Lecture 2

Pixels and Filters

Administrative

A0 is out.

- It is ungraded
- Meant to help you with python and numpy basics
- Learn how to do homeworks and submit them on gradescope.

Grading policy - Assignments

- **Assignment 0** (Using Colabs, Python basics)
  - Recommended Due by Jan 14 (Ungraded)
- **Assignment 1** (Filters, Convolutions, Edges)
  - Due Jan 24, 11:59 PST
- **Assignment 2** (Keypoints, Panaromas, Seam Carving)
  - Due Feb 7, 11:59 PST
- **Assignment 3** (Cameras, Clustering, Segmentation)
  - Due Feb 21, 11:59 PST
- **Assignment 4** (kNN, PCA, LDA, Detection)
  - Due Mar 7, 11:59 PST
- **Assignment 5** (Optical Flow, Tracking, Machine Learning)
  - Due Mar 15, 11:59 PST

Grading policy - assignments

- Most assignments will have an extra credit worth 1% of your total grade.

- Late policy
  - 5 free late days – use them in your ways
  - Maximum of 2 late days per assignment
  - Afterwards, 25% off per day late

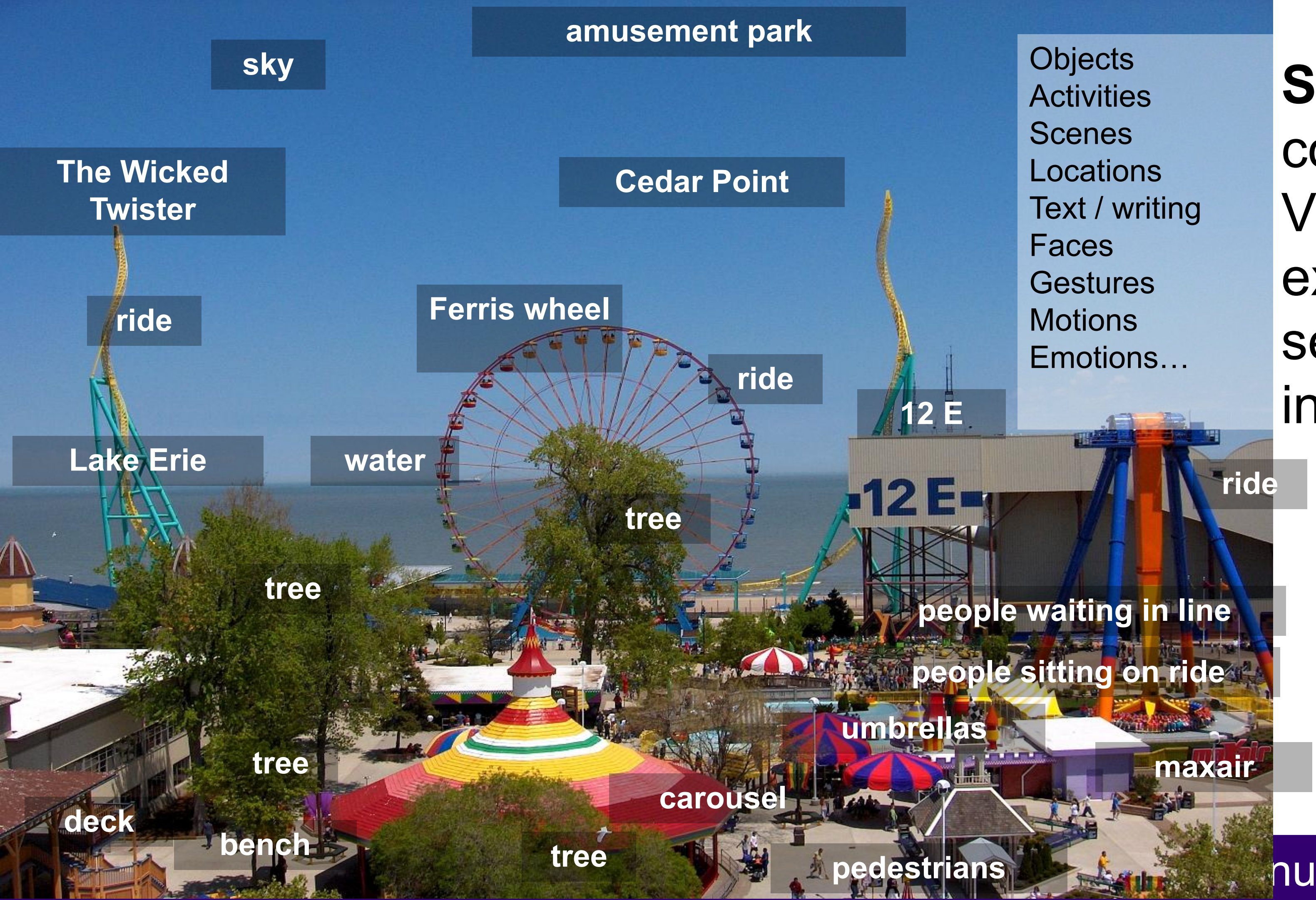
- Collaboration policy
  - Read the student code book, understand what is ‘collaboration’ and what is ‘academic infraction’
  - We have links to this on the course webpage

Administrative

Recitations (2 options)

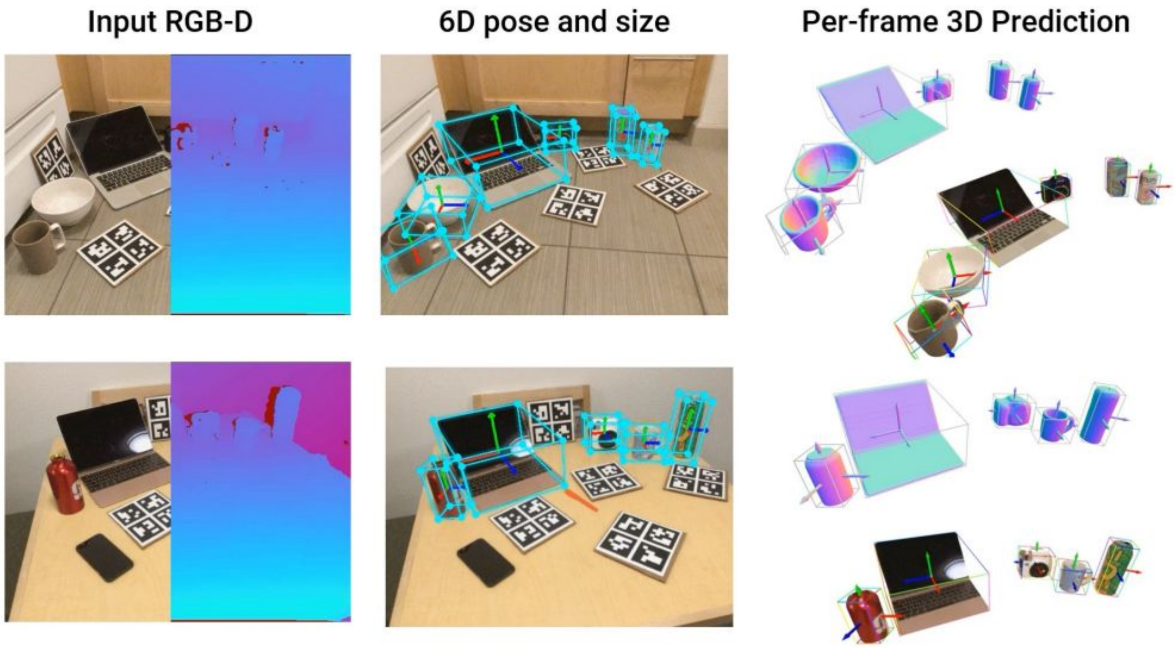
- Friday mornings 9:30-10:20am @ MGH 231
- Friday afternoons 12:30-1:20pm @ CSE2 G01

This week:
We will go over Linear algebra basics



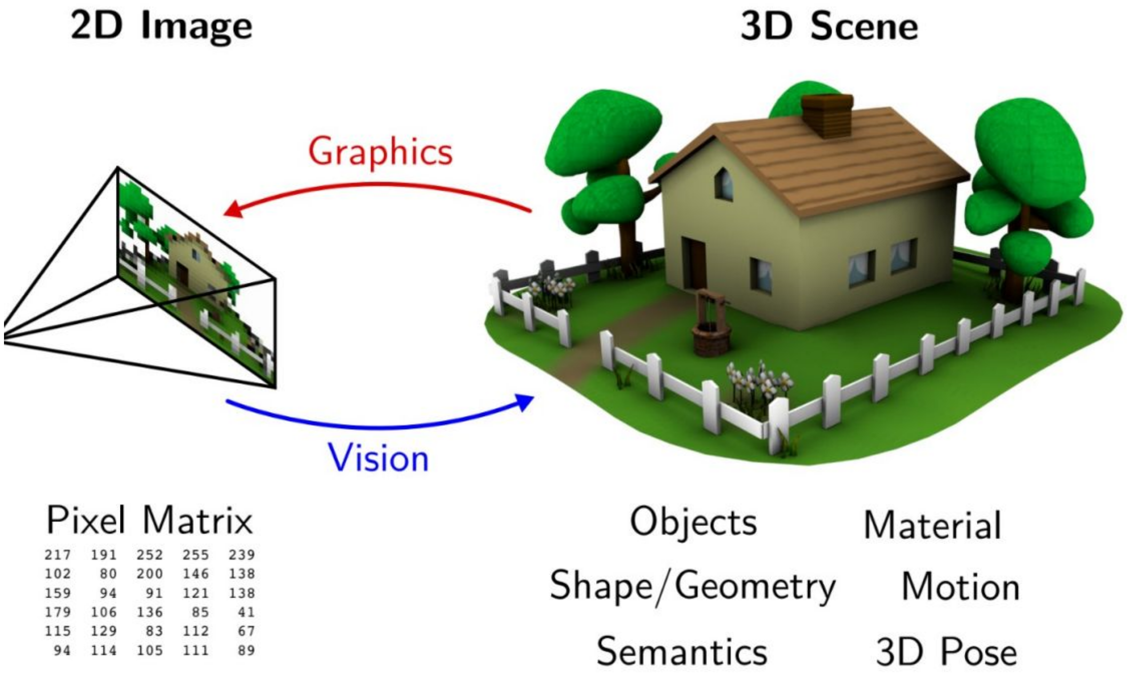
So far: computer Vision extracts semantic information

