
Hierarchical Modeling

CSE 457, Autumn 2003

Graphics

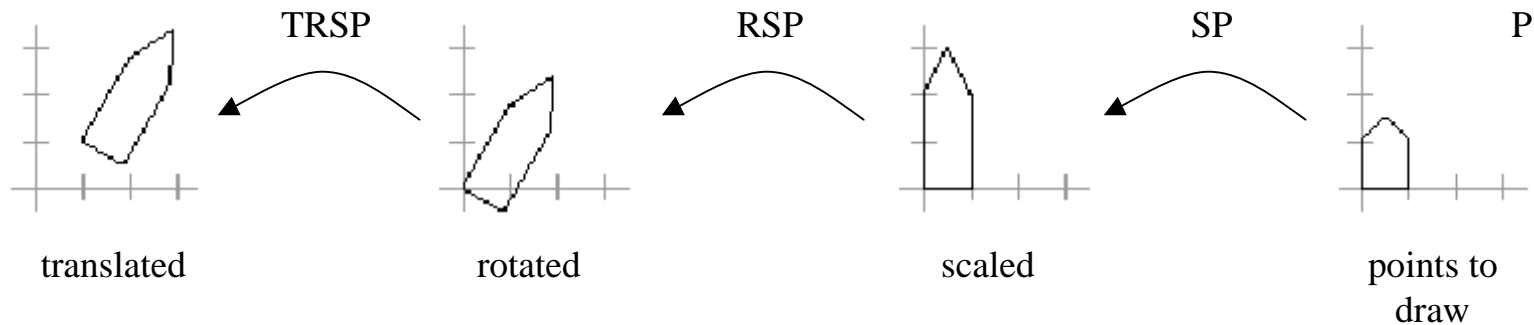
<http://www.cs.washington.edu/education/courses/457/03au/>

References

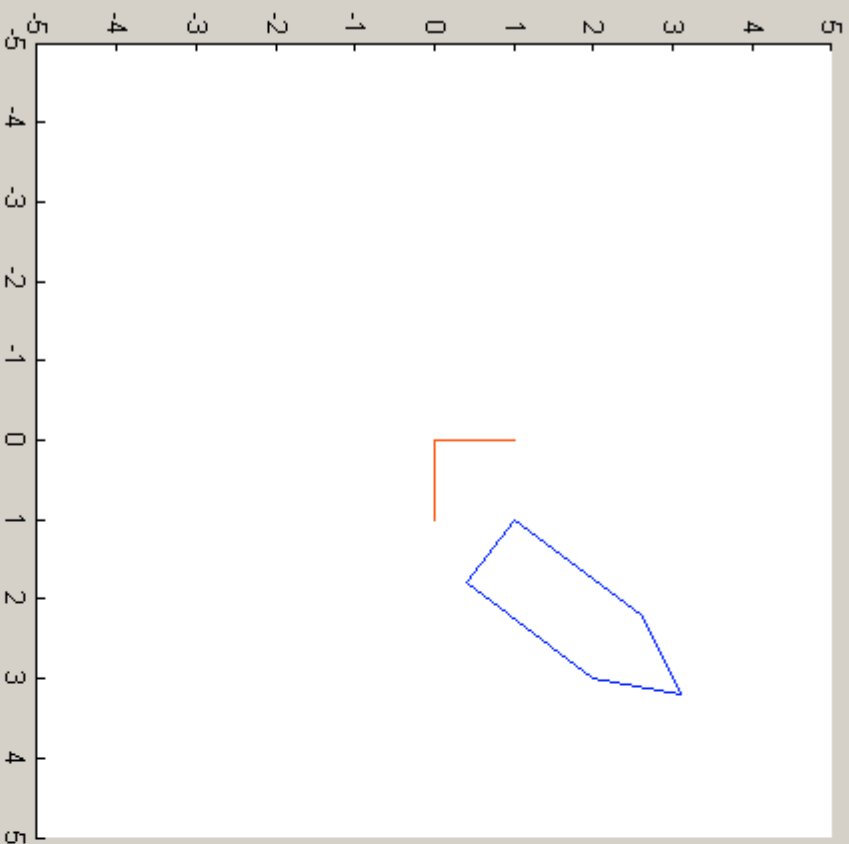
- *OpenGL Programming Guide*, The Red Book, chapter 3
- *Interactive Computer Graphics, A Top Down Approach with OpenGL*, E. Angel, sections 8.1 - 8.6

