## Reading

## Suggested:

- Watt (2 $2^{\text {nd }}$ ed.), Chapter 14.


## 4. Color

## Outline

## Spectra and photopigments

Color matching
The CIE XYZ color space
Color spaces for computer graphics

## The radiant energy spectrum

We can think of light as waves, instead of rays.
Wave theory allows a nice arrangement of electromagnetic radiation (EMR) according to wavelength:


## Emission spectra

A light source can be characterized by an emission spectrum:


Emission spectra for daylight and a tungsten lightbulb (Wandell, 4.4)

The spectrum describes the energy at each wavelength.

## What is color?

The eyes and brain turn an incoming emission spectrum into a discrete set of values.

The signal sent to our brain is somehow interpreted as color.

Color science asks some basic questions:
-When are two colors alike?

- How many pigments or primaries does it take to match another color?

One more question: why should we care?

## Photopigments

Photopigments are the chemicals in the rods and cones that react to light. Can respond to a single photon!

Rods contain rhodopsin, which has peak sensitivity at about 500 nm .


Rods are active under low light levels, I.e., they are responsible for scotopic vision.

## Univariance

Principle of univariance: For any single photoreceptor, no information is transmitted describing the wavelength of the photon.


Measuring photoreceptor photocurrent (Wandell, 4.15)


Photocurrents measured for two light stimuli: 550 nm (solid) and 659 nm (gray). The brightnesses of the stimuli are different, but the shape of the response is the same. (Wandell 4.17)

## The color matching experiment

We can construct an experiment to see how to match a given test light using a set of lights called primaries with power control knobs.


The color matching experiment (Wandell, 4.10)

The primary spectra are $a(\lambda), b(\lambda), c(\lambda), \ldots$
The power knob settings are $A, B, C, \ldots$

## Rods and "color matching"

A rod responds to a spectrum through its spectral sensitivity function, $p(\lambda)$. The response to a test light, $t(\lambda)$, is simply:

$$
P_{t}=\int t(\lambda) p(\lambda) d \lambda
$$

How many primaries are needed to match the test light?
$a(\lambda)=0.25$ at 505 nm
$\mathrm{t}(\lambda)=1.0$ at $430 \mathrm{~nm} \rightarrow \mathrm{~A}=$
$\mathrm{t}(\lambda)=1.0$ at $455 \mathrm{~nm} \rightarrow \mathrm{~A}=$
$\mathrm{t}(\lambda)=1.0$ at $505 \mathrm{~nm} \rightarrow \mathrm{~A}=$


What does this tell us about rod color discrimination?

## Cone photopigments

Cones come in three varieties: $\mathrm{L}, \mathrm{M}$, and S .


Cone photopigment absorption (Glassner, 1.1)

Cones are active under high light levels, l.e., they are responsible for photopic vision.

## Cones and color matching

Color is perceived through the responses of the cones to light.

The response of each cone can be written simply as:

$$
\begin{aligned}
L_{t} & =\int t(\lambda) l(\lambda) d \lambda \\
M_{t} & =\int t(\lambda) m(\lambda) d \lambda \\
S_{t} & =\int t(\lambda) s(\lambda) d \lambda
\end{aligned}
$$

These are the only three numbers used to determine color.
Any pair of stimuli that result in the same three numbers will be indistinguishable.
How many primaries do you think we'll need to match $t$ ?

## Color matching

Let's assume that we need 3 primaries to perform the color matching experiment.

Consider three primaries, $a(\lambda), b(\lambda), c(\lambda)$, with three emissive power knobs, $A, B, C$.

The three knobs create spectra of the form:

$$
e(\lambda)=A a(\lambda)+B b(\lambda)+C c(\lambda)
$$

What is the response of the l-cone?

$$
\begin{aligned}
L_{a b c} & =\int e(\lambda) l(\lambda) d \lambda \\
& =\int[A a(\lambda)+B b(\lambda)+C c(\lambda)] l(\lambda) d \lambda \\
& =\int A a(\lambda) l(\lambda) d \lambda+\int B b(\lambda) l(\lambda) d \lambda+\int C c(\lambda) l(\lambda) d \lambda \\
& =A \int a(\lambda) l(\lambda) d \lambda+B \int b(\lambda) l(\lambda) d \lambda+C \int c(\lambda) l(\lambda) d \lambda \\
& =A L_{a}+B L_{b}+C L_{c}
\end{aligned}
$$

How about the m - and s -cones?

