

# Lecture 7

## Detectors and Descriptors

Slide Credit: Ranjay Krishna

# So far: General approach for search

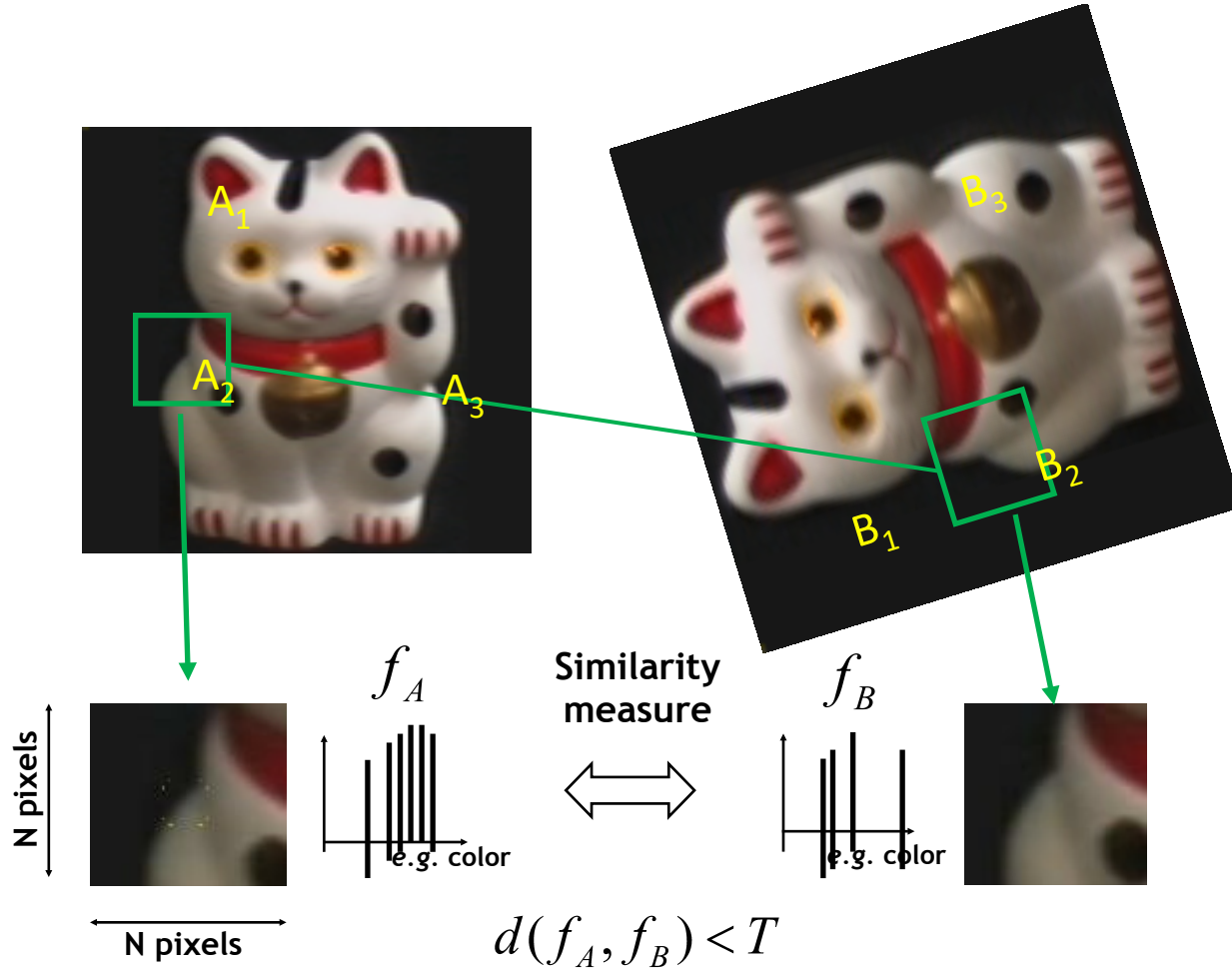
1. Find a set of distinctive **key-points**

2. Define a region/**patch** around each keypoint

3. **Normalize** the region content

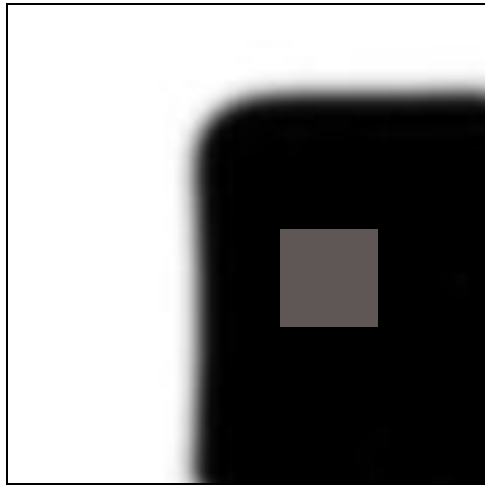
4. Compute a local **descriptor** from the normalized region

5. **Match** local descriptors

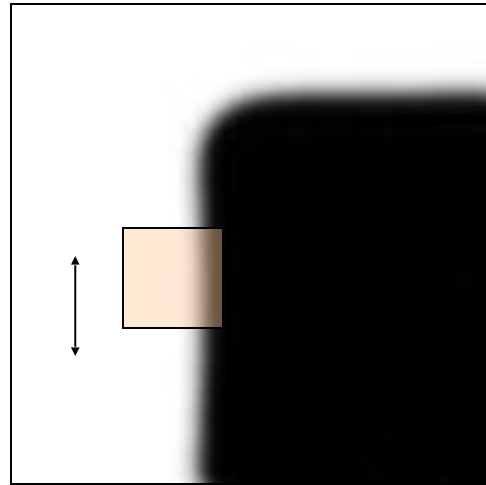


# So far: Corners as key-points

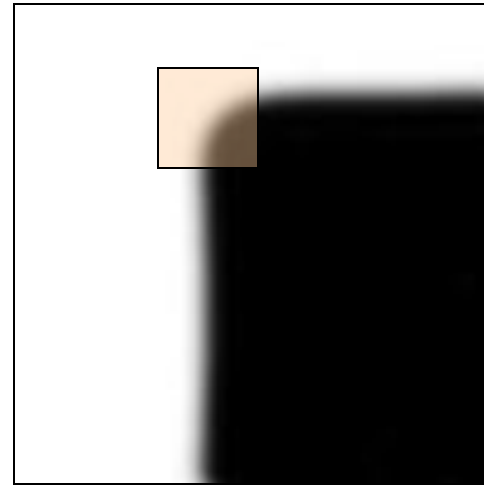
- We should easily recognize the corner point by looking through a small window (*locality*)
- Shifting the window in *any direction* should give a *large change* in intensity (*good localization*)



**“flat”** region:  
no change in  
all directions



**“edge”**:  
no change along  
the edge direction



**“corner”**:  
significant change  
in all directions

# So far: Harris Corner Detector [Harris88]

- Compute second moment matrix (autocorrelation matrix)

$$M(\sigma_I, \sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

$\sigma_D$ : for Gaussian in the derivative calculation  
 $\sigma_I$ : for Gaussian in the windowing function

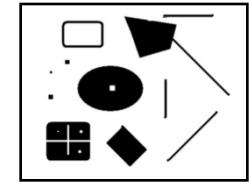
2. Square of derivatives

3. Gaussian filter  $g(\sigma_I)$

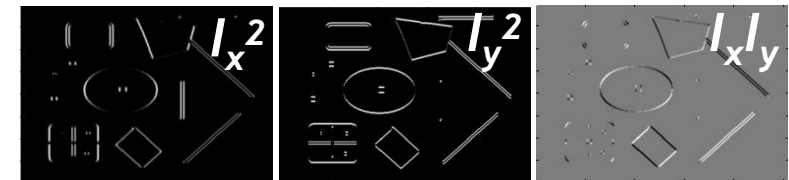
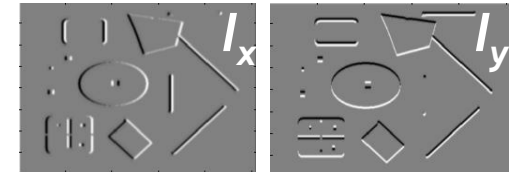
4. Cornerness function - two strong eigenvalues

$$\begin{aligned} \theta &= \det[M(\sigma_I, \sigma_D)] - \alpha [\text{trace}(M(\sigma_I, \sigma_D))]^2 \\ &= g(I_x^2)g(I_y^2) - [g(I_x I_y)]^2 - \alpha [g(I_x^2) + g(I_y^2)]^2 \end{aligned}$$

5. Perform non-maximum suppression



1. Image derivatives

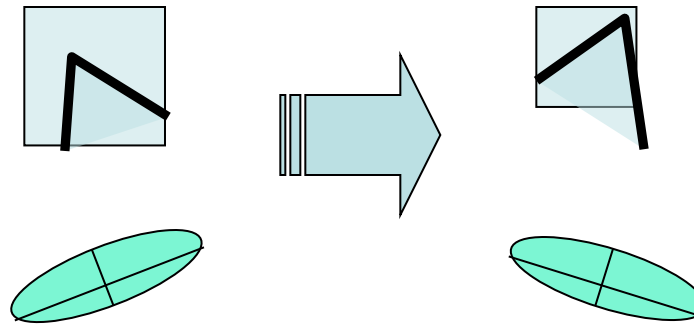


# So far: Harris Detector Properties

- Translation invariance?

# So far: Harris Detector Properties

- Translation invariance
- Rotation invariance?

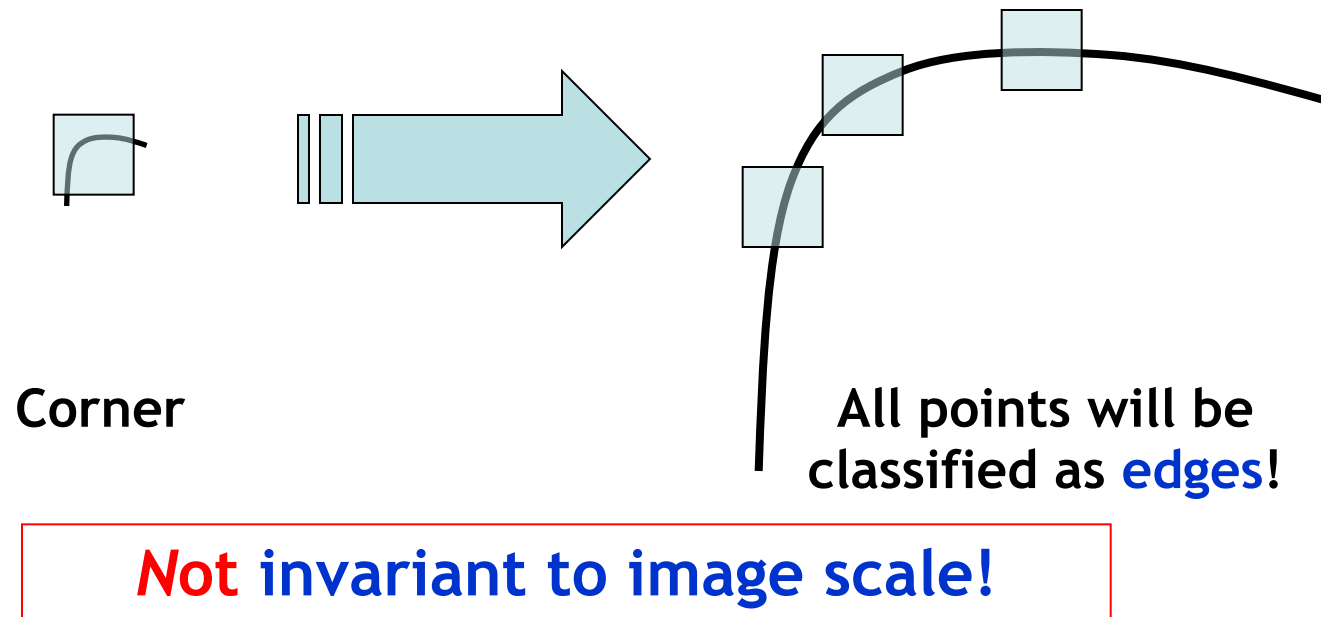


Ellipse rotates but its shape (i.e. eigenvalues) remains the same

***Corner response  $\theta$  is invariant to image rotation***

# So far: Harris Detector Properties

- Translation invariance
- Rotation invariance
- Scale invariance?



# Today's agenda

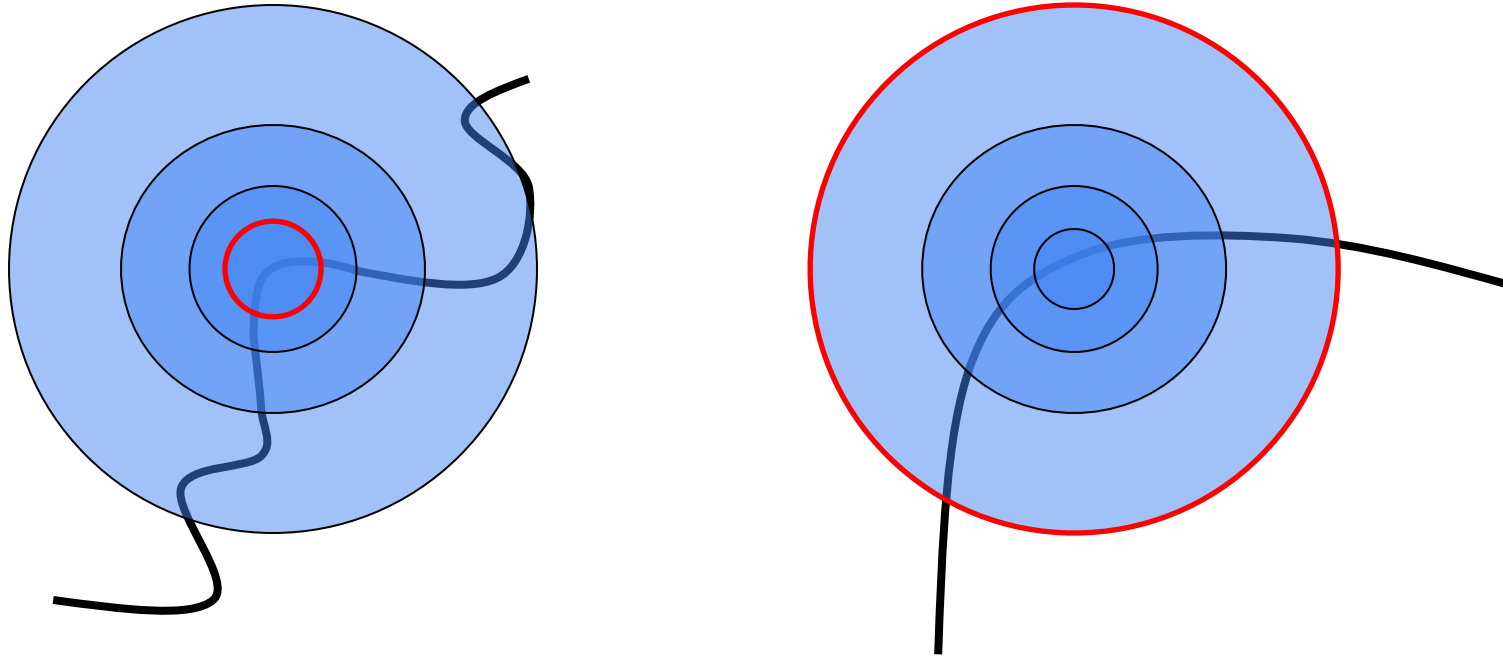
- Scale invariant keypoint detection
- Local detectors (SIFT)
- Local descriptors (SIFT)
- Global descriptors (HoG)

# What will we learn today?

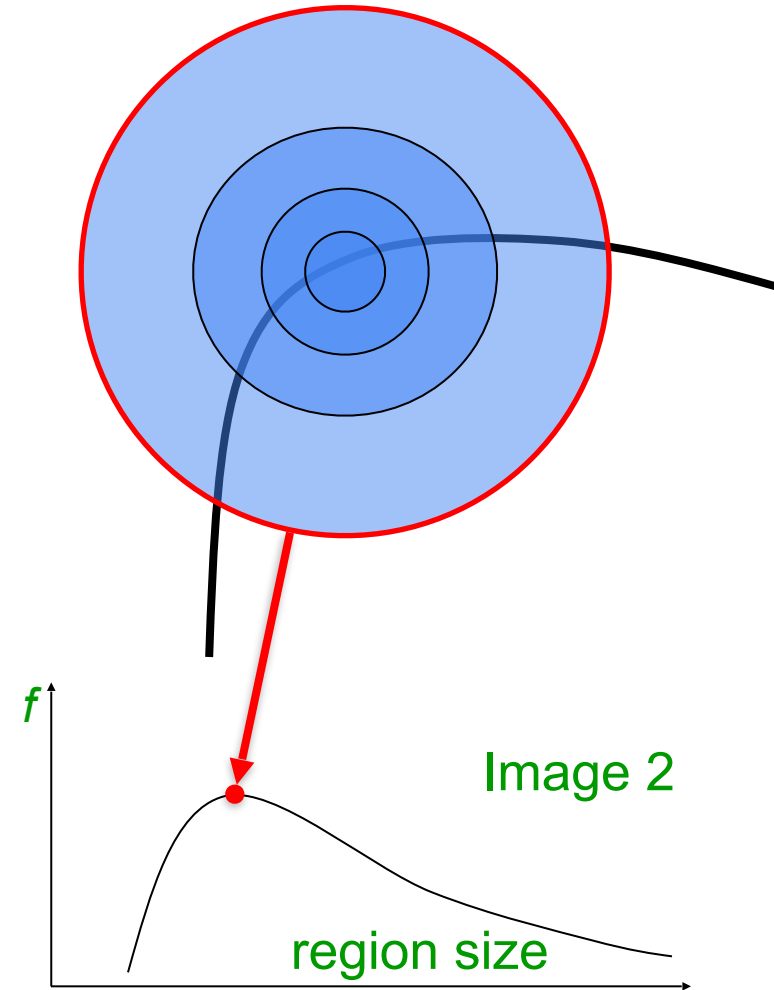
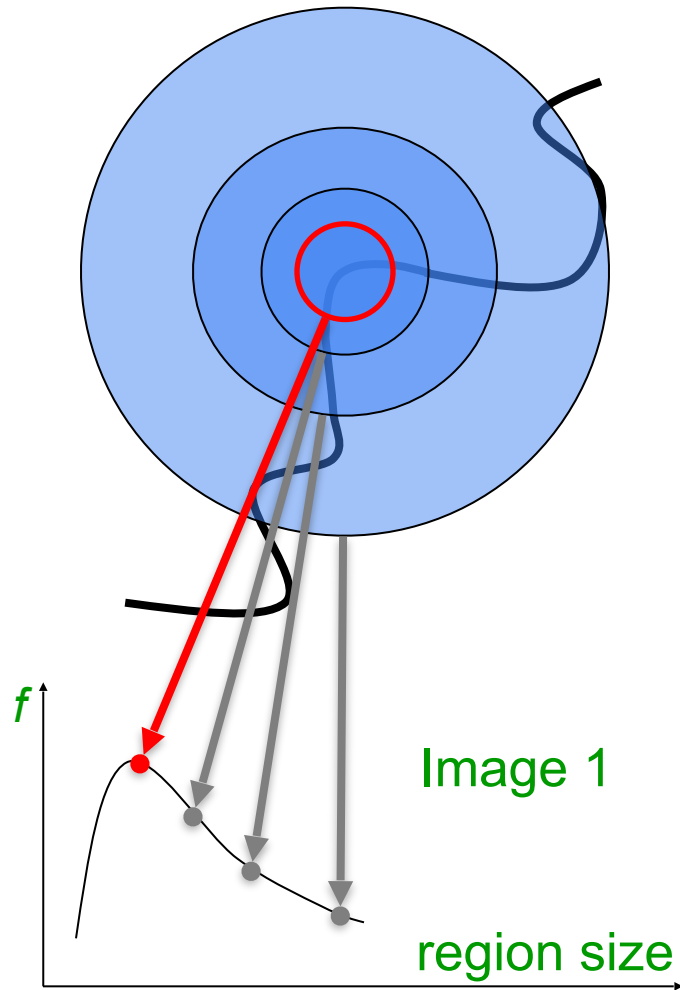
- Scale invariant keypoint detection
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# Scale Invariant Detection

- Consider regions (e.g. circles) of different sizes around a point
- What region size do we choose, so that the regions look the same in both images?

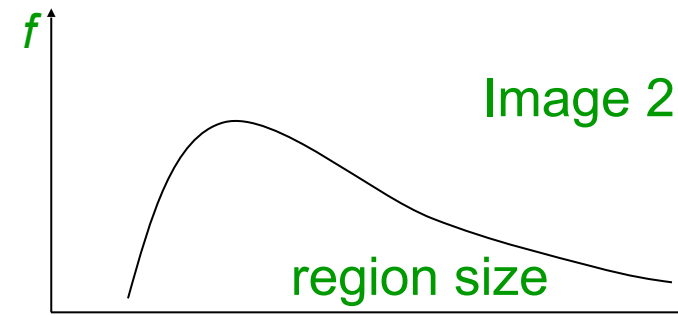
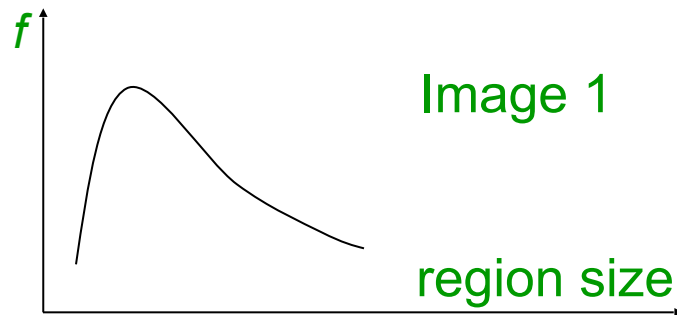


**Problem:** How do we choose region sizes **independently** in each image?



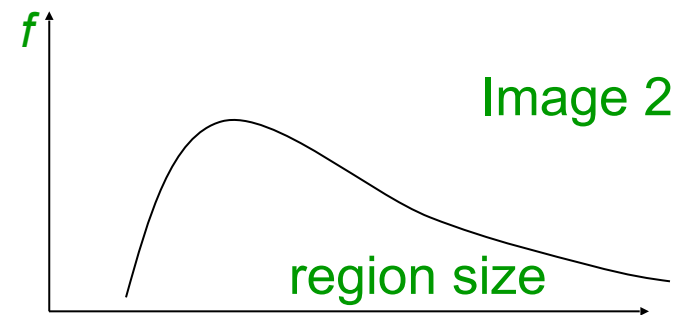
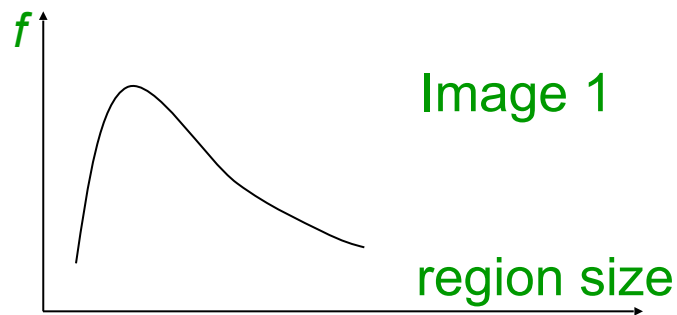
# Solution: design a “scale-invariant” detector

- Assume that the detector is made up of a **series of functions**,
  - each function depends on the pixel values and the **region’s size**
- The function on the region should have the same value even if the keypoints are at different scales