So far: Computer vision extracts geometric 3D information from 2D images



TRI & GATech's ShaPO (ECCV'22): https://zubair-irshad.github.io/projects/ShAPO.html

So far: why is computer vision hard?



It is an ill posed problem

Today’s agenda

- Color spaces
- Image sampling and quantization
- Image histograms
- Images as functions
- Filters
- Properties of systems

Some background reading:
Forsyth and Ponce, Computer Vision, Chapter 7

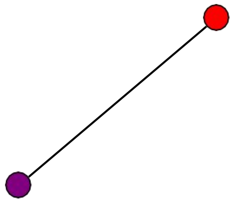
Today’s agenda

- Color spaces
- Image sampling and quantization
- Image histograms
- Images as functions
- Filters
- Properties of systems

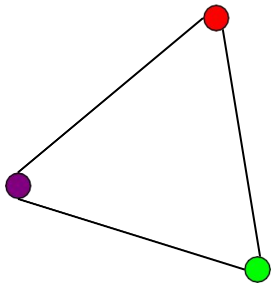
Some background reading:
Forsyth and Ponce, Computer Vision, Chapter 7

Linear color spaces

- Defined by a choice of three *primaries*
- The coordinates of a color are given by the weights of the primaries used to match it

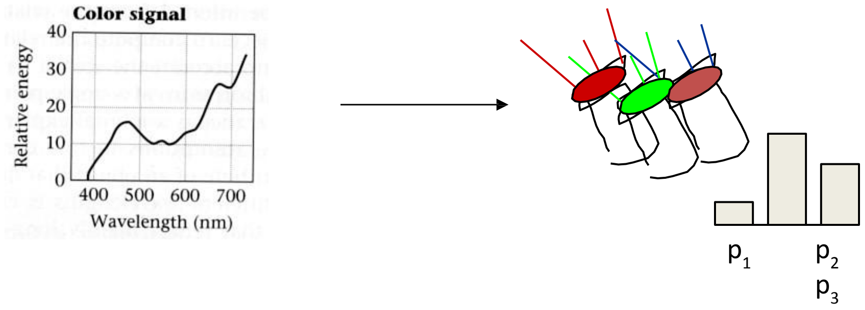


mixing two lights produces colors that lie along a straight line in color space



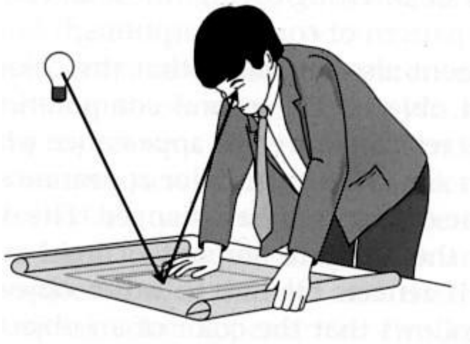
mixing three lights produces colors that lie within the triangle they define in color space

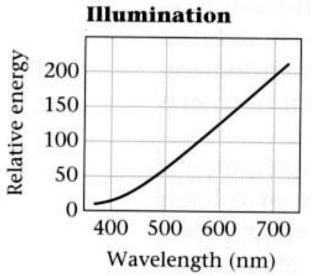
How to compute the weights of the primaries to match any spectral signal



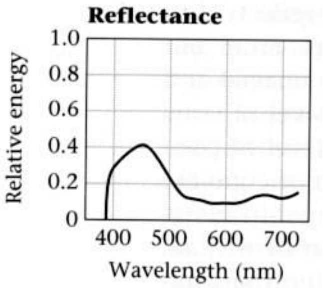
Matching functions: the amount of each primary needed to match a monochromatic light source at each wavelength