Symbols and instances

- Most graphics APIs support a few geometric **primitives**:
 - » spheres, cubes, cylinders
 - » these procedures define points for you, but they're still just points **P**
- These symbols are **instanced** using an **instance transformation**.
 - » the points are originally defined in a local coordinate system (eg, unit cube)



- **Q:** What is the matrix for the instance transformation above?



$$\begin{bmatrix} 0.8 & 1.2 & 1.0 \\ -0.6 & 1.6 & 1.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} .8 & .6 & 0 \\ -.6 & .8 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

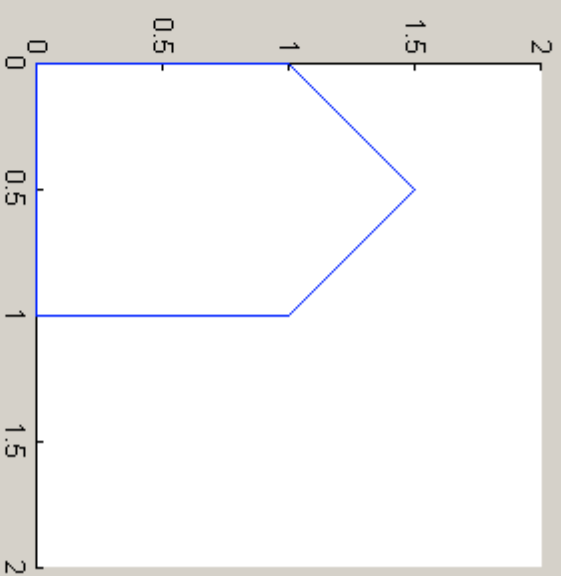
1



1



1



```

x  1.0  1.0  0.5  0.0  0.0  1.0
y  0.0  1.0  1.5  1.0  0.0  0.0
z  1.0  1.0  1.0  1.0  1.0  1.0
    
```

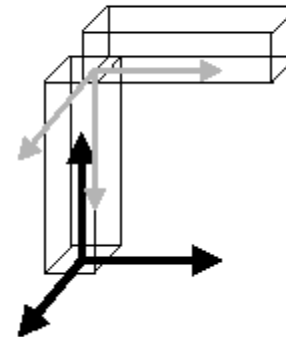
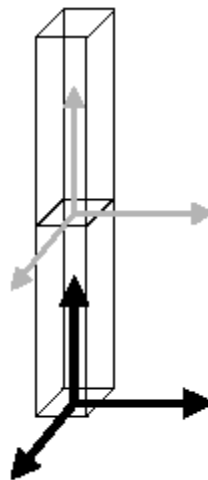
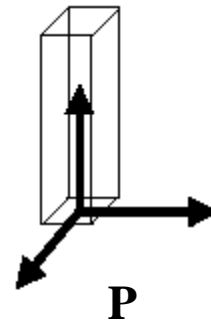
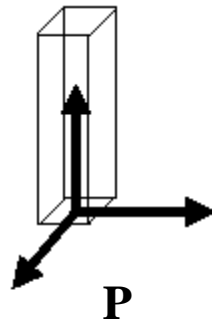
Draw Points