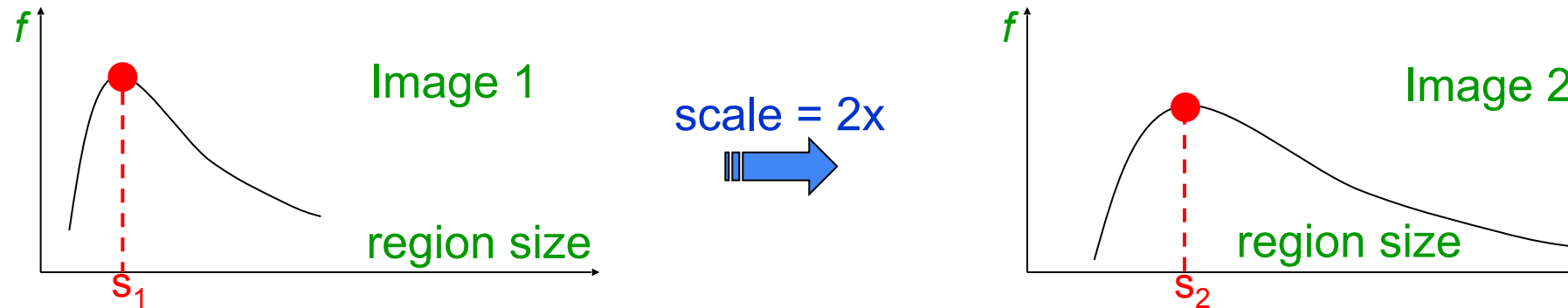
# Scale Invariant Detection

- Common approach to choose scale:
  - Take a local maximum of this function
- **Important:** this scale invariant region size is found in each image for each corner!
- **Observation:** the region size at the maximum should be *correlated* to the keypoint's scale. In other words, the size is correlated with the size of the corner



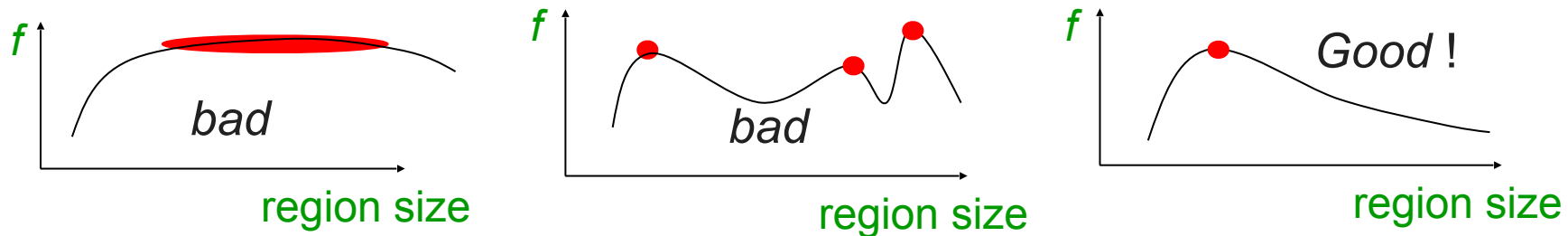
# Scale Invariant Detection

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# Scale Invariant Detection

- A “good” function for scale selection has one stable sharp peak



- For usual images: a good function would be one which responds to contrast (sharp local intensity change)

# Why we care about knowing the keypoint patch size??

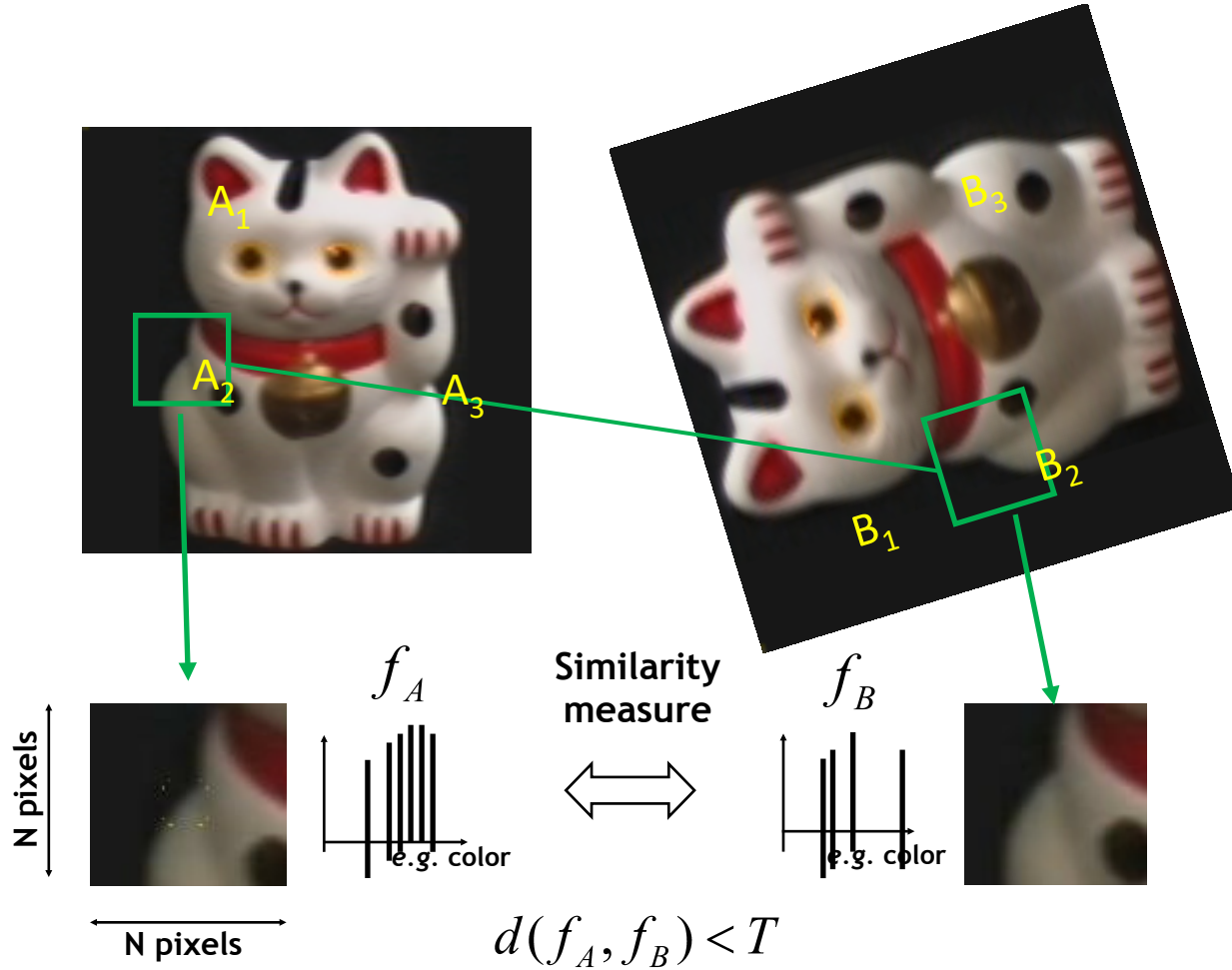
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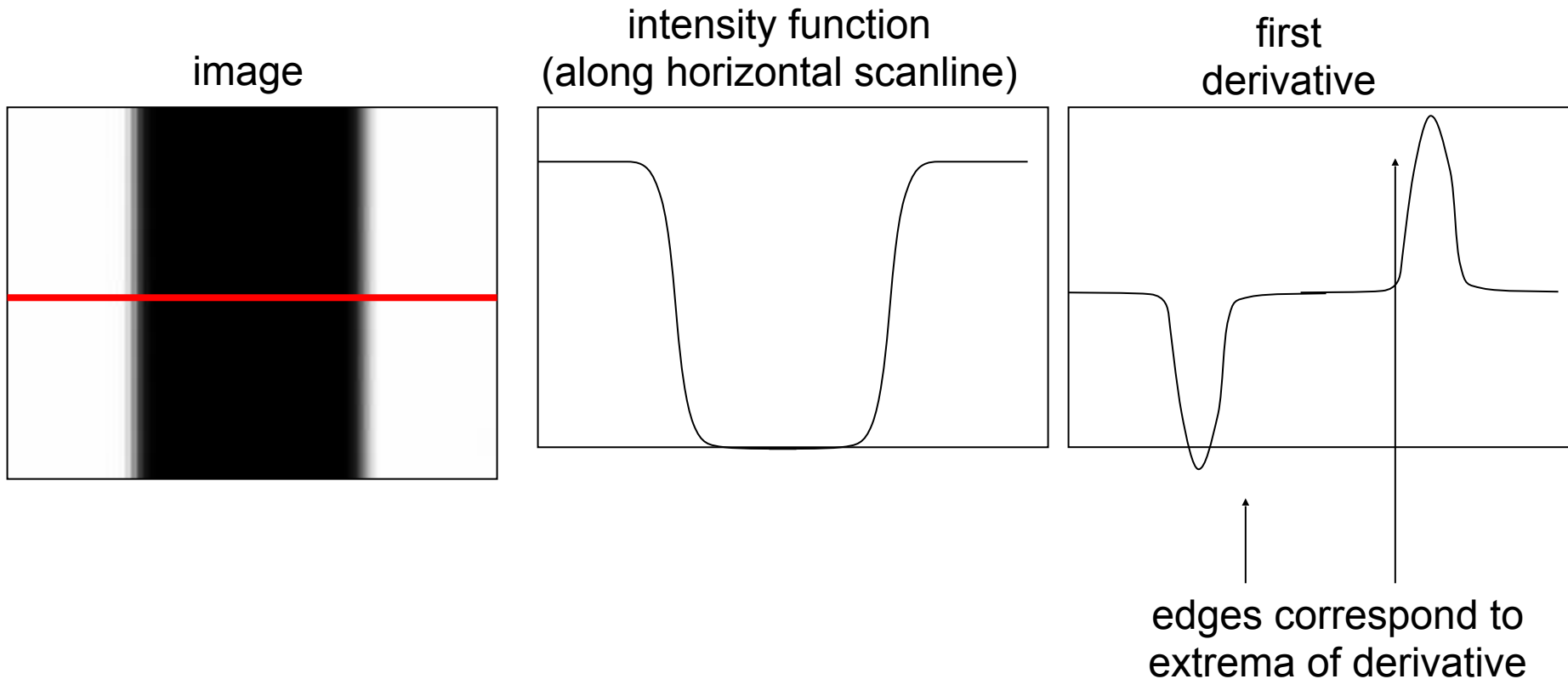
4. Compute a local **descriptor** from the normalized region

5. **Match** local descriptors



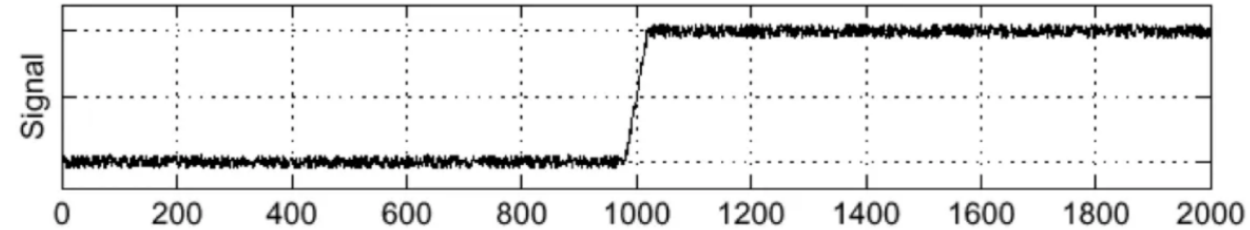
# Before we design this function, let's review: Characterizing edges

An edge is a place of rapid change in the image intensity function

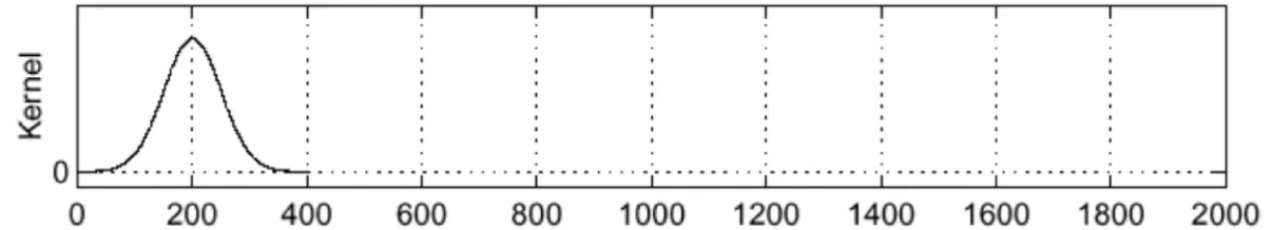


# Review: detecting edges

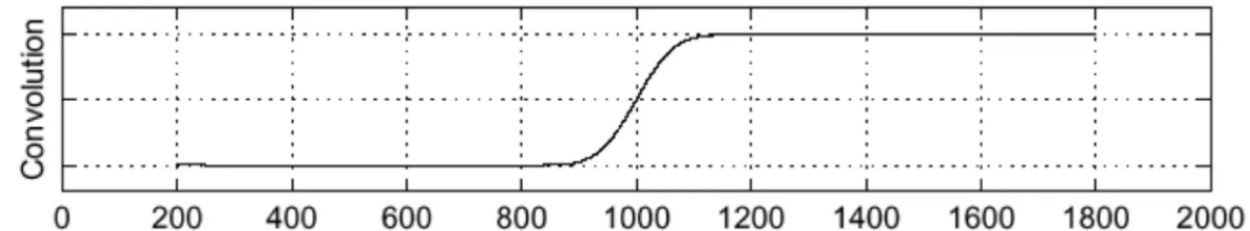
Image  $f$



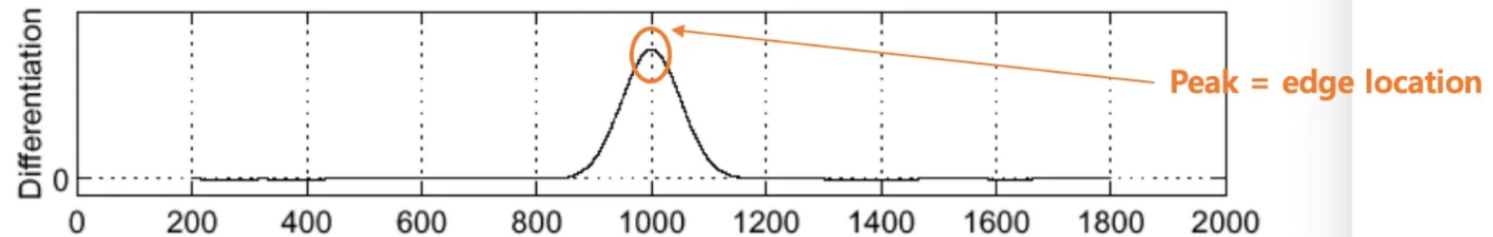
Gaussian Filter  $h$



Convolution  $h \star f$



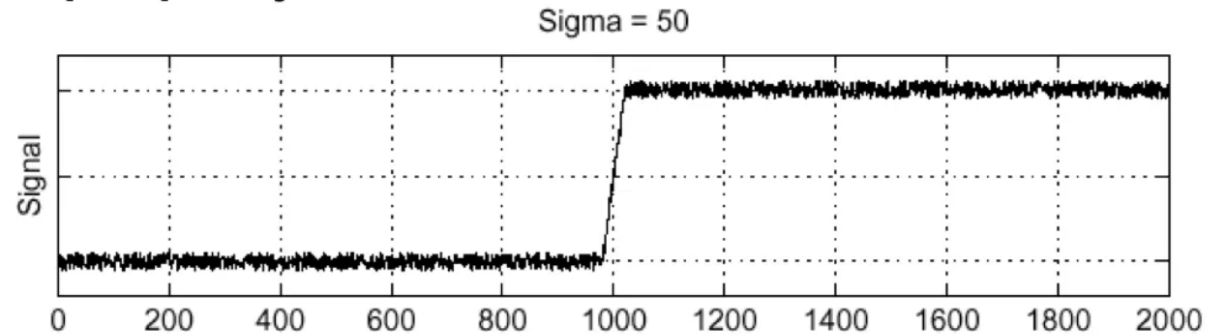
Derivative  $\frac{\partial}{\partial x}(h \star f)$



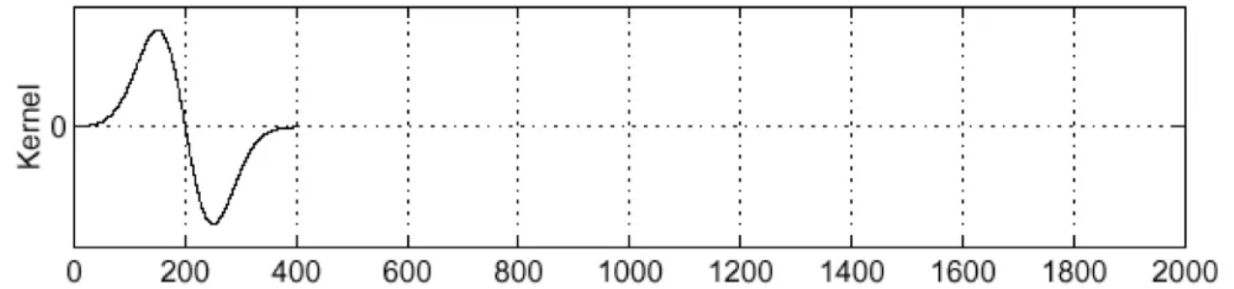
# Review: Because convolutions are linear:

$$\frac{\partial}{\partial x}(h \star f) = \left(\frac{\partial}{\partial x}h\right) \star f$$

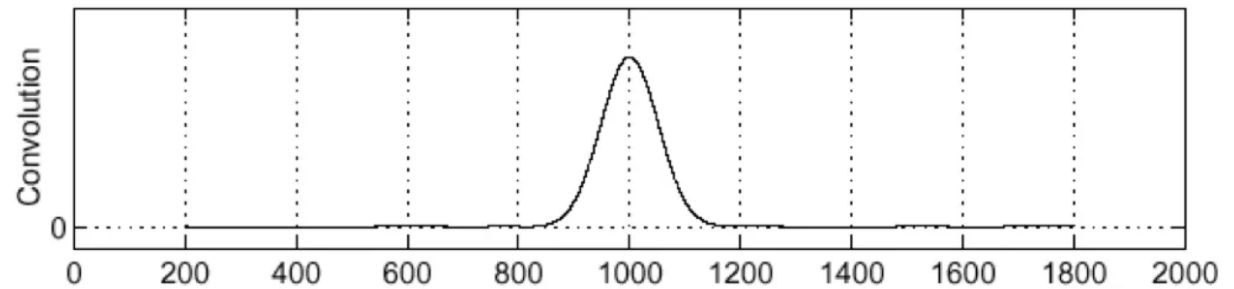
$f$



$\frac{\partial}{\partial x}h$



$\left(\frac{\partial}{\partial x}h\right) \star f$



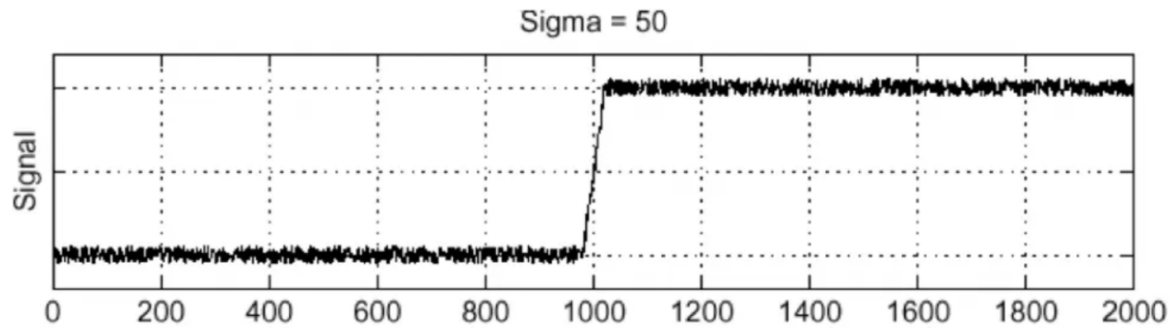
# Another similar filter: The Laplacian

$$\text{Laplacian } \nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$$

0	-1	0
-1	4	-1
0	-1	0

# Another similar filter: The **Laplacian** (second derivative) of a Gaussian

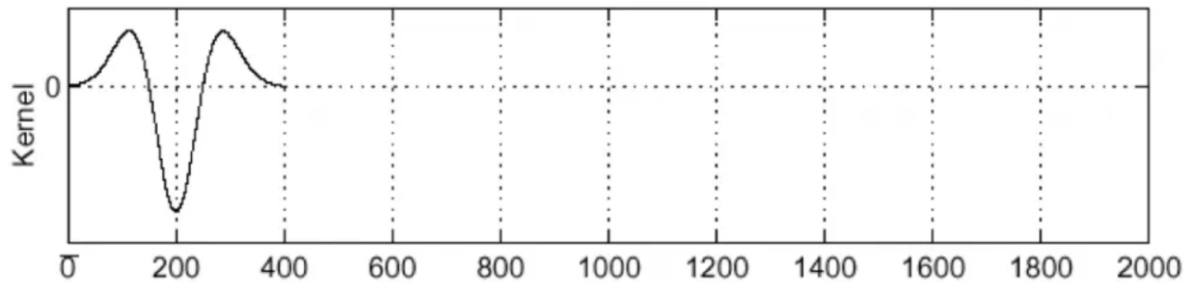
$f$



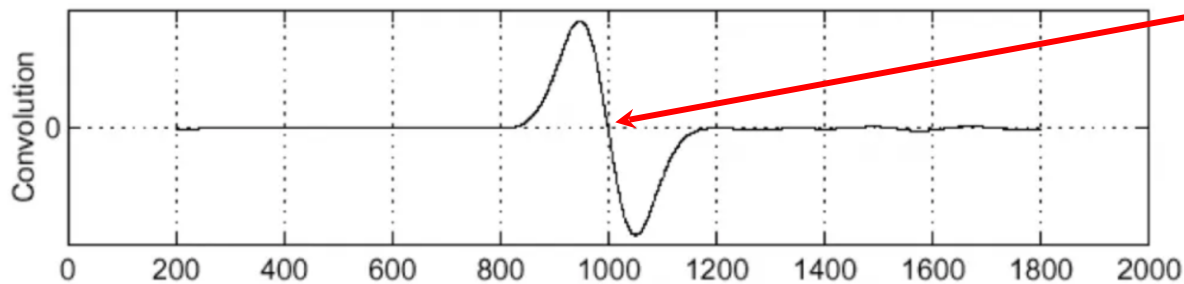
0	-1	0
-1	4	-1
0	-1	0

$\frac{\partial^2}{\partial x^2} h$

Laplacian of Gaussian

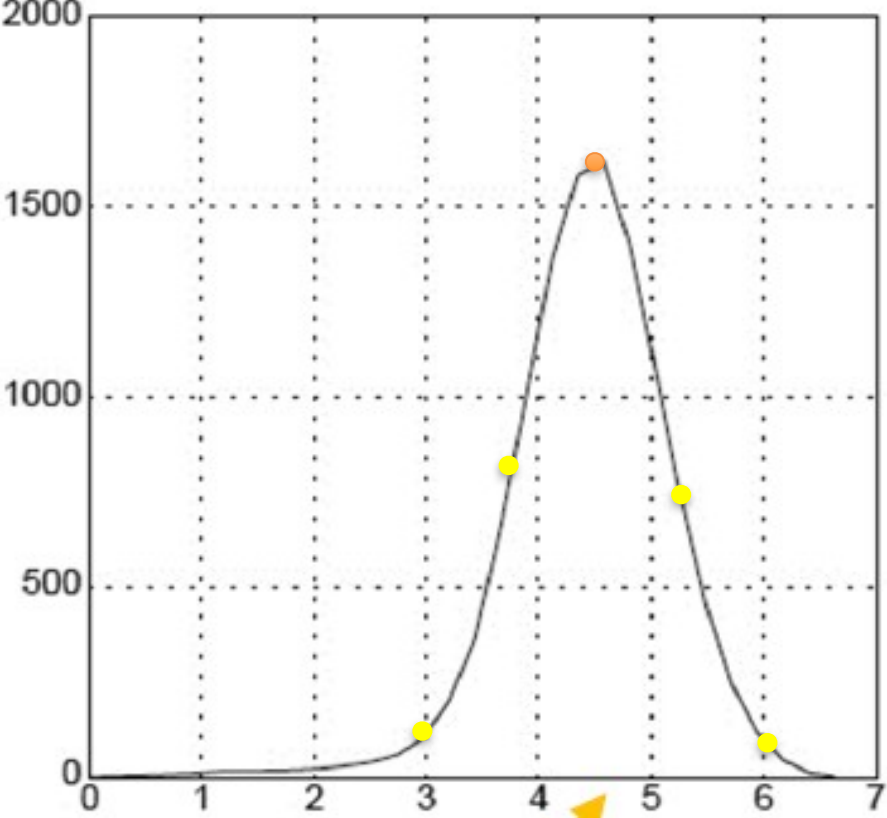
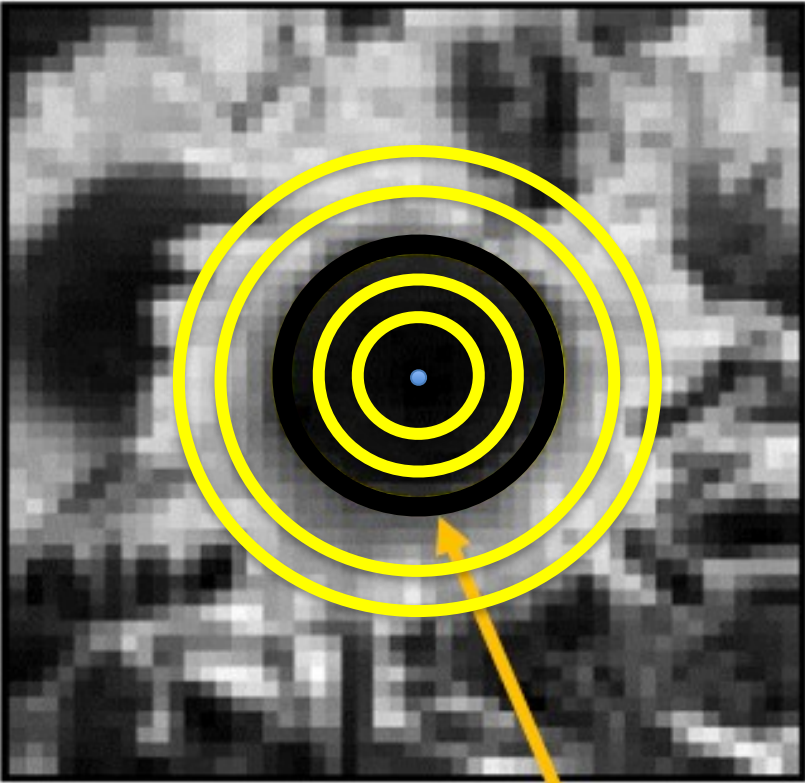