Explaining Color - Simplified

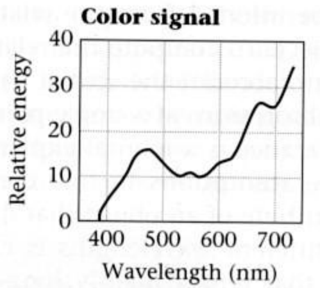




.*



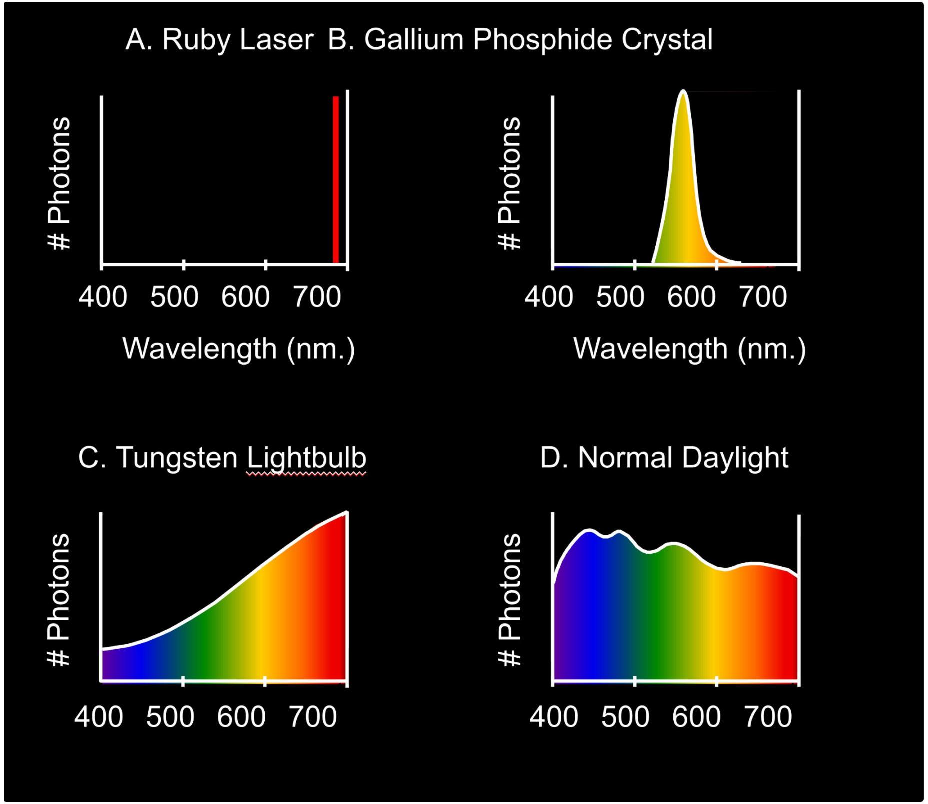
=



Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

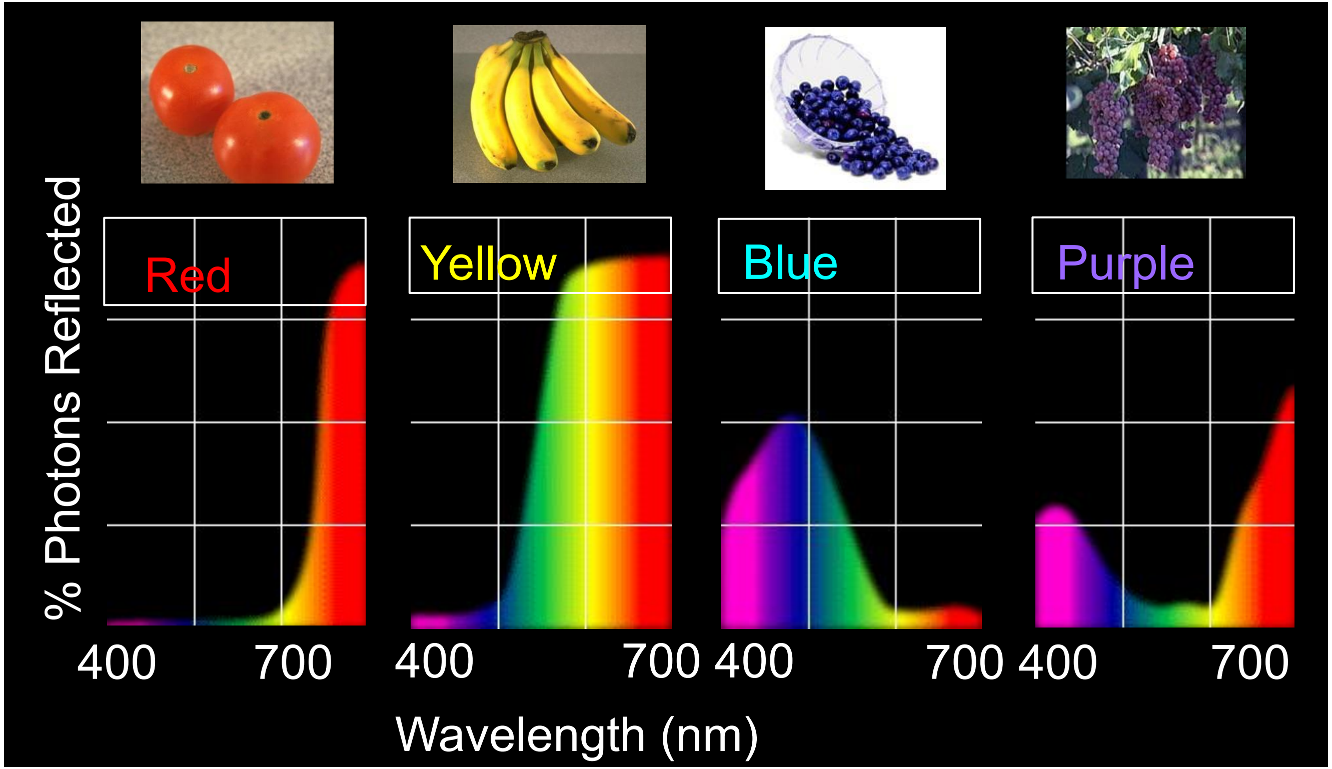
The Physics of Light Sources

Some examples of the spectra of light sources

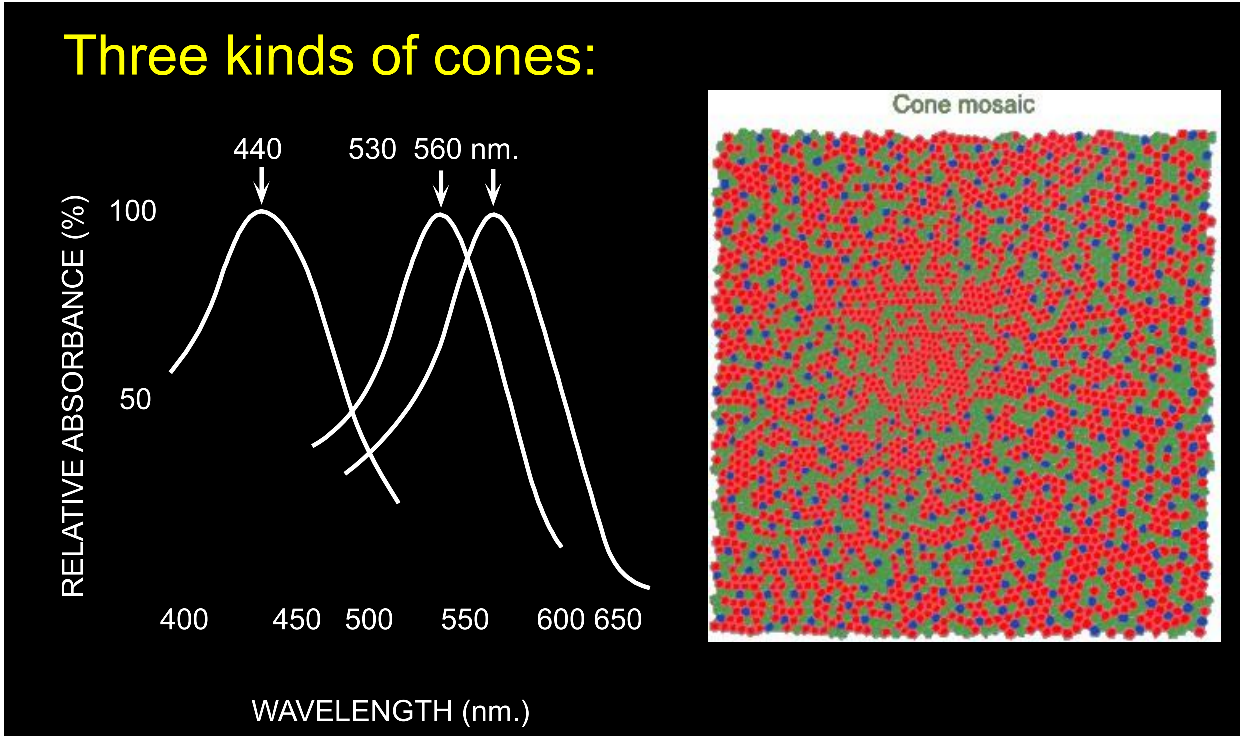


The Physics of Reflectance

Some examples of the <u>reflectance</u> spectra of <u>surfaces</u>



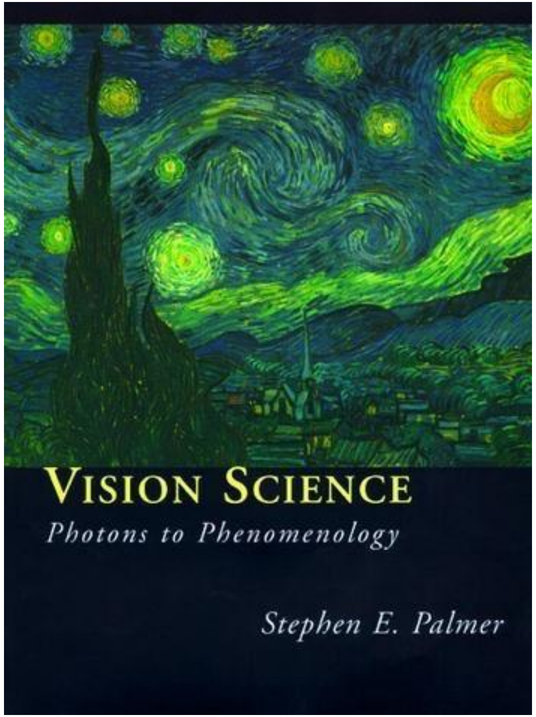
Physiology of Human Vision



Color is a psychological phenomenon

- The result of interaction between physical light in the environment and our visual system.

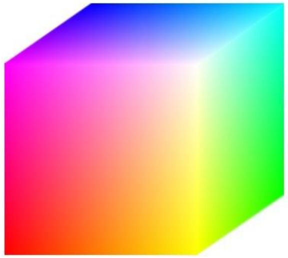
- A *psychological property* of our visual experiences when we look at objects and lights, *not a physical property* of those objects or lights.



RGB space

Primaries are monochromatic lights (for monitors, they correspond to the three types of phosphors)