Clear Points

Clear Image

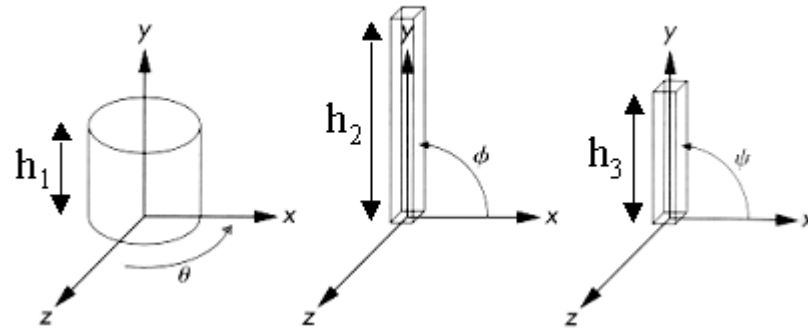
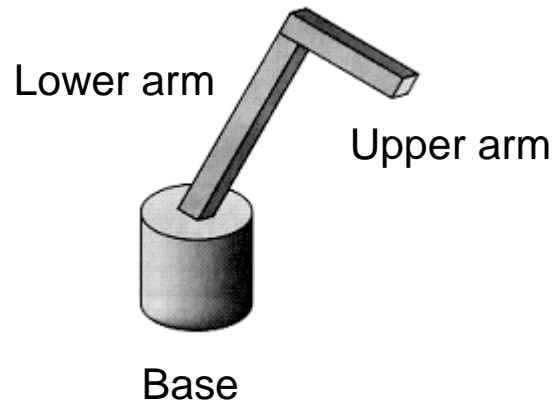
Use Homogeneous

Connecting primitives



3D Example: A robot arm

- Consider this robot arm with 3 degrees of freedom:
 - » Base rotates about its vertical axis by θ
 - » Lower arm rotates in its xy -plane by ϕ
 - » Upper arm rotates in its xy -plane by ψ



- **Q:** What matrix do we use to transform
 - » the base? the upper arm? the lower arm?

Base Transformation

| | | | |
|------|------|-------|------|
| 0.60 | 0.00 | -0.80 | 0.00 |
| 0.00 | 1.00 | 0.00 | 0.00 |
| 0.80 | 0.00 | 0.60 | 0.00 |
| 0.00 | 0.00 | 0.00 | 1.00 |

=

M base

| | | | |
|------|------|-------|------|
| 0.60 | 0.00 | -0.80 | 0.00 |
| 0.00 | 1.00 | 0.00 | 0.00 |
| 0.80 | 0.00 | 0.60 | 0.00 |
| 0.00 | 0.00 | 0.00 | 1.00 |

R theta



-53

Theta

Lower Arm Transformation

| | | | |
|-------|------|-------|------|
| 0.52 | 0.30 | -0.80 | 0.00 |
| -0.50 | 0.87 | 0.00 | 2.00 |
| 0.69 | 0.40 | 0.60 | 0.00 |
| 0.00 | 0.00 | 0.00 | 1.00 |

=

M lower

| | | | |
|------|------|-------|------|
| 0.60 | 0.00 | -0.80 | 0.00 |
| 0.00 | 1.00 | 0.00 | 0.00 |
| 0.80 | 0.00 | 0.60 | 0.00 |
| 0.00 | 0.00 | 0.00 | 1.00 |

R theta

| | | | |
|------|------|------|------|
| 1.00 | 0.00 | 0.00 | 0.00 |
| 0.00 | 1.00 | 0.00 | 2.00 |
| 0.00 | 0.00 | 1.00 | 0.00 |
| 0.00 | 0.00 | 0.00 | 1.00 |

T h1

| | | | |
|-------|------|------|------|
| 0.87 | 0.50 | 0.00 | 0.00 |
| -0.50 | 0.87 | 0.00 | 0.00 |
| 0.00 | 0.00 | 1.00 | 0.00 |
| 0.00 | 0.00 | 0.00 | 1.00 |

R phi



-30

Phi

Upper Arm Transformation

| | | | |
|-------|-------|-------|------|
| -0.30 | 0.52 | -0.80 | 0.90 |
| -0.87 | -0.50 | 0.00 | 4.60 |
| -0.40 | 0.69 | 0.60 | 1.20 |
| 0.00 | 0.00 | 0.00 | 1.00 |

=

M upper

| | | | |
|------|------|-------|------|
| 0.60 | 0.00 | -0.80 | 0.00 |
| 0.00 | 1.00 | 0.00 | 0.00 |
| 0.80 | 0.00 | 0.60 | 0.00 |
| 0.00 | 0.00 | 0.00 | 1.00 |

