Lecture 10

Segmentation and clustering

Administrative

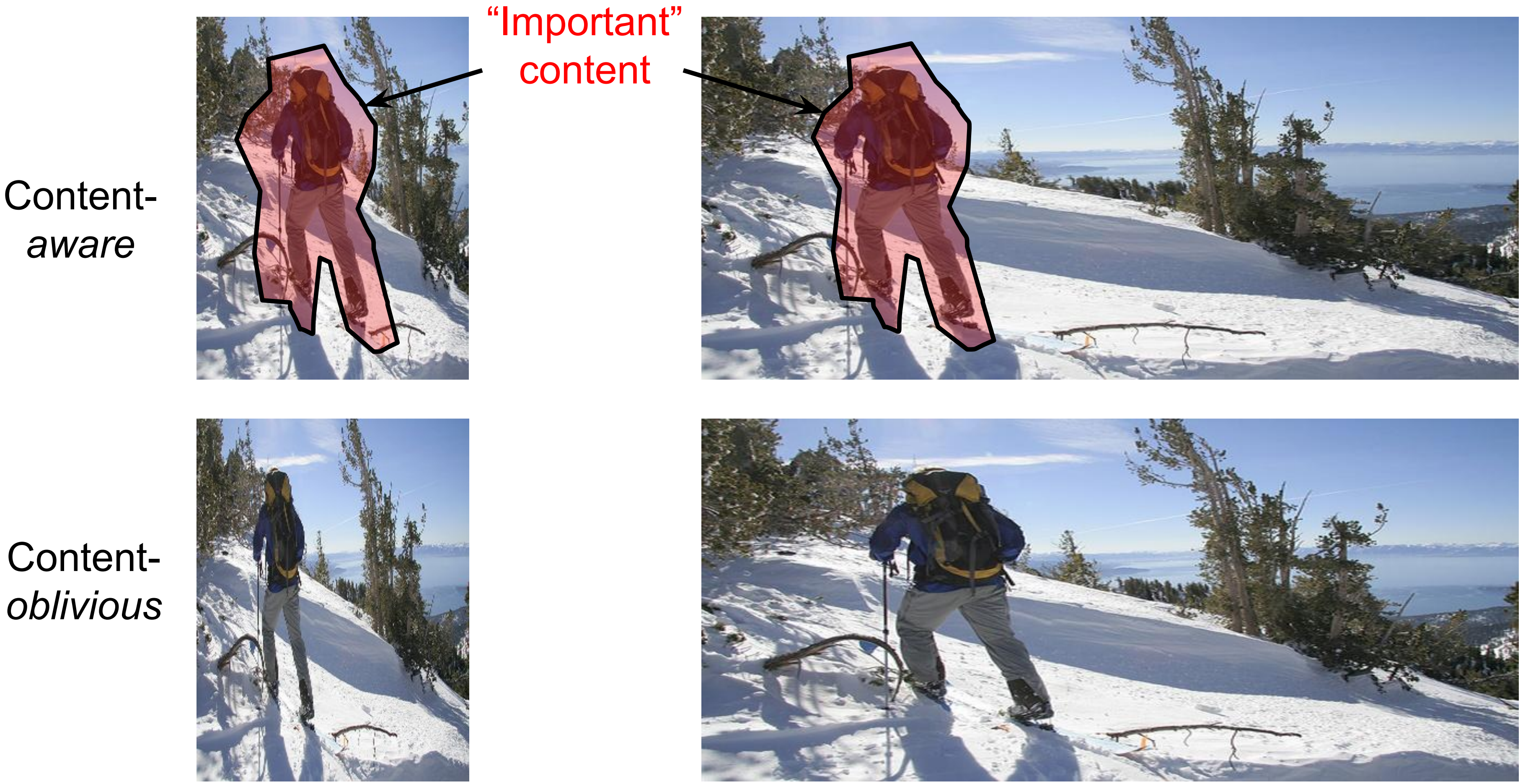
A2 is out
- Due Feb 7th

A3 is out
- Due Feb 21st

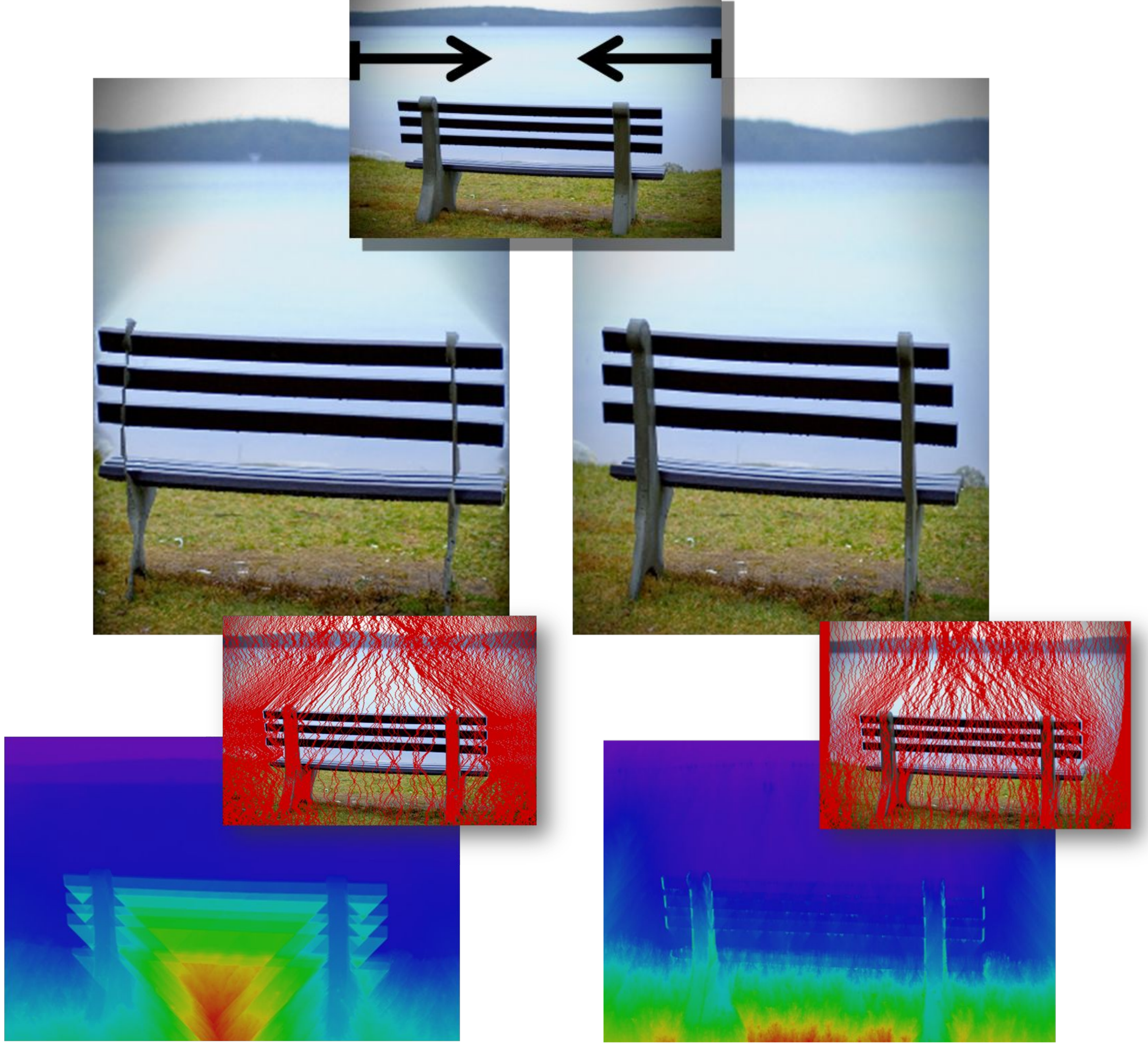
Administrative

Recitation
- Geometric transformations

Content-aware Retargeting Operators

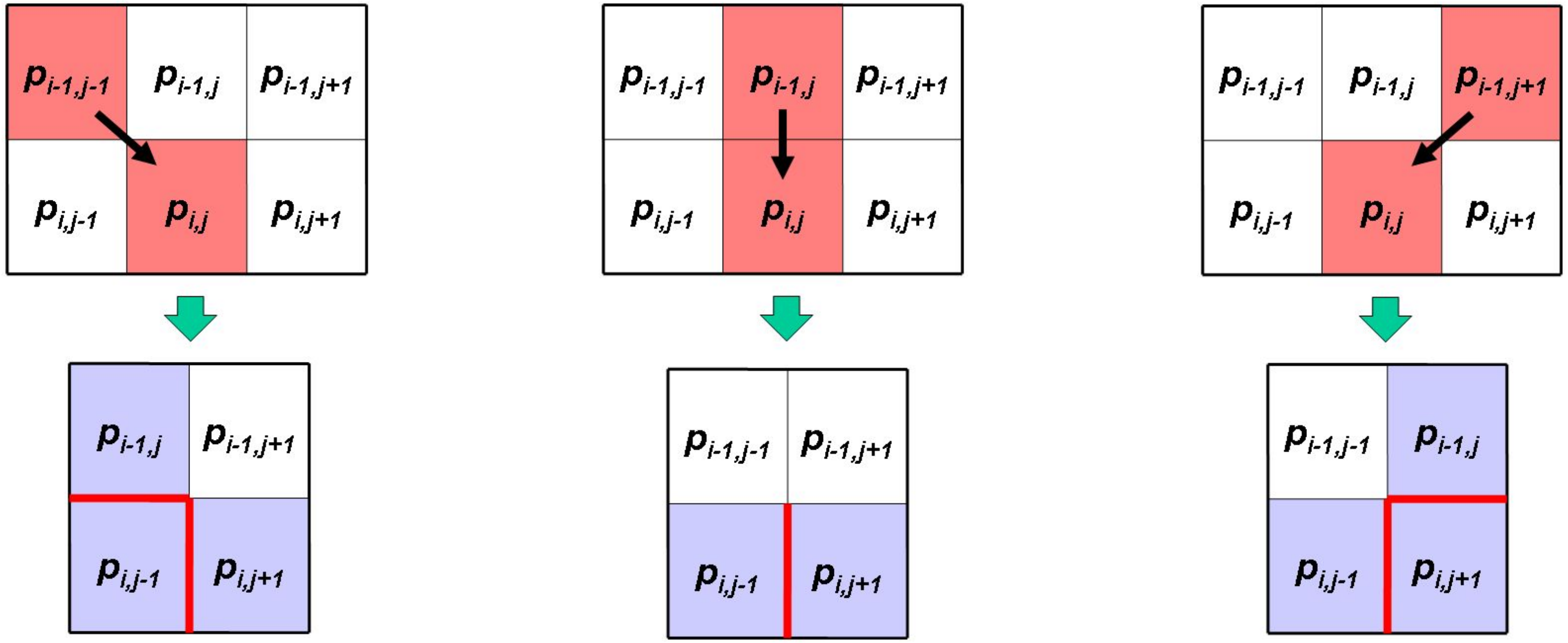


So far



So far: Seam carving with pixel energies

$$M(i,j) = E(i,j) + min \begin{cases} M(i-1,j-1) + C_L(i,j) \\ M(i-1,j) + C_V(i,j) \\ M(i-1,j+1) + C_R(i,j) \end{cases}$$



Retargeting in Both Dimensions

- Let T(r,c) denote a new cost matrix of obtaining an image of size (n-r)x(m-c).

$$\begin{aligned}\mathbf{T}(r,c) \quad = \quad & min(\mathbf{T}(r-1,c) + E(\mathbf{s^x}(\mathbf{I_{n-r-1\times m-c}})), \\ & \mathbf{T}(r,c-1) + E(\mathbf{s^y}(\mathbf{I_{n-r\times m-c-1}})))\end{aligned}$$

where $E(\mathbf{s^x}(\mathbf{I_{n-r-1\times m-c}})$ is the cost of removing a horizontal

seam from the image $\mathbf{I_{n-r-1\times m-c}}$

Today's agenda

- Introduction to segmentation and clustering
- Gestalt theory for perceptual grouping
- Graph-based oversegmentation
- Agglomerative clustering

Reading:
Szeliski, 2nd edition, Chapter 7.5

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Q. What do you see?

