Parametric surfaces

CSE 457, Autumn 2003
Graphics

http://www.cs.washington.edu/education/courses/457/03au/

Readings and References

Readings

- Sections 2.1.4, 3.4-3.5, *3D Computer Graphics*, Watt

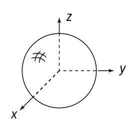
Other References

- Section 3.6, *3D Computer Graphics*, Watt.
- *An Introduction to Splines for use in Computer Graphics and Geometric Modeling*, Bartels, Beatty, and Barsky.

Mathematical surface representations

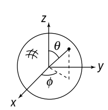
Explicit $z=f(x,y)$ (a.k.a., a "height field")
what if the curve isn't a function?

Implicit $g(x,y,z) = 0$



Parametric $S(u,v)=(x(u,v),y(u,v),z(u,v))$
For the sphere:

$x(u,v) = r \cos 2\pi v \sin \pi u$
$y(u,v) = r \sin 2\pi v \sin \pi u$
$z(u,v) = r \cos \pi u$



As with curves, we'll focus on parametric surfaces.

Surfaces of revolution

Idea: rotate a 2D **profile curve** around an axis.
What kinds of shapes can you model this way?

Constructing surfaces of revolution

Given: A curve $C(u)$ in the xy-plane:

$$C(u) = \begin{bmatrix} c_x(u) \\ c_y(u) \\ 0 \\ 1 \end{bmatrix}$$

Let $R_x(\theta)$ be a rotation about the x-axis.

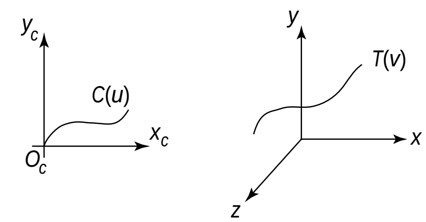
Find: A surface $S(u,v)$ which is $C(u)$ rotated about the x-axis.

Solution:

General sweep surfaces

The **surface of revolution** is a special case of a **swept surface**.

Idea: Trace out surface $S(u,v)$ by moving a **profile curve** $C(u)$ along a **trajectory curve** $T(v)$.



More specifically:

- » Suppose that $C(u)$ lies in an (x_c, y_c) coordinate system with origin O_c.
- » For every point along $T(v)$, lay $C(u)$ so that O_c coincides with $T(v)$.

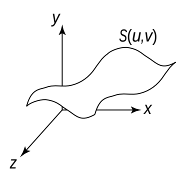
Orientation

The big issue:

- » How to orient $C(u)$ as it moves along $T(v)$?

Here are two options:

1. **Fixed** (or **static**): Just translate O_c along $T(v)$.



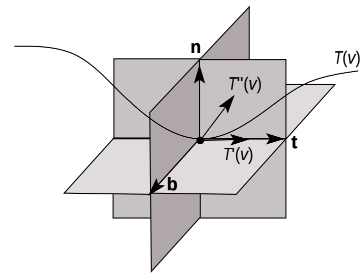
2. Moving. Use the **Frenet frame** of $T(v)$.

   Allows smoothly varying orientation.

   Permits surfaces of revolution, for example.

Frenet frames

Motivation: Given a curve $T(v)$, we want to attach a smoothly varying coordinate system.



$$\mathbf{t}(v) = \text{normalize}[T'(v)]$$
$$\mathbf{b}(v) = \text{normalize}[T'(v) \times T''(v)]$$
$$\mathbf{n}(v) = \mathbf{b}(v) \times \mathbf{t}(v)$$

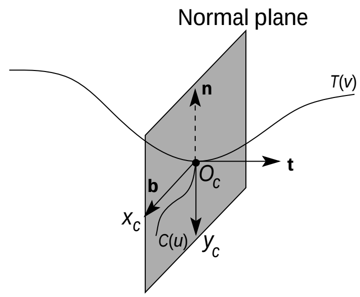
To get a 3D coordinate system, we need 3 independent direction vectors.

As we move along $T(v)$, the Frenet frame (t,b,n) varies smoothly.

Frenet swept surfaces

Orient the profile curve $C(u)$ using the Frenet frame of the trajectory $T(v)$:

- » Put $C(u)$ in the **normal plane** .
- » Place O_c on $T(v)$.
- » Align x_c for $C(u)$ with **b**.
- » Align y_c for $C(u)$ with **-n**.



