Computer Graphics	Instructor: Doug Johnson
CSE 457	Autumn 2004

Homework #2

Projections, Hidden Surfaces, Shading, Ray Tracing,

Texture Mapping, Parametric Curves

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Assigned: Monday, November 8^{th}
Due: Wednesday, November 24^{th}
at the beginning of class

Directions: Please provide short written answers to the questions in the space provided. If you require extra space, you may staple additional pages to the back of your assignment. Feel free to discuss the problems with classmates, but please ***answer the questions on your own*****.**

Name:____

Problem 1. Projections (7 Points)

Perspective Projections

True or False – Briefly justify your answer

1. Size varies inversely with distance

2. Distance and angles are preserved

3. Perspective projections make z-buffers more imprecise than orthographic projections

Parallel Projections

True or False – Briefly justify your answer

4. More realistic looking than perspective projections

5. Bad for exact measurements

6. Parallel lines do not remain parallel

7. Lengths vary with distance to the eye

Problem 2. Shading *Justify each answer* (6 points)

a. By increasing the number of polygons, can you can make the difference between Gouraud interpolation and Phong interpolation to be arbitrarily small (so that there is no perceivable difference?

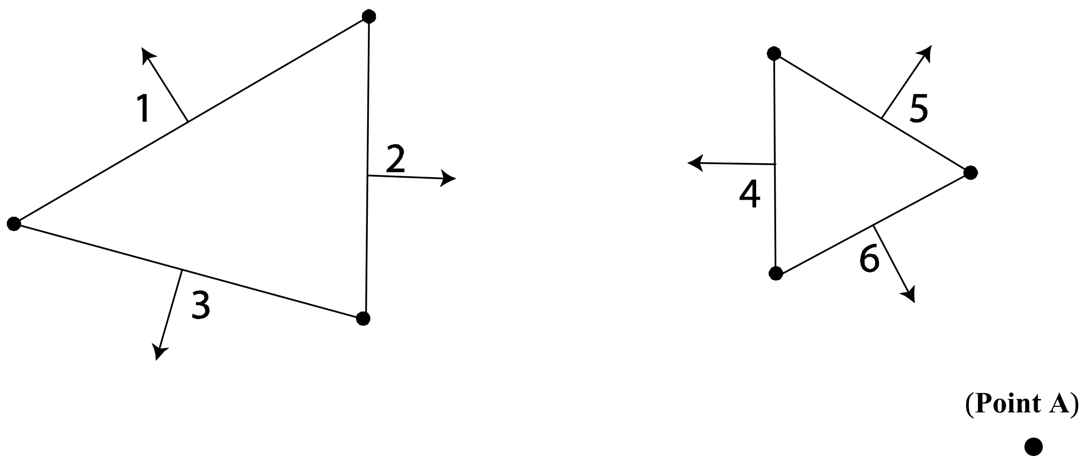
b. Does Gouraud interpolation produce specular highlights? How?

c. Can the intensity of a light source be represented by a single scalar value? When would this not be appropriate?

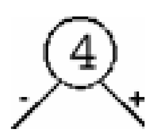
Problem 3. BSP Trees (8 points)

Recall that Binary Space Partitioning (BSP) trees break the world up into a tree of positive and negative half-spaces. As spatial partitioning data structures they are very versatile and can be applied to aid in hidden surface removal, constructive solid geometry, and even robot motion planning.

Below is a world in two dimensions described by a set of numbered line segments. Note that each segment normal (shown as arrows) points into the positive (+) half-space of its segment.



a. (4 points) Draw a diagram of a BSP tree that could be generated from this scene using segment #4 as the root. Note that while many binary tree configurations are valid, you should try to make your tree reasonably balanced.



b. (4 points) Given the viewpoint marked **Point A** in the scene, traverse your BSP tree to list the polygons in the order they would be rendered for hidden surface removal so that no Z-buffer is required (i.e., using the "Painter's algorithm"). For your answer, you only need to provide the list of primitives in the order in which they will be drawn.