$(\frac{\partial^2}{\partial x^2} h) \star f$



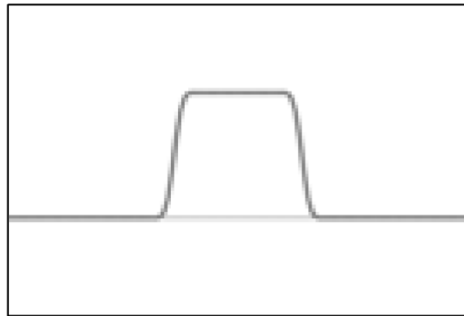
Edge at zero crossing

# Laplacian of a Gaussian

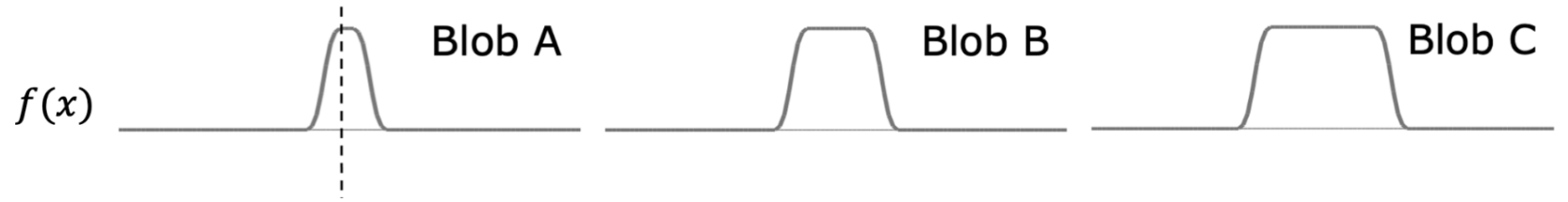


Characteristic scale

LoG is very good to detecting not just edges or corners but any “**blob**”

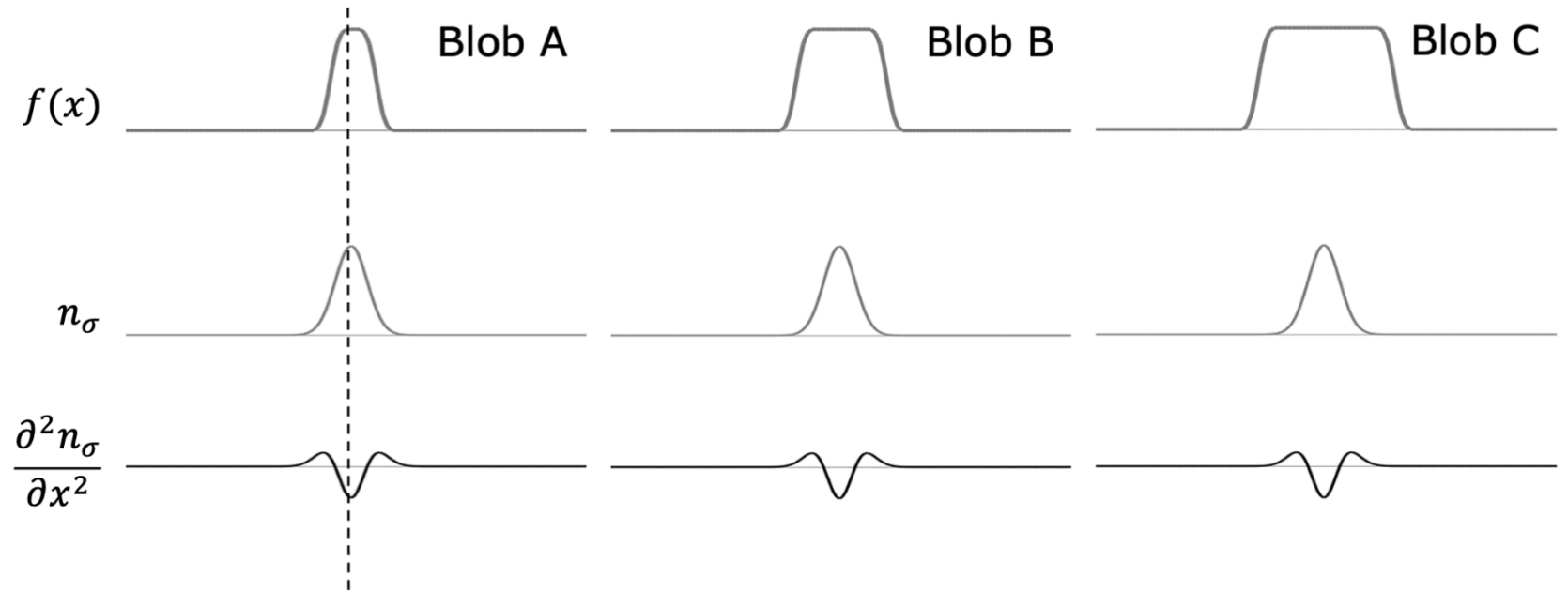


1D  
example  
of how  
blobs are  
detected  
with LoG

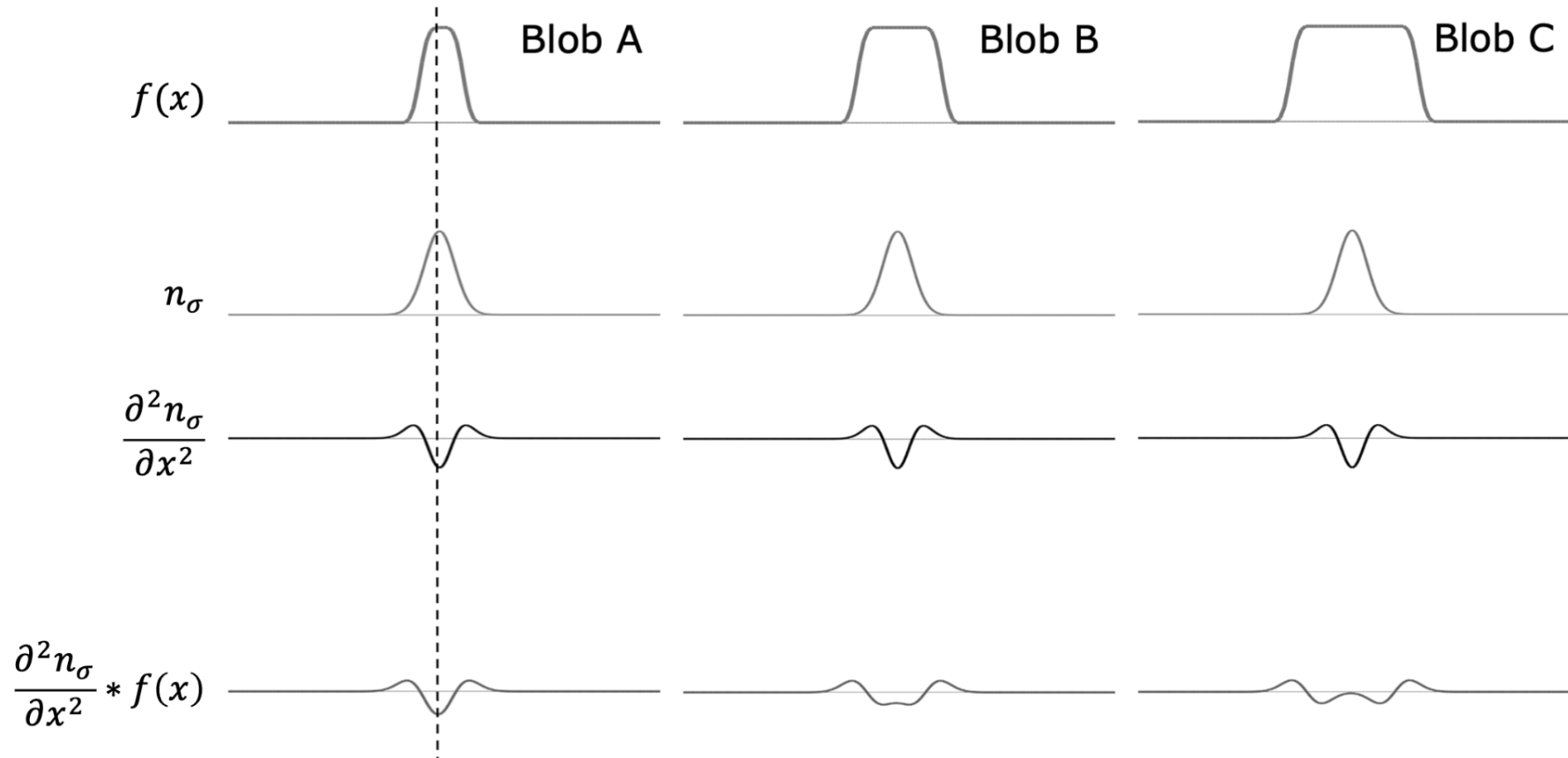


Blob B is 2x as wide as blob A  
Blob C is 3x as wide as blob B

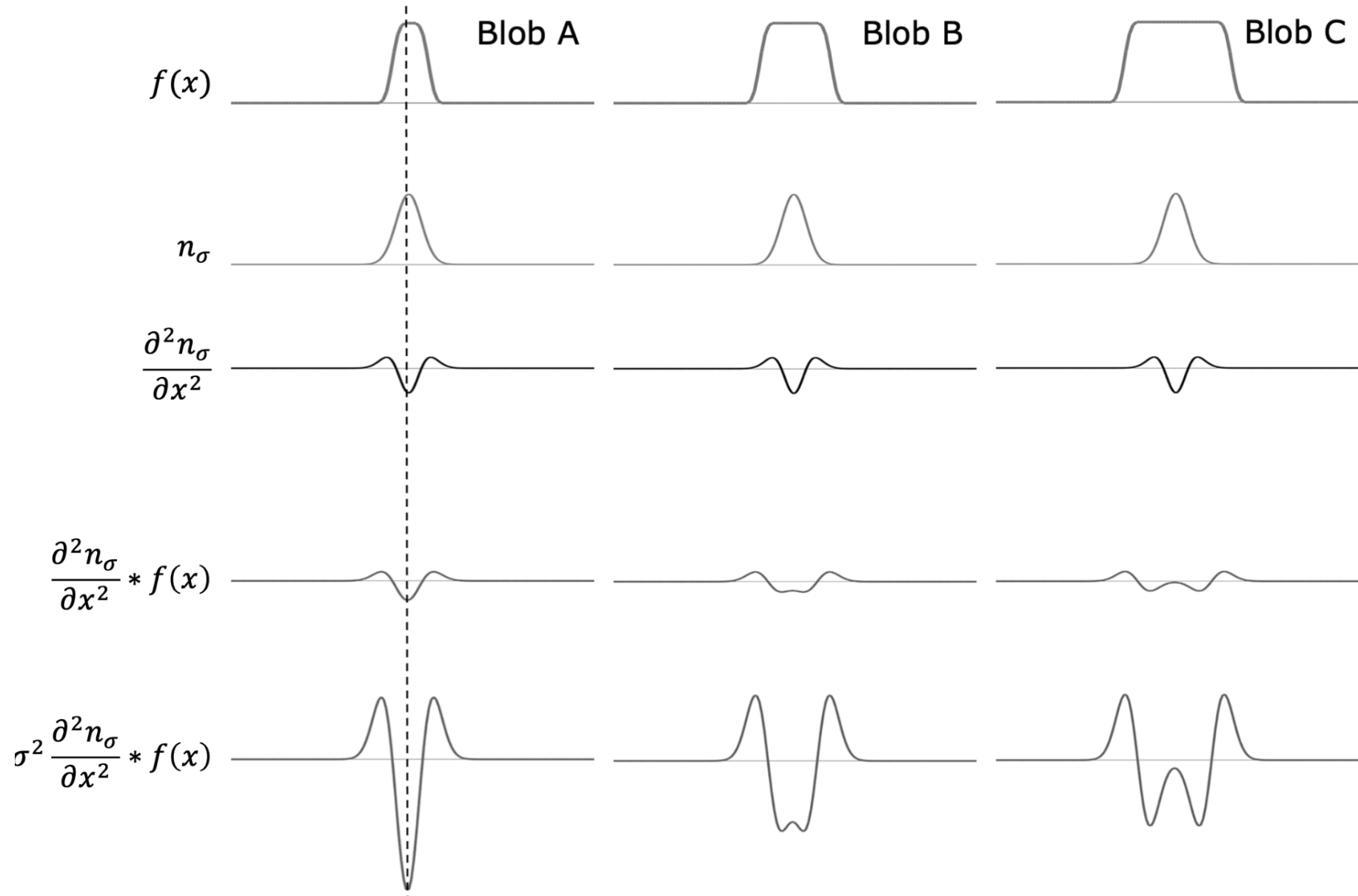
1D  
example  
of how  
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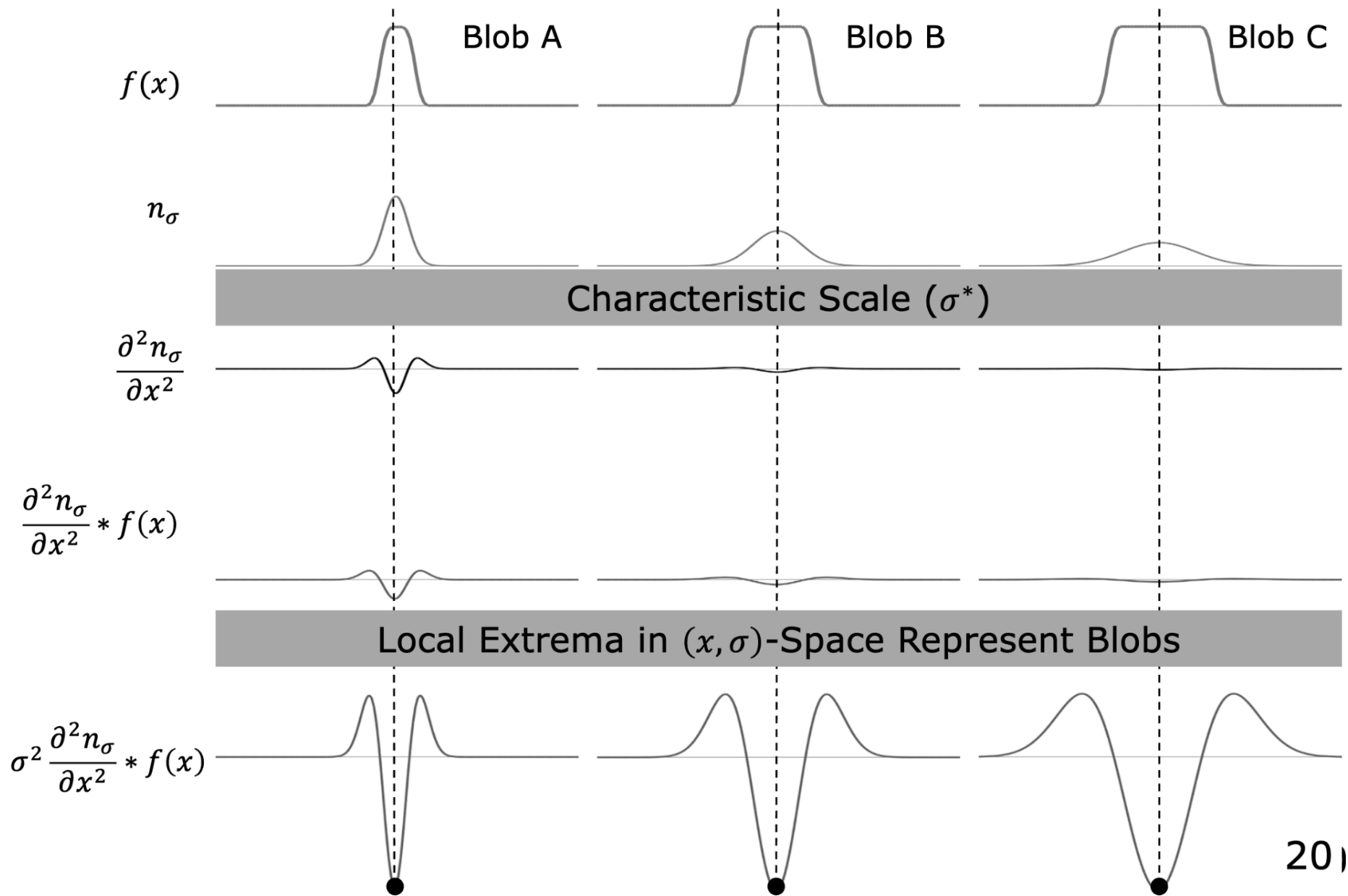
1D  
example  
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1D  
example  
of how  
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with LoG



By increasing sigma, we can detect blobs of different sizes



Given: 1D signal  $f(x)$

Compute:  $\sigma^2 \frac{\partial^2 n_\sigma}{\partial x^2} * f(x)$  at many scales  $(\sigma_0, \sigma_1, \sigma_2, \dots, \sigma_k)$ .

Find:  $(x^*, \sigma^*) = \arg \max_{(x, \sigma)} \left| \sigma^2 \frac{\partial^2 n_\sigma}{\partial x^2} * f(x) \right|$

$x^*$ : Blob Position

$\sigma^*$ : Characteristic Scale (Blob Size)

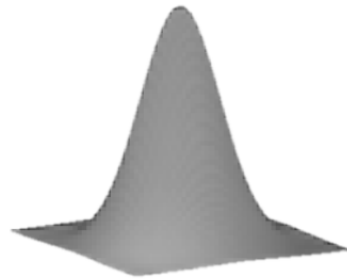
# Example in 2D

Normalized LoG (NLoG) is used to find blobs in images

Laplacian

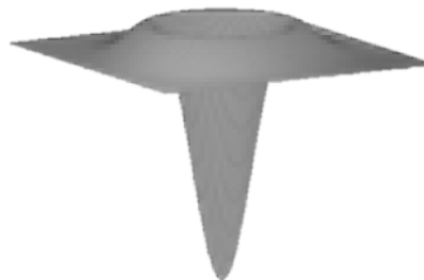
$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Gaussian



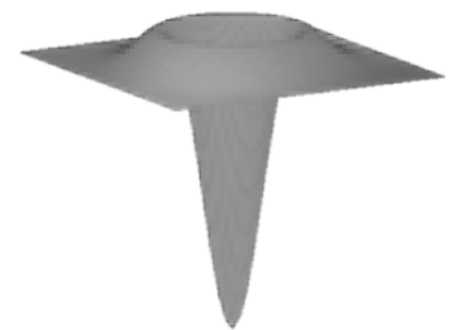
$n_\sigma$

LoG



$\nabla^2 n_\sigma$

NLoG



$\sigma^2 \nabla^2 n_\sigma$

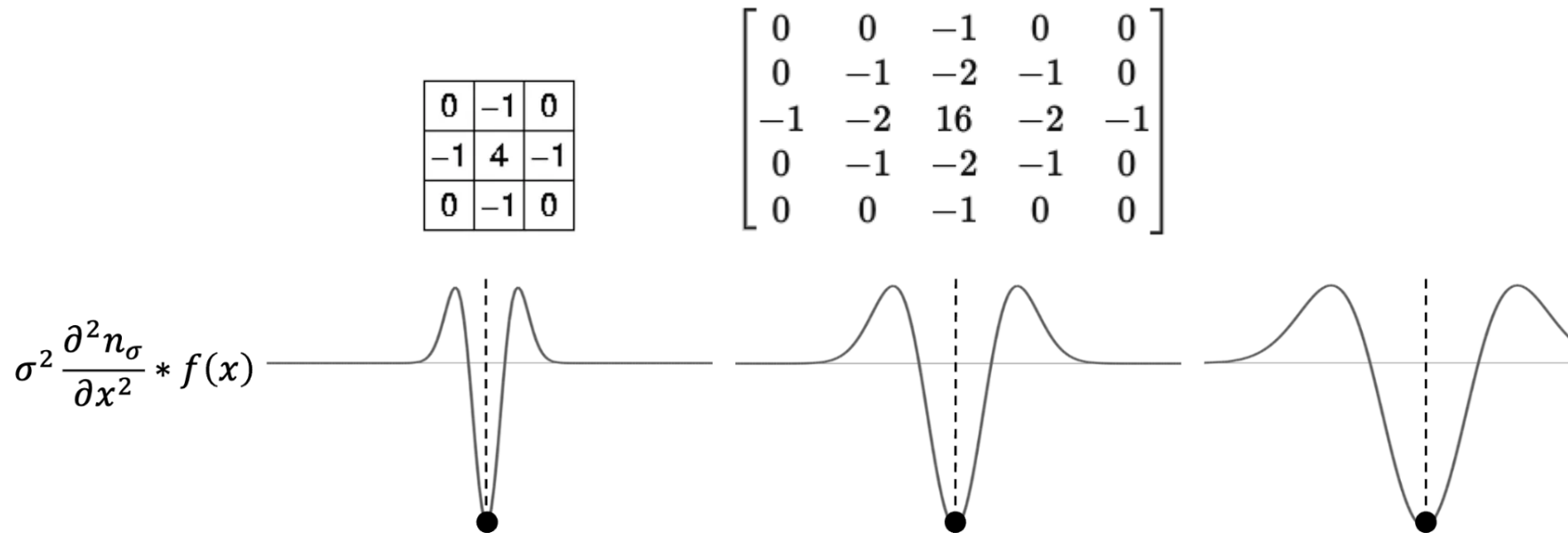
Location of Blobs identified by Local maxima after applying NLoG at many scales.

# What do laplacian filters look like?

The size of the filter increases with increasing sigma.

Meaning that larger blobs require a larger filter.