RGB primaries

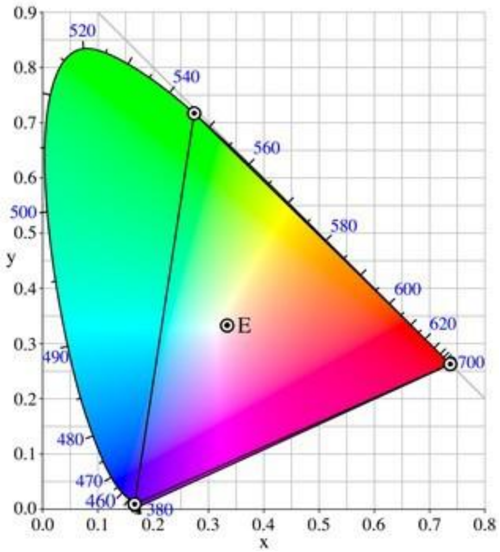


$p_1 = 645.2$ nm
$p_2 = 525.3$ nm
$p_3 = 444.4$ nm

Linear color spaces: CIE XYZ

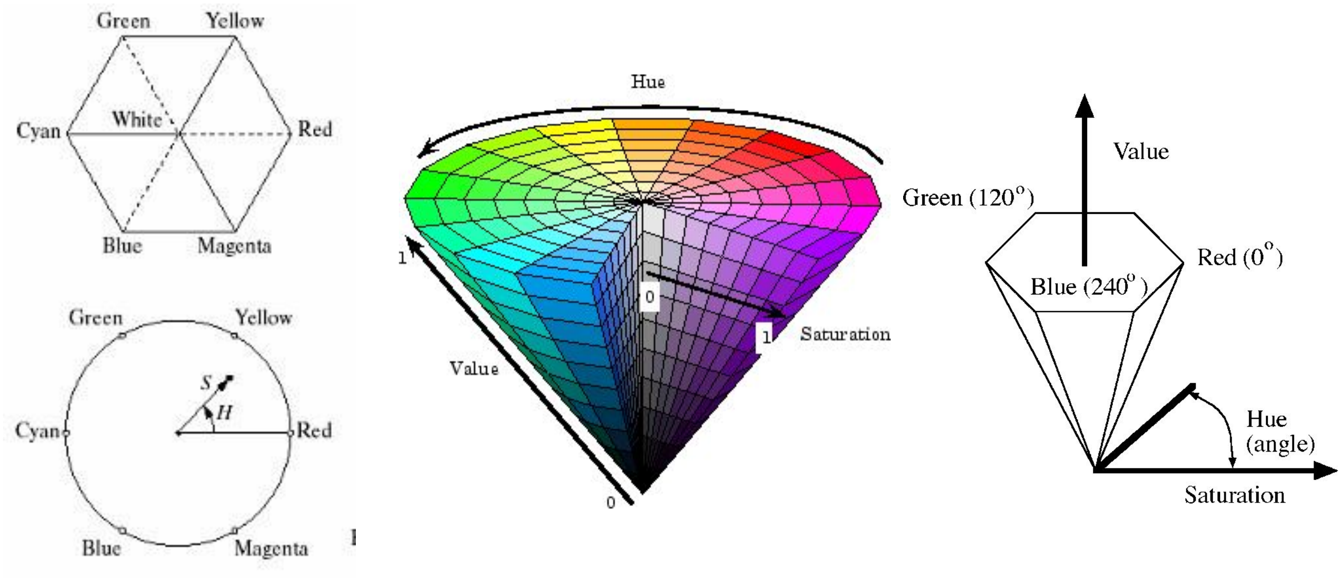
- Primaries (X, Y and Z) are imaginary
  - X: Represents a mix of red and green.
  - Y: Represents luminance (brightness).
  - Z: Represents a mix of blue and green.

- 2D visualization: draw (x,y), where
  $x = X/(X+Y+Z)$, $y = Y/(X+Y+Z)$



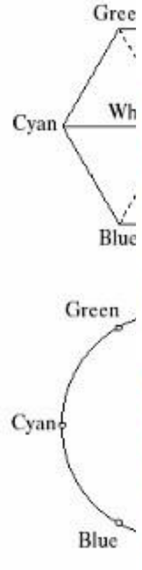
http://en.wikipedia.org/wiki/CIE_1931_color_space

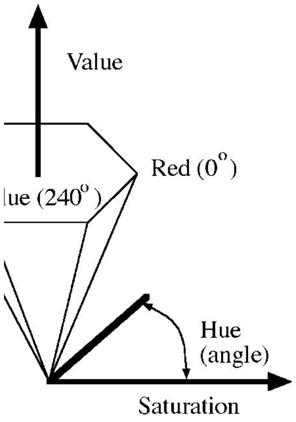
Nonlinear color spaces: HSV



- Perceptually meaningful dimensions: Hue, Saturation, Value (Intensity)

Nonlinear color spaces: HSV





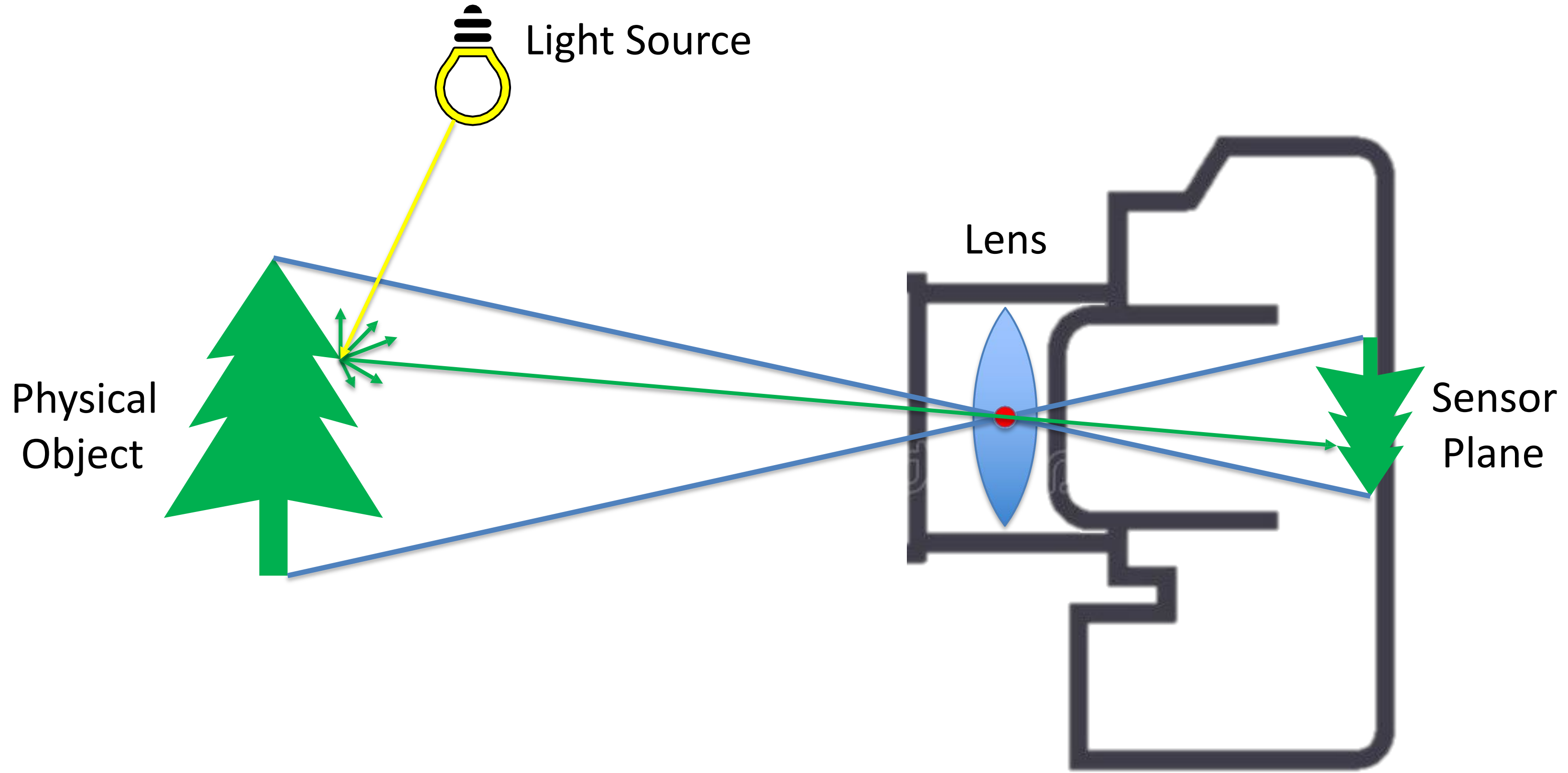
- Perceptually me tensity)

Today’s agenda

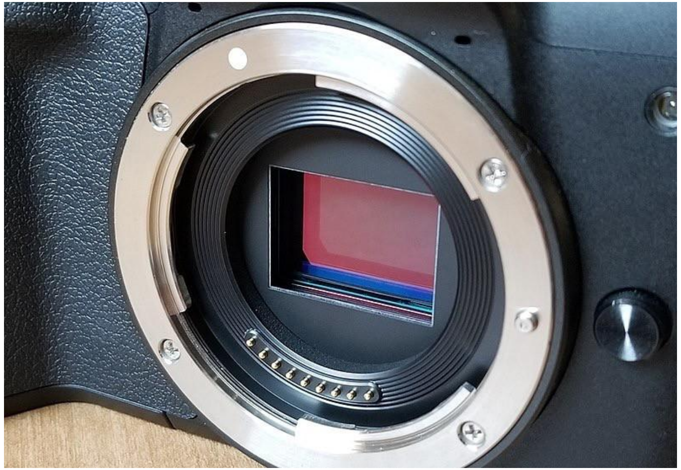
- Color spaces
- Image sampling and quantization
- Image histograms
- Images as functions
- Filters
- Properties of systems