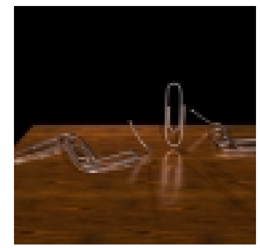
If $T(v)$ is a circle, you get a surface of revolution exactly!

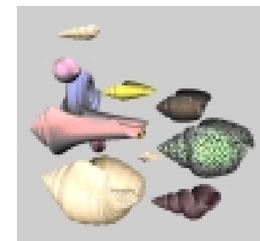
What happens at inflection points, i.e., where curvature goes to zero?

Variations

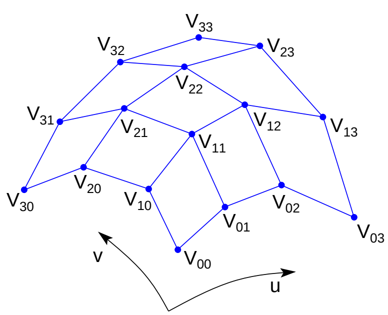
Several variations are possible:

- » Scale $C(u)$ as it moves, possibly using length of $T(v)$ as a scale factor.
- » Morph $C(u)$ into some other curve $\tilde{C}(u)$ as it moves along $T(v)$.
- » …





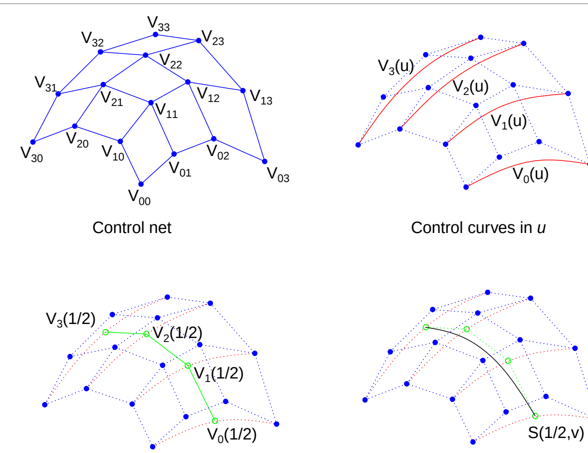
Tensor product Bézier surfaces



Given a grid of control points V_{ij}, forming a **control net**, construct a surface $S(u,v)$ by:

- » treating rows of V (the matrix consisting of the V_{ij}) as control points for curves $V_0(u),\ldots, V_n(u)$.
- » treating $V_0(u),\ldots, V_n(u)$ as control points for a curve parameterized by v.

Tensor product Bézier surfaces, cont.



Control net

Control curves in *u*

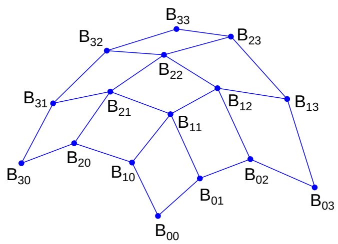
Control polygon at *u*=1/2

Curve at *S*(1/2,*v*)

Which control points are interpolated by the surface?

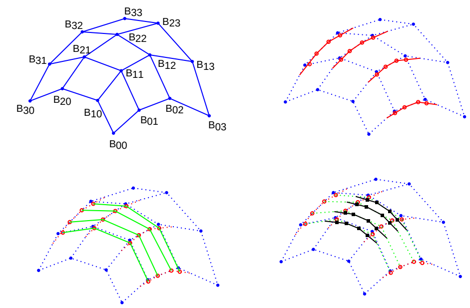
Tensor product B-spline surfaces

As with spline curves, we can piece together a sequence of Bézier surfaces to make a spline surface. If we enforce C^2 continuity and local control, we get B-spline curves:



» treat rows of *B* as control points to generate Bézier control points in *u*.
» treat Bézier control points in *u* as B-spline control points in *v*.
» treat B-spline control points in *v* to generate Bézier control points in *u*.

Tensor product B-spline surfaces, cont.



- Which B-spline control points are interpolated by the surface?

Summary

- What to take home:
  » How to construct swept surfaces from a profile and trajectory curve:
    - with a fixed frame
    - with a Frenet frame
  » How to construct tensor product Bézier surfaces
  » How to construct tensor product B-spline surfaces