Q. Why is this a problem?



# This is a very expensive algorithm!

Given an image  $I(x, y)$

Convolve the image using NLoG at many scales  $\sigma$

Find:

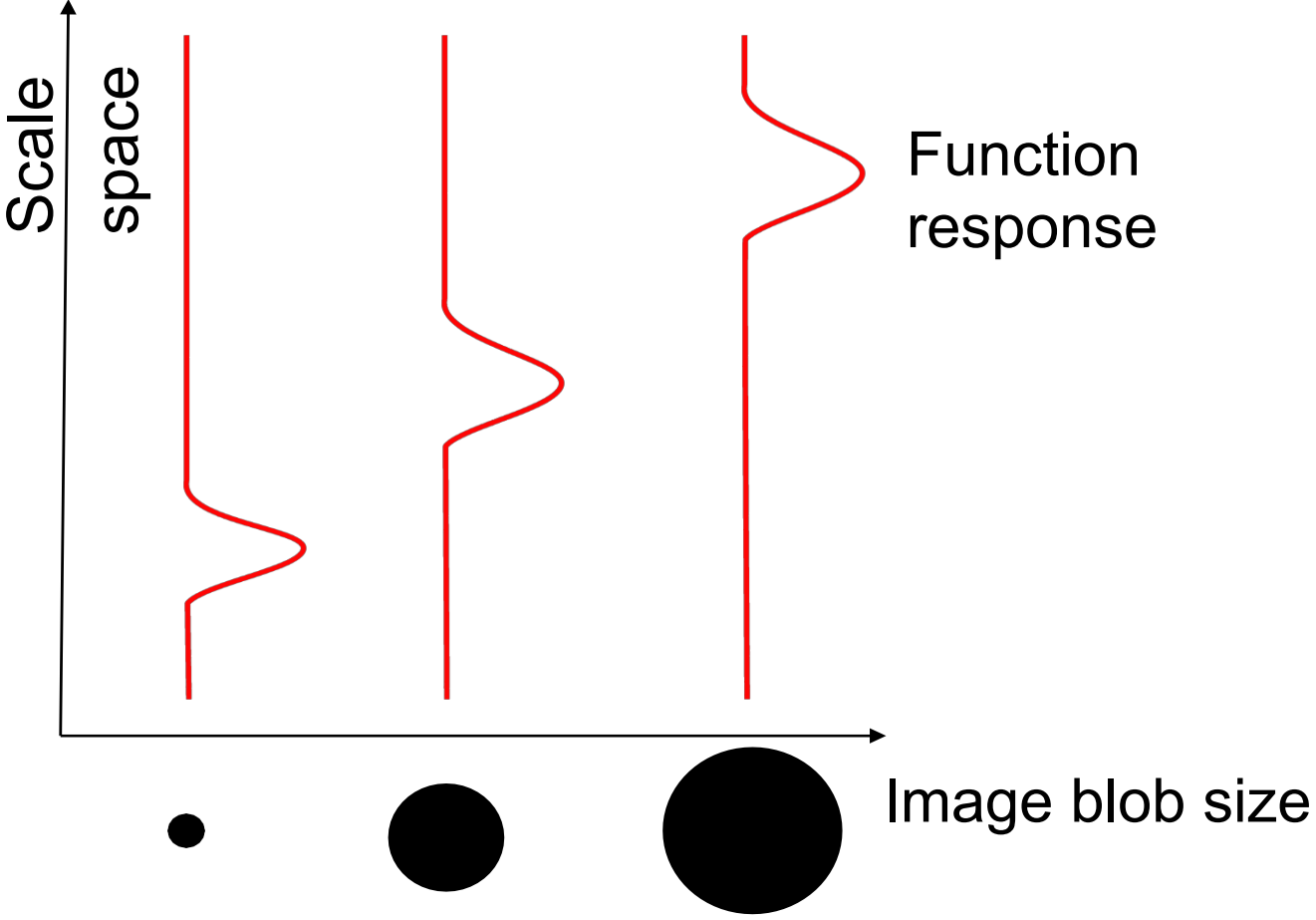
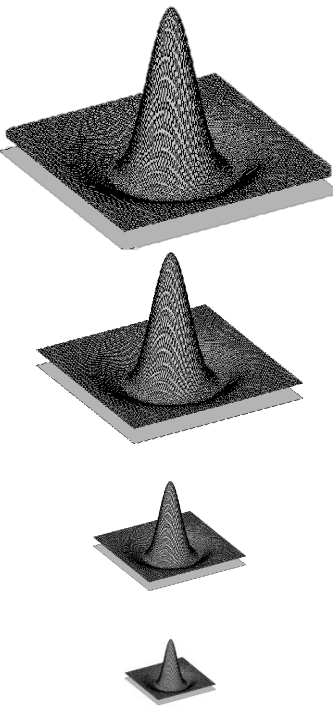
$$(x^*, y^*, \sigma^*) = \arg \max_{(x, y, \sigma)} |\sigma^2 \nabla^2 n_\sigma * I(x, y)|$$

$(x^*, y^*)$ : Position of the blob

$\sigma^*$ : Size of the blob

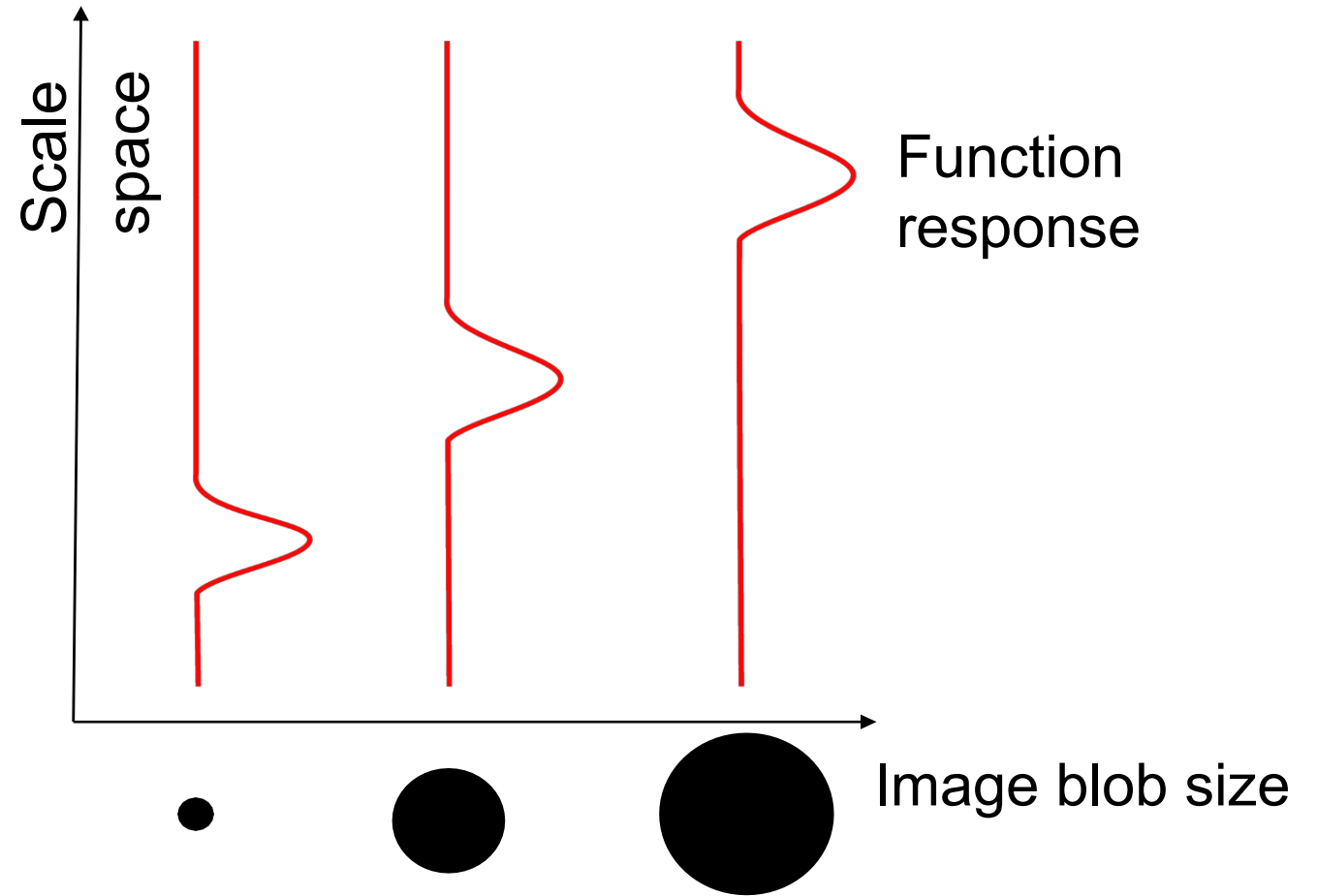
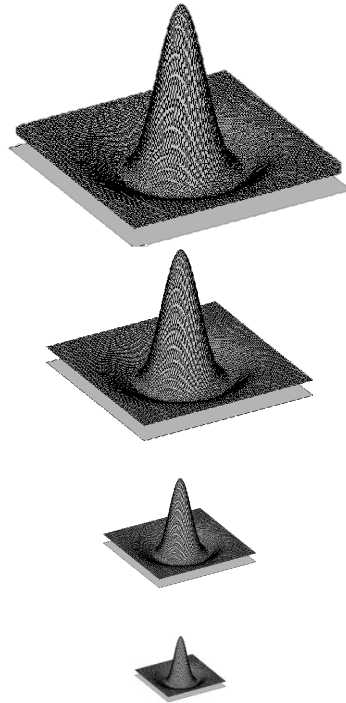
# Laplacian of a Gaussian

Laplacian (2<sup>nd</sup> derivative) of Gaussian (LoG)



**Problem:** We have to convolve multiple filters of different sized laplacians to find all blobs. This is computationally expensive!

Laplacian (2<sup>nd</sup> derivative) of Gaussian (LoG)



# Today's agenda

- Scale invariant keypoint detection
- **Local detectors (SIFT)**
- Local descriptors (SIFT)
- Global descriptors (HoG)

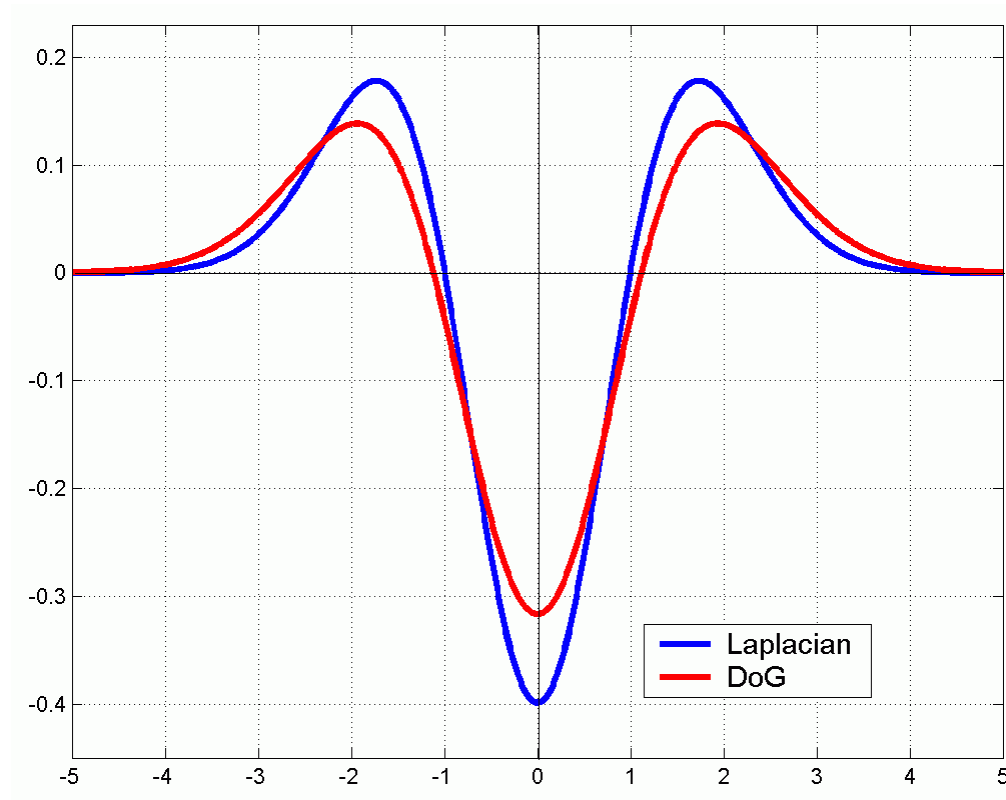
# The LoG is very similar to the difference of Gaussians (DoG)



$$f * g(1) - f * g(2) = f * (g(1) - g(2))$$

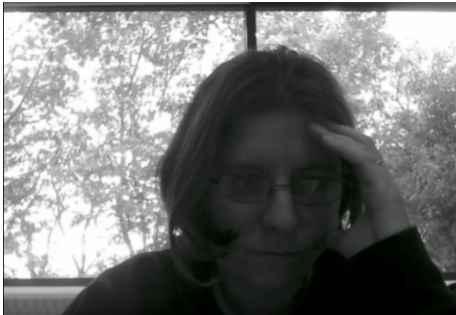


# LoG and DoG are very similar



Note: both filters are invariant to *scale* and *rotation*

# Why does this approximation matter?



Original video



Blurred with a  
Gaussian kernel



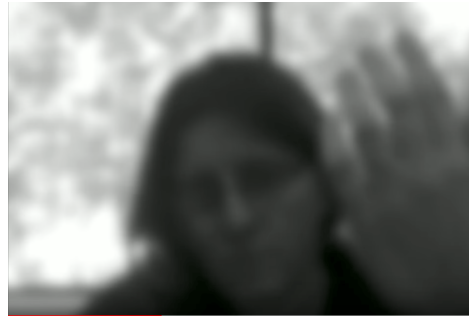
Blurred with a different  
Gaussian kernel

Q. What happens if you subtract one blurred image from another?

# Difference of Gaussians (DoG) example



Original video



Blurred with a  
Gaussian kernel:  $k_1$



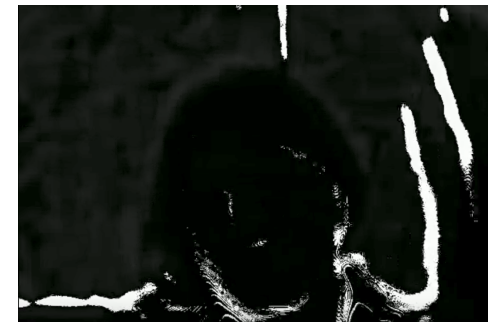
Blurred with a different  
Gaussian kernel:  $k_2$



DoG:  $k_1 - k_2$



DoG:  $k_1 - k_3$



DoG:  $k_1 - k_4$

# Example in 2D



$S(x, y, \sigma_0)$



$S(x, y, \sigma_1)$



$S(x, y, \sigma_2)$



$S(x, y, \sigma_3)$

...

Increasing  $\sigma$ , Higher Scale, Lower Resolution

**Scale Space:** Stack of images created by filtering an image with Gaussians of different sigma values

$$S(x, y, \sigma) = n(x, y, \sigma) * I(x, y)$$

# Example in 2D



$S(x, y, \sigma_0)$



$S(x, y, \sigma_1)$



$S(x, y, \sigma_2)$



$S(x, y, \sigma_3)$

...

Increasing  $\sigma$ , Higher Scale, Lower Resolution

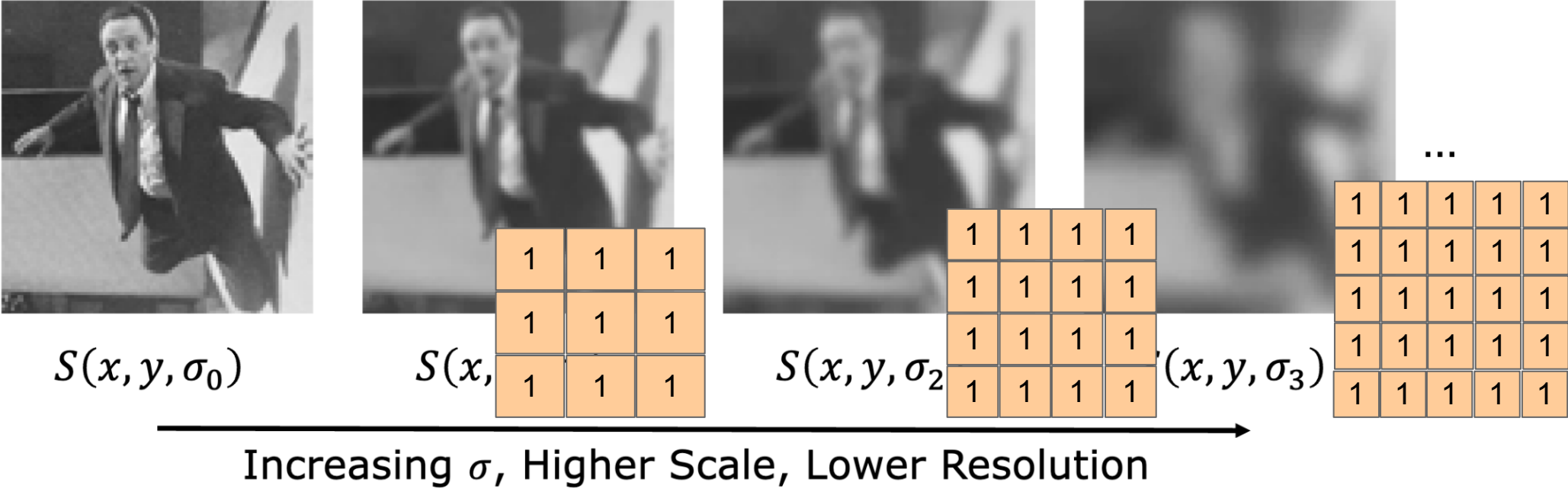
Selecting sigmas to generate the scale-space:

$$\sigma_k = \sigma_0 s^k \quad k = 0, 1, 2, 3, \dots$$

$s$ : Constant multiplier

$\sigma_0$ : Initial Scale

# sigma represents size of a filter



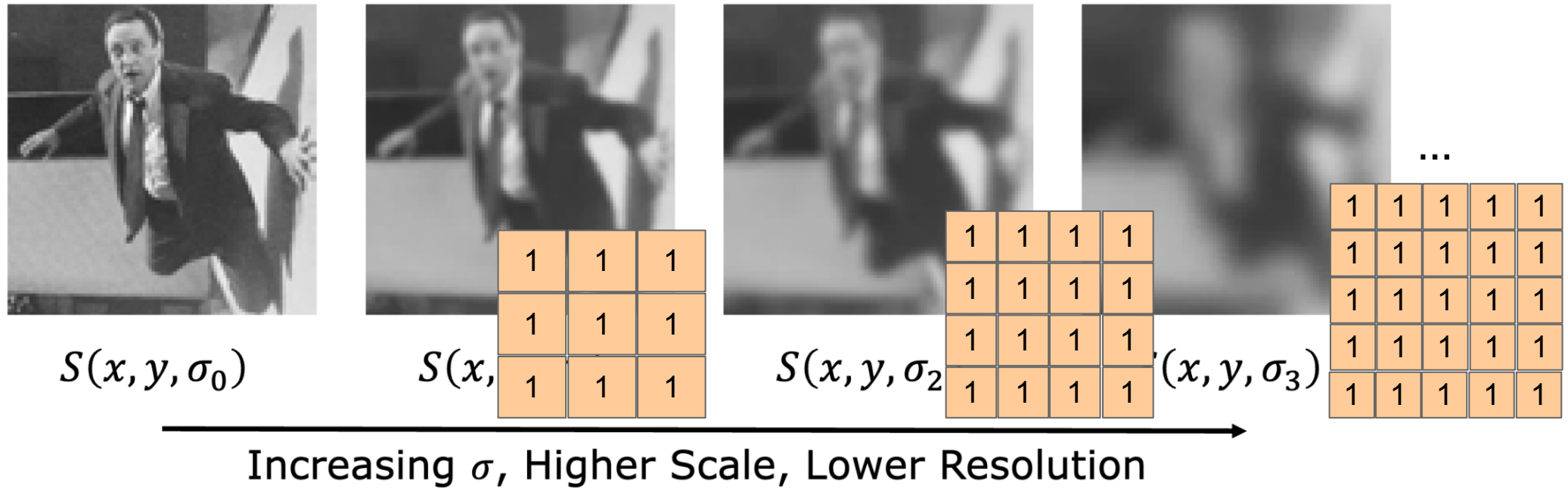
Selecting sigmas to generate the scale-space:

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But wait, aren't we again using larger filter? Doesn't this mean that using Gaussian filters is computationally expensive too?



Selecting sigmas to generate the scale-space:

$$\sigma_k = \sigma_0 s^k \quad k = 0, 1, 2, 3, \dots$$

$s$ : Constant multiplier

$\sigma_0$ : Initial Scale

# Remember from A1: Gaussian kernels are separable

Convolving with two 1D convolution filters = convolving with a large 2D filter

1
1
1
1
1

1	1	1	1	1
---	---	---	---	---

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

# What do laplacian filters look like?

**Laplacian**

0	-1	0
-1	4	-1
0	-1	0

$$\begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & -2 & -1 & 0 \\ -1 & -2 & 16 & -2 & -1 \\ 0 & -1 & -2 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

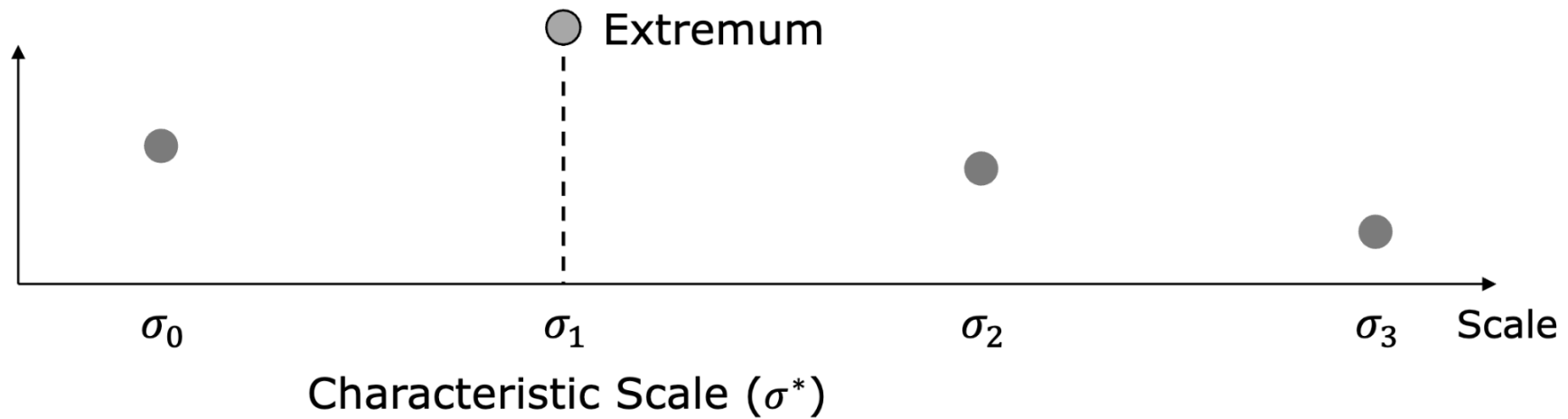
**Gaussian**

$$\frac{1}{256} \cdot \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

# Example in 2D



$$\sigma^2 \nabla^2 S(x, y, \sigma)$$



# Example in 2D



$S(x, y, \sigma_0)$



$S(x, y, \sigma_1)$

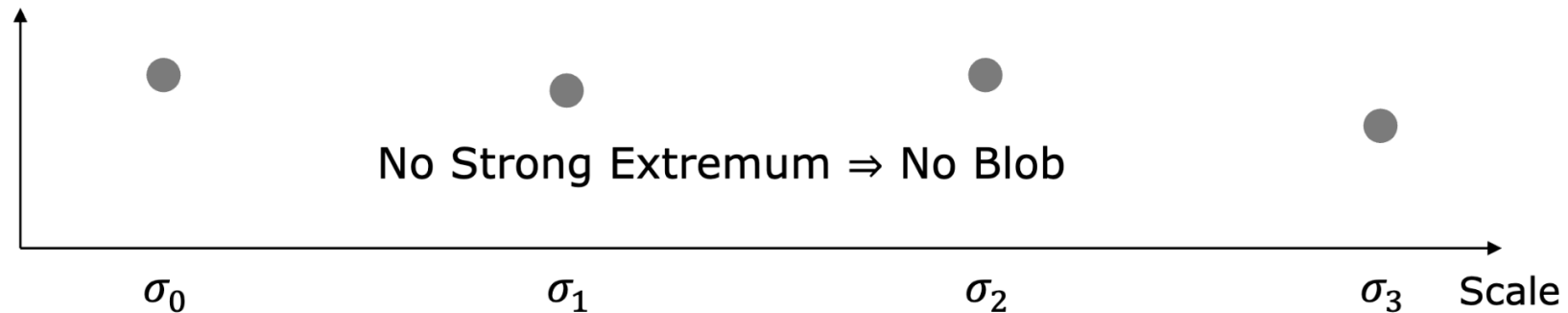


$S(x, y, \sigma_2)$

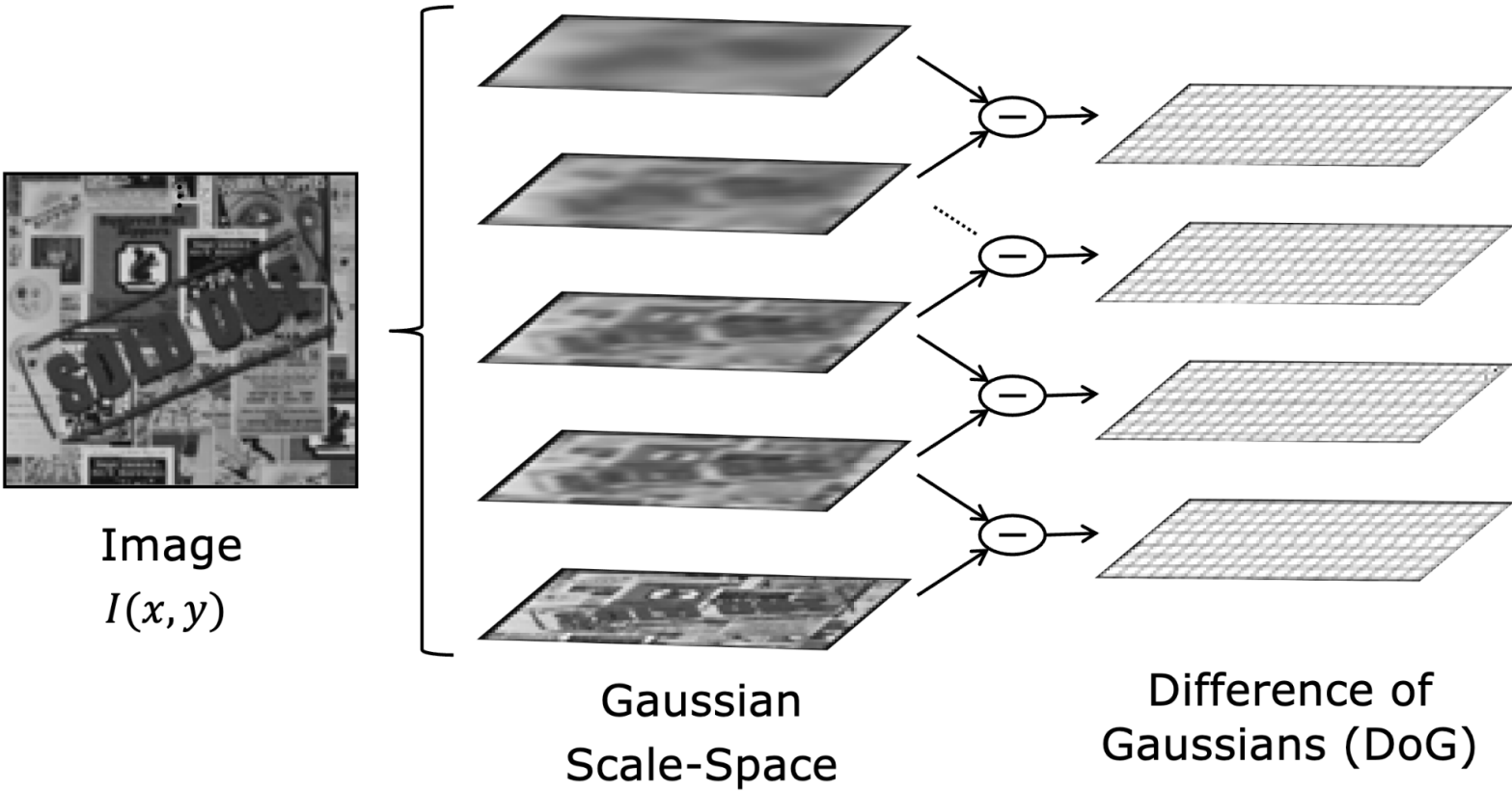


$S(x, y, \sigma_3)$

...