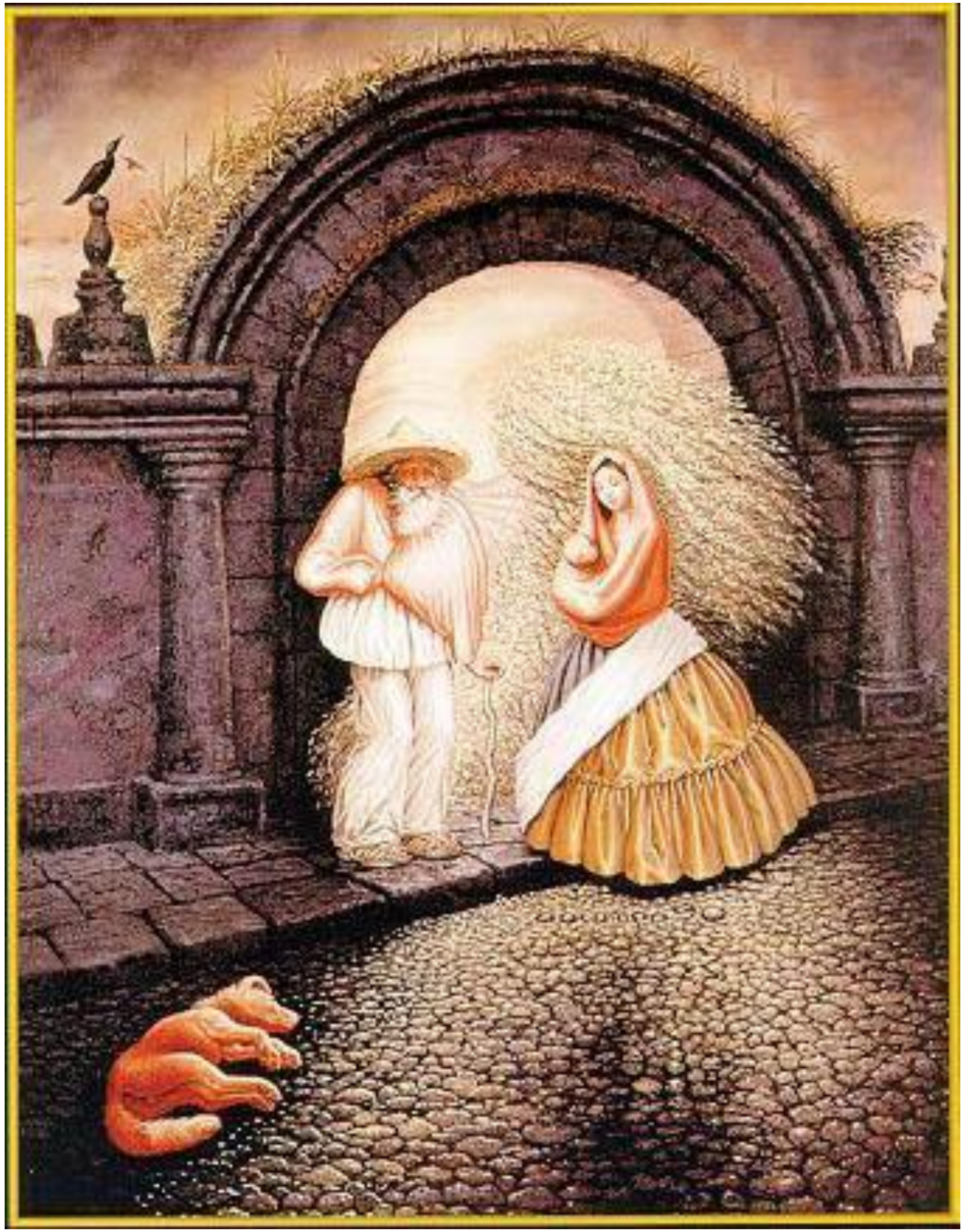
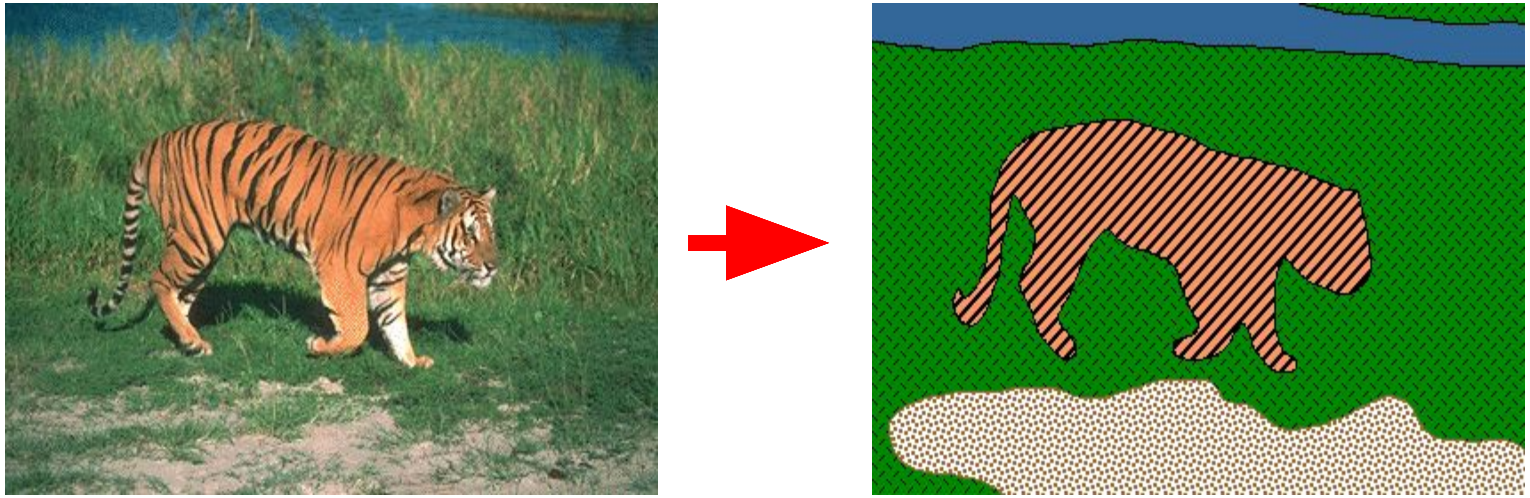


Image Segmentation

- Goal: identify groups of pixels that go together

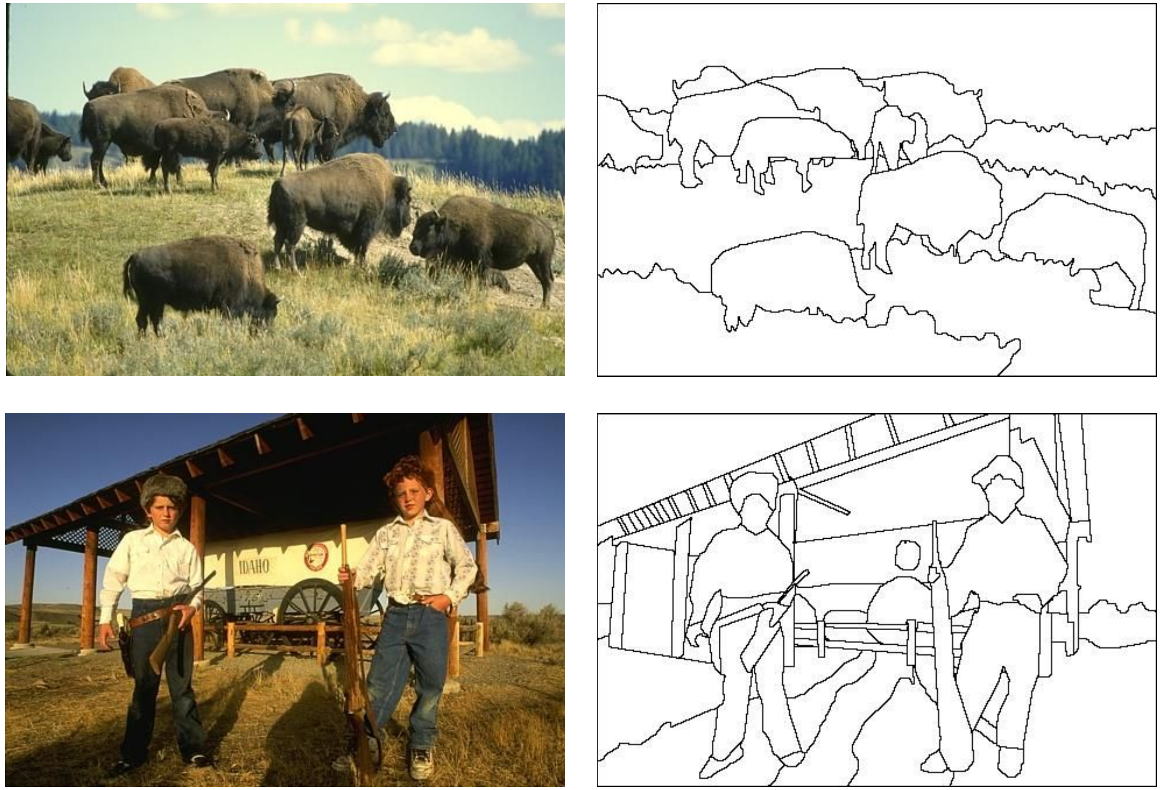


Slide credit: Steve Seitz, Kristen Grauman

The Goals of Segmentation

- Separate image into coherent “objects”