Some background reading:
Forsyth and Ponce, Computer Vision, Chapter 7

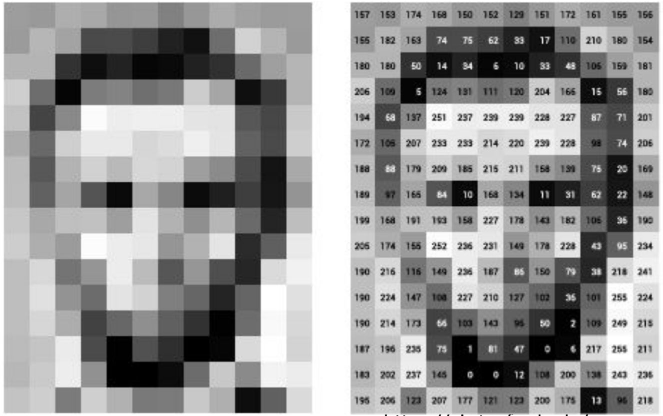
Image Formation



Camera sensors produce discrete outputs



https://commons.wikimedia.org/wiki/File:Mirrorless_Camera_Sensor.jpg



157	153	174	168	150	152	129	151	172	161	156	156
156	182	163	74	75	62	33	17	110	210	180	154
180	180	50	14	34	6	10	33	48	106	159	181
206	109	5	124	131	111	120	204	166	15	56	180
194	68	137	251	237	239	239	228	227	87	71	201
172	105	207	233	233	214	220	239	228	98	74	206
188	88	179	209	185	215	211	158	139	75	20	169
189	97	165	84	10	168	134	11	31	62	22	148
199	168	191	193	158	227	178	143	182	106	36	190
205	174	155	252	236	231	149	178	228	43	95	234
190	216	116	149	236	187	86	150	79	38	218	241
190	224	147	108	227	210	127	102	36	101	255	224
190	214	173	66	103	143	96	50	2	109	249	215
187	196	235	75	1	81	47	0	6	217	255	211
183	202	237	145	0	0	12	108	200	138	243	236
195	206	123	207	177	121	123	200	175	13	96	218