Problem 4. Phong Shading (10 points)

The Phong shading model for a scene illuminated by a global ambient light and a single directional light can be summarized by the following equation from the lecture slides:

$$I = k_e + k_a L_a + \sum_j f_{atten}(d_j) L_j [k_d (\mathrm{N} \bullet \mathrm{L}_j)_+ + k_s \ (\mathrm{V} \bullet \mathrm{R}_j)_+)_+^{n_s}]$$

Imagine a scene consisting of a sphere illuminated by white global ambient light and a single white directional light. Assume the directional light is pointing in the same direction as the viewer. Describe the effect of the following conditions on the shading of the sphere. At each incremental step, assume that all the preceding steps have been applied first. Justify all of your answers.

a. The directional light is off. The ambient light is on. How does the shading vary over the surface of the object?

b. Now turn the directional light on. The specular reflection coefficient k_s and the specular exponent n_s of the material are both zero, and the diffuse reflection coefficient k_d is nonzero. How does the shading vary over the surface of the object?

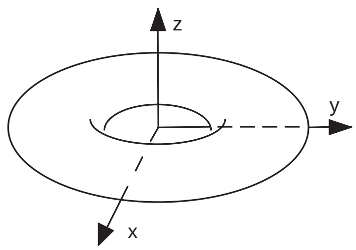
c. Now set the specular exponent, n_s, to be nonzero. Will the shading change? If so, how?

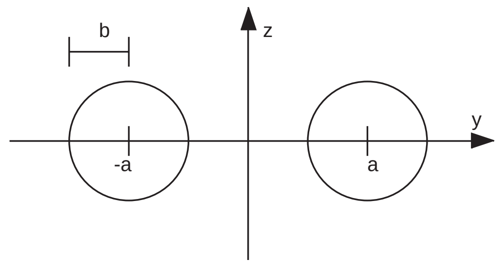
d. Now translate the sphere towards the viewer. You should notice that the shading on the sphere doesn't change. Could you change anything from the Phong equation to make the shading change as the sphere approaches you? If so, what?

e. Translate the sphere back to the origin, and undo any changes you might have done in part d. Make the specular reflection coefficient nonzero. What happens?

Problem 5. Ray tracing of implicit surfaces (10 points)

There are many ways to represent a surface. One such way is to define a function of the form $f(x, y, z) = 0$. Such a function is called an *implicit surface* representation. For example, the equation $f(x, y, z) = x^2 + y^2 + z^2 - r^2 = 0$ defines a sphere of radius *r*. Suppose we wanted to ray trace a torus. Tori are described by the equation $(x^2 + y^2 + z^2 - (a^2 + b^2))^2 - 4a^2 \cdot (b^2 - z^2) = 0$, with a and b being the major and minor radii, respectively, as pictured in the y-z slice below.





For this problem, we will use the torus with major radius *a* = 2 and minor radius *b* = 1. This gives us the equation $(x^2 + y^2 + z^2 - 5)^2 - 16 \cdot (1 - z^2) = 0$.

a. (5 points) Consider the ray $P + t\mathbf{d}$, where $P = (0 \quad 0 \quad 0)$ and $\mathbf{d} = (0 \quad 0 \quad 1)$. Does the ray intersect the torus? What about when $\mathbf{d} = (0 \quad 1 \quad 0.5)$? If it does intersect, which values does the ray intersect at? Which values do we care about when ray tracing and why?

Problem 5 (cont'd)

b. (2 points) Any ray substituted into the equation for the torus will result in a 4^{th} degree polynomial in *t*. This means that the resulting polynomial will have 4 roots, although some of them may be complex, and some of them may be repeated. Answer the following questions about your roots from part a.

i) How many distinct, real roots were there for both **d**'s?

