

Homework 1

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Assigned: Friday, April 6, 2001
Due: Friday, April 20, 2001

DIRECTIONS

Directions: Please provide short written answers to the questions in the space provided. If you require extra space, you may staple additional pages to the back of your assignment. Feel free to talk over the problems with classmates, but please *answer the questions on your own*.

NAME: _____

Problem 2. Image Filters (16 points)

Each of the matrices below is actually a convolution filter for image processing. In addition, we have included a median filter. Next to each filter, identify the image (next page) that would result from applying the filter. In addition, write one or more characteristics that apply. Possible characteristics include:

- mean blurring
- gaussian blurring
- edge-preserving blurring
- blurring in x
- blurring in y
- gradient in x
- gradient in y
- edge enhancing
- rotating
- translating
- identity (no effect).

(a) $\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

Image: _____

(b) $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

Image: _____

(c) $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

Image: _____

(d) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Image: _____

(e) $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

Image: _____

(f) $\frac{1}{25} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

Image: _____

(g) $\frac{1}{5} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Image: _____

(h) Median filter over 4x4 region.

Image: _____



1. Original image



2



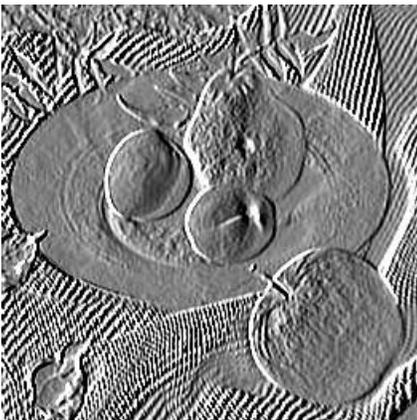
3



4



5



6 [after adding 128 to each pixel]



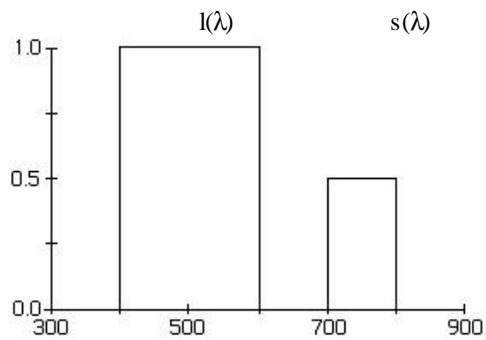
7



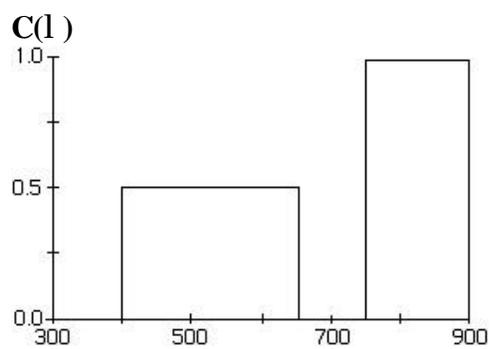
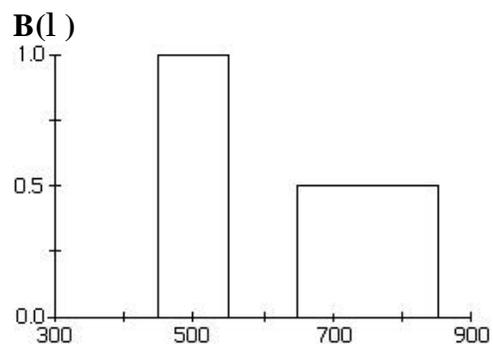
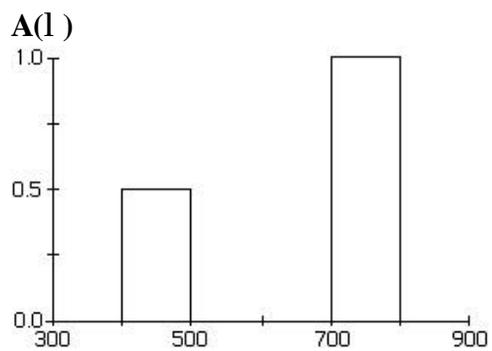
8 [after adding 128 to each pixel]

Problem 3. Color (8 Points)

Prakites, aliens from the planet Prakon-7, have two types of cones, $s(\lambda)$ and $l(\lambda)$, with spectral sensitivities as shown below:



Prakon-7 is part of a three-star system, the stars having emission spectrums as shown below. Determine which, if any, of these stars appear as metamers for the Prakons.