Image | Human segmentation

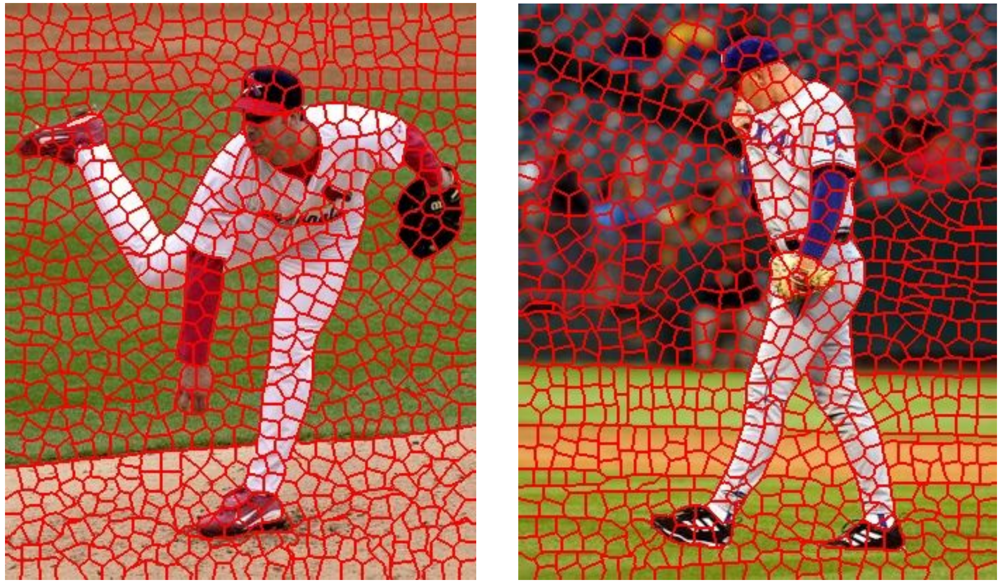


Slide credit: Svetlana Lazebnik

The Goals of Segmentation

- Separate image into coherent "objects"
- Group together similar-looking pixels for efficiency of further processing

"superpixels"



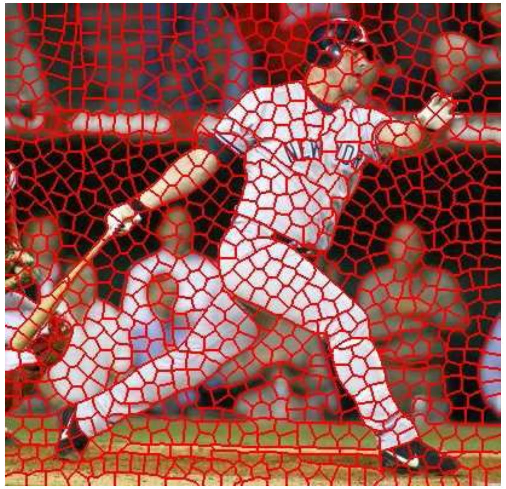
X. Ren and J. Malik. Learning a classification model for segmentation. ICCV 2003.

Segmentation for feature support

Slide: Derek Hoiem

Segmentation for efficiency

[Felzenszwalb and Huttenlocher 2004]



[Hoiem et al. 2005, Mori 2005]

[Shi and Malik 2001]

Slide: Derek Hoiem

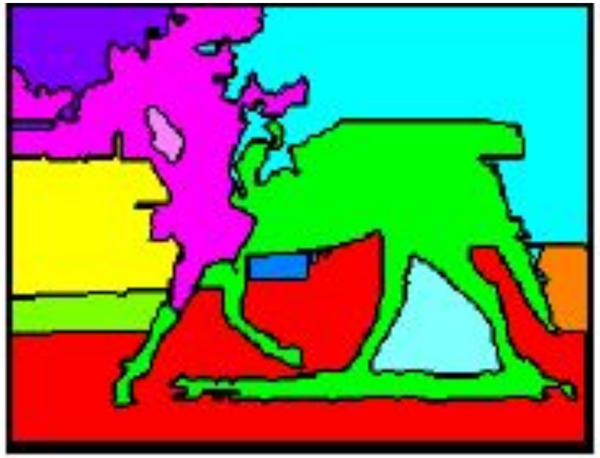
Segmentation is used in Adobe photoshop to remove background

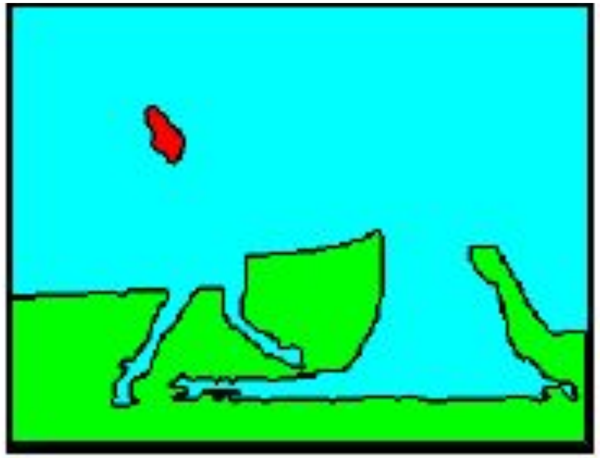
Rother et al. 2004

[Segment Anything](#) [2023]

Levels of segmentations

Over-segmentation





Under-segmentation

One way to think about "segmentation" is clustering

Clustering: group together similar data points and represent them with a single token

Key Challenges:

1) What makes two points/images/patches similar?
2) How do we compute an overall grouping from pairwise similarities?

Slide: Derek Hoiem

Why do we cluster?

- **Summarizing data**
  - Look at large amounts of data
  - Find clusters of pixels
  - Represent each cluster of pixels with a HoG feature
- **Counting**
  - Histograms of texture, color, SIFT vectors
- **Foreground-background separation**
  - Separate the image into different regions
- **Prediction**
  - Images in the same cluster may have the same labels

Slide: Derek Hoiem

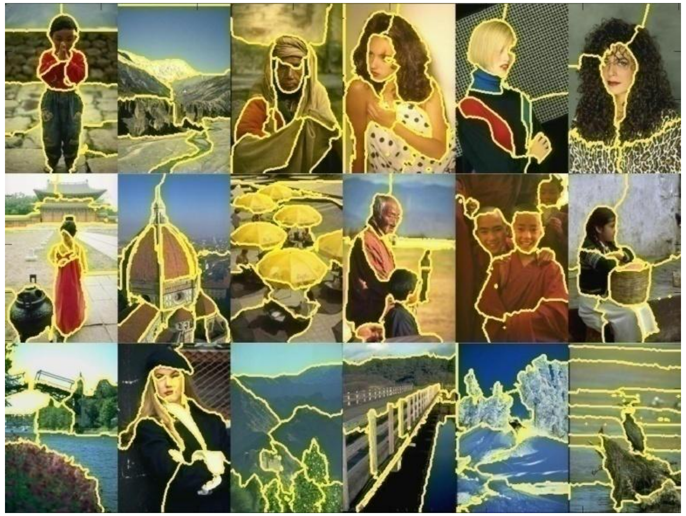
How do we cluster?

- **Agglomerative clustering**
  - Start with each point as its own cluster and iteratively merge the closest clusters
- **K-means**
  - Iteratively re-assign points to the nearest cluster center
- **Mean-shift clustering**
  - Estimate modes of pdf

General ideas

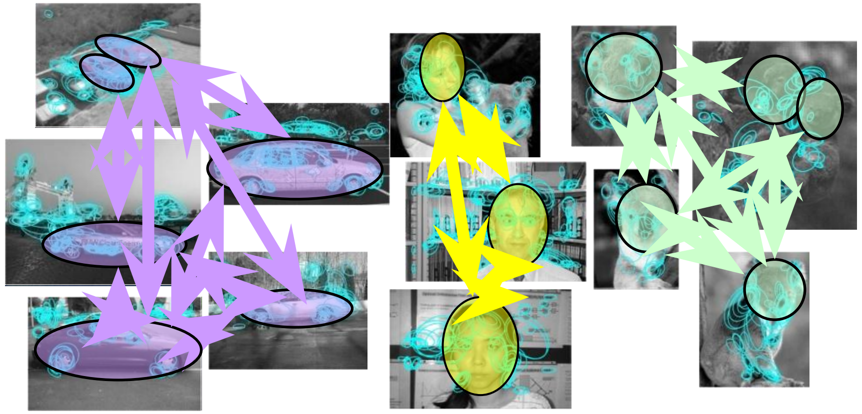
- Tokens
  - Things that can be grouped together
  - (e.g. pixels, points, surface elements, etc., etc.)
- Bottom up clustering
  - tokens belong together because they are locally coherent
- Top down clustering
  - tokens belong together because they lie on the same visual entity (object, scene…)
- > These two are not mutually exclusive

Examples of Grouping in Vision



Determining image regions

Grouping video frames into shots



Object-level grouping

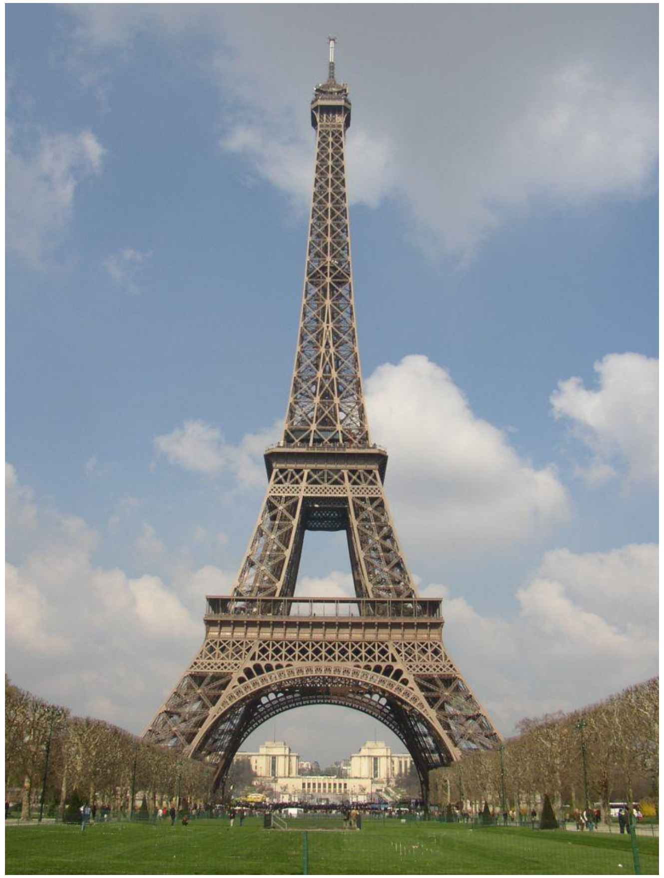
Figure-ground

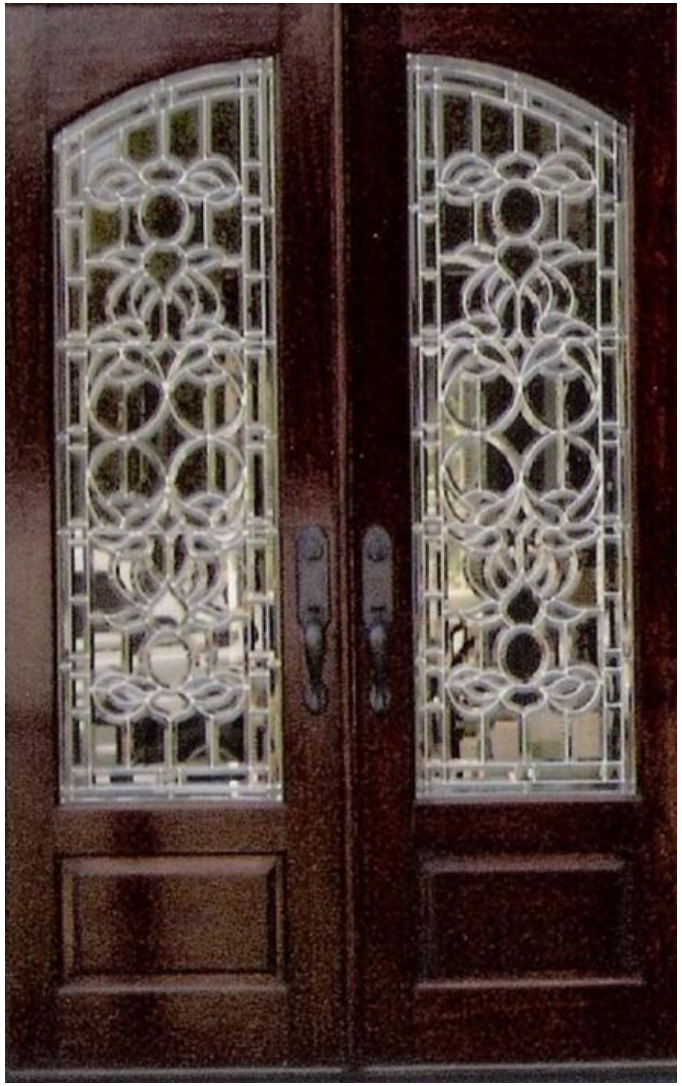
Similarity

What things should be grouped?