https://ai.stanford.edu/~syyeung/cvweb/Pictures1/imagematrix.png

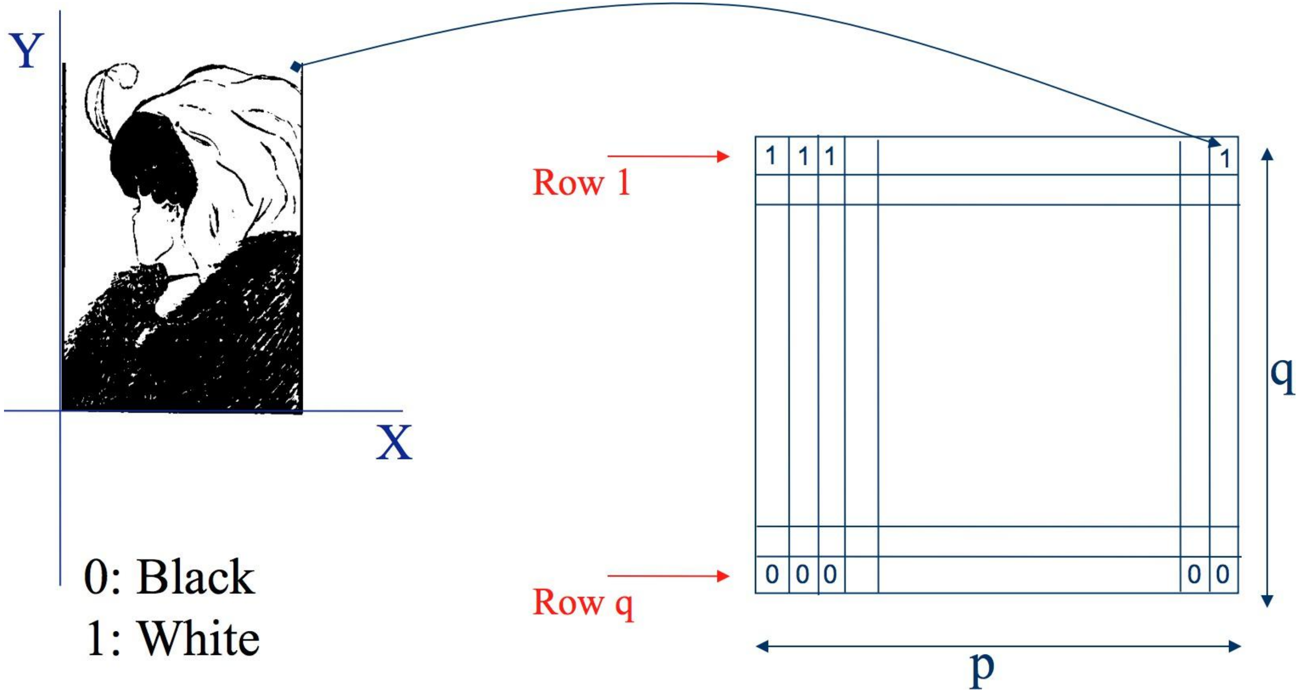
Types of Images

Binary

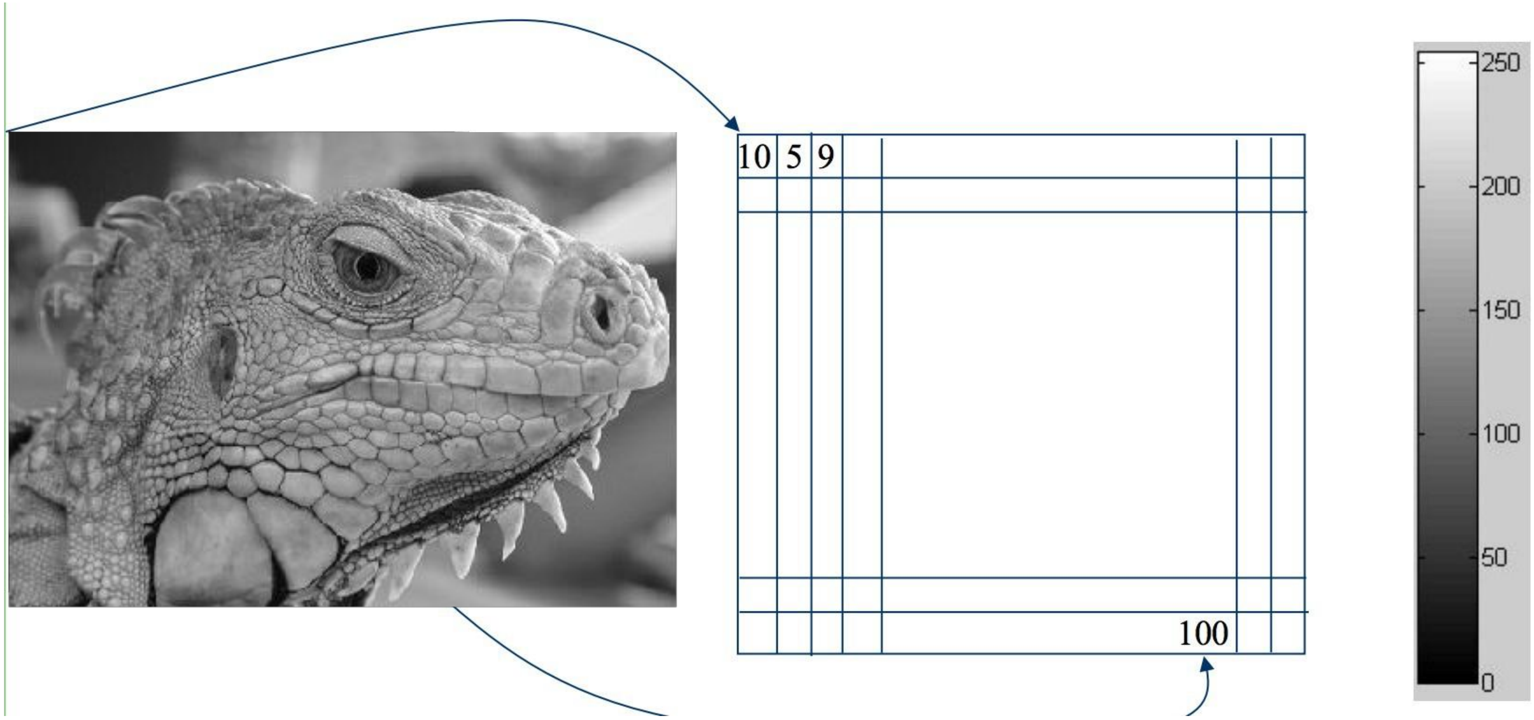
Grayscale

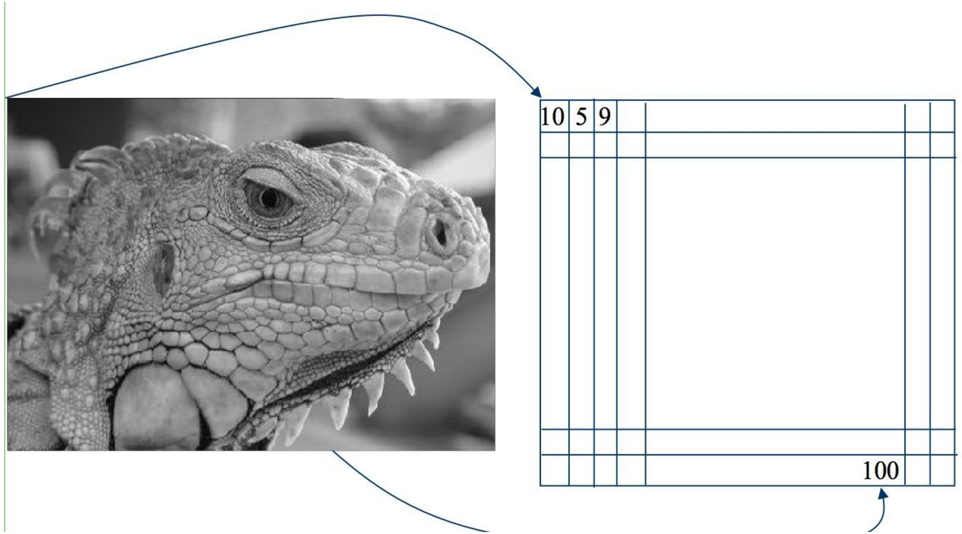
Color

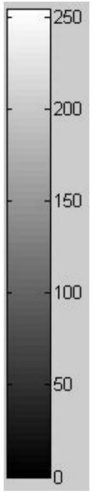
Binary image representation

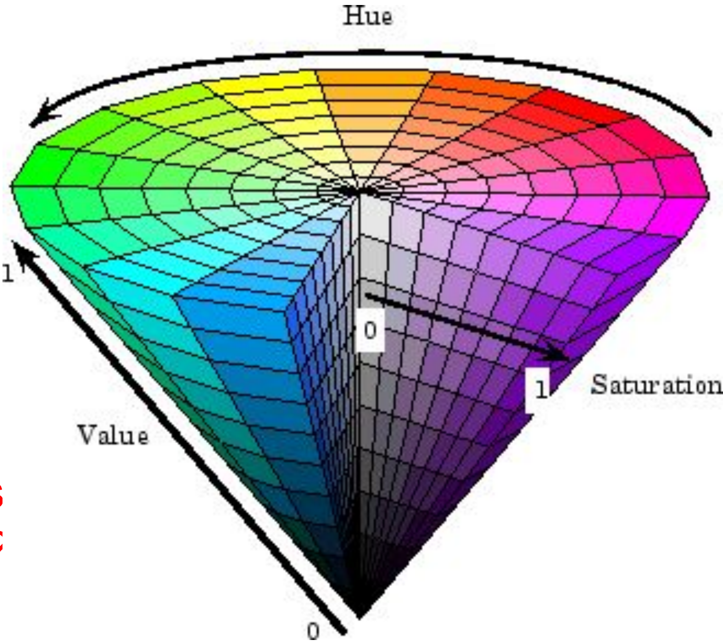


Grayscale image representation









Q. If you used HSV to represent grayscale images, is the slider representing hue? Or saturation? Or value?

Color image representation

B channel

G channel

R channel

Color image - one channel

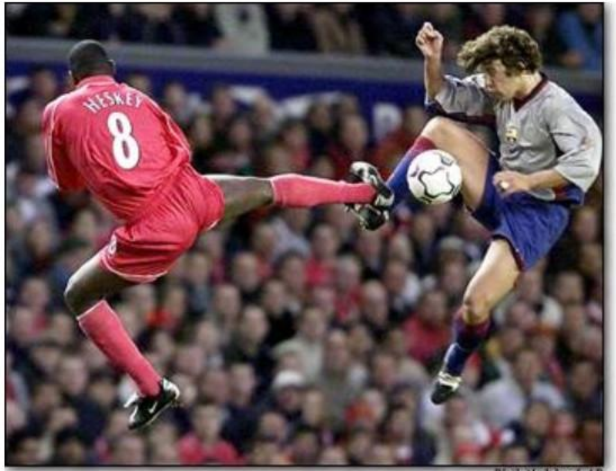
R channel

Types of Images

Binary

Grayscale

Color

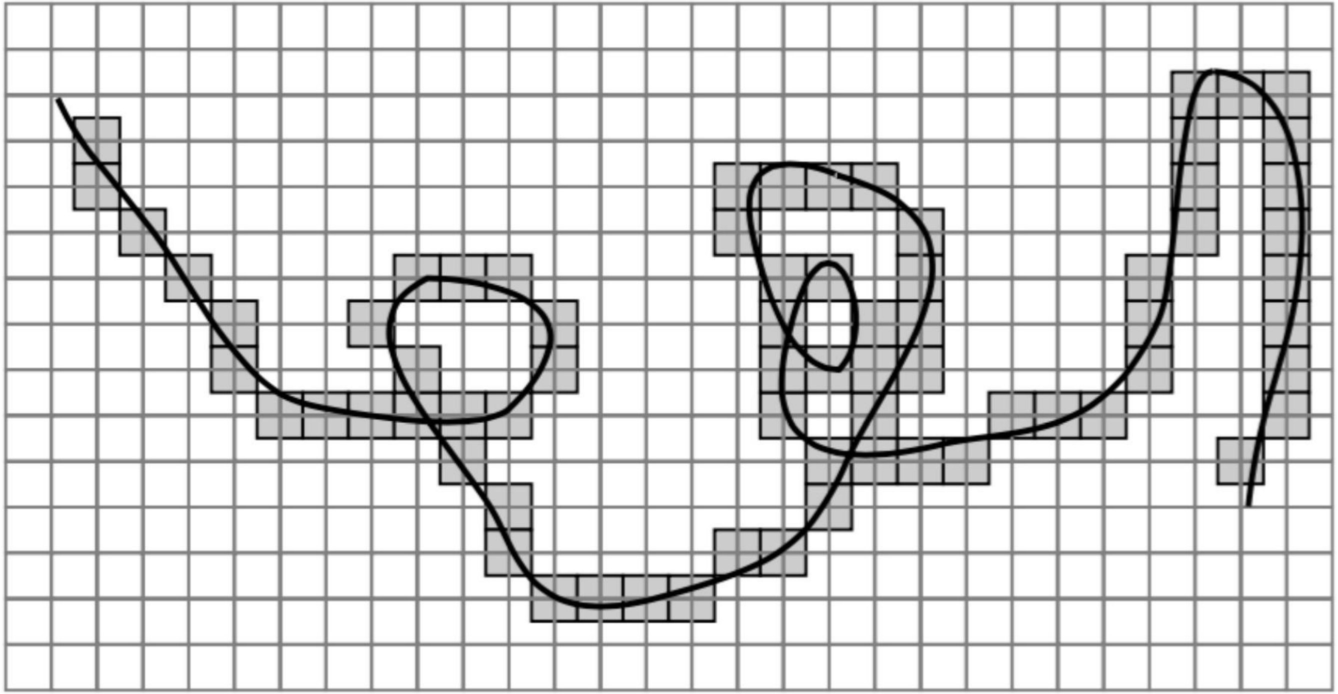


Phil Noble / AP

Digital Images are sampled

What happens when we zoom into the images we capture?

Errors due to Sampling

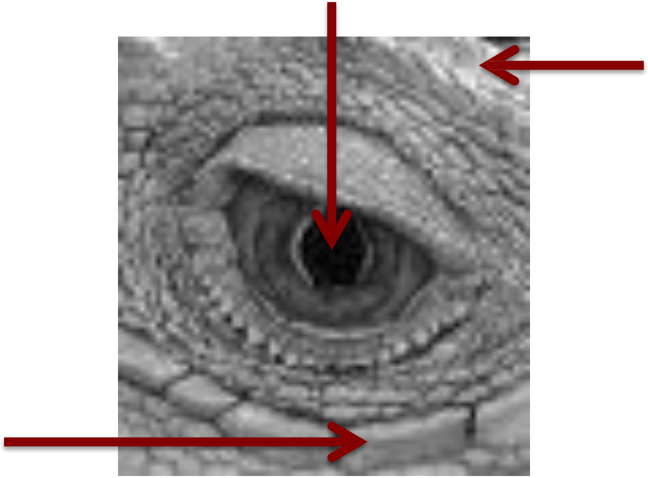


Resolution

is a **sampling** parameter, defined in dots per inch (DPI) or equivalent measures of spatial pixel density

Images are Sampled and Quantized

- An image contains discrete number of pixels
  - –Pixel value:
    - •“grayscale”
      (or “intensity”): [0,255]



Images are Sampled and Quantized

- An image contains discrete number of pixels
  - –Pixel value:
    - •“grayscale”
      (or “intensity”): [0,255]
    - •“color”
      - –RGB: [R, G, B]

With this loss of information (from sampling and quantization),

Can we still use images for useful tasks?

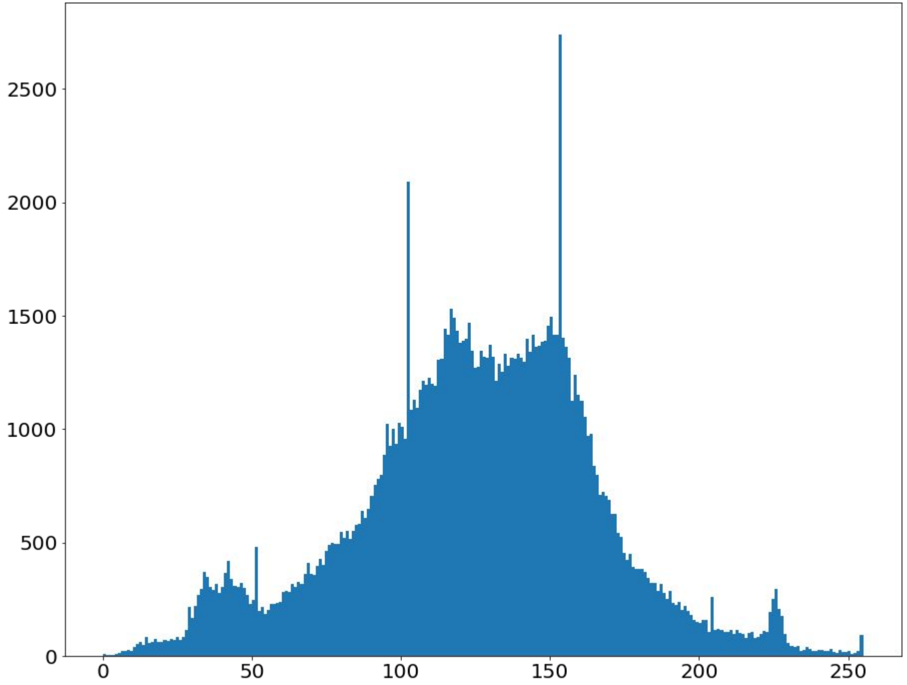
Today’s agenda

- Color spaces
- Image sampling and quantization
- Image histograms
- Images as functions
- Filters
- Properties of systems