## Color matching, cont'd

We end up with similar relations for all the cones:

$$
\begin{aligned}
L_{a b c} & =A L_{a}+B L_{b}+C L_{c} \\
M_{a b c} & =A M_{a}+B M_{b}+C M_{c} \\
S_{a b c} & =A S_{a}+B S_{b}+C S_{c}
\end{aligned}
$$

We can re-write this as a matrix:

$$
\left[\begin{array}{c}
L_{a b c} \\
M_{a b c} \\
S_{a b c}
\end{array}\right]=\left[\begin{array}{ccc}
L_{a} & L_{b} & L_{c} \\
M_{a} & M_{b} & M_{c} \\
S_{a} & S_{b} & S_{c}
\end{array}\right]\left[\begin{array}{l}
A \\
B \\
C
\end{array}\right]
$$

and then solve for the knob settings:

$$
\left[\begin{array}{l}
A \\
B \\
C
\end{array}\right]=\left[\begin{array}{ccc}
L_{a} & L_{b} & L_{c} \\
M_{a} & M_{b} & M_{c} \\
S_{a} & S_{b} & S_{c}
\end{array}\right]^{-1}\left[\begin{array}{c}
L_{a b c} \\
M_{a b c} \\
S_{a b c}
\end{array}\right]
$$

In other words, we can choose the knob settings to cause the cones to react as we please!
Well, one little "gotcha" - we may need to set the knob values to be negative.

## Choosing Primaries

The primaries could be three color (monochromatic) lasers.

But, they can also be non-monochromatic, e.g., monitor phosphors:


Emission spectra for RGB monitor phosphors (Wandell B.3)

## Emission Spectrum is not Color

Recall how much averaging the eye does. Light is infinite dimensional!

Different light sources can evoke exactly the same colors. Such lights are called metamers.


## Colored Surfaces

So far, we've discussed the colors of lights. How do surfaces acquire color?


Subtractive colour mixing (Wasserman 2.2)
A surface's reflectance, $\rho(\lambda)$, is its tendency to reflect incoming light across the spectrum.

Reflectance is combined "subtractively" with incoming light. Actually, the process is multiplicative:

$$
I(\lambda)=\rho(\lambda) t(\lambda)
$$

## Subtractive Metamers



Surfaces that are metamers under only some lighting conditions (Wasserman 3.9)

Reflectance adds a whole new dimension of complexity to color perception.

The solid curve appears green indoors and out. The dashed curve looks green outdoors, but brown under incandescent light.

## Illustration of Color Appearance



How light and reflectance become cone responses (Wandell, 9.2)

## Lighting design

When deciding the kind of "feel" for an architectural space, the spectra of the light sources is critical.

Lighting design centers have displays with similar scenes under various lighting conditions.

For example:


We have one such center on Capitol Hill: The Northwest Lighting Design Lab.
http://www.northwestlighting.com/
Go visit in person sometime - it's really cool!!

## The CIE XYZ System

A standard created in 1931 by CIE, defined in terms of three color matching functions.


The XYZ color matching functions (Wasserman 3.8)
These functions are related to the cone responses as roughly:

$$
\begin{aligned}
& \bar{x}(\lambda) \approx k_{1} s(\lambda)+k_{2} l(\lambda) \\
& \bar{y}(\lambda) \approx k_{3} m(\lambda) \\
& \bar{z}(\lambda) \approx k_{4} s(\lambda)
\end{aligned}
$$

## CIE Coordinates

Given an emission spectrum, we can use the CIE matching functions to obtain the $X, Y$ and $Z$ coordinates.

$$
\begin{aligned}
X & =\int \bar{x}(\lambda) t(\lambda) d \lambda \\
Y & =\int \bar{y}(\lambda) t(\lambda) d \lambda \\
Z & =\int \bar{z}(\lambda) t(\lambda) d \lambda
\end{aligned}
$$

Using the equations from the previous page, we can see that XYZ coordinates are closely related to LMS responses.

## The CIE Colour Blob



Different views of the CIE color space (Foley II.1)


The chromaticity diagram (a kind of slice through CIE space, Wasserman 3.7)

## Gamuts

Not every output device can reproduce every color. A device's range of reproducible colors is called its gamut.


## RGB

## Color Spaces for Computer Graphics

In practice, there's a set of more commonly-used color spaces in computer graphics:

- RGB for display
- CMY (or CMYK) for hardcopy
- HSV for user selection

Perhaps the most familiar color space, and the most convenient for display on a CRT.

What does the RGB color space look like?


## CMY

A subtractive color space used for printing.
Involves three subtractive primaries:

- Cyan - subtracts red
- Magenta - subtracts green
- Yellow - subtracts blue

Mixing two pigments subtracts their opposites from white.

CMYK adds black ink rather than using equal amounts of all three.

## RGB vs. CMY



## Summary

Here's what you should take home from this lecture:

- All the boldfaced terms.
- How to compute cone responses
- The difference between emissive and reflective color
- What the CIE XYZ color standard and chromaticity diagram are
- The color spaces used in computer graphics