What cues indicate groups?

Symmetry





Slide credit: Kristen Grauman

Common Fate

Image credit: Arthus-Bertrand (via F. Durand)

Slide credit: Kristen Grauman

Proximity

What will we learn today?

- Introduction to segmentation and clustering
- Gestalt theory for perceptual grouping
- Graph-based oversegmentation
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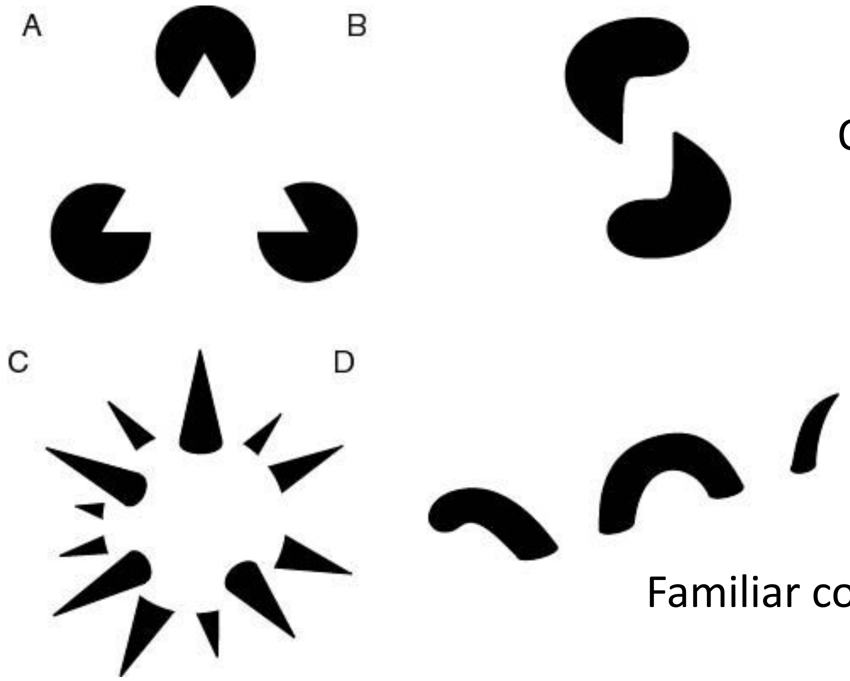
The Gestalt School

- Grouping is key to visual perception
- Elements in a collection can have properties that result from different **relationships (space, affordance, etc.)**
  - "The whole is greater than the sum of its parts"



Illusory/subjective contours

Occlusion

Familiar configuration

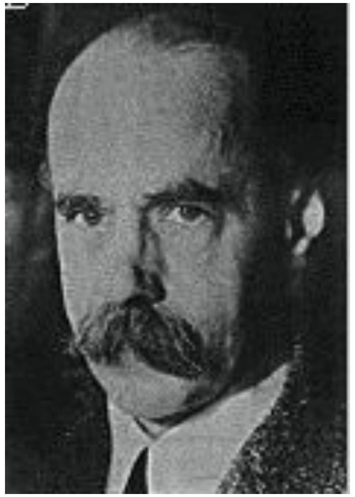
http://en.wikipedia.org/wiki/Gestalt_psychology

Gestalt Theory

- Gestalt: whole or group
  - Whole is greater than sum of its parts
  - Relationships among parts can yield new properties/features
- Psychologists identified series of factors that predispose set of elements to be grouped (by human visual system)

"I stand at the window and see a house, trees, sky. Theoretically I might say there were 327 brightnesses and nuances of colour. Do I have "327"? No. I have sky, house, and trees."

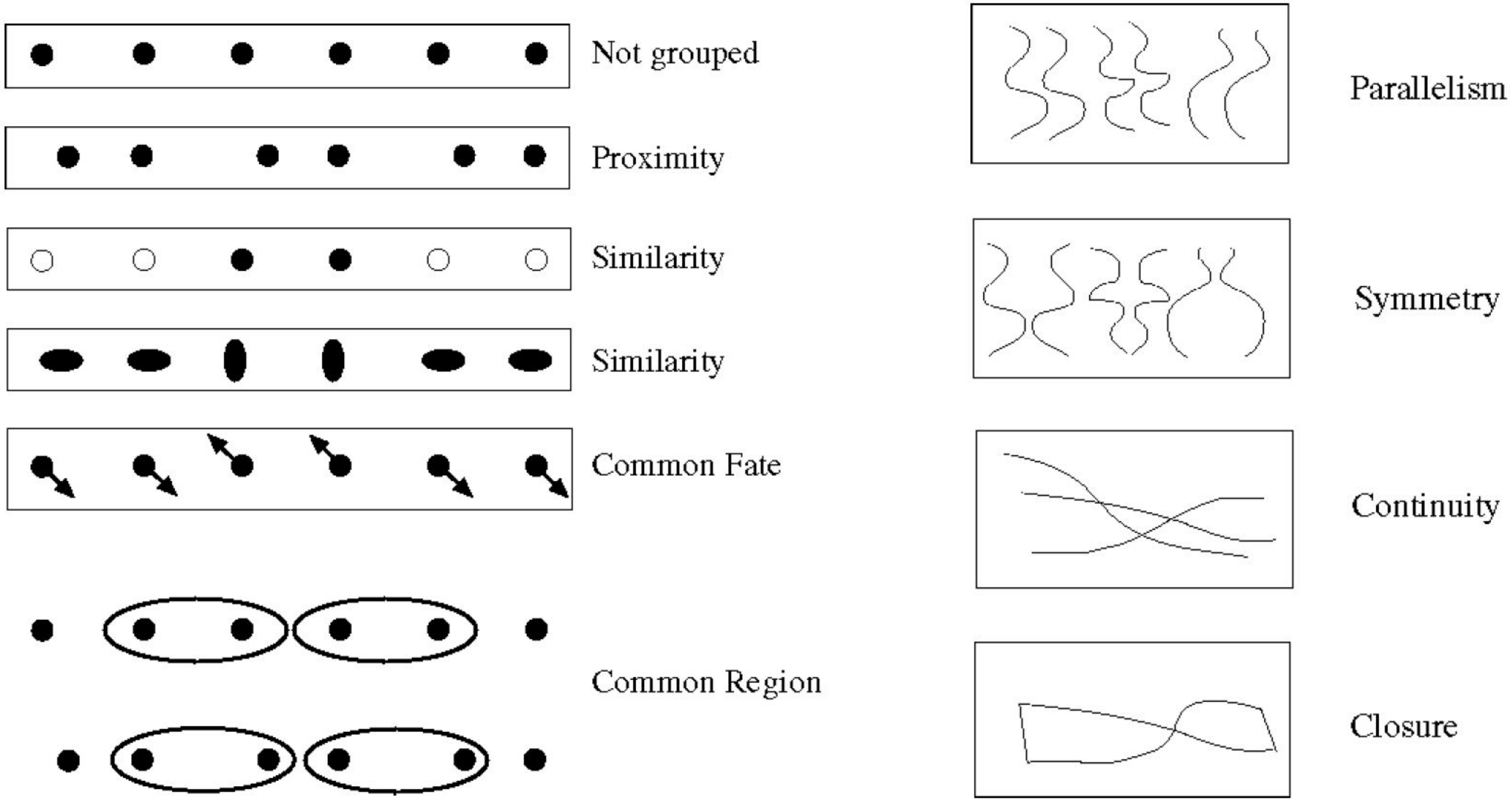
Max Wertheimer (1880-1943)



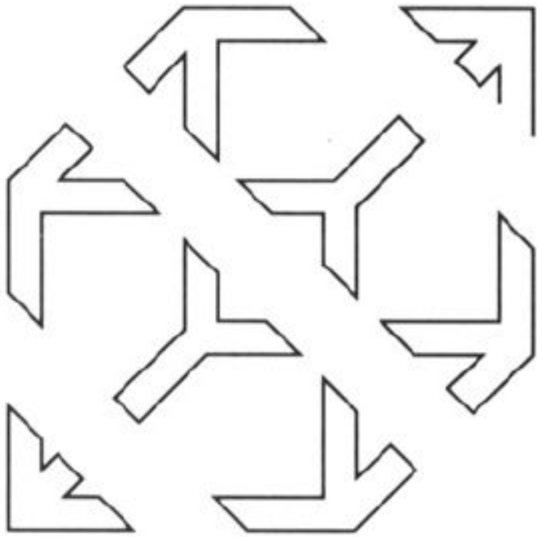
Untersuchungen zur Lehre von der Gestalt,
Psychologische Forschung, Vol. 4, pp. 301-350, 1923
http://psy.ed.asu.edu/~classics/Wertheimer/Forms/forms.htm

These factors make intuitive sense, but are very difficult to translate into algorithms.