$\mathbf{d} = (0 \quad 0 \quad 1)$

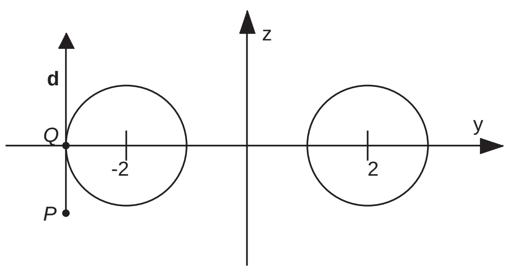
$\mathbf{d} = (0 \quad 1 \quad 0.5)$

ii) Did any of these roots have multiplicity greater than one? If so, how many, and what multiplicity?

$\mathbf{d} = (0 \quad 0 \quad 1)$

$\mathbf{d} = (0 \quad 1 \quad 0.5)$

c. (3 points) Consider the following ray passing through *P* in the direction **d** in the x-z plane and intersecting the torus at *Q*:



Answer the following questions about the 4 roots that will result when this ray is substituted into the torus equation. (Note: You do not have to give values for the roots, or show any math to derive your answers. Just reason from what you learned about roots and intersections when we studied ray-sphere intersections in class.)

i) How many distinct, real roots are there?

ii) Do any of these roots have multiplicity greater than one? If so, how many, and what multiplicity?

iii) How many complex roots does this leave?

Problem 6. Texture mapping (8 points)

When a texture map is applied to a surface, points that are distinct in the rectangular texture map may be mapped to the same place on the object. For example, when a texture map is applied to a cylinder, the left and right edges of the texture map are mapped to the same place. We will call a mapping “allowed” if it does not map two points of different colors to the same point on the object to be texture mapped. For each of the primitives below, describe the **specific** requirements for a mapping to be allowed, and draw at least one example of an allowed mapping, and at least one example of a mapping that is not allowed. These drawings should reflect the specific requirements, i.e., drawing a rectangle that is one solid color for the allowed mapping is not acceptable. On the mapping that is not allowed, mark the places that violate the same color rule.

Uncapped cylinder (top and bottom are open):

Sphere:

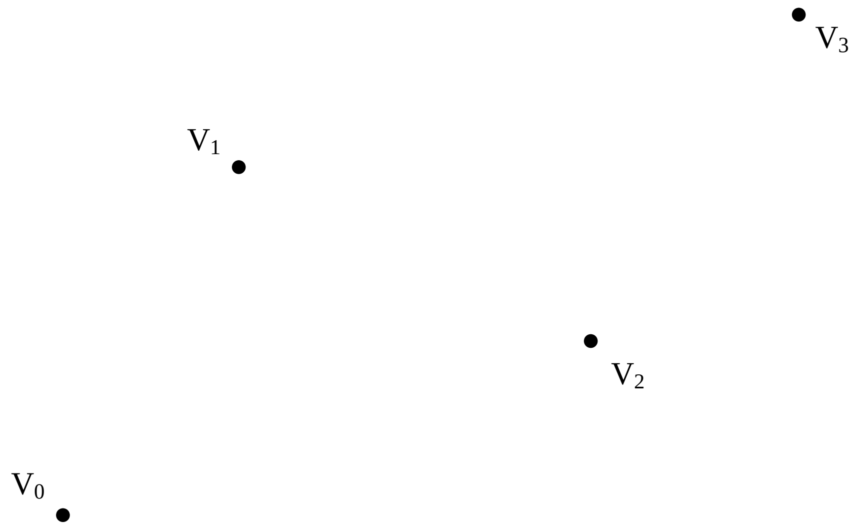
Uncapped cone (bottom is open):

Torus:

