Hidden Surface Algorithms

Reading

Reading:

- Angel 5.6, 9.10.3

Optional reading:

- Foley, van Dam, Feiner, Hughes, Chapter 15
- I. E. Sutherland, R. F. Sproull, and R. A. Schumacker, A characterization of ten hidden surface algorithms, *ACM Computing Surveys* 6(1): 1-55, March 1974.

Introduction

In the previous lecture, we figured out how to transform the geometry so that the relative sizes will be correct if we drop the *z* component.

But, how do we decide which geometry actually gets drawn to a pixel?

Known as the **hidden surface elimination problem** or the **visible surface determination problem**.

There are dozens of hidden surface algorithms.

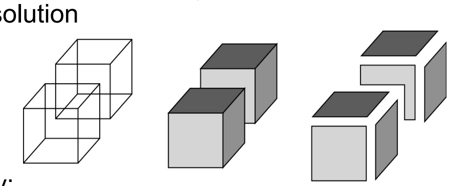
They can be characterized in at lease three ways:

- Object-precision vs. image-precision (a.k.a., object-space vs. image-space)
- Object order vs. image order
- Sort first vs. sort last

Object-precision algorithms

Basic idea:

- Operate on the geometric primitives themselves. (We'll use "object" and "primitive" interchangeably.)
- Objects typically intersected against each other
- Tests performed to high precision
- Finished list of visible objects can be drawn at any resolution



Complexity:

- For n objects, can take $O(n^2)$ time to compute visibility.
- For an mxm display, have to fill in colors for m^2 pixels.
- Overall complexity can be $O(k_{obj}\, n^2 + k_{disp}\, m^2)$.

Implementation:

- Difficult to implement
- Can get numerical problems

Image-precision algorithm

Basic idea:

- Find the closest point as seen through each pixel
- Calculations performed at display resolution
- Does not require high precision

Complexity:

- Naïve approach checks all n objects at every pixel. Then, $O(n\ m^2)$.
- Better approaches check only the objects that *could* be visible at each pixel. Let's say, on average, *d* objects project to each pixel (a.k.a., depth complexity). Then, $O(d\ m^2)$.

Implementation:

- Very simple to implement.
  - Used a lot in practice.

Object order vs. image order

Object order:

- Consider each object only once, draw its pixels, and move on to the next object.
- Might draw to the same pixel multiple times.

Image order:

- Consider each pixel only once, find nearest object, and move on to the next pixel.
- Might compute relationships between objects multiple times.

Sort first vs. sort last

Sort first:

- Find some depth-based ordering of the objects relative to the camera, then draw back to front.
- Build an ordered data structure to avoid duplicating work.

Sort last:

- Determine depth observed at each pixel and draw the color corresponding to the closest depth
- Can be done by considering all depths together or by "lazily" keeping track of depths as they arrive.

Three hidden surface algorithms

- Z-buffer
- Ray casting
- Binary space partitioning (BSP) trees

Z-buffer

The **Z-buffer** or **depth buffer** algorithm [Catmull, 1974] is probably the simplest and most widely used.

Here is pseudocode for the Z-buffer hidden surface algorithm:

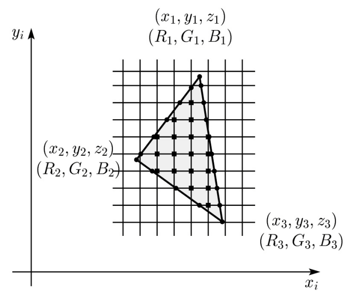
```
for each pixel (i,j) do
    Z-buffer [i,j] ← FAR
    Framebuffer[i,j] ← <background color>
end for
for each polygon A do
    for each pixel in A do
        Compute depth z and shade s of A at (i,j)
        if z > Z-buffer [i,j] then
            Z-buffer [i,j] ← z
            Framebuffer[i,j] ← s
        end if
    end for
end for
```

Q: What should FAR be set to?

Rasterization

The process of filling in the pixels inside of a polygon is called **rasterization**.

During rasterization, the *z* value and shade *s* can be computed incrementally (fast!).



Curious fact:

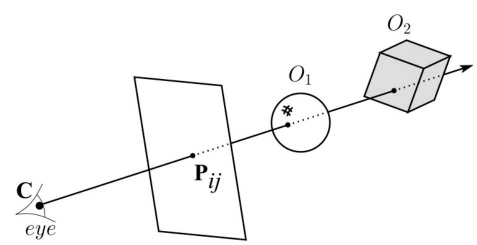
- Described as the "brute-force image space algorithm" by [SSS]
- Mentioned only in Appendix B of [SSS] as a point of comparison for huge memories, but written off as totally impractical.

Today, Z-buffers are commonly implemented in hardware.

Z-buffer: Analysis

- Classification?
- Easy to implement?
- Easy to implement in hardware?
- Incremental drawing calculations (uses coherence)?
- Pre-processing required?
- On-line (doesn't need all objects before drawing begins)?
- If objects move, does it take more work than normal to draw the frame?
- If the viewer moves, does it take more work than normal to draw the frame?
- Typically polygon-based?
- Efficient shading (doesn't compute colors of hidden surfaces)?
- Handles transparency?
- Handles refraction?