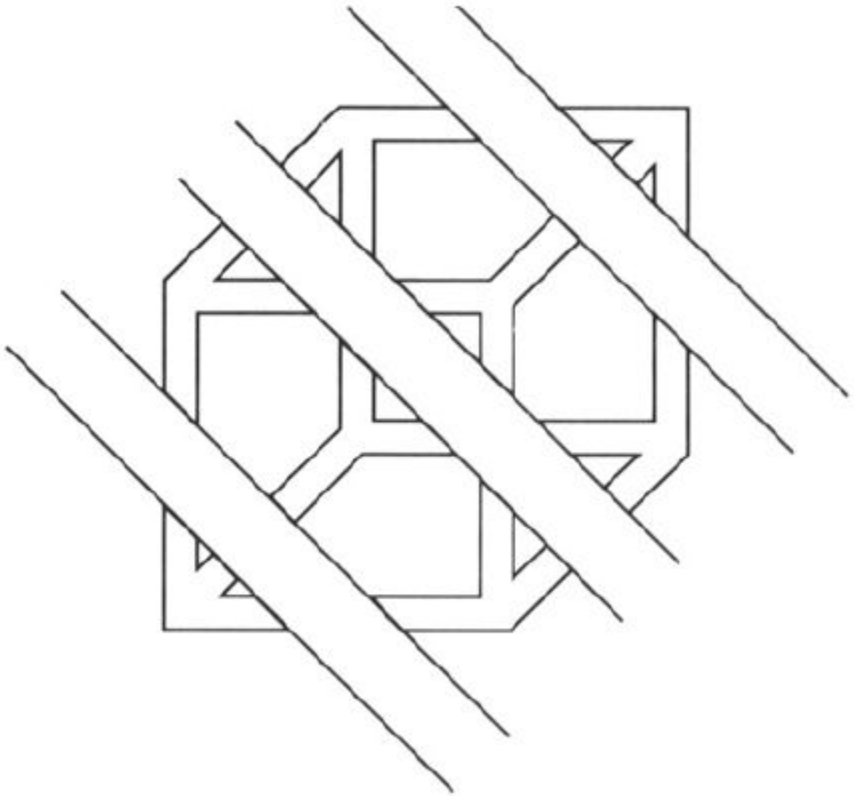
Gestalt Factors



Continuity through Occlusion Cues

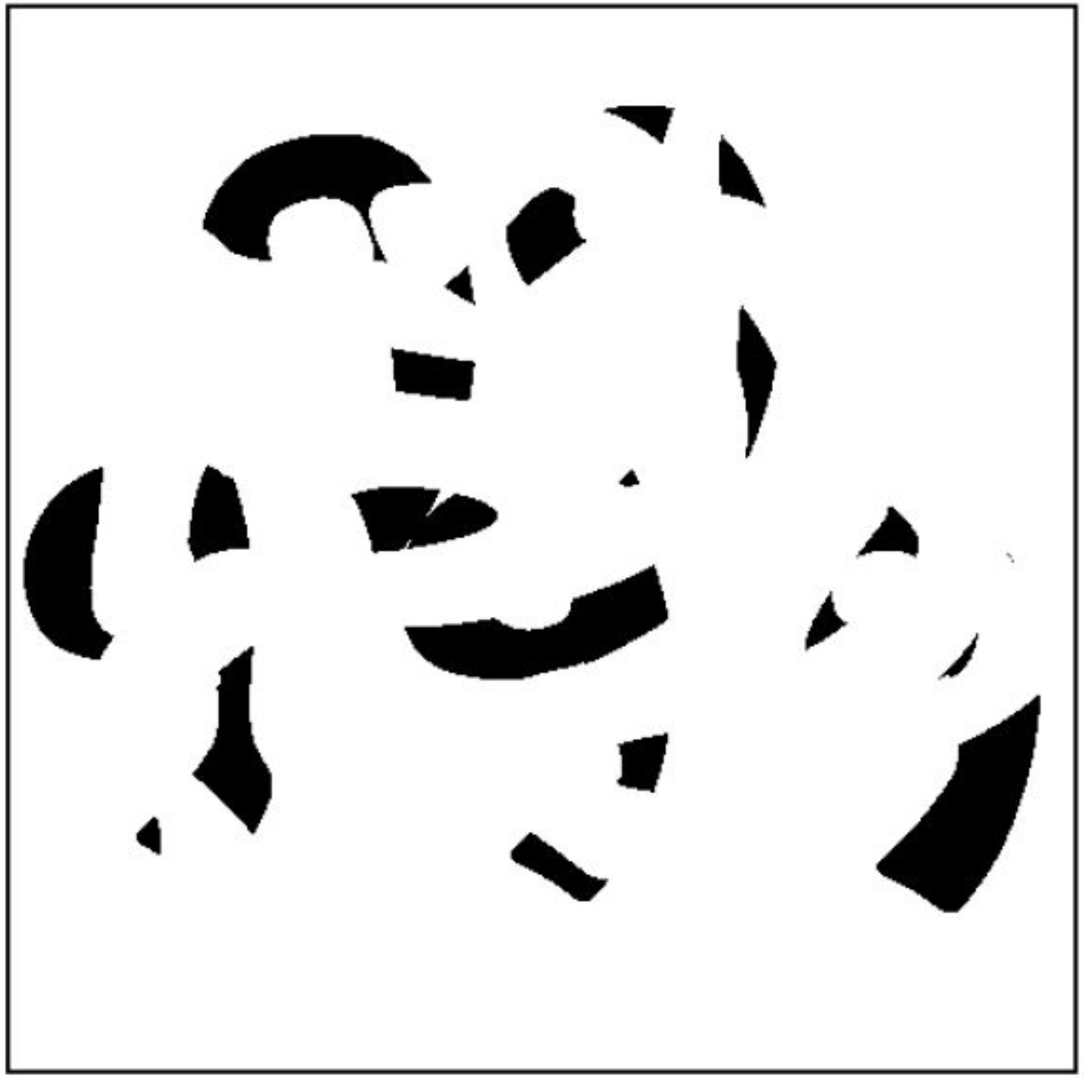


Continuity through Occlusion Cues



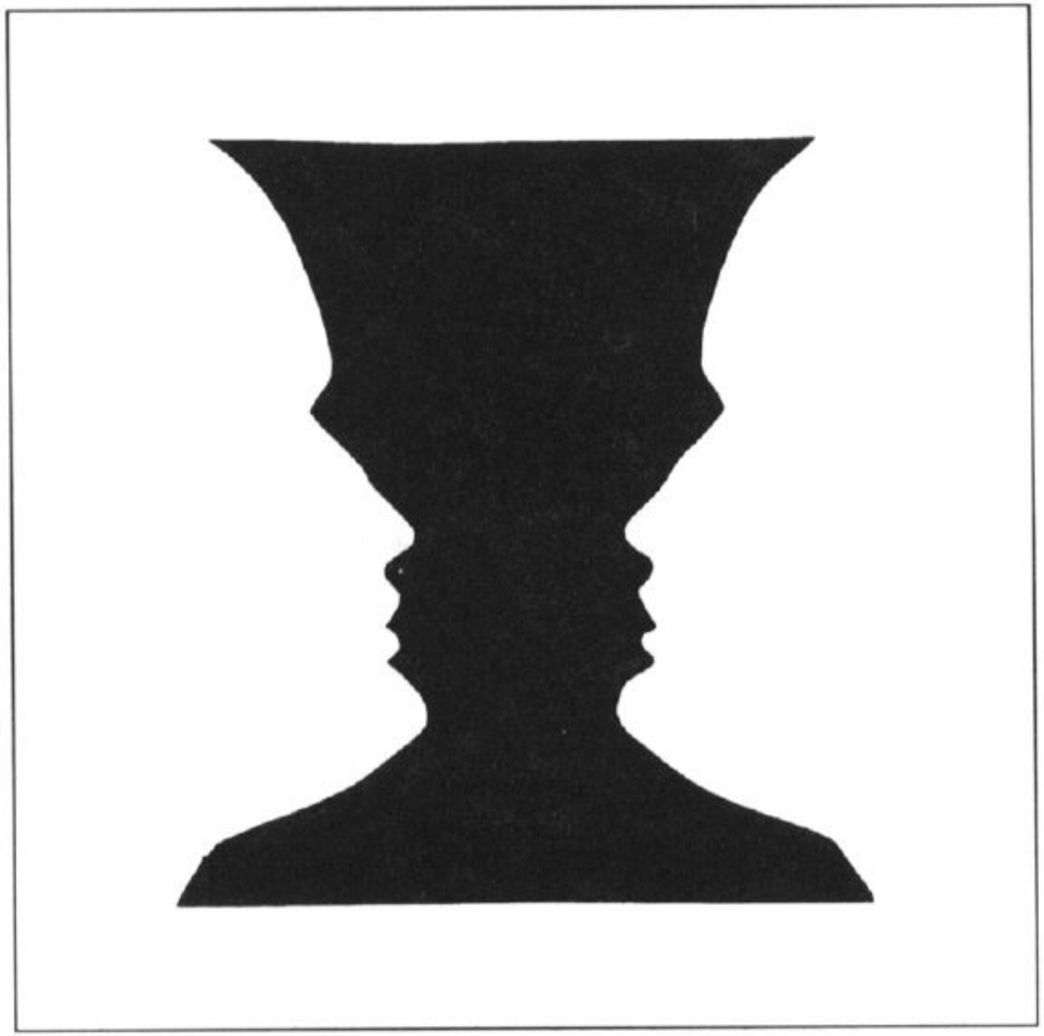
Continuity, explanation by occlusion

Continuity through Occlusion Cues



Continuity through Occlusion Cues

Figure-Ground Discrimination



The Ultimate Gestalt?

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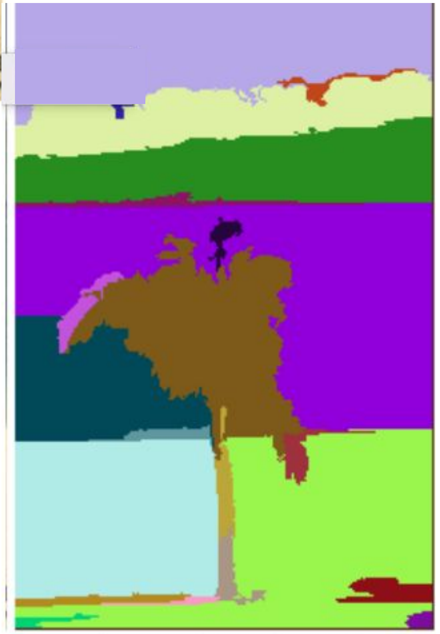
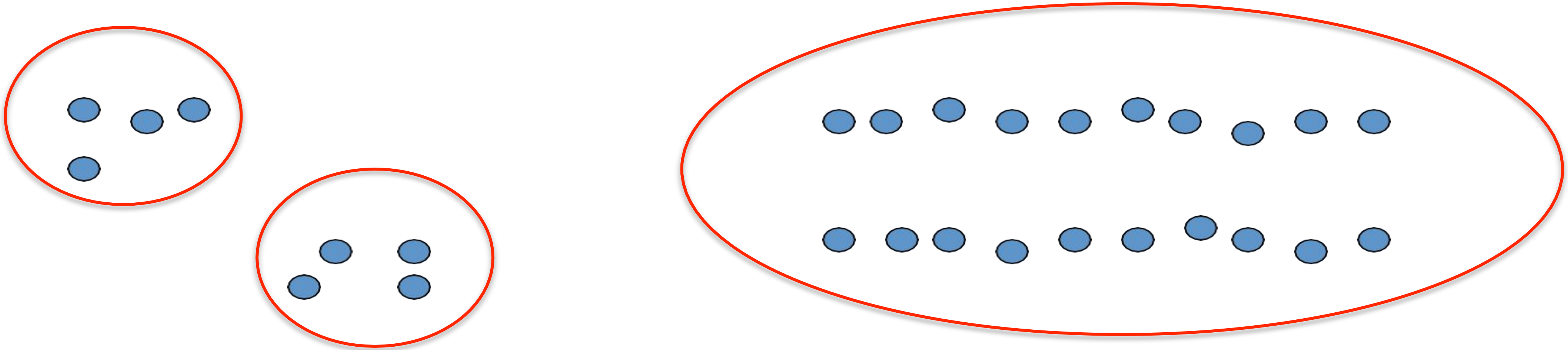


Over-segmenting images

- Graph-based clustering for Image Segmentation
  - Introduced by *Felzenszwalb and Huttenlocher* in the paper titled *Efficient Graph-Based Image Segmentation.*

Imagine you have a set of pixels, how should you clustering them?