Problem 4. Convolution (6 Points)

Consider the following “impulse” function $h(x,y)$:

$$h(x, y) = \begin{cases} 0 & \text{if } x \neq 0 \text{ OR } y \neq 0 \\ 1 & \text{if } x = 0 \text{ AND } y = 0 \end{cases}$$

a) (2 points) If $f(x,y)$ is a grayscale input image, explain in two or three sentences why the following convolution produces an image $g(x,y)$ that is identical to $f(x,y)$.

$$g(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x', y') h(x - x', y - y') dx' dy'$$

d) (4 points) Consider an image processing operation that takes an input image $f(x,y)$ and produces an output image $g(x,y)$ that is just the original image shifted by u units in the x dimension and v units in the y dimension. This operation can be represented as a convolution using an impulse function $h'(x,y)$ that is similar to $h(x,y)$ described above. Given the following convolution definition, write down the function $h'(x,y)$ in terms of u and v . **Hint:** Your function definition should look very similar to the definition given for $h(x,y)$.

$$g(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x', y') h'(x - x', y - y') dx' dy'$$

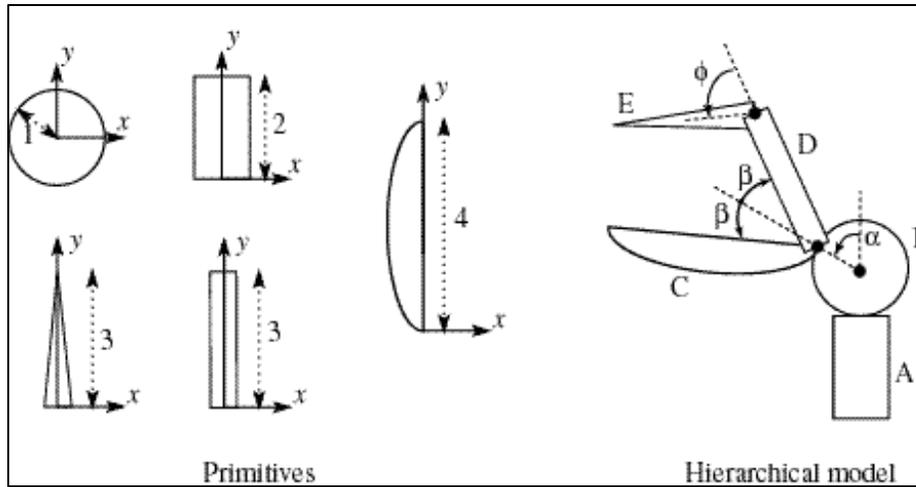
Problem 5. Affine Transformations (20 Points)

Using the Euler angle rotation matrices $R_x(\alpha)$, $R_y(\alpha)$ and $R_z(\alpha)$ (parameterized by the angle α) as building blocks, specify how one could build a transformation matrix which rotates around the arbitrary axis unit vector $\mathbf{v}=(x,y,z)$ located at the origin, by angle q . In words and drawings, describe all parts of the construction. You don't need to compute exact formulas for the rotation angles, but you must describe how to compute each of the rotation angles.