R theta

| | | | |
|------|------|------|------|
| 1.00 | 0.00 | 0.00 | 0.00 |
| 0.00 | 1.00 | 0.00 | 2.00 |
| 0.00 | 0.00 | 1.00 | 0.00 |
| 0.00 | 0.00 | 0.00 | 1.00 |

T h1

| | | | |
|-------|------|------|------|
| 0.87 | 0.50 | 0.00 | 0.00 |
| -0.50 | 0.87 | 0.00 | 0.00 |
| 0.00 | 0.00 | 1.00 | 0.00 |
| 0.00 | 0.00 | 0.00 | 1.00 |

R phi

| | | | |
|------|------|------|------|
| 1.00 | 0.00 | 0.00 | 0.00 |
| 0.00 | 1.00 | 0.00 | 3.00 |
| 0.00 | 0.00 | 1.00 | 0.00 |
| 0.00 | 0.00 | 0.00 | 1.00 |

T h2

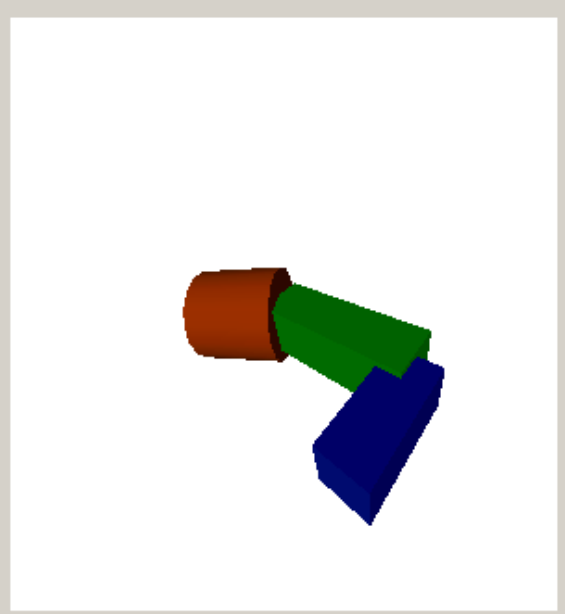
| | | | |
|-------|------|------|------|
| 0.00 | 1.00 | 0.00 | 0.00 |
| -1.00 | 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 1.00 | 0.00 |
| 0.00 | 0.00 | 0.00 | 1.00 |

R psi



-90

Psi



Robot arm implementation

The robot arm could be displayed by using a global matrix and recomputing it at each step:

```
Matrix M_model;
```

```
main() {  
    . . .  
    robot_arm();  
    . . .  
}
```

```
robot_arm() {  
    M_model = R_y(theta);  
    base();  
    M_model = R_y(theta)*T(0,h1,0)*R_z(phi);  
    upper_arm();  
    M_model = R_y(theta)*T(0,h1,0)*R_z(phi)*T(0,h2,0)*R_z(psi);  
    lower_arm();  
}
```

Do the matrix computations seem just a tad wasteful?

Robot arm implementation, better

Instead of recalculating the global matrix each time, we could just update it as we go along:

```
Matrix M_model;

main() {
    . . .
    M_model = Identity();
    robot_arm();
    . . .
}

robot_arm() {
    M_model *= R_y(theta);
    base();
    M_model *= T(0,h1,0)*R_z(phi);
    upper_arm();
    M_model *= T(0,h2,0)*R_z(psi);
    lower_arm();
}
```

Robot arm implementation, OpenGL

OpenGL maintains a global state matrix called the **model-view matrix**.

```
main() {
    . . .
    glMatrixMode( GL_MODELVIEW );
    glLoadIdentity();
    robot_arm();
    . . .
}

robot_arm() {
    glRotatef( theta, 0.0, 1.0, 0.0 );
    base();
    glTranslatef( 0.0, h1, 0.0 );
    glRotatef( phi, 0.0, 0.0, 1.0 );
    upper_arm();
    glTranslatef( 0.0, h2, 0.0 );
    glRotatef( psi, 0.0, 0.0, 1.0 );
    lower_arm();
}
```


Hierarchical modeling

- Hierarchical models can be composed of instances using trees or DAGs:

- edges contain geometric transformations
- nodes contain geometry (and possibly drawing attributes)