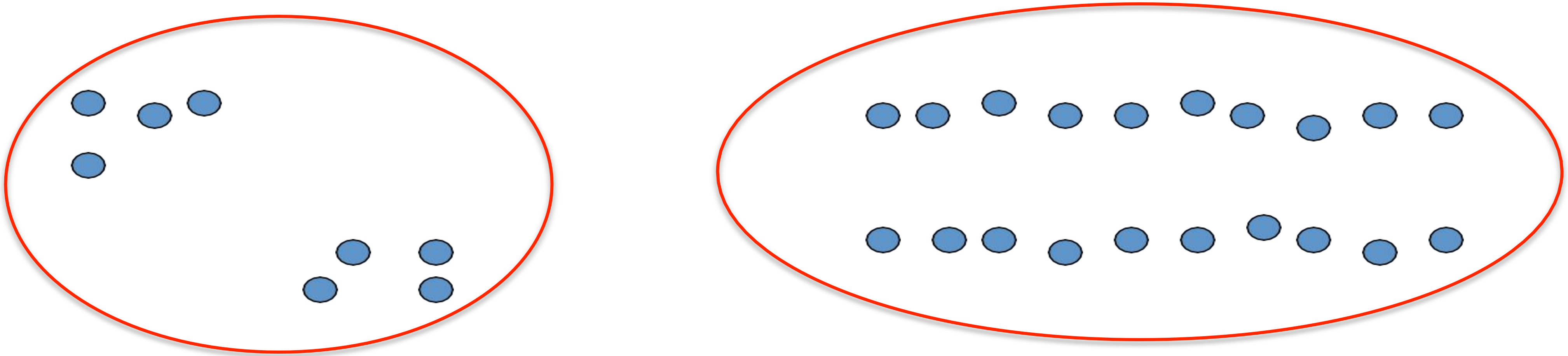


Basic idea: group together similar instances

Q. how do you measure similarity?

Q. do you need to measure similarity between every two pixels?

Imagine you have a set of pixels, how should you clustering them?

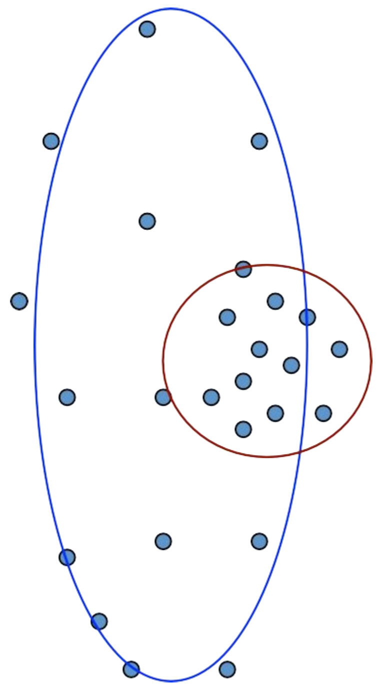


Basic idea: group together similar instances

Q. how do you measure similarity?

Q. do you need to measure similarity between every two pixels?

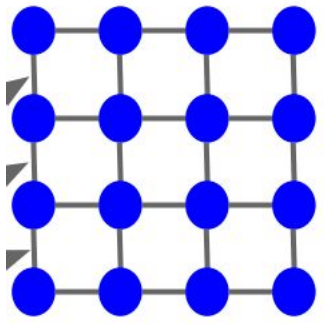
Distances calculated using only (x,y) location of each pixel can be a bad idea



- Clusters may overlap
- Some clusters may be “wider” than others
- Distances can be deceiving!

Image as a Graph - Features and weights

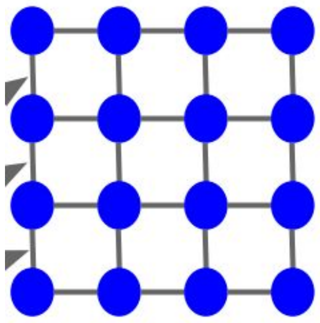
- Every pixel is connected to its 8 neighboring pixels
- The edges between neighbors have weights that are determined by the distance between them.
- Edge weights between pixels are determined using dist(x, x') distance in feature space.
  - where x and x' are two neighboring pixels



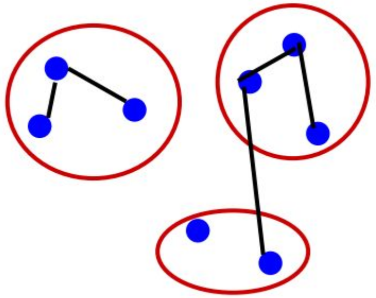
- Q. What is a good feature space?

What are good pixel features?

- Use RGB values?
  - v = [r, g, b]
  - It is 3-dimensional
- Use location?
  - v = [x, y]
  - 2-dim
- Use RGB + location?
  - v = [x, y, r, g, b]
  - 5-dim
- Use gradient magnitude?
  - v = [df/dx, df/dy]
  - 2-d



Problem Formulation



- Graph G = (V, E)
- V is set of nodes (i.e. pixels)
- E is a set of undirected edges between pairs of pix
- dist(v_i , v_j) is the weight/distance of the edge between nodes v_i and v_j.

- S is a segmentation of a graph G such that G' = (V, E') where E' ⊂ E.
  - That is, **we keep all vertices**, but **select a subset E'** from all initial edges E.
- S divides G into G' such that it contains distinct clusters C.

Weights of edges: distance measure

Clustering is an unsupervised learning method. Given items $v_1, v_2, \ldots, v_n \in \mathcal{R}^D$, the goal is to group them into clusters.

We need a pairwise distance/similarity function between items, and sometimes the desired number of clusters.

When data (e.g. images, objects, documents) are represented by feature vectors, commonly used measures are:

- *Euclidean distance.*
- *Cosine similarity.*

Defining Distance Measures

Let x and x' be two objects from the universe of possible objects.
The distance (or similarity) between x and x' is a real number:

- The Euclidean distance is defined as $$dist(v_1, v_2) = \sqrt{\sum_i (v_{1i} - v_{2i})^2}$$
- In contrast, the cosine similarity measure would be

$$\begin{aligned} dist(v_1, v_2) &= 1 - cos(v_1, v_2) \\ &= 1 - \frac{v_1^T v_2}{||v_1|| \cdot ||v_2||} \end{aligned}$$

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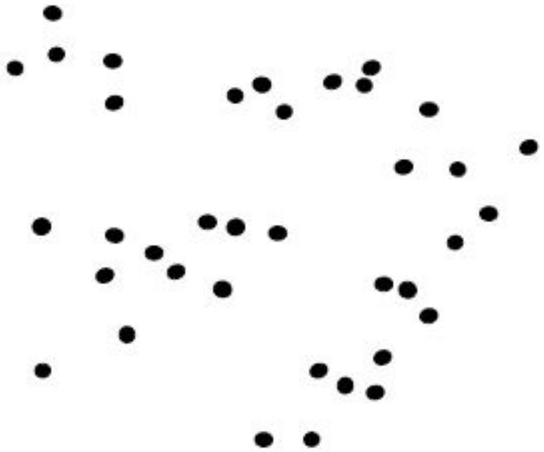
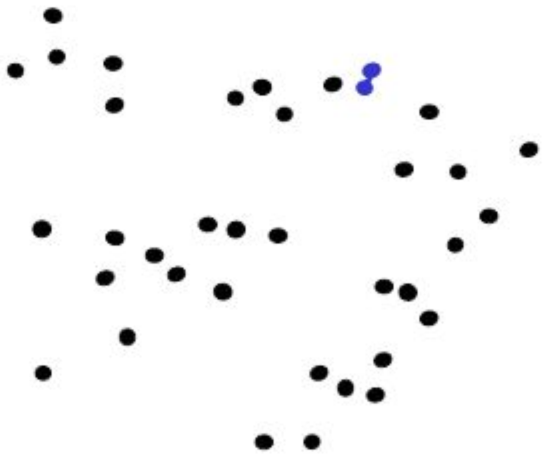
Agglomerative clustering



1. Say "Every point is its own cluster"

Slide credit: Andrew Moore

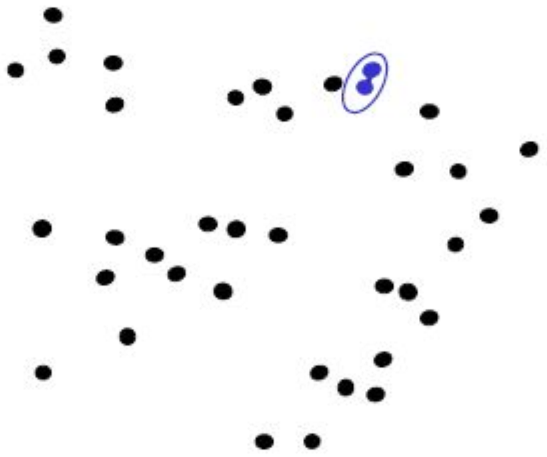
Agglomerative clustering



1. Say "Every point is its own cluster"
2. Find "most similar" pair of clusters

Slide credit: Andrew Moore

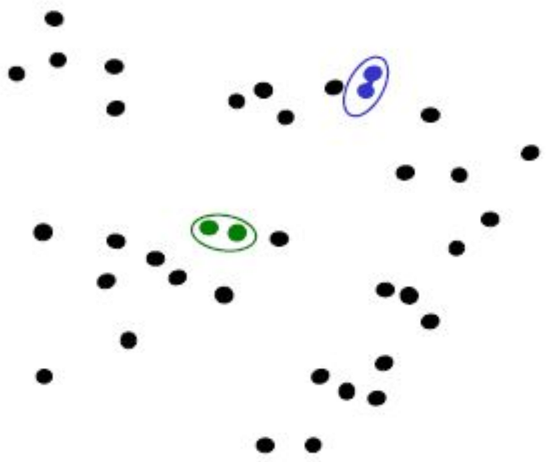
Agglomerative clustering



1. Say "Every point is its own cluster"
2. Find "most similar" pair of clusters
3. Merge it into a parent cluster

Slide credit: Andrew Moore

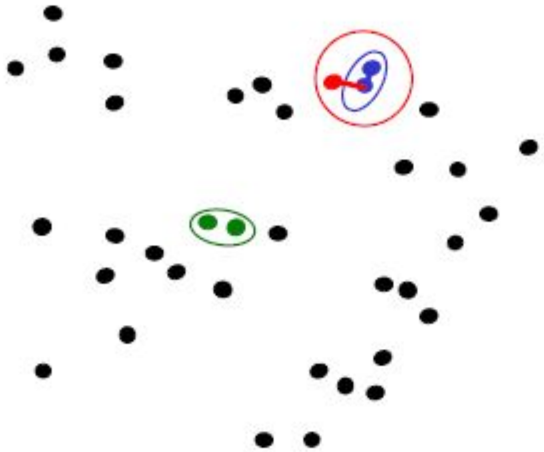
Agglomerative clustering



1. Say "Every point is its own cluster"
2. Find "most similar" pair of clusters
3. Merge it into a parent cluster
4. Repeat

Slide credit: Andrew Moore

Agglomerative clustering



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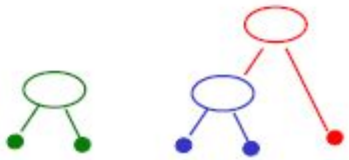
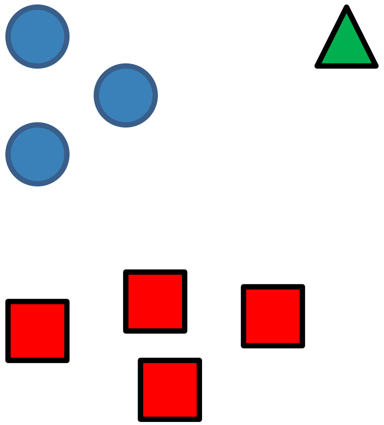


Slide credit: Andrew Moore

Agglomerative clustering

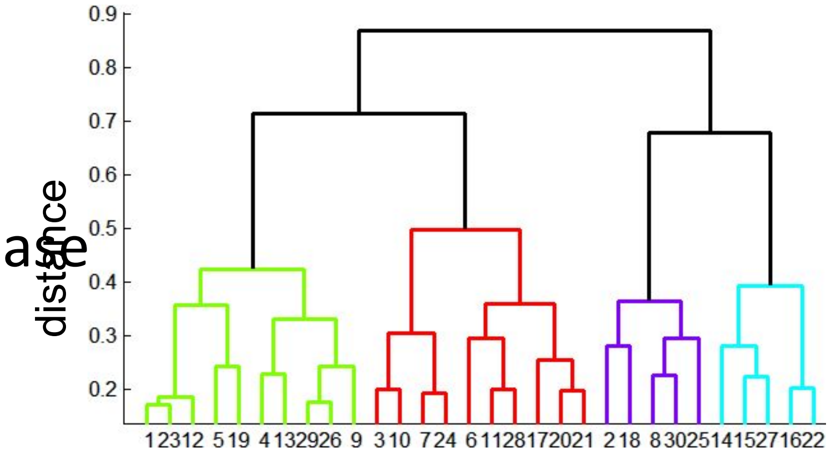
How to define cluster similarity?