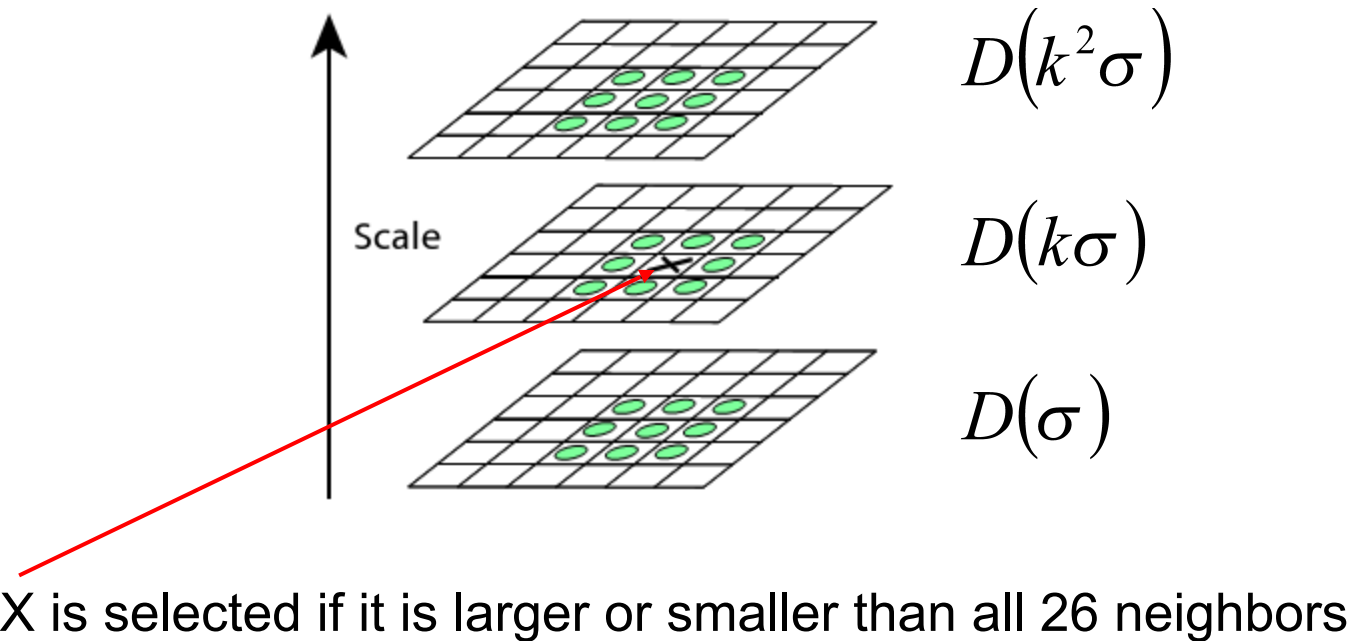


# Overall SIFT detector algorithm



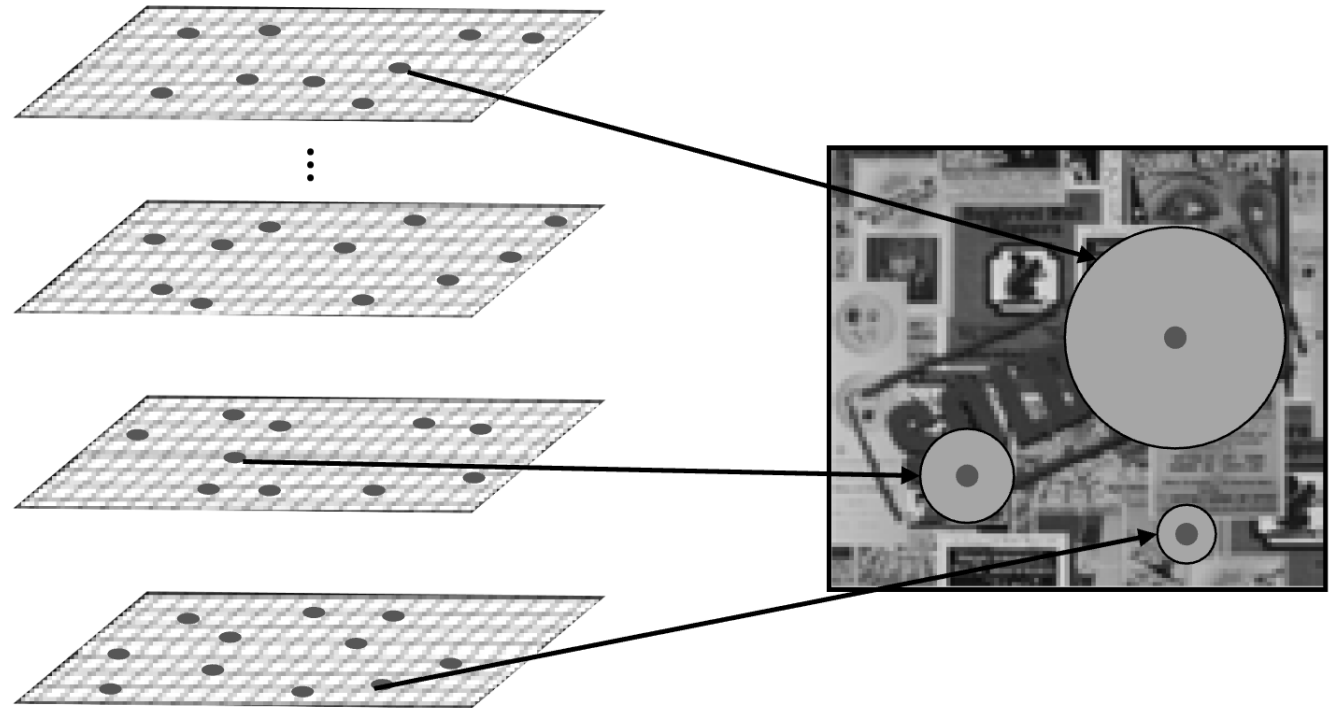
# Extracting SIFT keypoints and scales

- Choose the maxima within 3x3x3 neighborhood.



# Extracting SIFT keypoints and scales

- Sigma value tells you how big the blob is

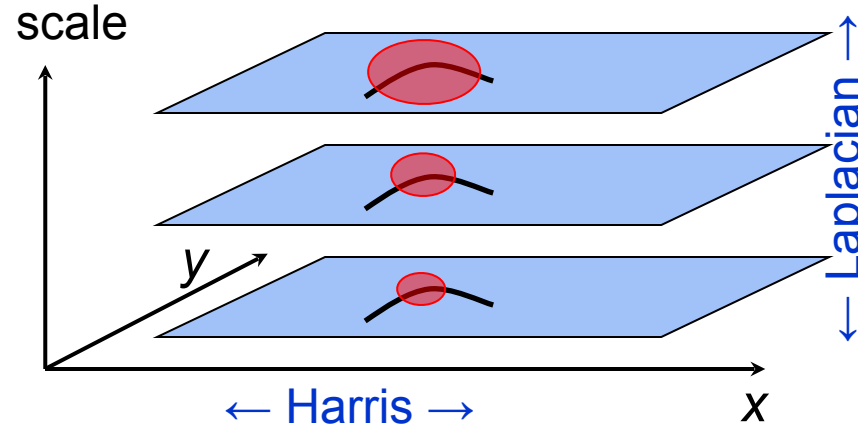


# Scale Invariant Detectors

- **Harris-Laplacian**<sup>1</sup>

*Find local maximum of:*

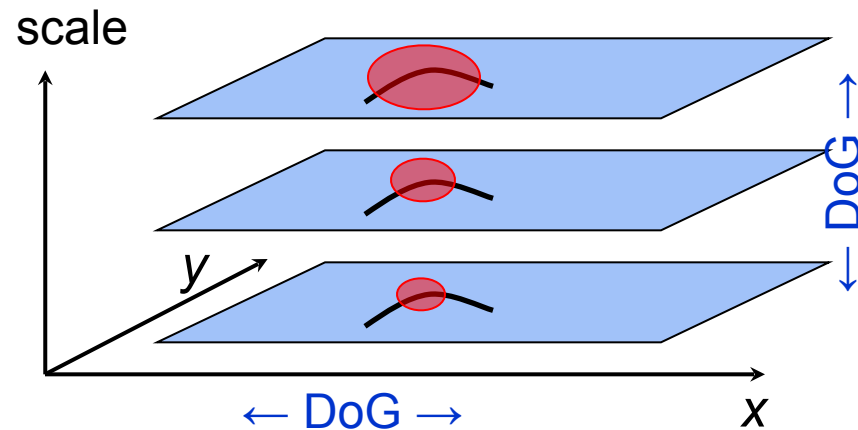
- Harris corner detector in space (image coordinates)
- Laplacian in scale



- **DoG (from SIFT by Lowe)**<sup>2</sup>

*Find local maximum of:*

- Difference of Gaussians in space and scale

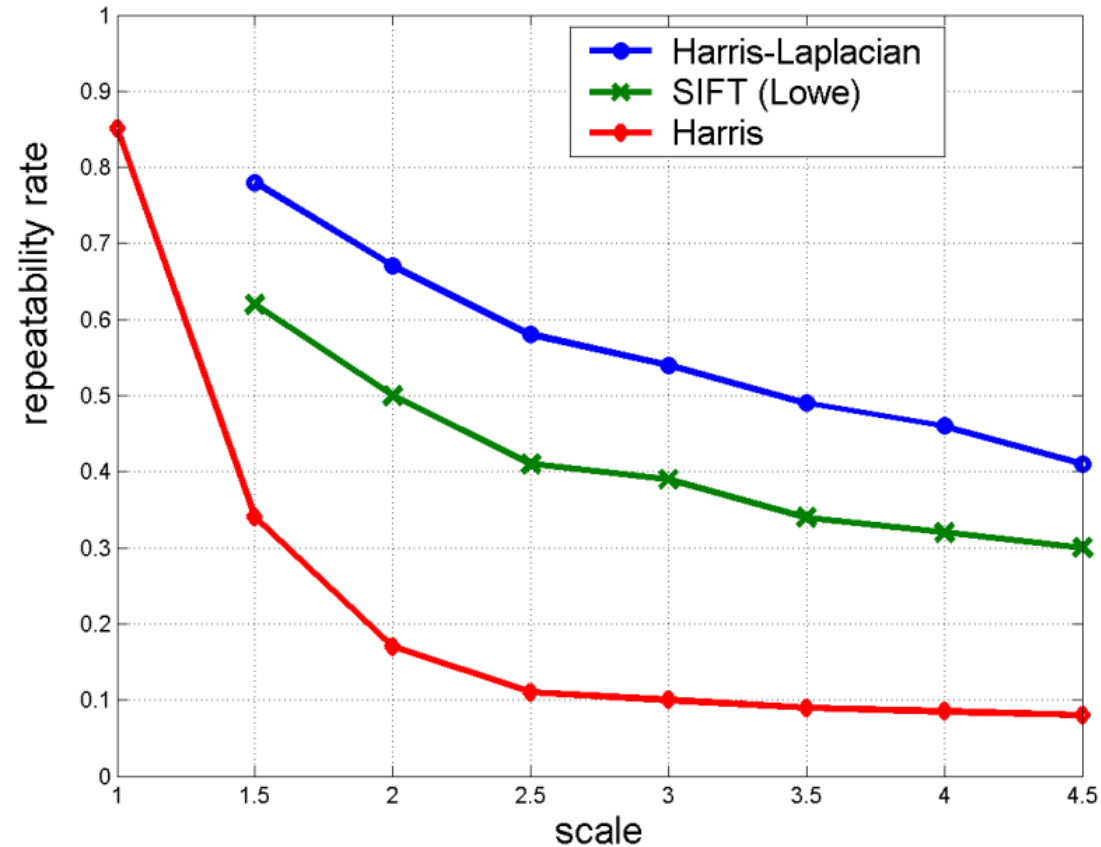
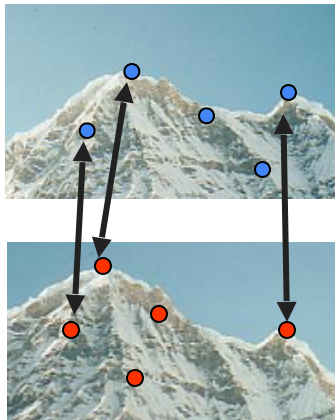


# Scale Invariant Detectors

- Experimental evaluation of detectors w.r.t. scale change

Repeatability rate:

$$\frac{\# \text{ correspondences}}{\# \text{ possible correspondences}}$$



# Scale Invariant Detection: Summary

- **Given:** two images of the same scene with a large *scale difference* between them
- **Goal:** find *the same* interest points *independently* in each image
- **Solution:** search for *maxima* of suitable functions in *scale* (DoG with different size) and in *space* (convolution over the image)

## Methods:

1. **Harris-Laplacian** [Mikolajczyk, Schmid]: maximize Laplacian over scale, Harris' measure of corner response over the image
2. **SIFT** [Lowe]: maximize Difference of Gaussians over scale and space

# Today's agenda

- Scale invariant keypoint detection
- Local detectors (SIFT)
- **Local descriptors (SIFT)**
- Global descriptors (HoG)

# What's next?

We now can detect keypoints at varying scales. But what can we do with those keypoints?

Things we would like to do:

- Search:
  - We would need to find similar key points in other images
- Panorama stitching
  - Match keypoints from one image to another.
- Etc...

For all such applications, we need a way of `describing` the keypoints.

# Why we care about knowing the keypoint patch size??

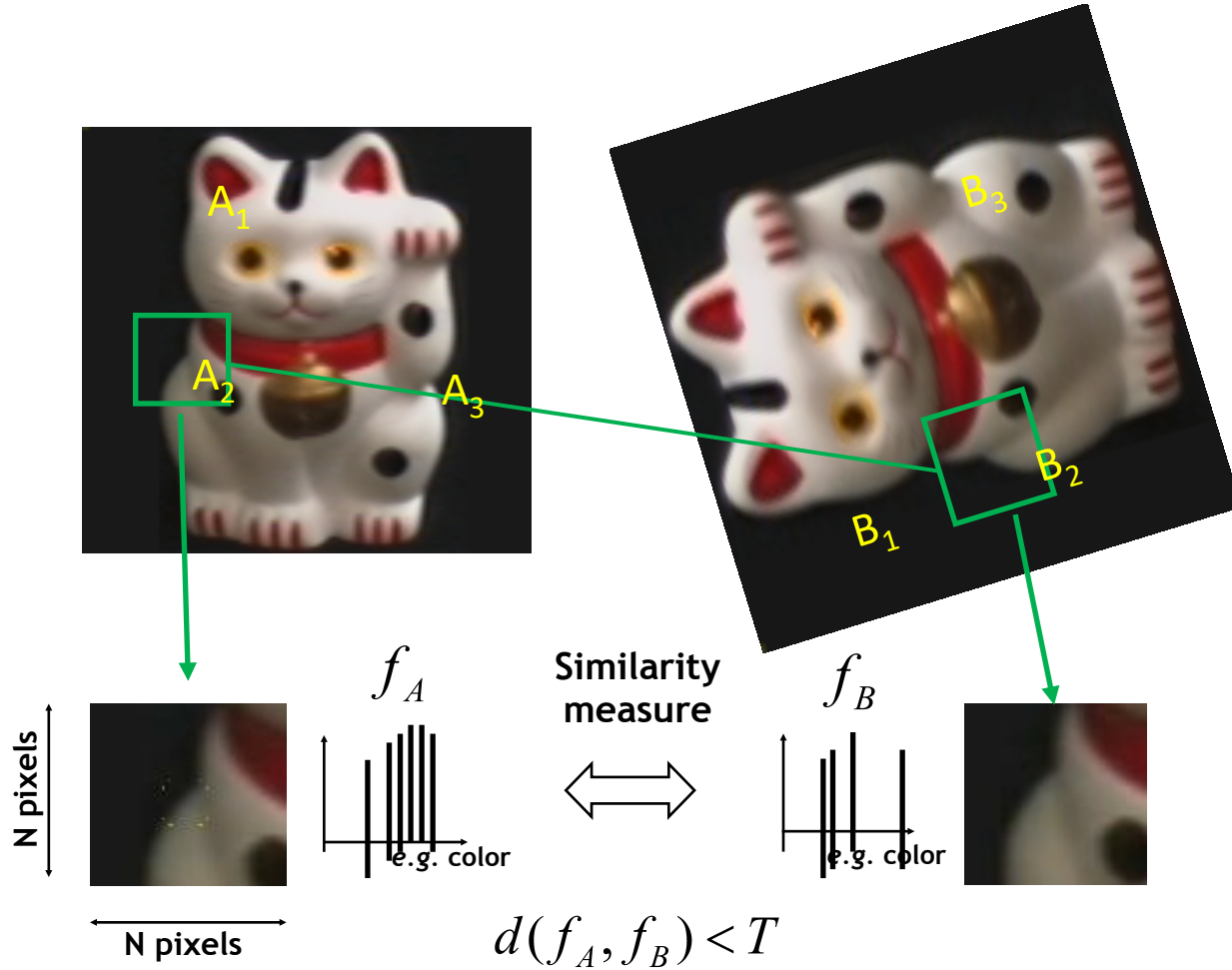
1. Find a set of distinctive **key-points**

2. Define a region/**patch** around each keypoint

3. **Normalize** the region content

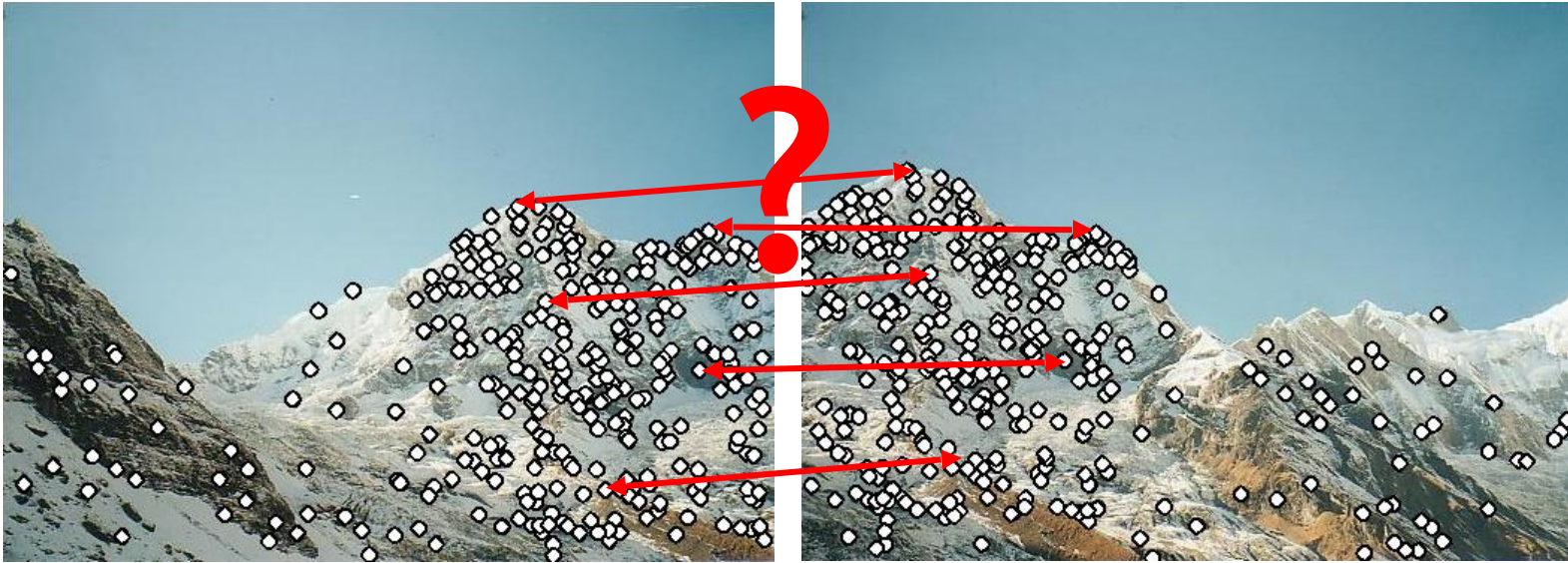
4. Compute a local **descriptor** from the normalized region

5. **Match** local descriptors



# Local Descriptors are vectors

- We know how to detect points
- Next question: How to describe them for matching?
- Descriptor: **Vector** that summarizes the content of the keypoint neighborhood.