Some background reading:
Forsyth and Ponce, Computer Vision, Chapter 7

Starting with grayscale images:

- Histogram captures the distribution of gray levels in the image.
- How frequently each gray level occurs in the image



Grayscale histograms in code

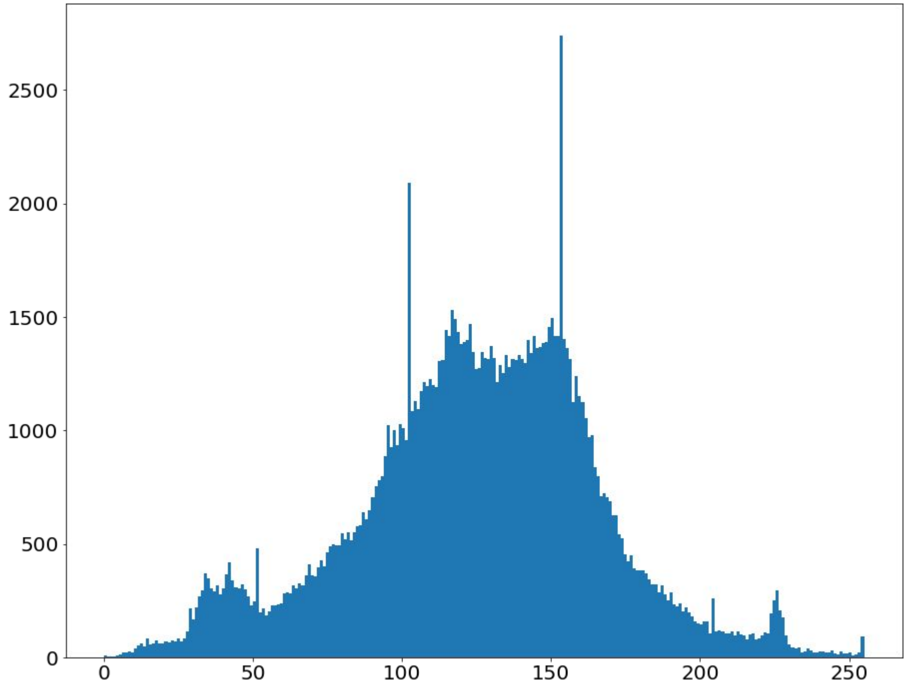
- Histogram of an image provides the frequency of the brightness (intensity) value in the image.

Here is an efficient implementation of calculating histograms:

```
def histogram(im):
    h = np.zeros(255)
    for row in im.shape[0]:
        for col in im.shape[1]:
            val = im[row, col]
            h[val] += 1
```

Visualizing h[:]

```
def histogram(im):
    h = np.zeros(255)
    for row in im.shape[0]:
        for col in im.shape[1]:
            val = im[row, col]
            h[val] += 1
```



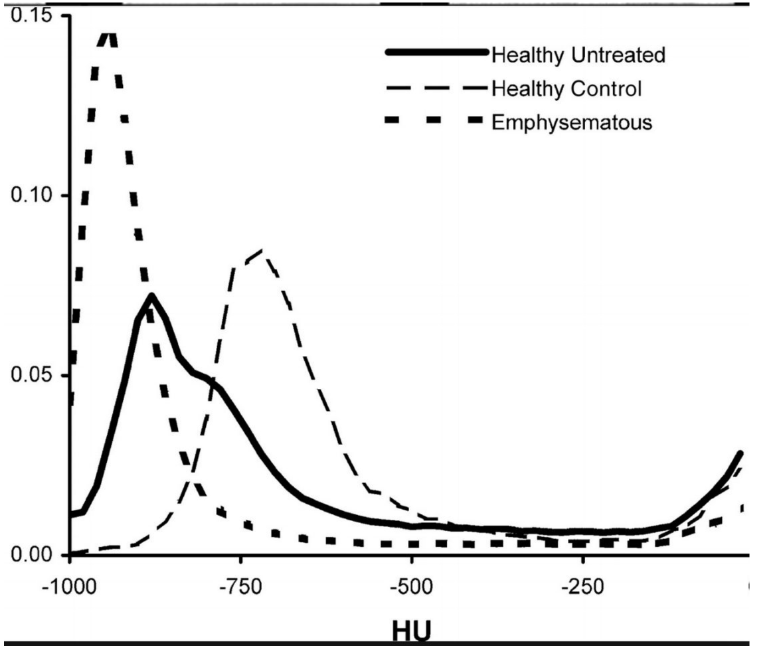
Visualizing Histograms for patches

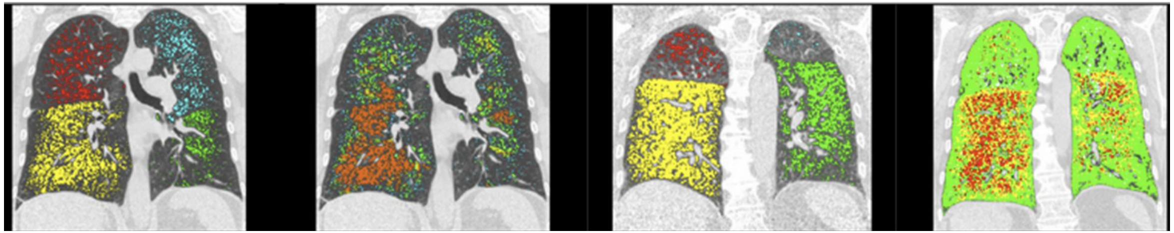
Slide credit: Dr. Mubarak Shah

Histogram – use case

In emphysema, the inner walls of the lungs' air sacs called alveoli are damaged, causing them to eventually rupture.

You can take a picture of the lung with special dye to mark the alveoli





Slide credit: Dr. Mubarak Shah

Histograms are a convenient representation to extract information

Can we develop better transformations than histograms?

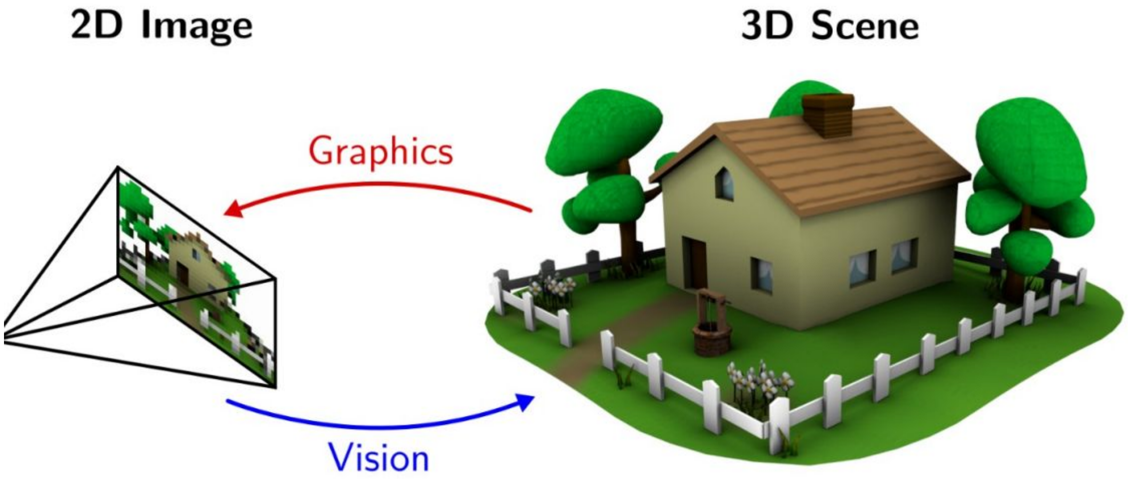
Today's agenda

- Color spaces
- Image sampling and quantization
- Image histograms
- Images as functions
- Filters
- Properties of systems

Some background reading:
Forsyth and Ponce, Computer Vision, Chapter 7

Images are a function!!!

This is a new formalism that will allow us to borrow ideas from signal processing to extract meaningful information.



At every pixel location, we get an intensity value for that pixel.