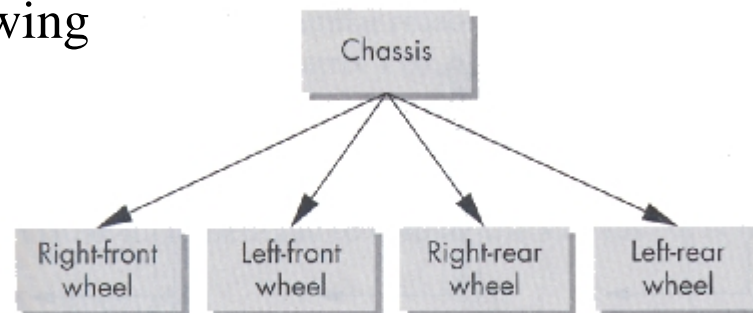
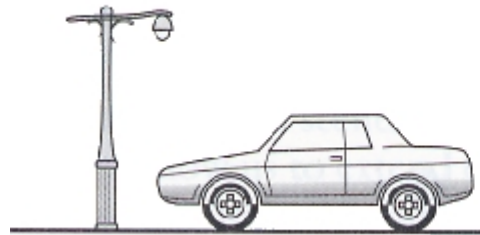


Figure 8.6 Tree structure for automobile.

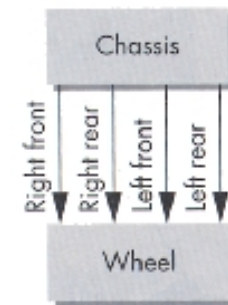


Figure 8.7 Directed-acyclic-graph (DAG) model of automobile.

figures from Angel

Another example: human figure

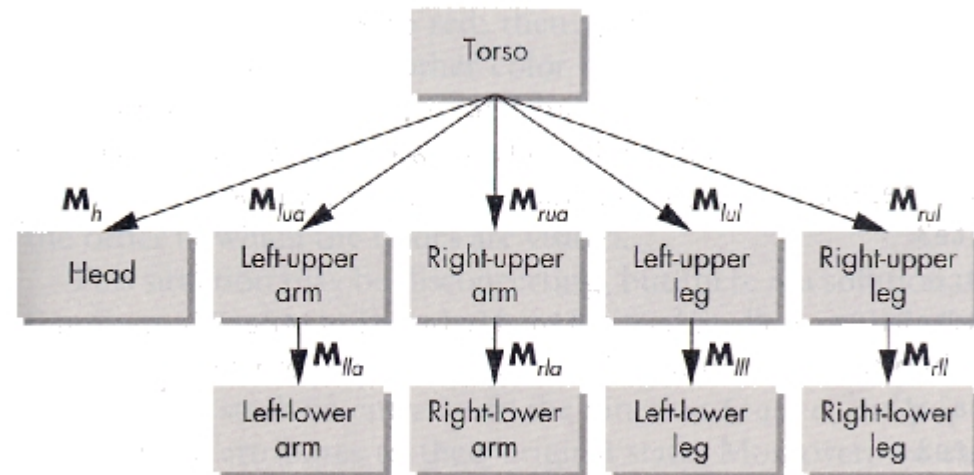
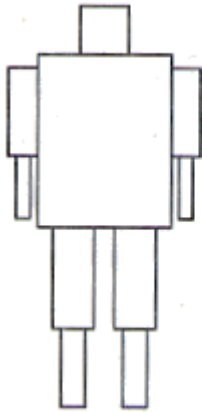


Figure 8.14 Tree with matrices.

Q: What's a sensible way to traverse this tree?