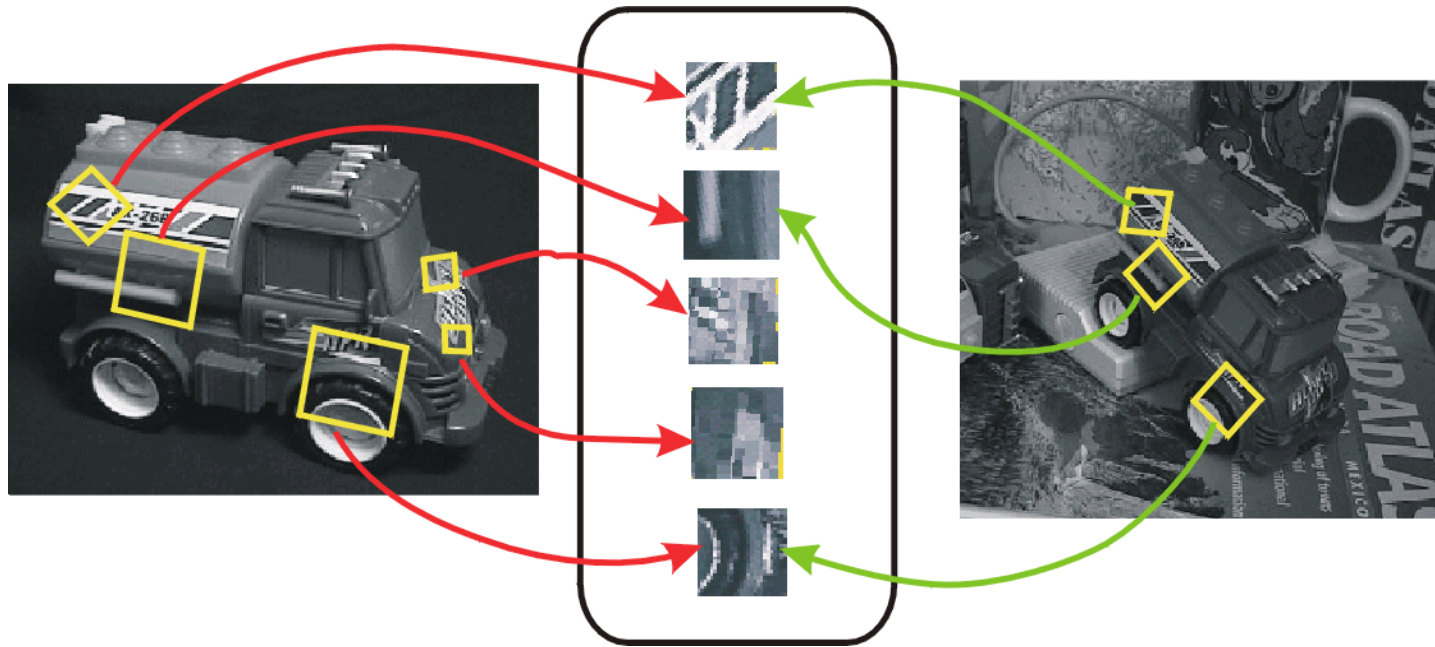


**Point descriptor should be:**

1. Invariant
2. Distinctive

# Invariant Local Descriptors

Image content is transformed into local feature coordinates that are **invariant** to **translation**, **rotation**, **scale**, and other imaging parameters

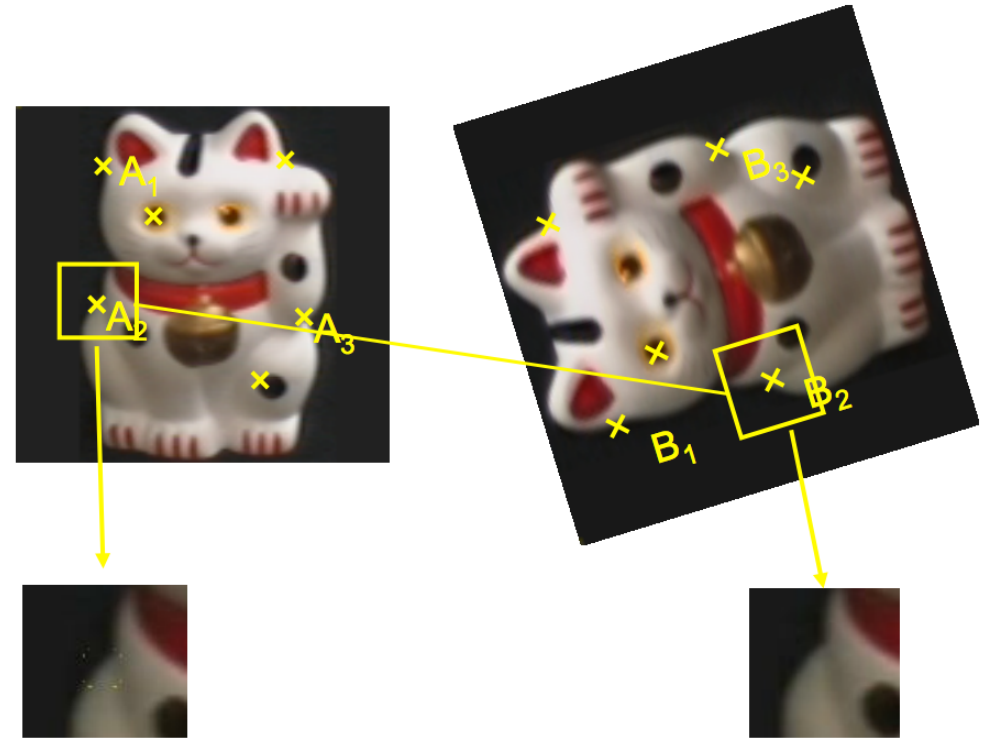


# Rotation invariant descriptors

So far, we have figured out the scale of the keypoints.

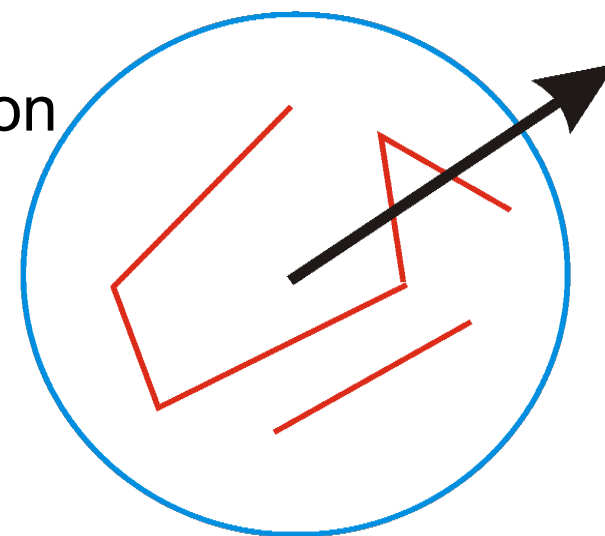
- So we can normalize them to be the same size.

Q. How do we re-orient the patches so that they are rotation invariant?



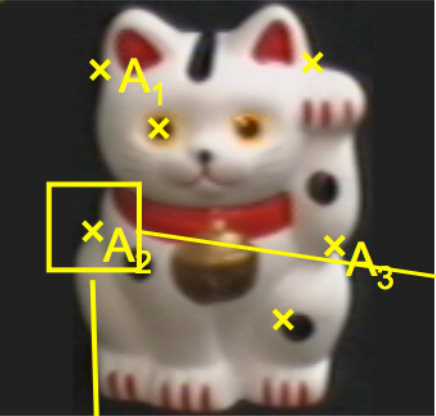
# Constructing a rotation invariant descriptor

- We are given a keypoint and its scale from **DoG**
- We will select the direction of maximum gradient as the orientation for the keypoint
- We will describe all features *relative* to this orientation

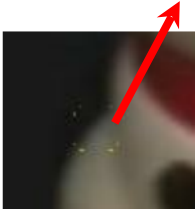


# Visualizing what that looks like

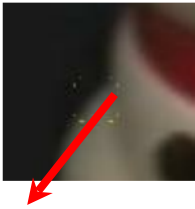
Q. Which one is the direction of the maximum gradient for this keypoint patch?



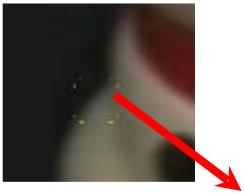
A)



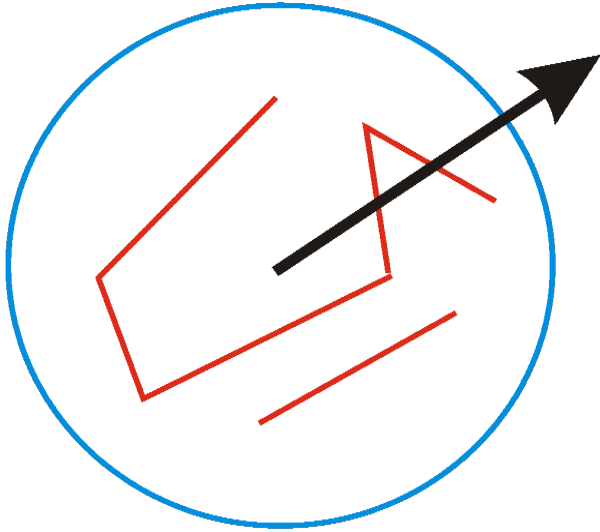
B)



C)

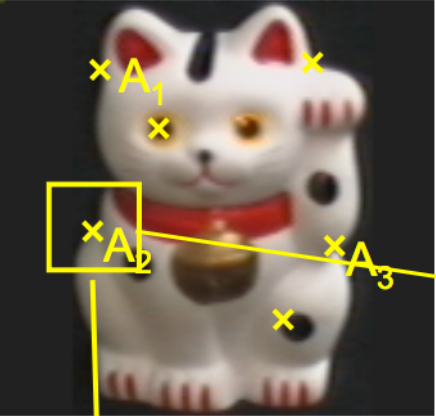


D)

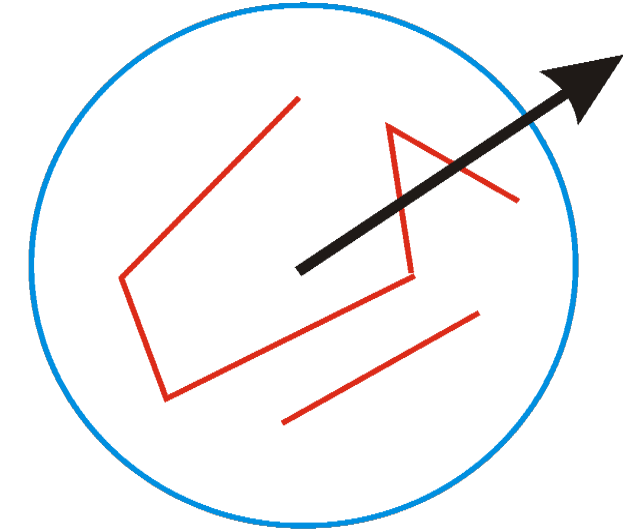
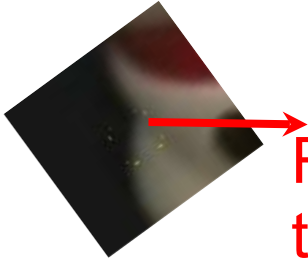


# Visualizing what that looks like

Q. Which one is the direction of the maximum gradient for this keypoint patch?



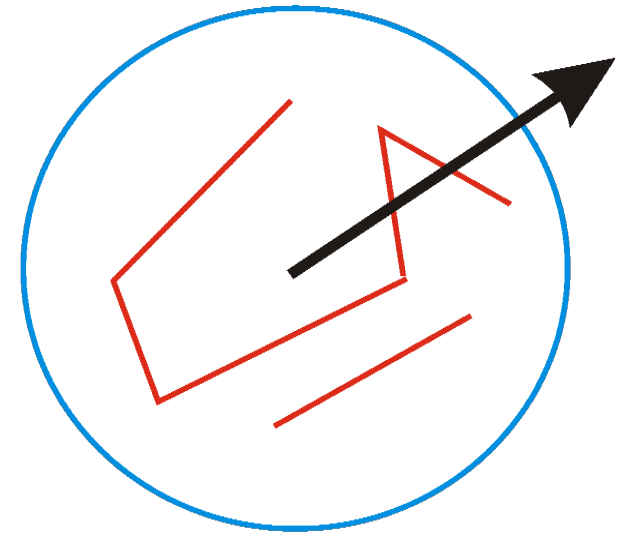
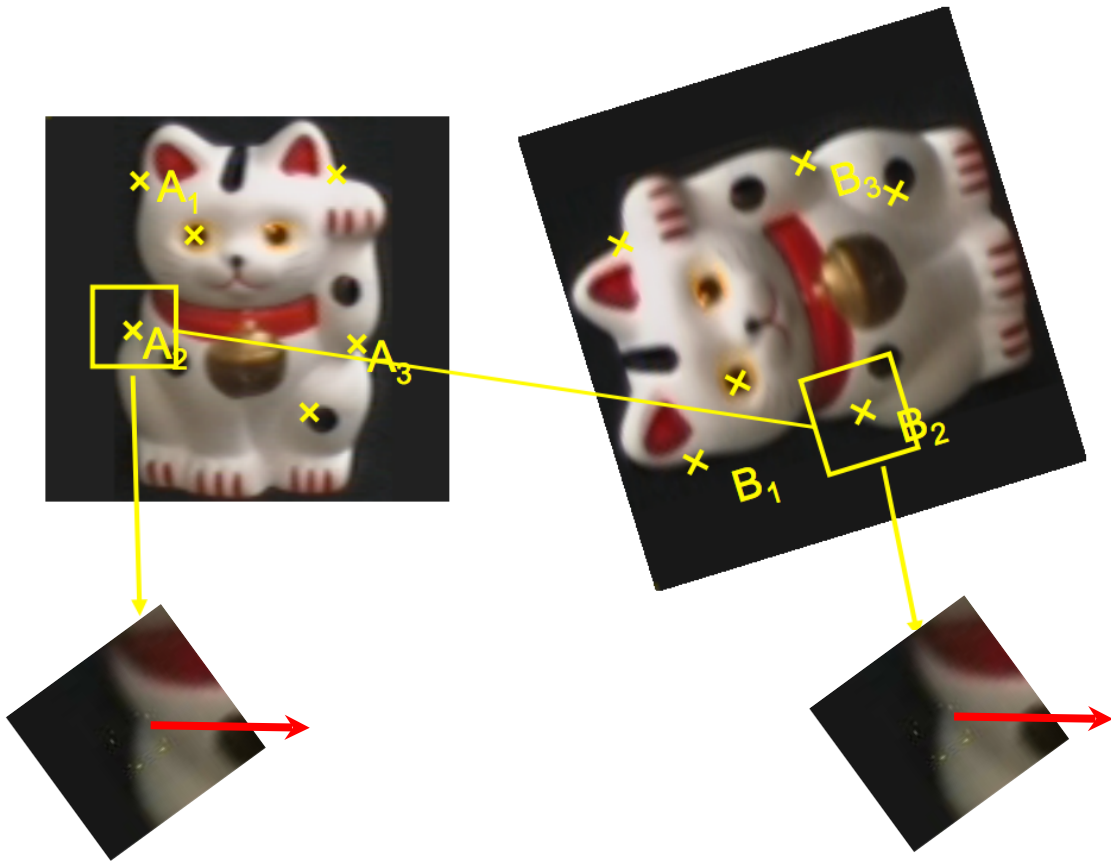
C)



Rotated patch to make sure the gradient  $\theta = 0$

# Feature descriptors become rotation invariant

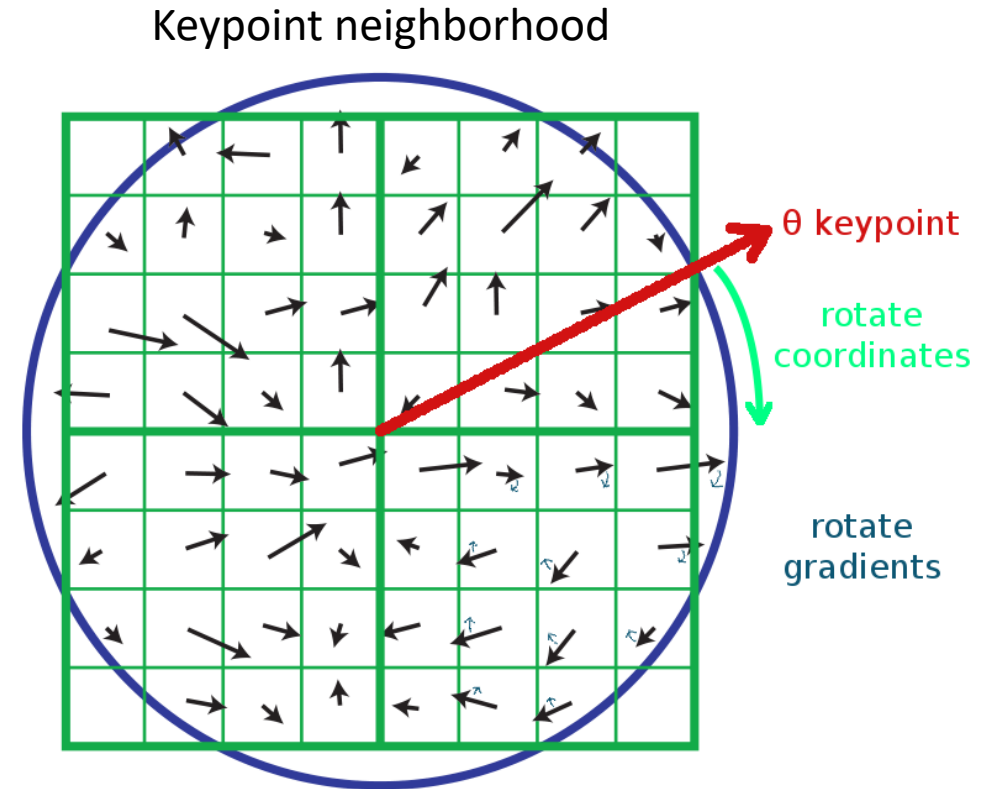
- If the keypoint appears rotated in another image, the features will be the same, because they're **relative** to the characteristic orientation



# SIFT descriptor (Scale-Invariant Feature Transform)

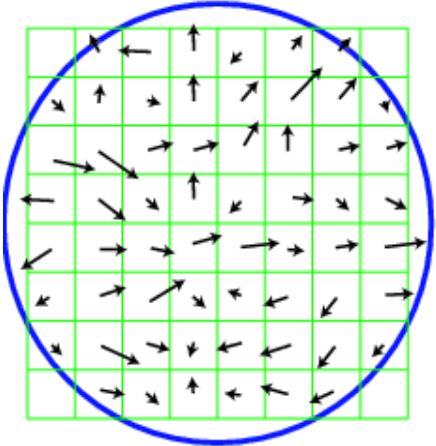
**Gradient-based** descriptor to capture texture in the keypoint neighborhood

1. Blur the keypoint's image patch to remove noise
2. Calculate image **gradients** over the neighborhood patch.
3. To become rotation invariant, rotate the gradients by  $-\theta$  (**- maximum direction**)
  - Now we've cancelled out rotation and have gradients expressed at locations relative to maximum direction  $\theta$
4. Generate a descriptor



# Generating the descriptor from rotated patch

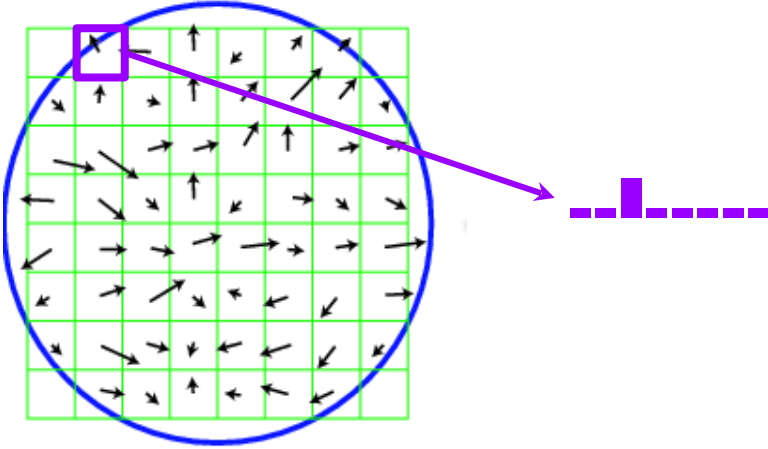
Keypoint neighborhood



- Q. How do we turn this into a vector?

# Generating the descriptor from rotated patch

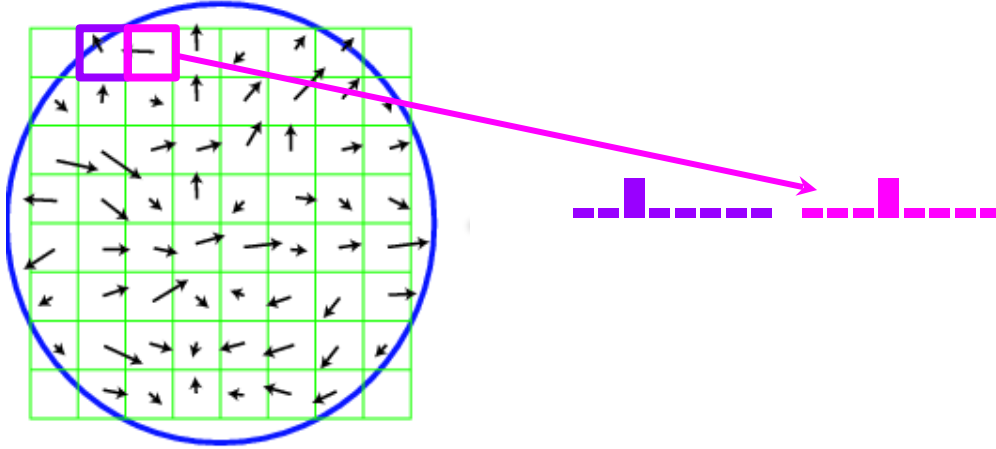
Keypoint neighborhood



- We can turn every pixel into a histogram
- Histogram contains 8 buckets, all of them zero except for one.
- Make the bucket of the direction of the gradient equal to 1

# Generating the descriptor from rotated patch

Keypoint neighborhood

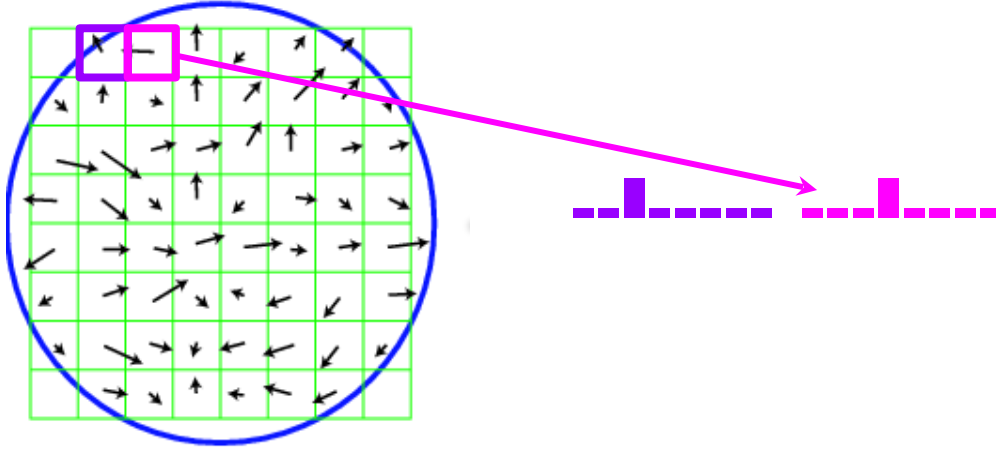


- Do this for every single pixel

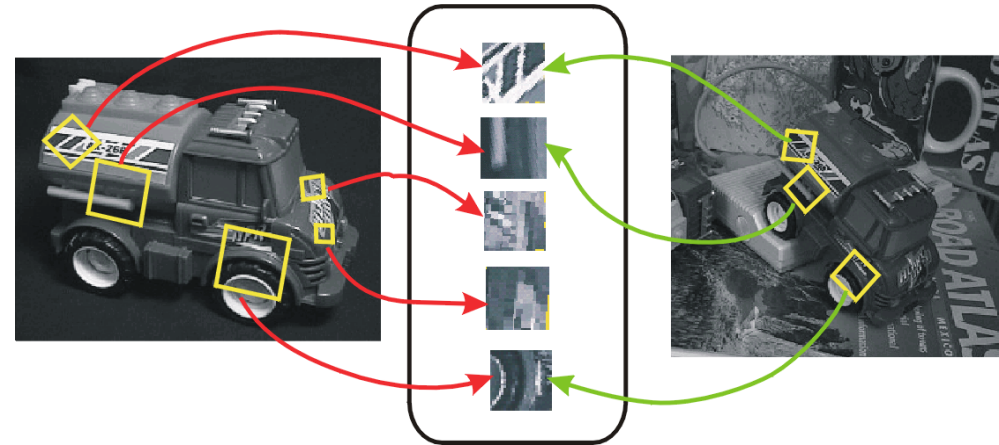
Q. What would the size of the keypoint vector be?