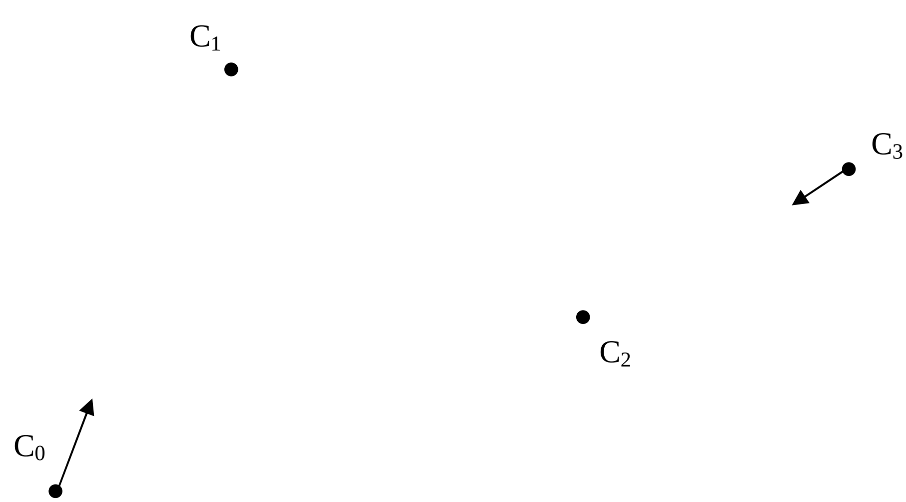
Problem 7. Parametric curves (12 points)

In this problem, we will explore the construction of parametric curves (a-c) and develop an understanding of parametric and geometric continuity for spline curves (d).

a. (4 points) Given the following Bezier control points, construct all of the de Casteljau lines and points needed to evaluate the curve at $u=1/4$. Mark this point on your diagram and then sketch the path the Bezier curve will take.

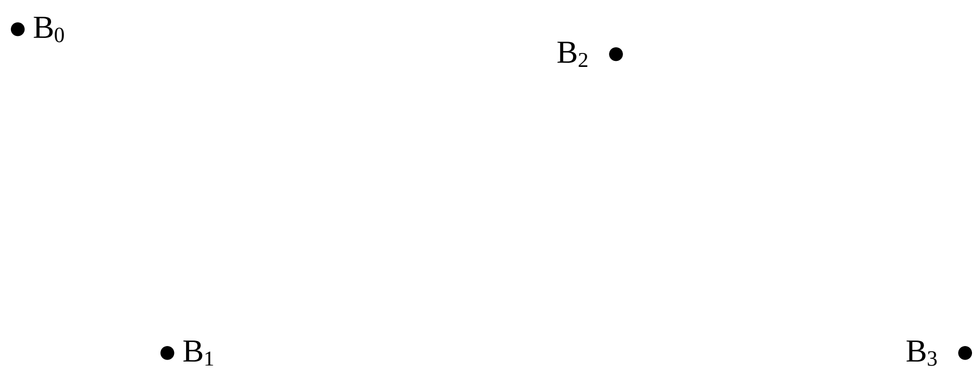


b. (4 points) Given the following Catmull-Rom control points, construct all of the lines and points needed to generate the Bezier control points for the Catmull-Rom curve. Use a tension value of $\tau=1$ and position the first control point after C_0 and the last control point before C_3 at the tips of the given arrows. You must mark each Bezier point with an X, but you do not need to label it (i.e., no need to give each Bezier point a name). Sketch the resulting spline curve.



Problem 7 (cont'd)

c. (4 points) Given the following de Boor points, construct all of the lines and points needed to generate the Bezier control points for the B-spline. Assume that the first and last points are repeated three times, so that the spline is endpoint interpolating. You must mark each Bezier point with an X, but you do not need to label it (i.e., no need to give each Bezier point a name). Sketch the resulting spline curve.



Problem 8. Continuity (13 points)

In this problem, we will explore the continuities between different curves.

a. (6 points) For each of the following curves, discuss whether the curve is interpolating, has local control, and is C^2 continuous.

B-Splines

Is interpolating?

Has local control?

Is C^2 continuous?

Catmull-Rom Curves

Is interpolating?

Has local control?

Is C^2 continuous?

b. (2 points) Can every third order Bezier curve be broken into two other third-order Bezier curves? If so, why?

c. (2 points) Is it possible to have a C2 continuous spline that is not also C1 continuous? Give an example or explain why it is not possible.

c. (3 points) Does a curve that is C^1 continuous imply that the curve is also G^1 continuous? If so, why? If not, give an example of a curve that would be C^1 continuous and G^0 continuous.