Problem 6. Transformation Hierarchies (12 points)

Suppose you want to create the hierarchical model shown below. The hierarchical model is made of five parts, labeled **A**, **B**, **C**, **D**, and **E** and each part is drawn as one of five primitives as given below (they are already scaled to the correct sizes). The following transformations are also available to you:

- $R(\theta)$ – rotate by θ degrees (counter clockwise)
- $T(a,b)$ – translate by (a,b)

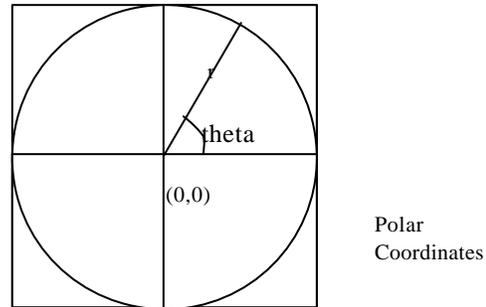
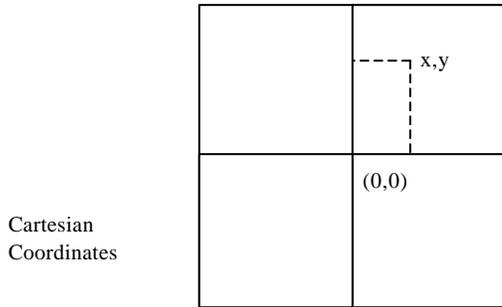


a) (8 points) Construct a tree to specify the hierarchical model where the nodes of the tree should be labeled **A**, **B**, **C**, **D**, and **E**. Along each edge, write out the product of matrices that should be performed when traversing that edge. Insert numerical values (i.e. for primitive sizes) where available.

b) (2 points) Write out the full transformation expression for the part labeled **E**.

Problem 7. Image Transformations (18 Points)

(A) Match the following image processing functions to their respective image pairs (next page), where each pair is of the form $f(x,y) \rightarrow f(X(x,y), Y(x,y))$ or $f(r, \theta) \rightarrow f(R, \text{THETA})$ and (r, θ) is polar coordinates. Note that repeats and/or "none of the above" are valid answers.

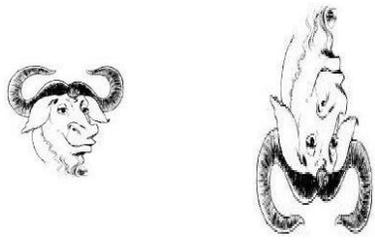


- ___ $X = x$
- ___ $X = x$
- ___ $X = x$
- ___ $X = 120.0 * \cos(x/40.0)$

- $Y = -2.0 * y$
- $Y = y / -2.0$
- $Y = -y^2$
- $Y = 120.0 * \sin(y/40.0)$

- ___ $R = \sqrt{r * 80.0}$
- ___ $R = r$
- ___ $R = r$
- ___ $R = r - 60.0$
- ___ $R = \theta$

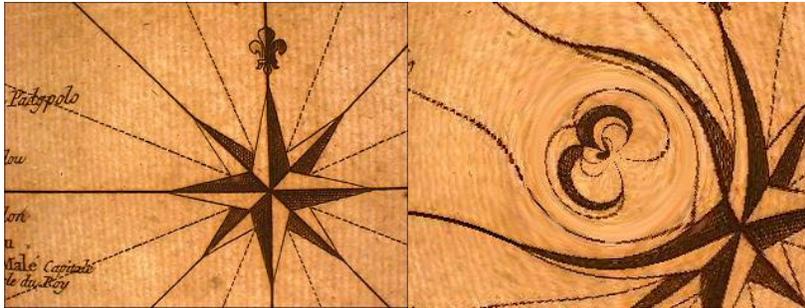
- $\text{THETA} = -\theta$
- $\text{THETA} = \theta + r/100.0$
- $\text{THETA} = \sqrt{\theta}$
- $\text{THETA} = \theta$
- $\text{THETA} = r$



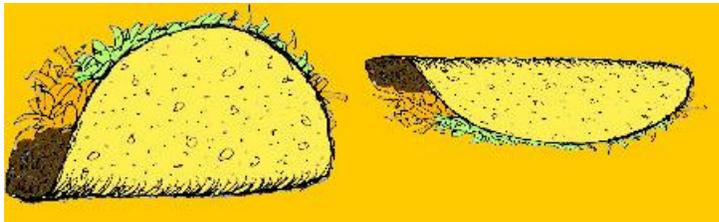
I.



II.



III.



IV.



V.



VI.