# Generating the descriptor from rotated patch

Keypoint neighborhood



- Do this for every single pixel

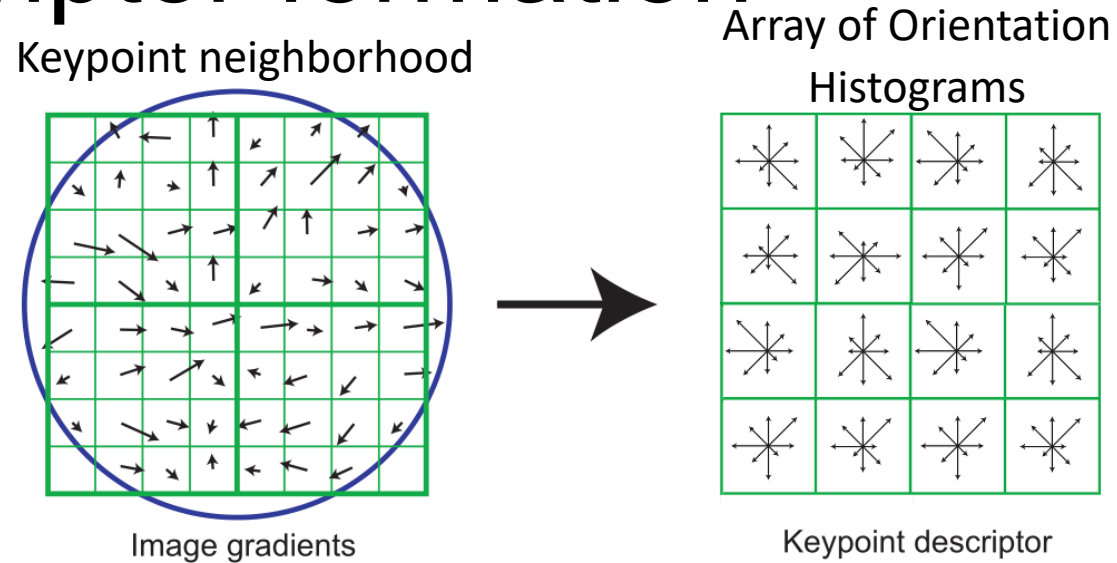


**Q. Why might this be a bad strategy? What could go wrong?**

Hint: think about how matching might fail



# SIFT descriptor formation



- Each cell gives us a histogram vector. We have a total of **4x4 vectors**
- Calculate the overall gradients in each patch into their local orientated histograms
  - Also, scale down gradient contributions for gradients far from the center
  - Each histogram is quantized into 8 directions (each 45 degrees)

# SIFT descriptor formation

Keypoint neighborhood

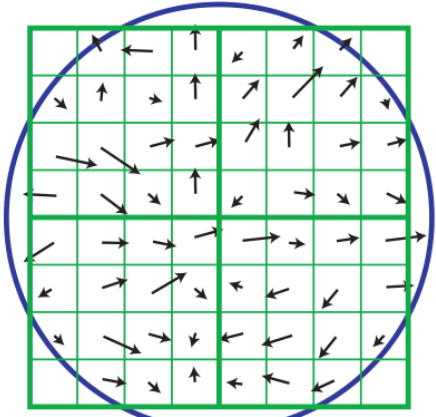
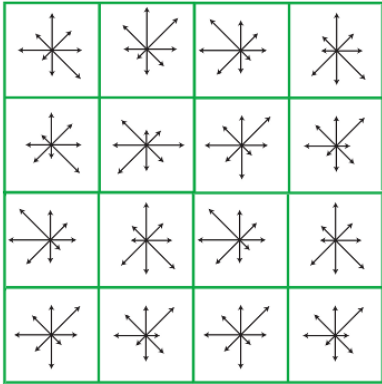


Image gradients



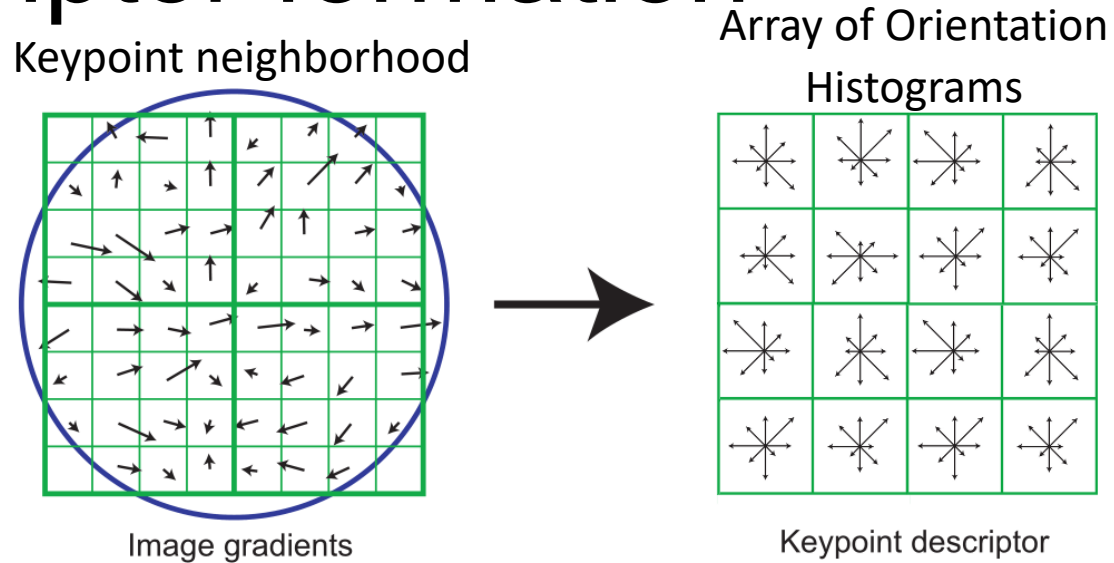
Array of Orientation Histograms



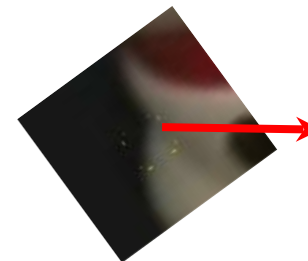
Keypoint descriptor

- Q. What is the size of the descriptor?

# SIFT descriptor formation

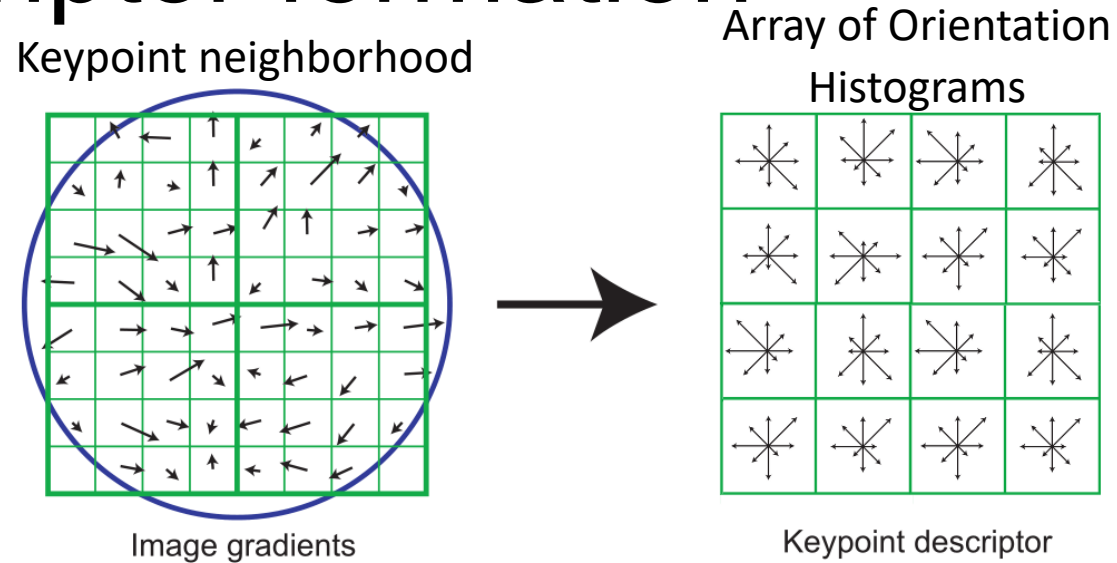


- 8 orientation bins per histogram,
- 4x4 histogram vectors,
- total is  $8 \times 4 \times 4 = 128$  numbers.
- So a SIFT descriptor is a length **128 vector**



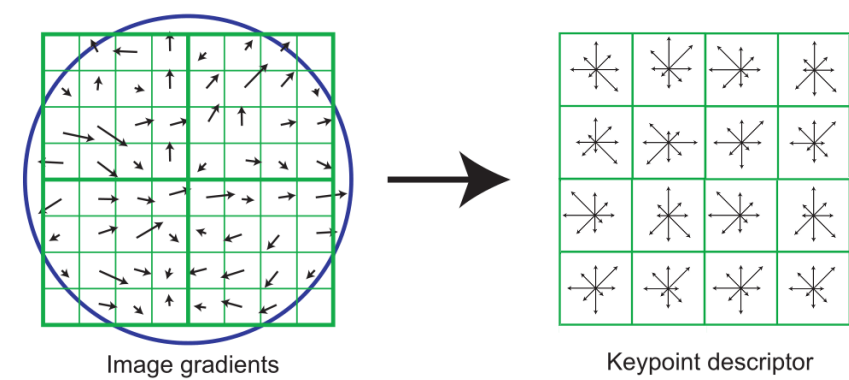
$$\mathcal{H}oG(k) = \begin{bmatrix} g_1 \\ g_2 \\ \dots \\ g_{128} \end{bmatrix}$$

# SIFT descriptor formation



- SIFT descriptor is invariant to **rotation** (because we rotated the patch) and **scale** (because we worked with the scaled image from DoG)
- We can compare each vector from image A to each vector from image B to find matching keypoints!
  - How do we match distances?

# Making descriptors robust



- Adding robustness to illumination changes:
- **Each descriptor is made of gradients** (differences between pixels),
  - It's already invariant to changes in brightness
  - (e.g. adding 10 to all image pixels yields the exact same descriptor)
- A **sharpening filter** applied to the image will increase the magnitude of gradients linearly.
  - To correct for contrast changes, **normalize the histogram** (scale to magnitude=1.0)
- **Very large image gradients** are usually from unreliable 3D illumination effects (glare, etc).
  - To reduce their effect, **clamp all values in the vector to be  $\leq 0.2$**  (an experimentally tuned value). Then normalize the vector again.
- Result is a vector which is fairly invariant to illumination changes.

# SIFT descriptor distances

Given keypoints  $k_1$  and  $k_2$ , we can calculate their HoG features:

$\mathcal{H}oG(k_1)$

$\mathcal{H}oG(k_2)$

We can calculate their matching score as:

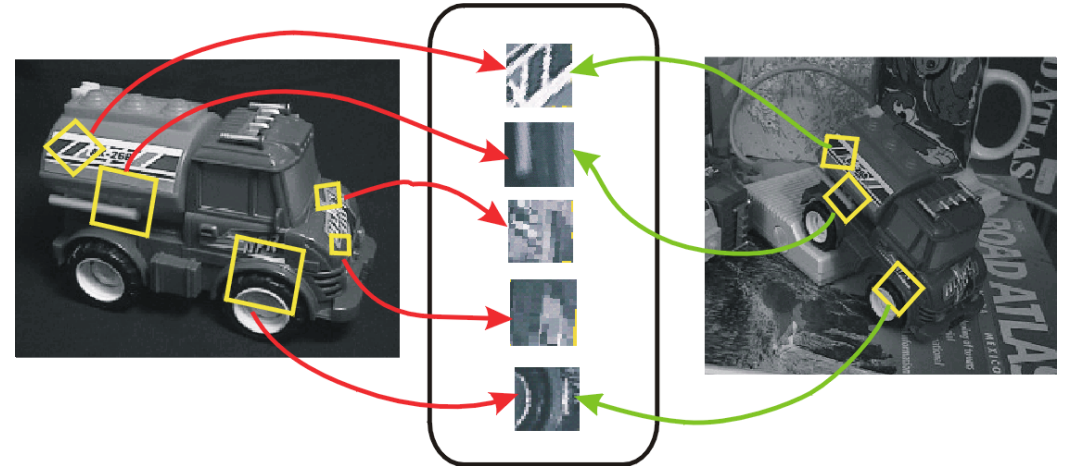
$$d_{\mathcal{H}oG}(k_1, k_2) = \sqrt{\sum_i (\mathcal{H}oG(k_1)_i - \mathcal{H}oG(k_2)_i)^2}$$

# Find nearest neighbor for each keypoint in image A in image B

Given keypoints  $k_1$  and  $k_2$ , we can calculate their HoG features:

$HoG(k_1)$

$HoG(k_2)$



We can calculate their matching score as:

$$d_{HoG}(k_1, k_2) = \sqrt{\sum_i (\mathcal{H}oG(k_1)_i - \mathcal{H}oG(k_2)_i)^2}$$



Figure 12: The training images for two objects are shown on the left. These can be recognized in a cluttered image with extensive occlusion, shown in the middle. The results of recognition are shown on the right. A parallelogram is drawn around each recognized object showing the boundaries of the original training image under the affine transformation solved for during recognition. Smaller squares indicate the keypoints that were used for recognition.

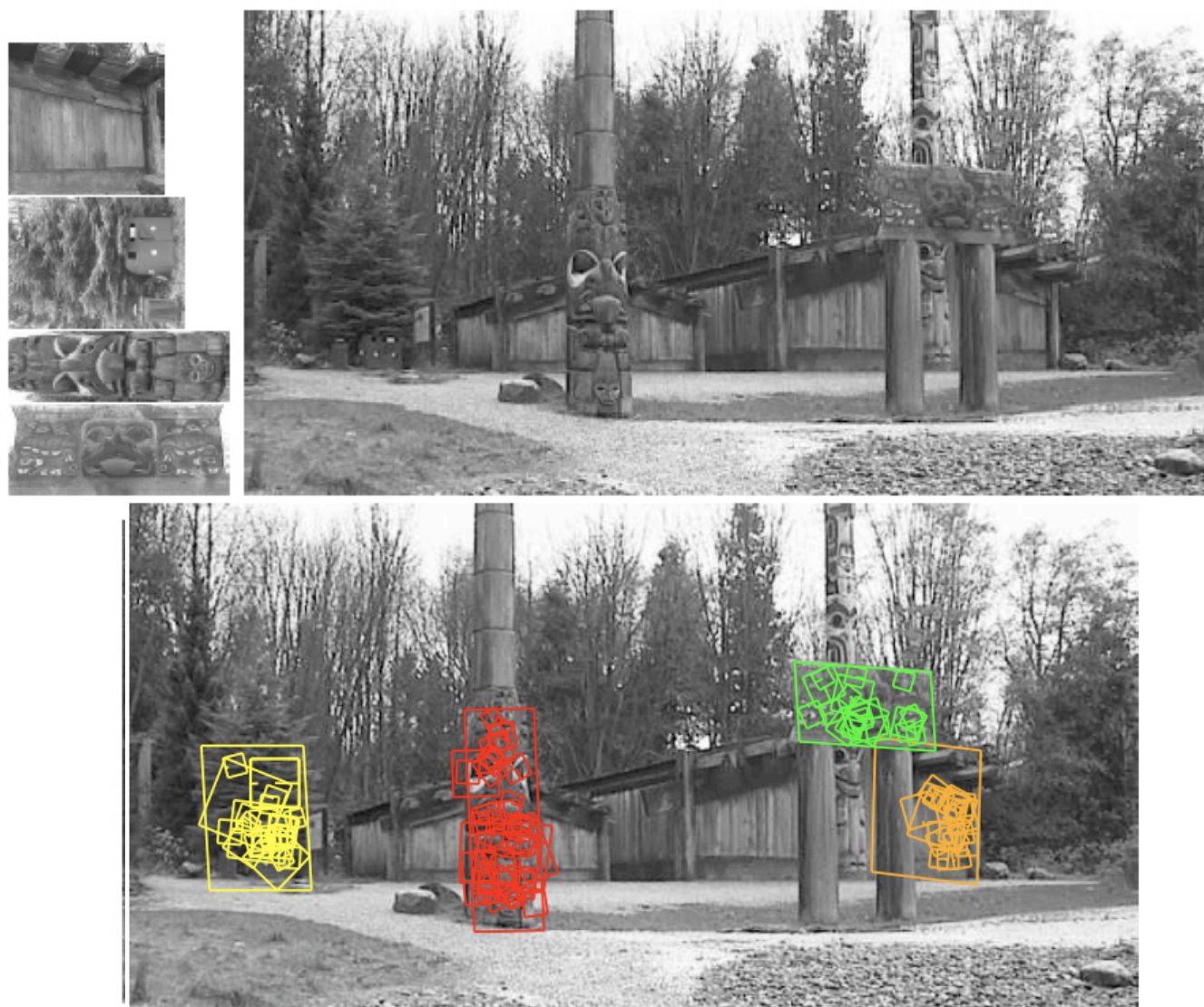


Figure 13: This example shows location recognition within a complex scene. The training images for locations are shown at the upper left and the 640x315 pixel test image taken from a different viewpoint is on the upper right. The recognized regions are shown on the lower image, with keypoints shown as squares and an outer parallelogram showing the boundaries of the training images under the affine transform used for recognition.

# Recognition of specific objects, scenes



Schmid and Mohr 1997



Sivic and Zisserman, 2003

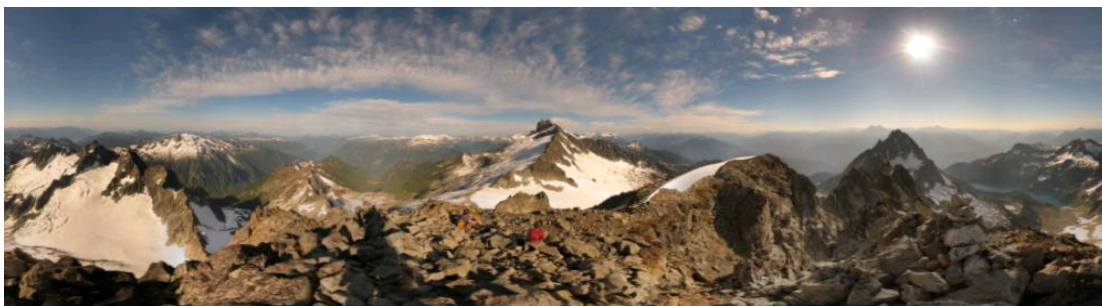
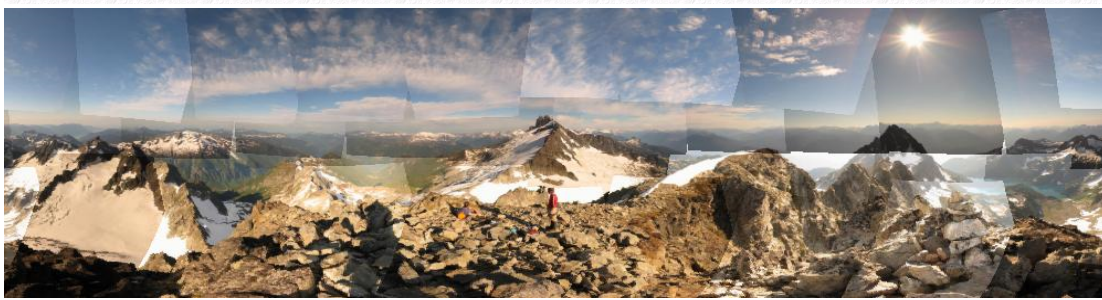
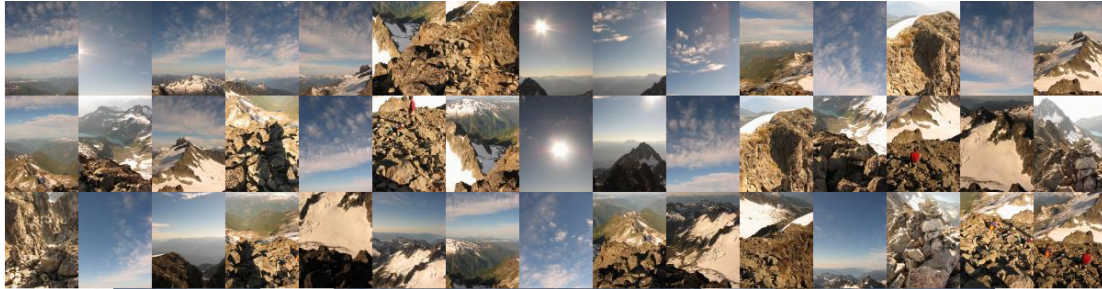


Rothganger et al. 2003



Lowe 2002

# Panorama stitching/Automatic image mosaic



<http://matthewalunbrown.com/autostitch/autostitch.html>

# Wide baseline stereo



# Even robust to extreme occlusions



# Applications of local invariant features

- Recognition
- Wide baseline stereo
- Panorama stitching
- Mobile robot navigation
- Motion tracking
- 3D reconstruction
- ...

# Today's agenda

- Scale invariant keypoint detection
- Local detectors (SIFT)
- Local descriptors (SIFT)
- Global descriptors (HoG)

# Global Feature descriptors

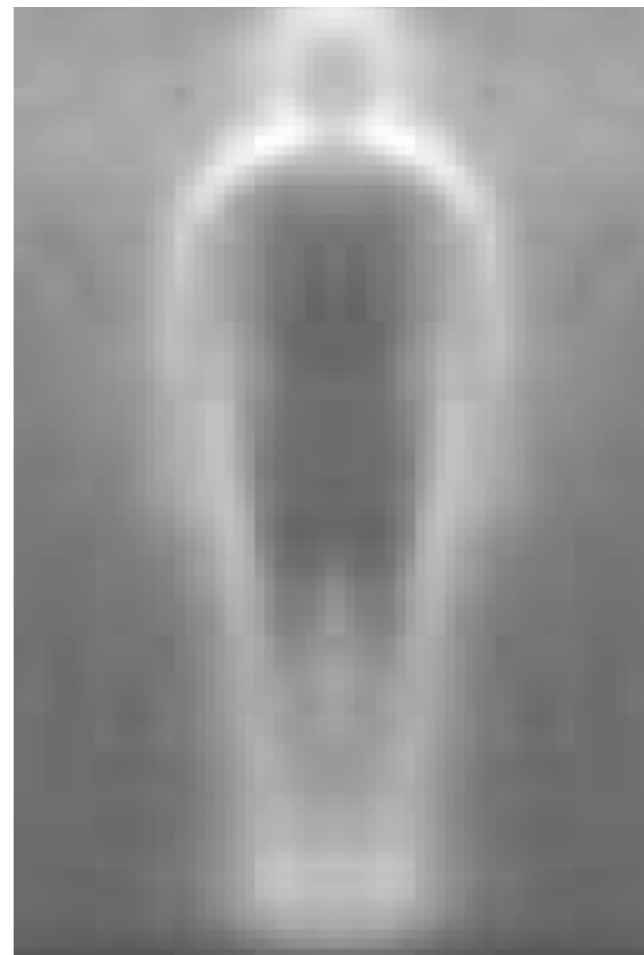
- Find robust feature set that allows **object shape to be recognized**.
- **Challenges**
  - Wide range of pose and large variations in appearances
  - Cluttered backgrounds under different illumination
  - Computation speed
- **Histogram of Oriented Gradients (HoG)**

[1] N. Dalal and B. Triggs. Histograms of Oriented Gradients for Human Detection. In CVPR, pages 886-893, 2005

[2] Chandrasekhar et al. CHoG: Compressed Histogram of Gradients - A low bit rate feature descriptor, CVPR 2009

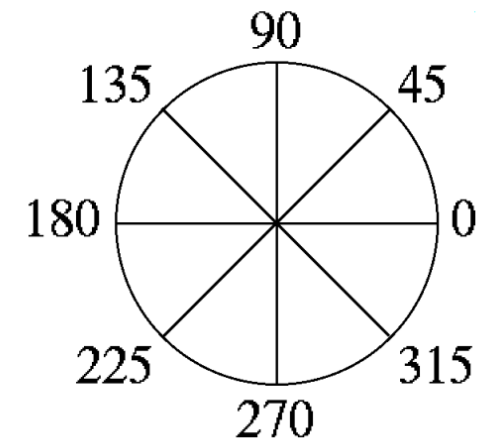
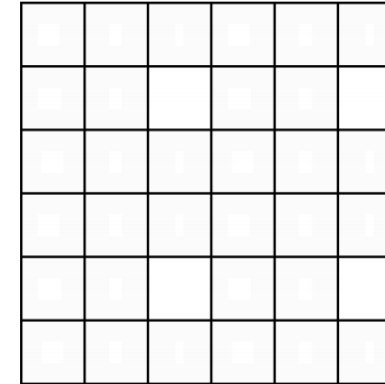
# Histogram of Oriented Gradients

- Local object appearance and shape can often be characterized well using gradients.
- Specifically, the distribution of local intensity gradients or edge directions.