Human figure implementation

- We can also design code for drawing the human figure, with a slight modification due to the branches in the tree:

```
figure() {
    torso();
    M_save = M_model;
    M_model *= T(. . .)*R(. . .);
    head();
    M_model = M_save;
    M_model *= T(. . .)*R(. . .);
    left_upper_arm();
    M_model *= T(. . .)*R(. . .);
    left_lower_arm();
    M_model = M_save;
    ...
}
```

Figure with hand



What if we add a hand?

```
figure() {  
    torso();  
    M_save = M_model;  
    M_model *= T(. . .)*R(. . .);  
    head();  
    M_model = M_save;  
    M_model *= T(. . .)*R(. . .);  
    left_upper_arm();  
    M_model *= T(. . .)*R(. . .);  
    left_lower_arm();  
    M_model *= T(. . .)*R(. . .);  
    left_hand();  
    M_save2 = M_model;  
    M_model *= T(. . .)*R(. . .);  
    left_thumb();  
    M_model = M_save2;  
    M_model *= T(. . .)*R(. . .);  
    left_forefinger();  
    M_model = M_save2;  
    . . .  
}
```

Is there a better way to keep track of piles of matrices that need to be saved, modified, and restored?

Push and pop

```
figure() {
  torso();
  push(M_model);
    M_model *= T(. . .)*R(. . .);
  head();
M_model = pop(M_model);
push(M_model);
  M_model *= T(. . .)*R(. . .);
  left_upper_arm();
  M_model *= T(. . .)*R(. . .);
  left_lower_arm();
  M_model *= T(. . .)*R(. . .);
  left_hand();
  push(M_model);
    M_model *= T(. . .)*R(. . .);
    left_thumb();
  M_model = pop(M_model);
  push(M_model);
    M_model *= T(. . .)*R(. . .);
    left_forefinger();
  M_model = pop(M_model);
  push(M_model);
  . . .
}
```


Push and pop, OpenGL

```
figure() {
    torso();
    glPushMatrix();
        glTranslate( ... );
        glRotate( ... );
        head();
    glPopMatrix();
    glPushMatrix();
        glTranslate( ... );
        glRotate( ... );
        left_upper_arm();
        glTranslate( ... );
        glRotate( ... );
        left_lower_arm();
        glTranslate( ... );
        glRotate( ... );
        left_hand();
        glPushMatrix();
            glTranslate( ... );
            glRotate( ... );
            left_thumb();
        glPopMatrix();
        glPushMatrix();
            glTranslate( ... );
            glRotate( ... );
            left_forefinger();
        glPopMatrix();
    . . .
}
```

Animation

- The above examples are called **articulated models**:
 - » rigid parts
 - » connected by joints
- They can be animated by specifying the joint angles (or other display parameters) as functions of time.

Kinematics and dynamics

- Definitions:
 - » **Kinematics:** how the positions of the parts vary as a function of the joint angles.
 - » **Dynamics:** how the positions of the parts vary as a function of applied forces.
- Questions:
- **Q:** What do the terms **inverse kinematics** and **inverse dynamics** mean?
- **Q:** Why are these problems more difficult?

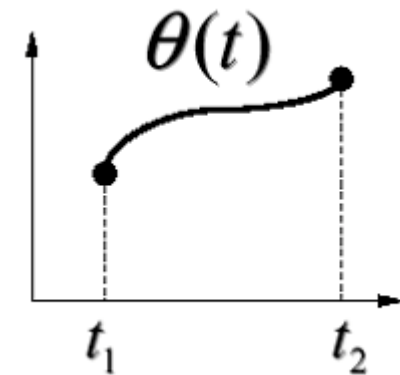
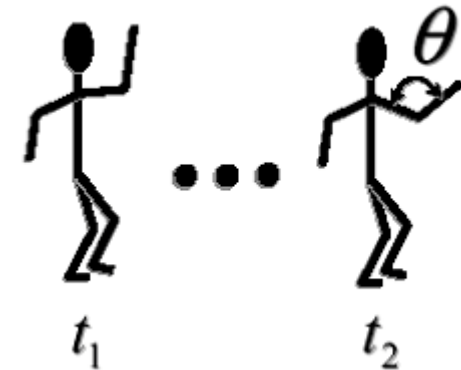
Key-frame animation

- The most common method for character animation in production is **key-frame animation**.