- Average distance between all pixels between the two cluster?
- Maximum distance?
- Minimum distance?
- Distance between means?



How many clusters?

- Clustering creates a dendrogram (a tree)
- Threshold based on max number of clusters or base
  between merges

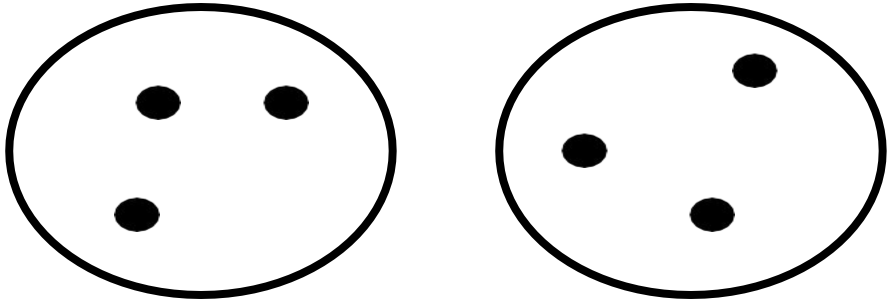


Agglomerative Hierarchical Clustering - Algorithm

Inputs:
- An input image
- Feature representation for each pixel
- Distance metric dist(-,-)

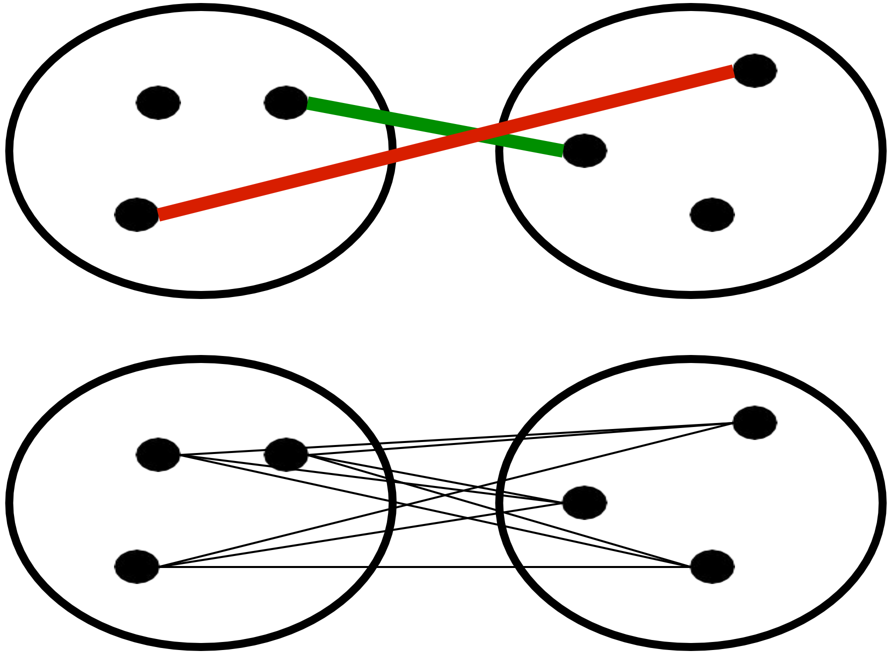
- ➢ Initially, each pixel $v_1, \ldots, v_n$ is its own cluster $C_1, \ldots, C_n$
- ➢ While True:
  - Find two nearest clusters according to dist(C_i, C_j)
  - Merge C = (C_i, C_j)
  - If only 1 cluster is left:
    - break

How should we define “closest” for clusters with multiple pixels already in it?



How should we define "closest" for clusters with multiple pixels already in it?

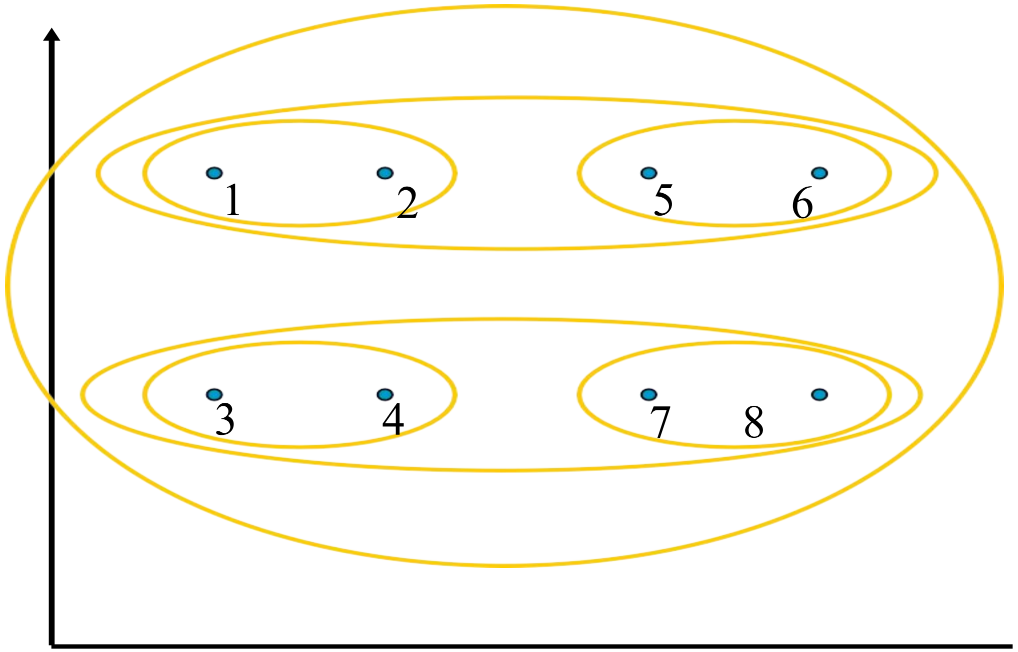
- Closest pair
  (single-link clustering)
- Farthest pair
  (complete-link clustering)
- Average of all pairs



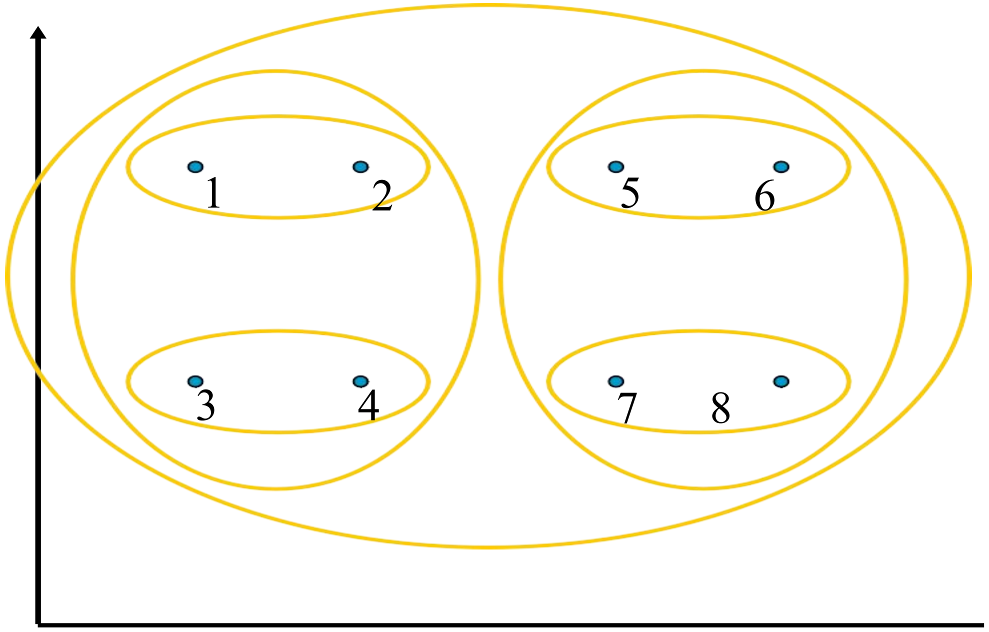
Different choices create different clustering behaviors

How should we define "closest" for clusters with multiple pixels already in it?

Closest pair
(single-link clustering)



Farthest pair
(complete-link clustering)

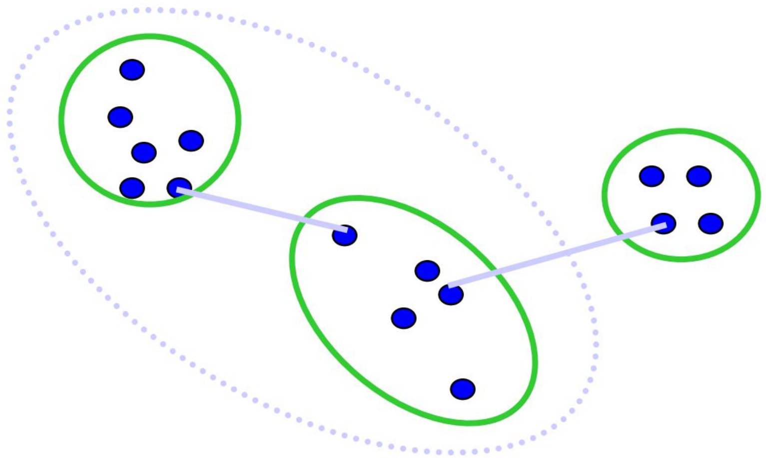


[Pictures from Thorsten Joachims]

Single Linkage distance measure

$$dist(C_i, C_j) = \min_{v_i \in C_i, v_j \in C_j, (C_i, C_j) \in E} dist(v_i, v_j)$$

Connects the clusters based on the distance of their closest pixels
It produces “long” clusters.