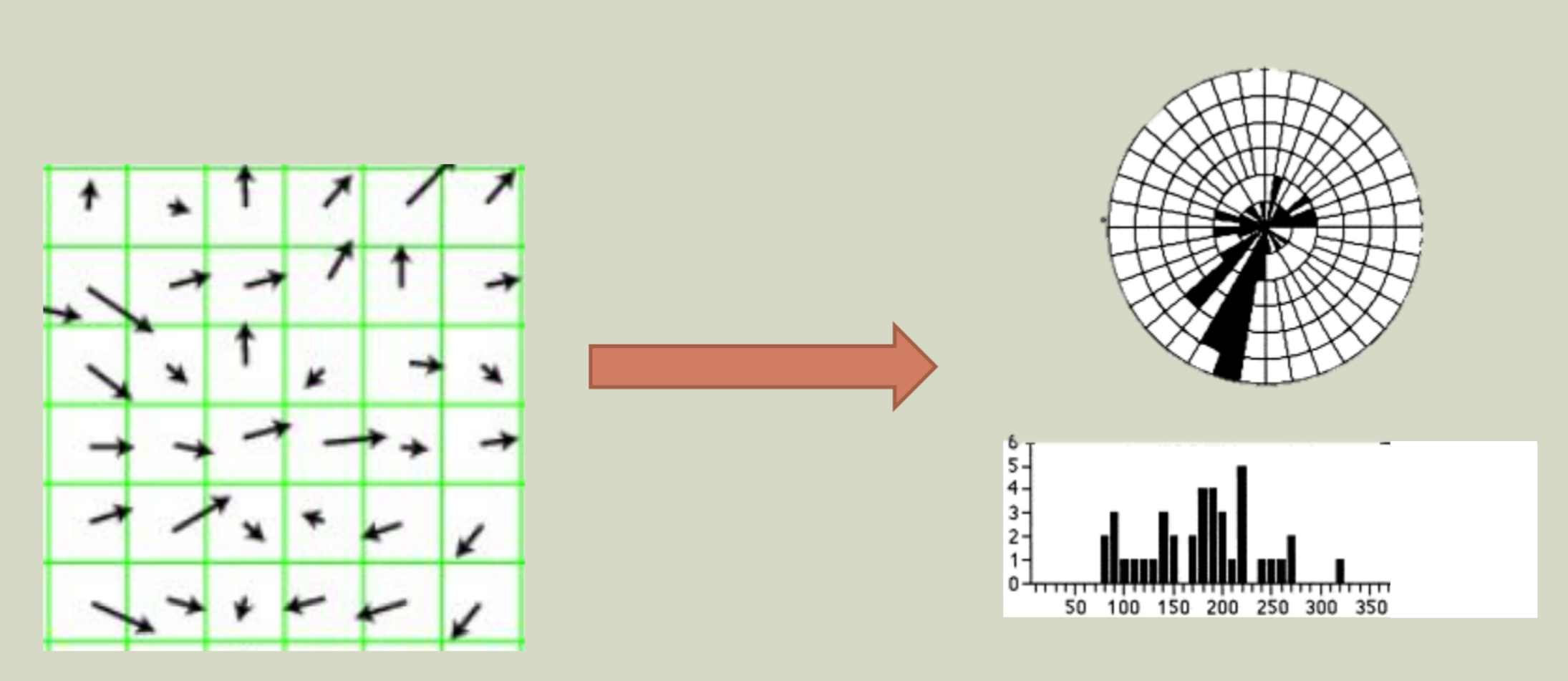


# Histogram of Oriented Gradients

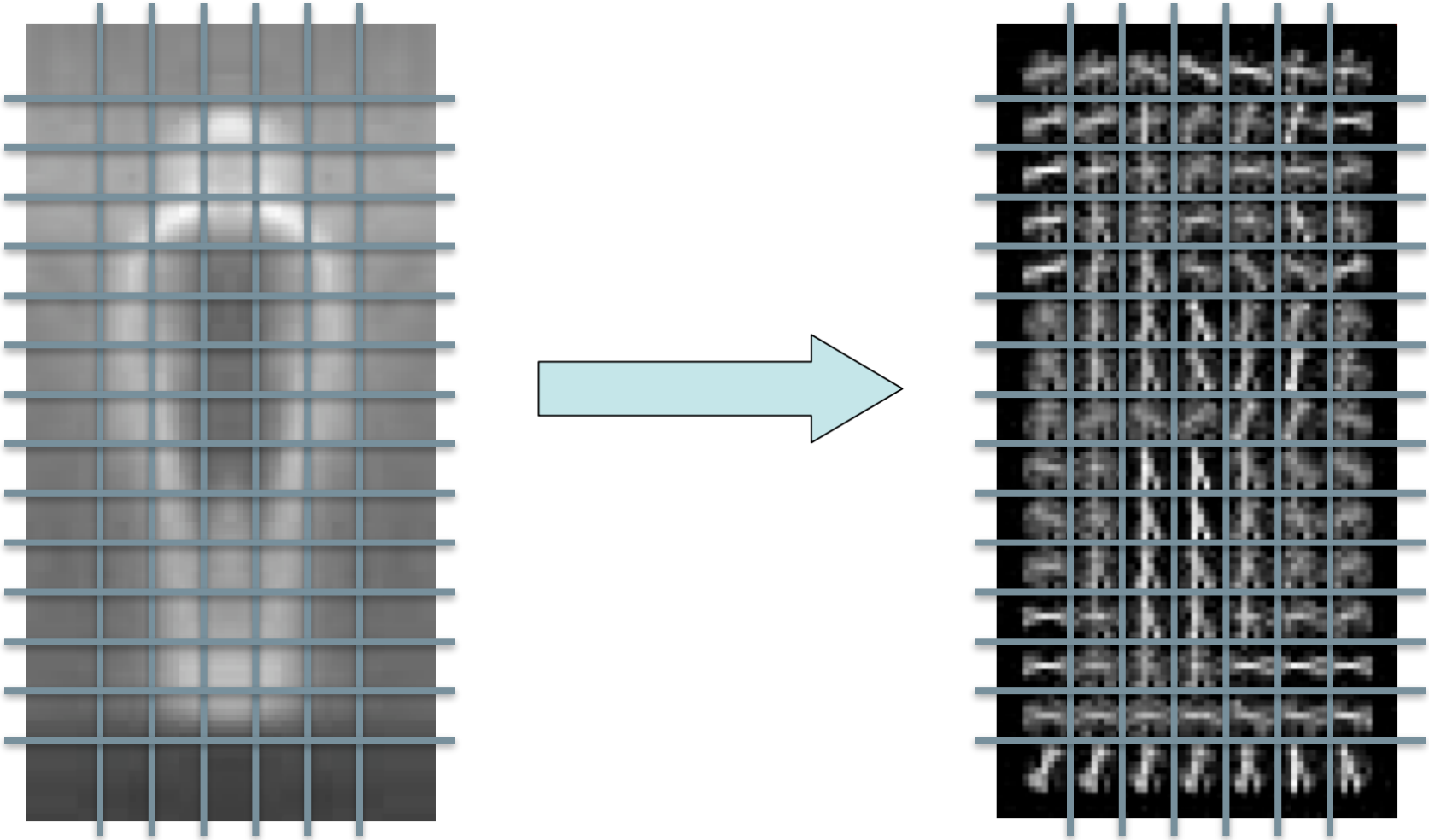
- Dividing the image window into small spatial regions (cells)
- Cells can be either rectangle or radial.
- Each window sums up local 1-D histogram of gradient directions over the pixels of the cell.



# Histogram of Oriented Gradients

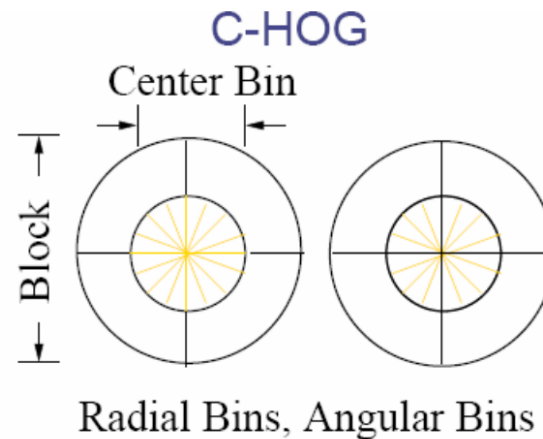
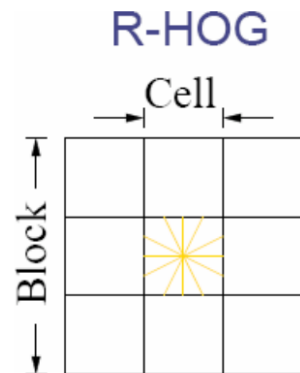


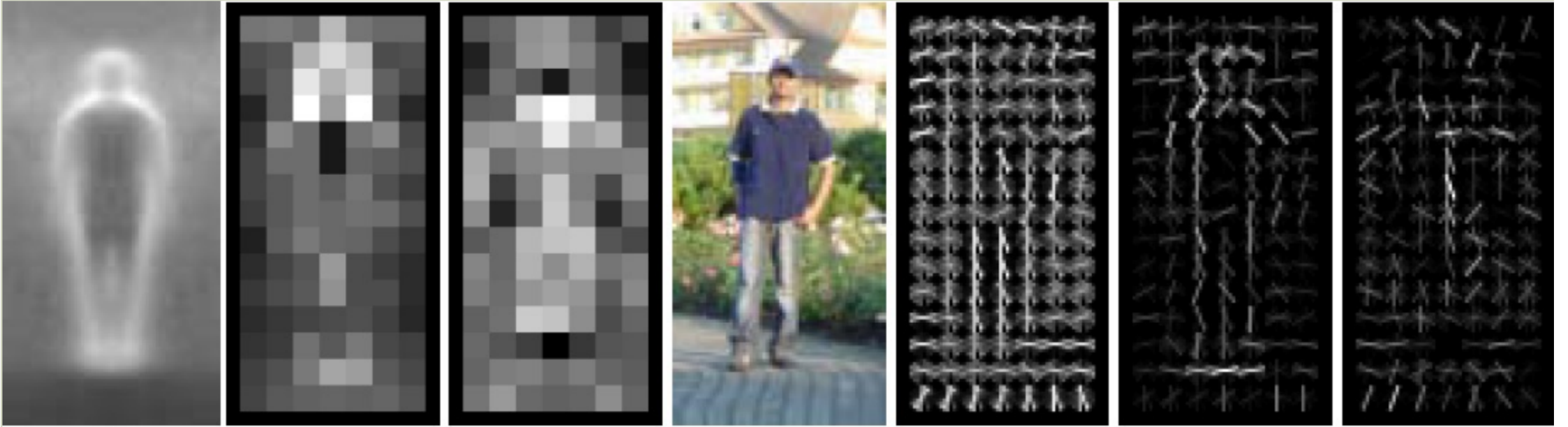
# Histogram of Oriented Gradients



# Normalization

- To make HoG invariant to illumination and shadows, it is useful to normalize the local responses
- Normalize each cell's histogram using histogram over a larger regions ("blocks").



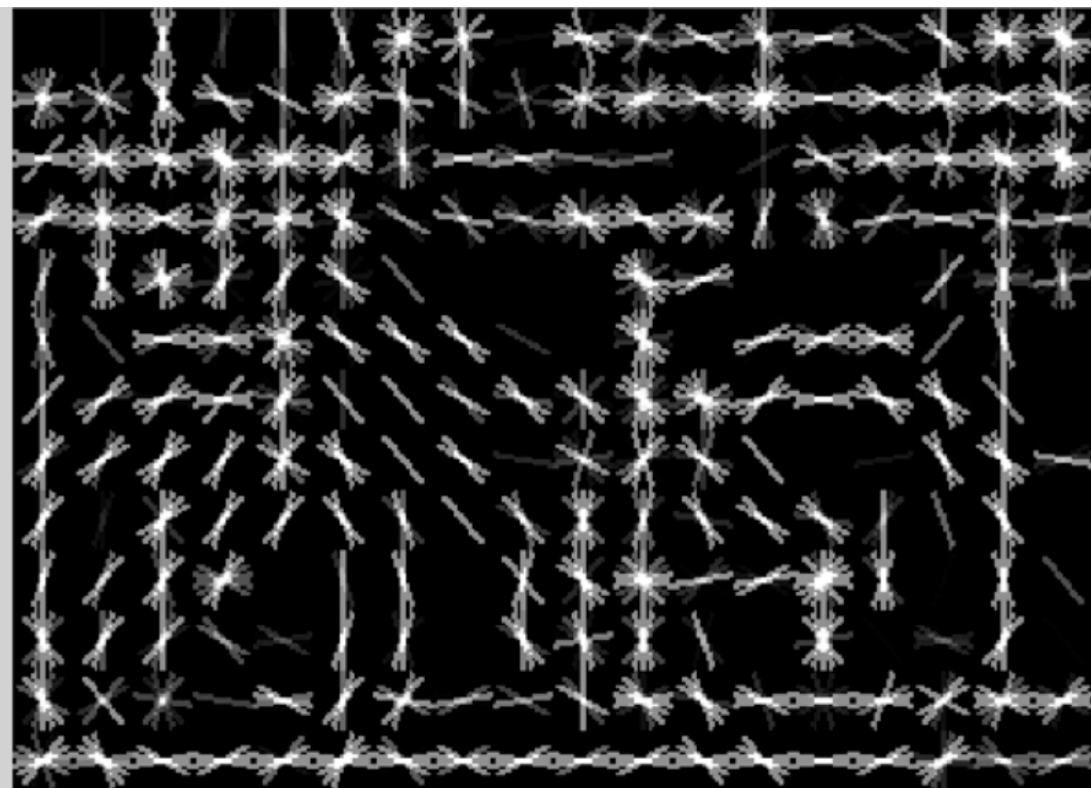


(a) (b) (c) (d) (e) (f) (g)

- a. Average gradient over example photo of a person
- b. “Positive” blocks that help match to other photos of people
- c. “Negative” blocks that do not match to photos of other people
- d. A test image
- e. It’s HOG descriptor visualized
- f. HOG descriptor weighted by positive weights
- g. HOG descriptor weighted by negative weights

# Visualizing HoG

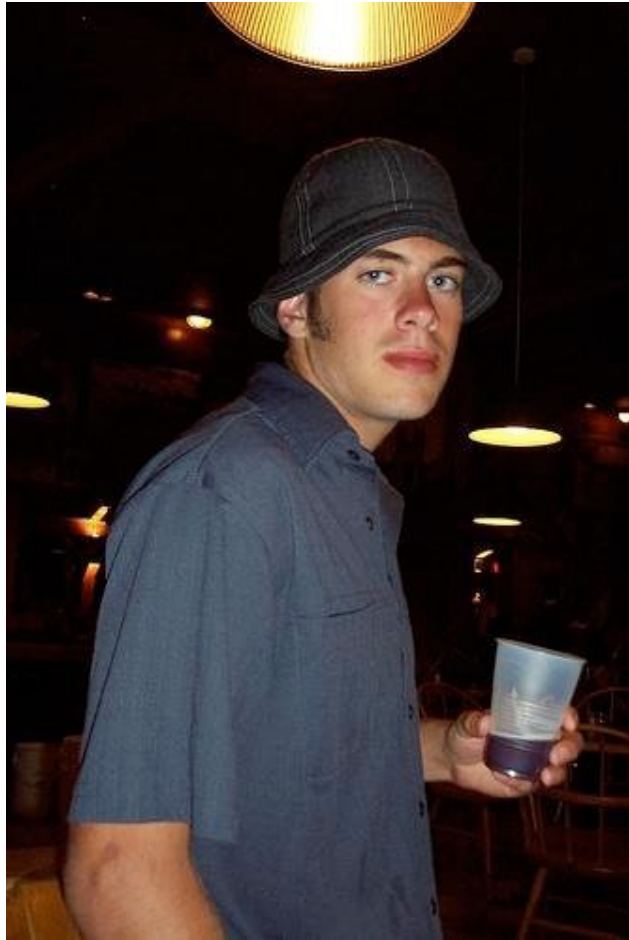
# Visualizing HoG



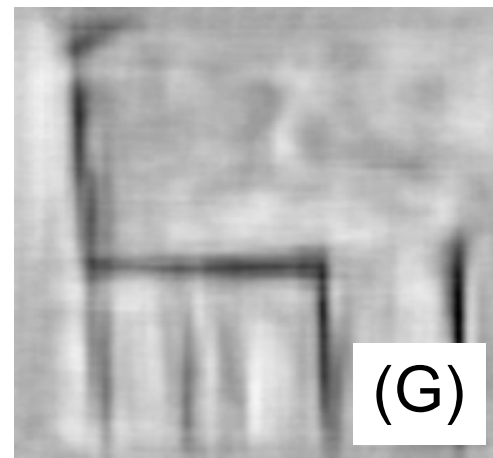
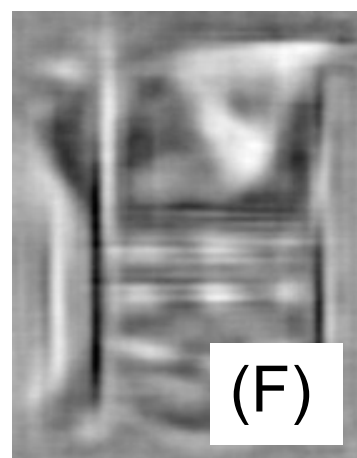
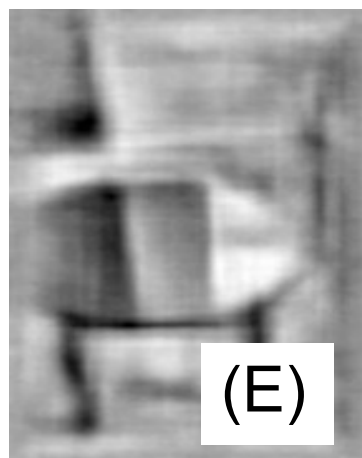
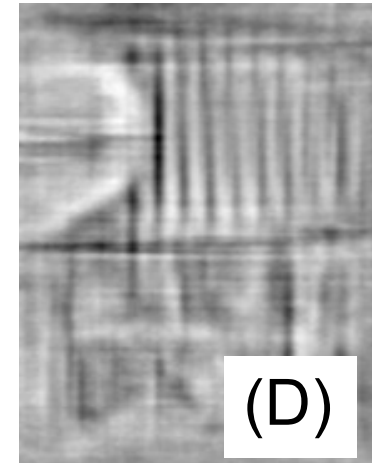
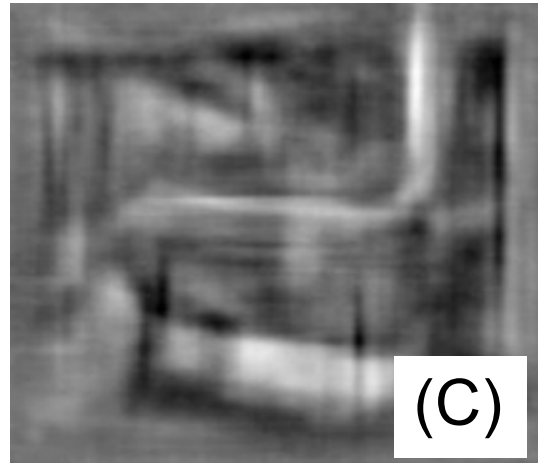
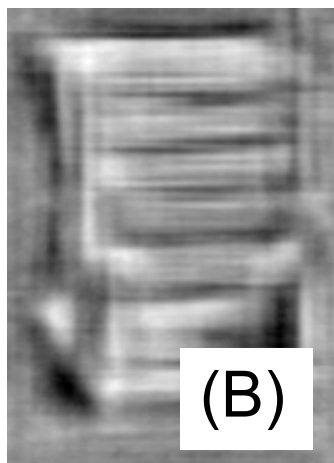
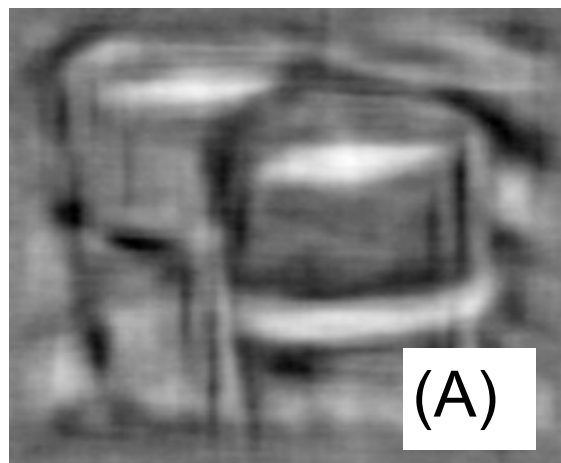
# Difference between HoG and SIFT

- HoG is usually used to describe larger image regions.
  - SIFT is used for key point matching
- 
- SIFT histograms are normalized with respect to the dominant gradient.
  - HoG gradients are normalized using neighborhood blocks.

HoG features are good but gradients are insufficient sometimes



# Chair Detections



# Chair Detections



# Car Detections



# Car Detections



# The HOGgles Challenge



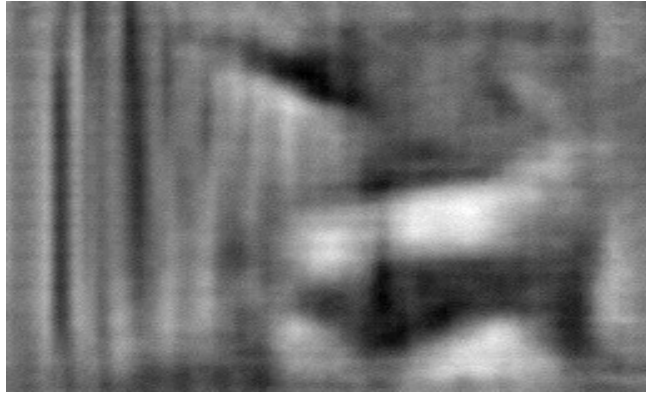
Clap your hands when you see a person

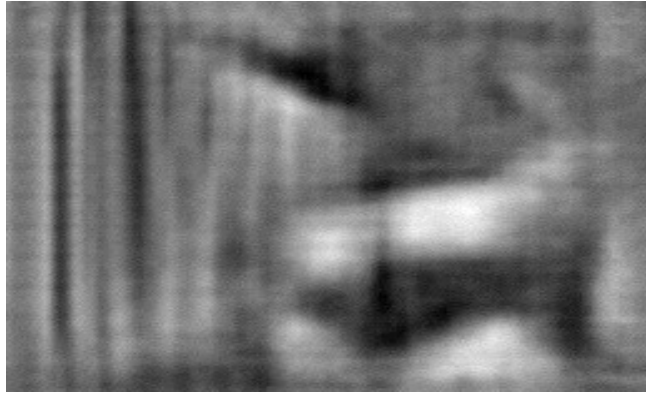












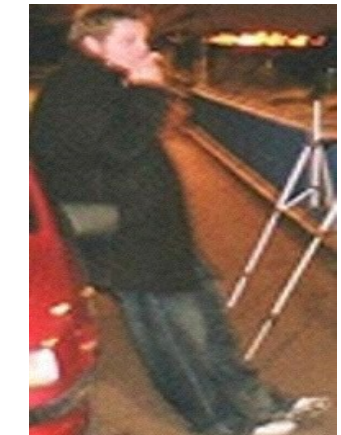
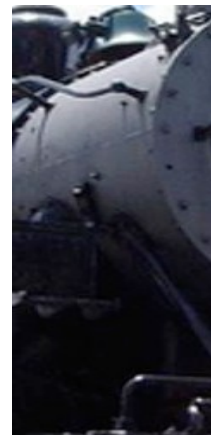
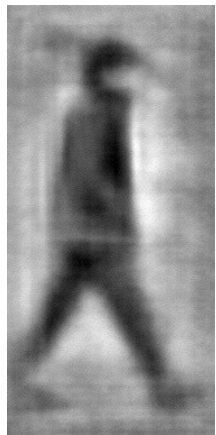








# The HOGgles Challenge



# Next time

Homographies