Each joint specified at various **key frames** (not necessarily the same as other joints)

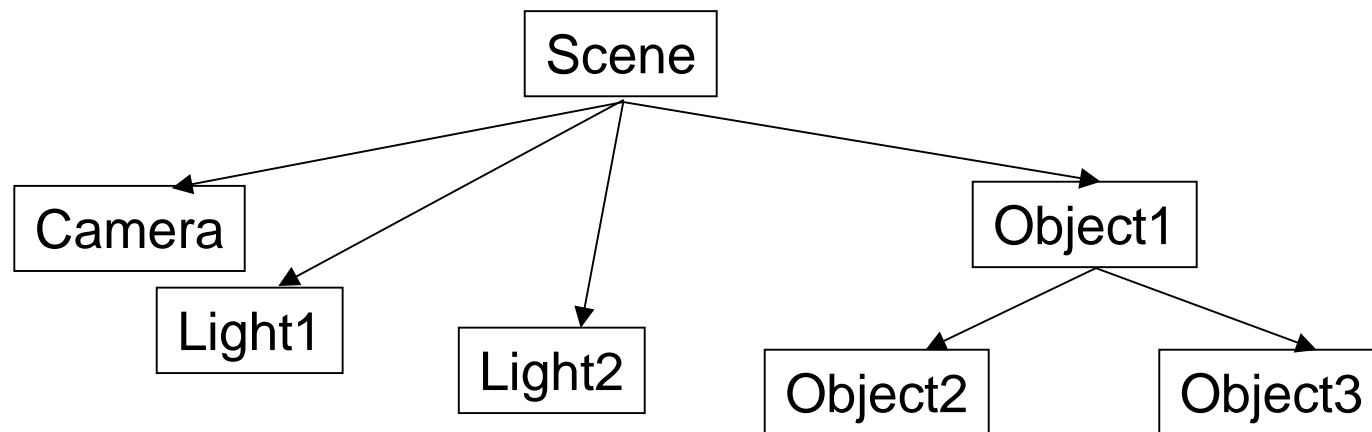
System does interpolation or **in-betweening**

- Doing this well requires:
 - A way of smoothly interpolating key frames:
splines
 - A good interactive system
 - A lot of skill on the part of the animator



Scene graphs

- The idea of hierarchical modeling can be extended to an entire scene, encompassing:
 - » many different objects
 - » lights
 - » camera position
- This is called a **scene tree** or **scene graph**.



Order of transformations

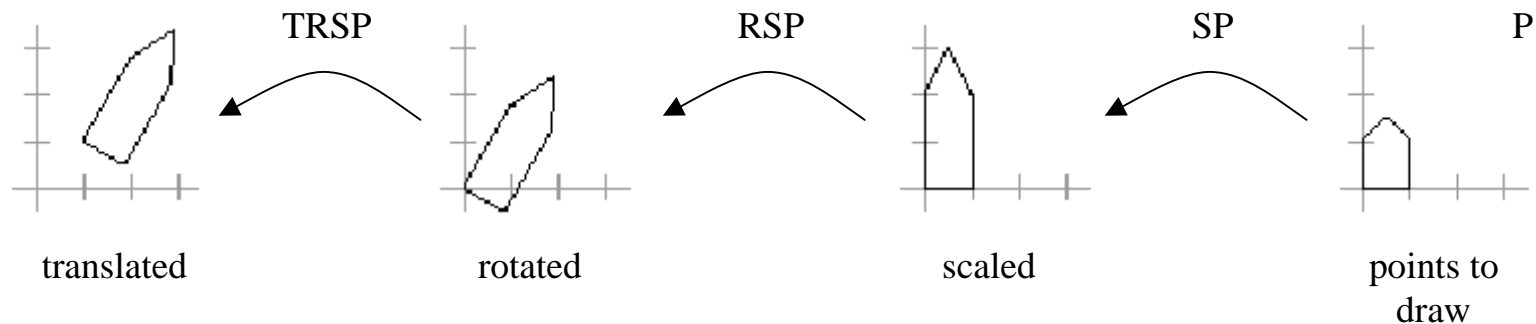
- Let's revisit the very first simple example in this lecture.
- To draw the transformed house, we would write OpenGL code like:

```
glMatrixMode( GL_MODELVIEW );  
glLoadIdentity();  
glTranslatef( ... );  
glRotatef( ... );  
glScalef( ... );  
house();
```

