computed incrementally (fast!).



<u>Curious fact:</u>

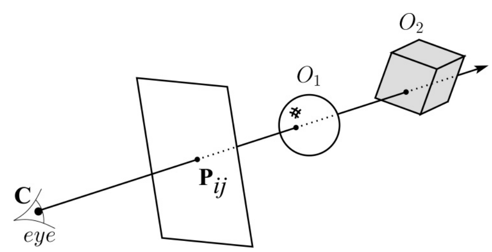
- Described as the "brute-force image space algorithm" by [SSS]
- Mentioned only in Appendix B of [SSS] as a point of comparison for <u>huge</u> memories, but written off as totally impractical.

Today, Z-buffers are commonly implemented in hardware.

Z-buffer: Analysis

- Classification?
- Easy to implement?
- Easy to implement in hardware?
- Incremental drawing calculations (uses coherence)?
- Pre-processing required?
- On-line (doesn't need all objects before drawing begins)?
- If objects move, does it take more work than normal to draw the frame?
- If the viewer moves, does it take more work than normal to draw the frame?
- Typically polygon-based?
- Efficient shading (doesn't compute colors of hidden surfaces)?
- Handles transparency?
- Handles refraction?

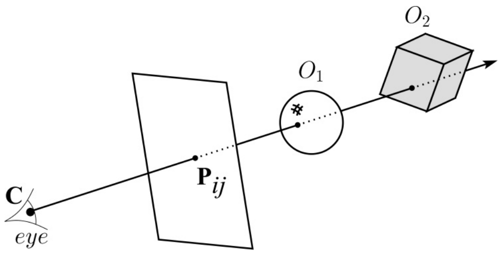
Ray casting



<u>Idea:</u> For each pixel center $\boldsymbol{P}_{ij}$

- Send ray from eye point (COP), **C**, through $\boldsymbol{P}_{ij}$ into scene.
- Intersect ray with each object.
- Select nearest intersection.

Ray casting, cont.



Implementation:

- Might parameterize each ray:

  $\mathbf{r}(t) = \mathbf{C} + t\,(\boldsymbol{P}_{ij} - \mathbf{C})$

- Each object O_k returns $t_k > 0$ such that first intersection with O_k occurs at $\mathbf{r}(t_k)$.

Q: Given the set $\{t_k\}$ what is the first intersection point?

Note: these calculations generally happen in world coordinates. No projective matrices are applied.

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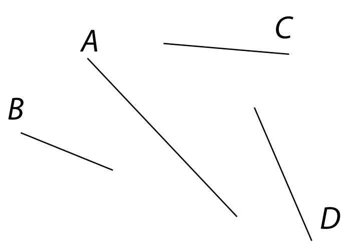
Binary-space partitioning (BSP) trees



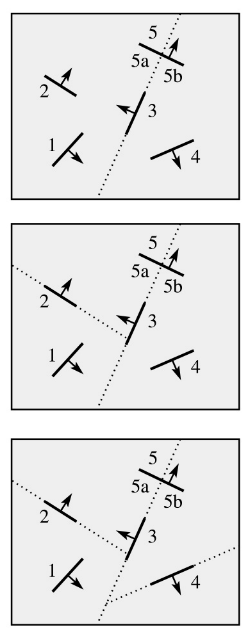
Idea:

- Do extra preprocessing to allow quick display from any viewpoint.

Key observation: A polygon *A* is painted in correct order if

- Polygons on far side of *A* are painted first
- *A* is painted next
- Polygons in front of *A* are painted last.

BSP tree creation



BSP tree creation (cont'd)

procedure *MakeBSPTree*:

takes *PolygonList L*

returns *BSPTree*

Choose polygon *A* from *L* to serve as root

Split all polygons in *L* according to *A*

node ← *A*

node.neg ← *MakeBSPTree*(Polygons on neg. side of A)

node.pos ← *MakeBSPTree*(Polygons on pos. side of A)

return node

end procedure

Note: Performance is improved when fewer polygons are split --- in practice, best of ~ 5 random splitting polygons are chosen.

Note: BSP is created in *world* coordinates. No projective matrices are applied before building tree.

BSP tree display

procedure *DisplayBSPTree:*

Takes *BSPTree T*

if *T* is empty **then return**

if viewer is in front (on pos. side) of *T.node*

DisplayBSPTree(T. ______)

Draw T.node

DisplayBSPTree(T.______)

else

DisplayBSPTree(T. ______)

Draw T.node

DisplayBSPTree(T. ______)

end if

end procedure

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Cost of Z-buffering

Z-buffering is ***the*** algorithm of choice for hardware rendering, so let's think about how to make it run as fast as possible…

The steps involved in the Z-buffer algorithm are:

1. Send a triangle to the graphics hardware.
2. Transform the vertices of the triangle using the modeling matrix.
3. Shade the vertices.
4. Transform the vertices using the projection matrix.
5. Set up for incremental rasterization calculations
6. Rasterize and update the framebuffer according to *z*.

What is the overall cost of Z-buffering?

Cost of Z-buffering, cont'd

We can approximate the cost of this method as:

$$k_{bus}\, v_{bus} + k_{xform}\, v_{xform} + k_{shade}\, v_{shade} + k_{setup}\, \Delta_{rast} + d\, m^2$$

Where:

k_{bus} = bus cost to send a vertex
v_{bus} = number of vertices sent over the bus
k_{xform} = cost of transforming a vertex
v_{xform} = number of vertices transformed
k_{shade} = cost of shading a vertex
v_{shade} = number of vertices shaded
k_{setup} = cost of setting up for rasterization
Δ_{rast} = number of triangles being rasterized
d = depth complexity (average times a pixel is covered)
m^2 = number of pixels in frame buffer

Visibility tricks for Z-buffers

Given this cost function:

$$k_{bus}\, v_{bus} + k_{xform}\, v_{xform} + k_{shade}\, v_{shade} + k_{setup}\, \Delta_{rast} + d\, m^2$$

what can we do to accelerate Z-buffering?

Accel method	v_{bus}	v_{xform}	v_{shade}	Δ_{rast}	d	m

Summary

What to take home from this lecture:

- Classification of hidden surface algorithms
- Understanding of Z-buffer, ray casting, and BSP tree hidden surface algorithms
- Familiarity with some Z-buffer acceleration strategies