## 9. Hidden Surface Algorithms

## Introduction

In the previous lecture, we figured out how to transform the geometry so that the relative sizes will be correct if we drop the $z$ component.

But, how do we decide which geometry actually gets drawn to a pixel?

Known as the hidden surface elimination problem or the visible surface determination problem.

There are dozens of hidden surface algorithms.

They can be characterized in at least three ways:

- Object-precision vs. image-precision (a.k.a, object-space vs. image-space)
- Object order vs. image order
- Sort first vs. sort last


## Object-precision algorithms

## Basic idea:

- Operate on the geometric primitives themselves. (We'll use "object" and "primitive" interchangeably.)
- Objects typically intersected against each other
- Tests performed to high precision
- Finished list of visible objects can be drawn at any resolution


## Complexity:

- For n objects, can take $O\left(n^{2}\right)$ time to compute visibility.
- For an $m \times m$ display, have to fill in colors for $m^{2}$ pixels.
- Overall complexity can be $O\left(k_{o b j} n^{2}+k_{\text {disp }} m^{2}\right)$


## Implementation:

- Difficult to implement
- Can get numerical problems


## Image-precision algorithms

## Basic idea:

- Find the closest point as seen through each pixel
- Calculations performed at display resolution
- Does not require high precision


## Complexity:

- Naïve approach checks all n objects at every pixel. Then, $O\left(n^{2}\right)^{2}$.
- Better approaches check only the objects that could be visible at each pixel. Let's say, on average, $d$ objects are visible at each pixel (a.k.a. depth complexity). Then, $O\left(d^{2}\right)$.

Implementation:

- Very simple to implement!
- Used a lot in practice!


## Object order vs. image order

## Object order:

- Consider each object only once, draw its pixels, and move on to the next object.
- Might draw to the same pixel multiple times.


## Image order:

- Consider each pixel only once, find nearest object, and move on to the next pixel.
- Might compute relationships between objects multiple times.


## Sort first vs. sort last

## Sort first:

- Find some depth-based ordering of the objects relative to the camera, then draw back to front.
- Means building an ordered data structure to avoid duplicating work.


## Outline of lecture

- Z-buffer
- Ray casting
- Binary space partitioning (BSP) trees


## Sort last:

- Sort implicitly as more information becomes available.


## Z-buffer

The Z-buffer or depth buffer algorithm [Catmull, 1974] is probably the simplest and most widely used.

Here is pseudocode for the Z-buffer hidden surface algorithm:

```
for each pixel ( \(i, j\) ) do
    Z-buffer \([i, j] \leftarrow-F A R\)
    Framebuffer \([i, j] \leftarrow\) <background color>
end for
for each polygon \(A\) do
    for each pixel in \(A\) do
        Compute depth \(z\) and shade \(s\) of \(A\) at \((i, j)\)
        if \(z>Z\)-buffer \([i, j]\) then
            Z-buffer[i,j] \(\leftarrow z\)
            Framebuffer[i,j] \(\leftarrow s\)
        end if
    end for
end for
```

Z-buffer $[i, j] \leftarrow-F A R$
Framebuffer $[i, j] \leftarrow$ <background color>
end for
for each polygon $A$ do
for each pixel in $A$ do
Compute depth $z$ and shade $s$ of $A$ at (i,j)
if $z>Z$-buffer $[i, j]$ then
Z-buffer[i,j] $\leftarrow z$
Framebuffer[i,j] $\leftarrow s$
end if
end for
end for

## Z-buffer (cont'd)

The process of filling in the pixels inside of a polygon is called rasterization.

During rasterization, the $z$ value and shade $s$ can be computed incrementally (fast!).


## Curious fact:

- Described as the " brute-force image space algorithm" by [SSS]
- Mentioned only in Appendix B of [SSS] as a point of comparison for huge memories, but written off as totally impractical.
Today, Z-buffers are commonly implemented in hardware.


## Z-buffer: Analysis

- Classification?
- Easy to implement?
- Easy to implement in hardware?
- Incremental drawing calculations (uses coherence)?
- Pre-processing required?
- On-line (doesn't need all objects before drawing begins)?
- If objects move, does it take extra work than normal to draw the frame?
- If the viewer moves, does it take extra work than normal to draw the frame?
- Typically polygon-based?
- Efficient shading (doesn't compute colors of hidden surfaces)?
- Handles transparency?
- Handles refraction?


## Ray casting



Idea: For each pixel center $\boldsymbol{p}_{i j}$

- Send ray from the eye point (COP), c, through $\boldsymbol{p}_{i j}$ into scene.
- Intersect ray with each object.
- Select nearest intersection.


## Ray casting (cont'd)



Implementation:

- Might parameterize each ray:

$$
\mathbf{r}(\mathrm{t})=\mathbf{c}+\mathrm{t}\left(\mathbf{p}_{i j}-\mathbf{c}\right)
$$

- Each object $O_{k}$ returns $t_{k}>1$ such that first intersection with $O_{k}$ occurs at $\mathbf{r}\left(t_{k}\right)$.
Q: Given the $t_{k}$ what is the first intersection point?

Note: these calculations generally happen in world coordinates.

## Ray casting: Analysis

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Binary-space partitioning (BSP) trees


## Idea:

- Do extra preprocessing to allow quick display from any viewpoint.

Key observation: A polygon $A$ is painted in correct order if

- Polygons on far side of $A$ are painted first.
- P is painted next.
- Polygons in front of $A$ are painted last.


## BSP tree creation



## BSP tree creation (cont'd)

## procedure MakeBSPTree:

takes PolygonList L
returns BSPTree
Choose polygon $A$ from $L$ to serve as root
Split all polygons in $L$ according to $A$
node $\leftarrow A$
node.neg $\leftarrow$ MakeBSPTree(polygons on neg. side of $A$ )
node.pos $\leftarrow$ MakeBSPTree(polygons on pos. side of $A$ )
return node
end procedure

Note: Performance is improved when fewer polygons are split - in practice, best of $\sim 5$ random splitting polygons are chosen.

Note: BSP is created in world coordinates.

## BSP tree display

procedure DisplayBSPTree:
Takes BSPTree T
if $T$ is empty then return
if viewer is in front (on pos. side) of T.node then
DisplayBSPTree(T.neg)
Draw T.node
DisplayBSPTree(T.pos)
else
DisplayBSPTree(T.pos)
Draw T.node
DisplayBSPTree(T.neg)
end if
end procedure

## BSP trees: Analysis

- Classification?
- Easy to implement?
- Easy to implement in hardware?
- Incremental drawing calculations (uses coherence)?
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- If objects move, does it take extra work than normal to draw the frame?
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## Summary

What to take home from this lecture:

- Classification of hidden surface algorithms
- Understanding of Z-buffer and ray casting hiddensurface algorithms
- Familiarity with BSP trees