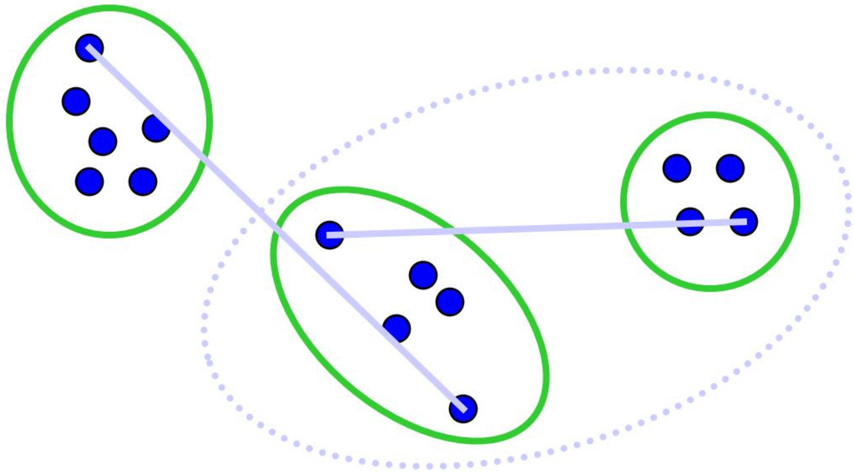


Long, skinny clusters

Complete Link distance measure

$$dist(C_i, C_j) = \max_{v_i \in C_i, v_j \in C_j, (C_i, C_j) \in E} dist(v_i, v_j)$$

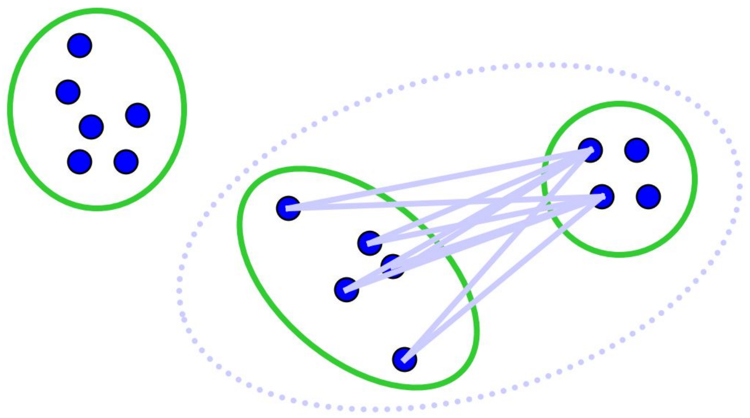
Produces compact clusters that are similar in diameter



Tight clusters

Average Link distance measures

$$dist(C_i, C_j) = \frac{\sum_{v_i \in C_i, v_j \in C_j, (C_i, C_j) \in E} dist(v_i, v_j)}{|C_i||C_j|}$$



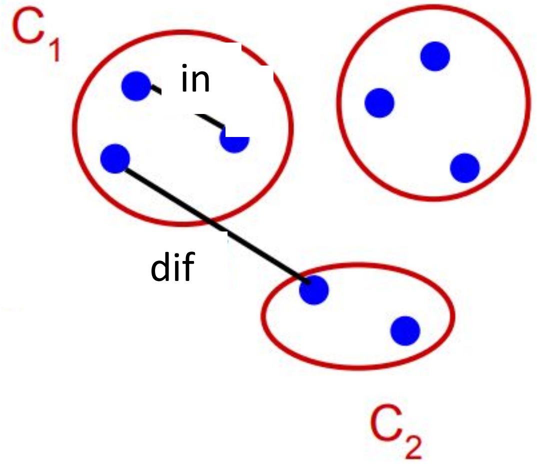
Robust against noise.

Inlier-outlier linkage distance measure

$$Merge(C_1, C_2) = \begin{cases} True & if\ dif(C_1, C_2) < in(C_1, C_2) \\ False & otherwise \end{cases}$$

Where

- dif(C1 , C2) is the difference between two clusters.
- in(C1 , C2) is the internal difference in the clusters C1 and C2



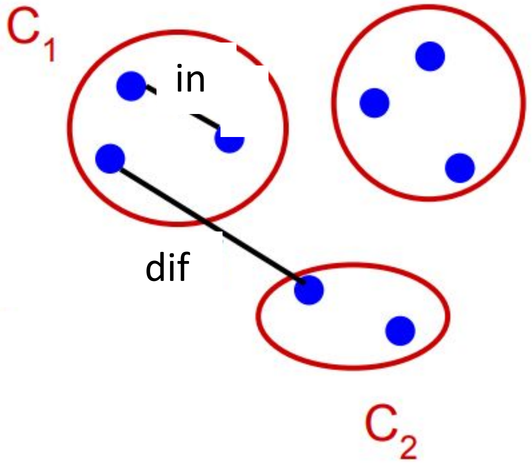
Inlier-outlier linkage distance measure

$$Merge(C_1, C_2) = \begin{cases} True & if\ dif(C_1, C_2) < in(C_1, C_2) \\ False & otherwise \end{cases}$$

$$dif(C_i, C_j) = \min_{v_i \in C_i, v_j \in C_j, (C_i, C_j) \in E} dist(v_i, v_j)$$

Where

- dif(C1 , C2) is the difference between two clusters.
- in(C1 , C2) is the internal difference in the clusters C1 and C2

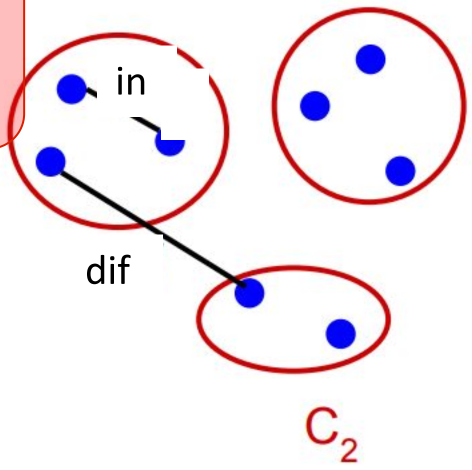


Inlier-outlier linkage distance measure

$$Merge(C_1, C_2) = \begin{cases} True & \text{if } dif(C_1, C_2) < in(C_1, C_2) \\ False & \text{otherwise} \end{cases}$$

$$dif(C_i, C_j) = \min_{v_i \in C_i, v_j \in C_j, (C_i, C_j) \in E} dist(v_i, v_j)$$

$$in(C_i, C_j) = \min_{C \in \{C_i, C_j\}} [\max_{v_i, v_j \in C} [dist(v_i, v_j) + \frac{k}{|C|}]]$$



Where

- dif(C1 , C2) is the difference between two clusters.
- in(C1 , C2) is the internal difference in the clusters C1 and C2

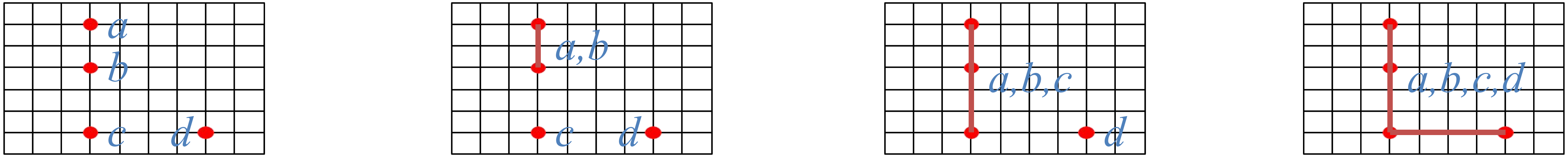
inlier-outlier linkage for Segmentation

- k/|C| sets the threshold by which the clusters need to be different from the internal pixels in a cluster.
- Effect of k:
  - **If k is large, it causes a preference for larger objects.**

Results

How to implement single-linkage efficiently

Euclidean Distance



(1) (2) (3)

	b	*c*	*d*
a	2	5	6
b		3	5
c			4

	b	*c*	*d*
a	2	5	6
b		3	5
c			4

	c	*d*
a, b	3	5
c		4

	d
a, b, c	4

Distance Matrix

Conclusions: Agglomerative Clustering

Pros:

- Simple to implement, widespread application.
- Clusters have adaptive shapes.
- Provides a hierarchy of clusters.
- No need to specify number of clusters **in advance**.

Cons:

- May have imbalanced clusters.
- Still have to choose number of clusters eventually for an application
- Does not scale well. Runtime of $O(n^3)$.
- Can get stuck at a local optima.

Today's agenda

- Introduction to segmentation and clustering
- Gestalt theory for perceptual grouping
- Graph-based oversegmentation
- Agglomerative clustering

Next time

K-means and mean shift

Other Kernels

A kernel is a function that satisfies the following requirements :

1. $\int_{R^d} \phi(x) = 1$
2. $\phi(x) \geq 0$

Some examples of kernels include :

1. Rectangular $\phi(x) = \begin{cases} 1 & a \leq x \leq b \\ 0 & else \end{cases}$
2. Gaussian $\phi(x) = e^{-\frac{x^2}{2\sigma^2}}$
3. Epanechnikov $\phi(x) = \begin{cases} \frac{3}{4}(1 - x^2) & if\ |x| \leq 1 \\ 0 & else \end{cases}$

source

Technical Details

Taking the derivative of: $\hat{f}_K = \frac{1}{nh^d} \sum_{i=1}^{n} K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h}\right)$

$$\nabla \hat{f}(\mathbf{x}) = \underbrace{\frac{2c_{k,d}}{nh^{d+2}} \left[\sum_{i=1}^{n} g\left(\left\| \frac{\mathbf{x} - \mathbf{x}_i}{h} \right\|^2 \right) \right]}_{\text{term 1}} \underbrace{\left[\frac{\sum_{i=1}^{n} \mathbf{x}_i g\left(\left\| \frac{\mathbf{x}-\mathbf{x}_i}{h} \right\|^2 \right)}{\sum_{i=1}^{n} g\left(\left\| \frac{\mathbf{x}-\mathbf{x}_i}{h} \right\|^2 \right)} - \mathbf{x} \right]}_{\text{term 2}}, \qquad (3)$$

where $g(x) = -k'(x)$ denotes the derivative of the selected kernel profile.

- Term1: this is proportional to the density estimate at x (similar to equation 1 from two slides ago).
- Term2: this is the mean-shift vector that points towards the direction of maximum density.

Comaniciu & Meer, 2002

Technical Details

Finally, the mean shift procedure from a given point x_t is:

1. Compute the mean shift vector **m**:

$$\left[\frac{\sum_{i=1}^{n} \mathbf{x}_i g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)} - \mathbf{x}\right]$$

2. Translate the density window:

$$\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \mathbf{m}(\mathbf{x}_i^t).$$

3. Iterate steps 1 and 2 until convergence.

$$\nabla f(\mathbf{x}_i) = 0.$$

Comaniciu & Meer, 2002

Technical Details

Given n data points $\mathbf{x}_i \in \mathbb{R}^d$, the multivariate kernel density estimate using a radially symmetric kernel[1] (e.g., Epanechnikov and Gaussian kernels), $K(\mathbf{x})$, is given by,

$$\hat{f}_K = \frac{1}{nh^d} \sum_{i=1}^{n} K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h}\right), \tag{1}$$

where h (termed the *bandwidth* parameter) defines the radius of kernel. The radially symmetric kernel is defined as,

$$K(\mathbf{x}) = c_k k(\|\mathbf{x}\|^2), \tag{2}$$

where c_k represents a normalization constant.

Comaniciu & Meer, 2002