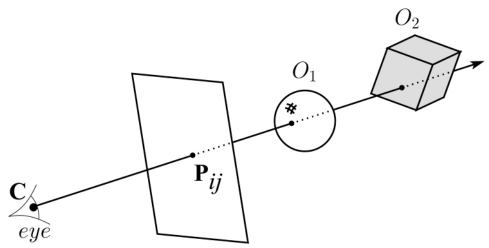
Ray casting



Idea: For each pixel center $\boldsymbol{P}_{ij}$

- Send ray from eye point (COP), **C**, through $\boldsymbol{P}_{ij}$ into scene.
- Intersect ray with each object.
- Select nearest intersection.

Ray casting, cont.



Implementation:

- Might parameterize each ray:

  $\mathbf{r}(t) = \mathbf{C} + t\,(\boldsymbol{P}_{ij} - \mathbf{C})$

- Each object O_k returns $t_k > 0$ such that first intersection with O_k occurs at $\mathbf{r}(t_k)$.

Q: Given the set $\{t_k\}$ what is the first intersection point?

Note: these calculations generally happen in world coordinates. No projective matrices are applied.

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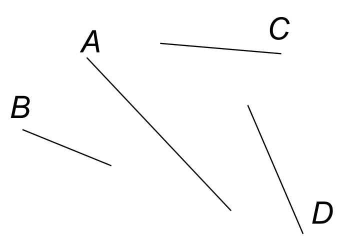
Binary-space partitioning (BSP) trees



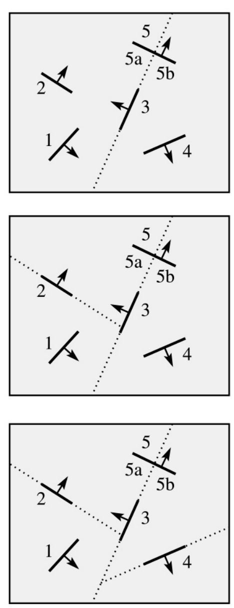
Idea:

- Do extra preprocessing to allow quick display from any viewpoint.

Key observation: A polygon *A* is painted in correct order if

- Polygons on far side of *A* are painted first
- *A* is painted next
- Polygons in front of *A* are painted last.

BSP tree creation



BSP tree creation (cont'd)

procedure *MakeBSPTree*:
takes *PolygonList L*
returns *BSPTree*

Choose polygon *A* from *L* to serve as root
Split all polygons in *L* according to *A*
node ← *A*
node.neg ← *MakeBSPTree*(Polygons on neg. side of A)
node.pos ← *MakeBSPTree*(Polygons on pos. side of A)
return node

end procedure

Note: Performance is improved when fewer polygons are split --- in practice, best of ~ 5 random splitting polygons are chosen.

Note: BSP is created in *world* coordinates. No projective matrices are applied before building tree.

BSP tree display

procedure *DisplayBSPTree:*
Takes *BSPTree T*

if *T* is empty **then return**
if viewer is in front (on pos. side) of *T.node*
DisplayBSPTree(T. _____)
Draw T.node
DisplayBSPTree(T._____)
else
DisplayBSPTree(T. _____)
Draw T.node
DisplayBSPTree(T. _____)
end if

end procedure

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Cost of Z-buffering

Z-buffering is ***the*** algorithm of choice for hardware rendering, so let's think about how to make it run as fast as possible…

The steps involved in the Z-buffer algorithm are:

- Send a triangle to the graphics hardware.
- Transform the vertices of the triangle using the modeling matrix.
- Shade the vertices.
- Transform the vertices using the projection matrix.
- Set up for incremental rasterization calculations
- Rasterize and update the framebuffer according to *z*.

What is the overall cost of Z-buffering?

Cost of Z-buffering, cont'd

We can approximate the cost of this method as:

$$k_{bus}\, v_{bus} + k_{xform}\, v_{xform} + k_{shade}\, v_{shade} + k_{setup}\, \Delta_{rast} + d\, m^2$$

Where:

k_{bus} = bus cost to send a vertex
v_{bus} = number of vertices sent over the bus
k_{xform} = cost of transforming a vertex
v_{xform} = number of vertices transformed
k_{shade} = cost of shading a vertex
v_{shade} = number of vertices shaded
k_{setup} = cost of setting up for rasterization
Δ_{rast} = number of triangles being rasterized
d = depth complexity (average times a pixel is covered)
m^2 = number of pixels in frame buffer

Visibility tricks for Z-buffers

Given this cost function:

$$k_{bus}\, v_{bus} + k_{xform}\, v_{xform} + k_{shade}\, v_{shade} + k_{setup}\, \Delta_{rast} + d\, m^2$$

what can we do to accelerate Z-buffering?

Accel method	v_{bus}	v_{xform}	v_{shade}	Δ_{rast}	d	m

Summary

What to take home from this lecture:

- Classification of hidden surface algorithms
- Understanding of Z-buffer, ray casting, and BSP tree hidden surface algorithms
- Familiarity with some Z-buffer acceleration strategies