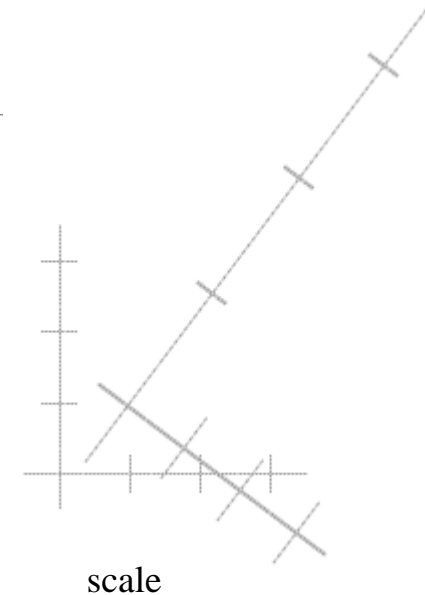
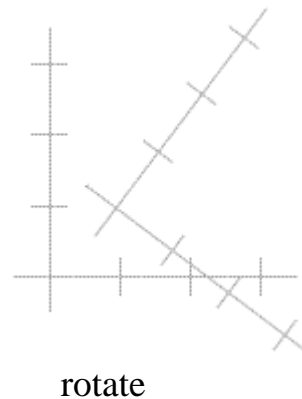
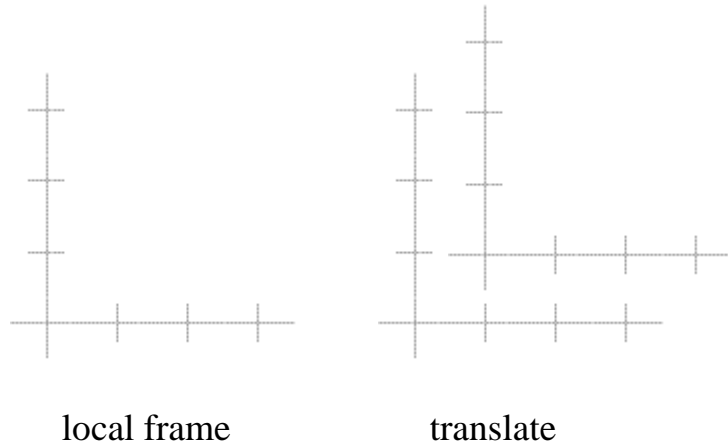
Note that we are building the composite transformation matrix by starting from the left and postmultiplying each additional matrix

Global, fixed coordinate system

- One way to think of transformations is as movement of points in a *global, fixed coordinate system*
 - » Build the transformation matrix sequentially from left to right: T, then R, then S
 - » Then apply the transformation matrix to the object points: multiply all the points in P by the composite matrix TRS
 - this transformation takes the points from original to final positions

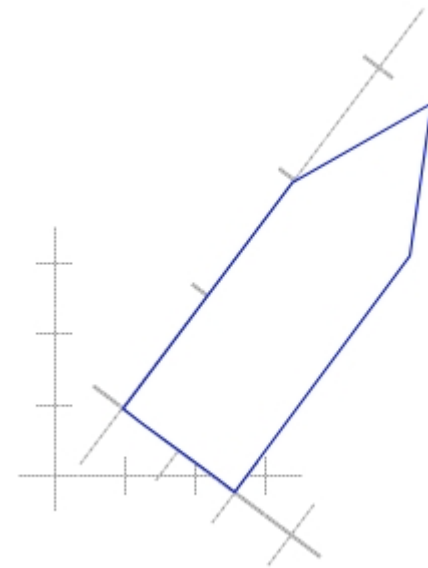


Local, changing coordinate system



- Another way to think of transformations is as affecting a *local coordinate system* that the primitive is eventually drawn in.
 - » This is EXACTLY the same transformation as on the previous page, it's just how you look at it.

Draw!



Summary

- Here's what you should take home from this lecture:
 - » All the **boldfaced terms**.
 - » How primitives can be instanced and composed to create hierarchical models using geometric transforms.
 - » How the notion of a model tree or DAG can be extended to entire scenes.
 - » How keyframe animation works.
 - » How transforms can be thought of as affecting either the geometry, or the coordinate system which it is drawn in.