The world captured by the image continues beyond the confines of the image

Images as discrete functions

- Digital images are usually discrete:
  - **Sample** the 2D space on a regular grid

- Represented as a matrix of integer values

pixel intensity

m

n

62	79	23	119	120	05	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
166	63	127	17	1	0	99	30

Images as discrete function f

- The input to the image function is a pixel location, [n m]
- The output to the image function is the pixel intensity

pixel intensity

m →

n ↓

62	79	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
166	63	127	17	1	0	99	30

Images as discrete function f

- The input to the image function is a pixel location, [n m]
- The output to the image function is the pixel intensity

Q1. What is f[0, 0]?

pixel intensity

m (→)

n (↓)

62	79	23	119	120	05	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
166	63	127	17	1	0	99	30

Images as discrete function f

- The input to the image function is a pixel location, [n m]
- The output to the image function is the pixel intensity

Q2. What is f[0, 4]?

pixel intensity

m

n

62	79	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
166	63	127	17	1	0	99	30

Images as discrete function f

- The input to the image function is a pixel location, [n m]
- The output to the image function is the pixel intensity

Q2. What is f[0, -8]?

pixel intensity

m →

n ↓

62	79	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
166	63	127	17	1	0	99	30

Images as coordinates

We can represent this function as f.

f[n, m] represents the pixel intensity at that value.

$$f[n,m] = \begin{bmatrix} \ddots & & \vdots & & \\ & f[-1,-1] & f[-1,0] & f[-1,1] & \\ \dots & f[0,-1] & \underline{f[0,0]} & f[0,1] & \dots \\ & f[1,-1] & f[1,0] & f[1,1] & \\ & & \vdots & & \ddots \end{bmatrix}$$

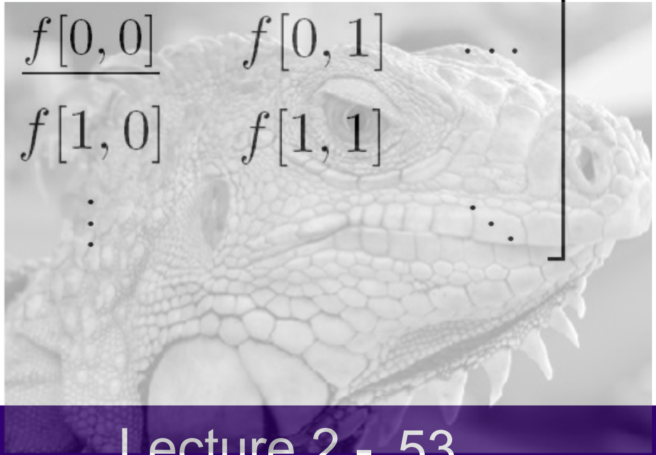
Notation for discrete functions

n and *m* can be any integer

Even negative!!

We don’t have the intensity values for negative indices

$$f[n,m] = \begin{bmatrix} \ddots & & \vdots & & \\ & f[-1,-1] & f[-1,0] & f[-1,1] & \\ \ldots & f[0,-1] & \underline{f[0,0]} & f[0,1] & \ldots \\ & f[1,-1] & f[1,0] & f[1,1] & \\ & & \vdots & & \ddots \end{bmatrix}$$

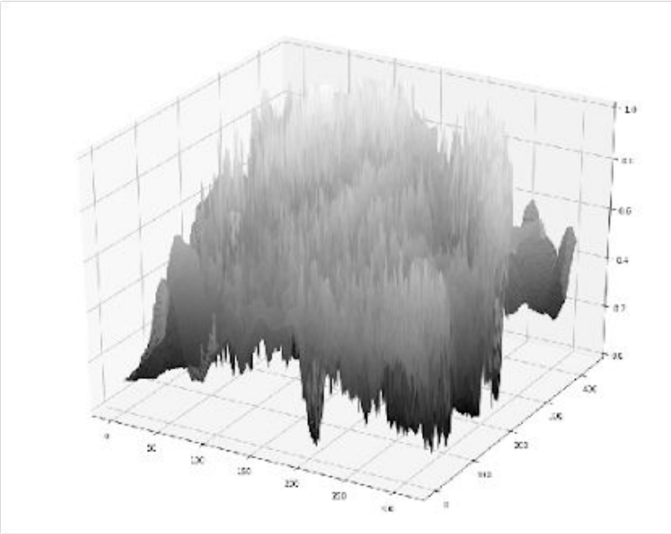


n and *m* can be any integer

Even negative!!

Images as functions

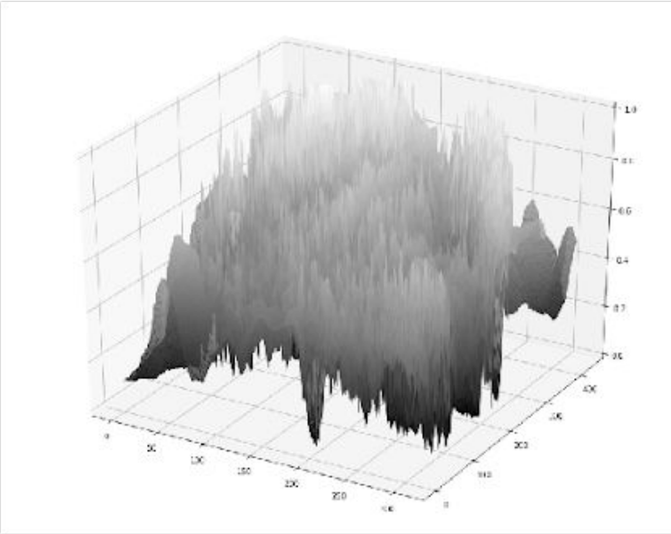
- **An Image** as a function *f* from R^2 to R^C:
  - if grayscale then C=1,
  - if color then C=3



Images as functions

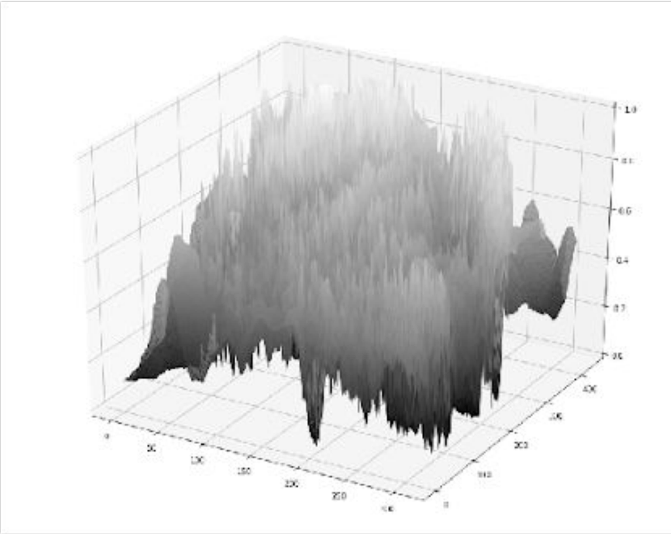
- **An Image** as a function *f* from R^2 to R^C:
  - if grayscale, C=1,
  - if color, C=3
  - f [n, m] gives the intensity at position [n, m]
  - Has values over a rectangle, with a finite range:

$$f: \underbrace{[0,H] \times [0,W]}_{\text{Domain support}} \rightarrow \underbrace{[0,255]}_{\text{range}}$$



Images as functions

- **An Image** as a function *f* from R^2 to R^C:
  - if grayscale, C=1,
  - if color, C=3
  - f [n, m] gives the intensity at position [n, m]
  - Has values over a rectangle, with a finite range:



f: $[0,H] \times [0,W] \rightarrow [0,255]$

Domain support — range

- Doesn't have values outside of the image rectangle

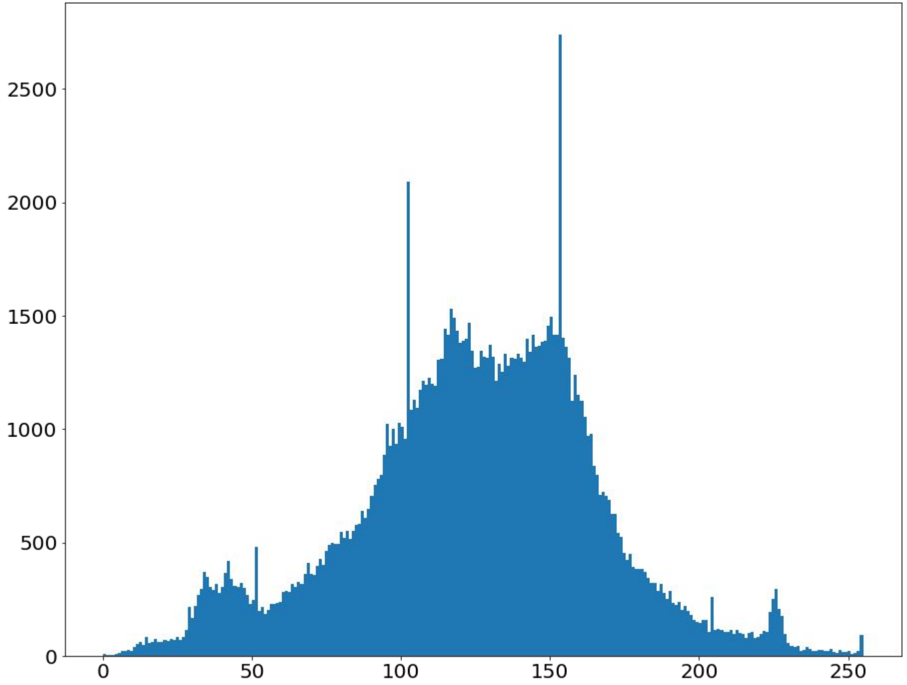
  f: $[-inf,inf] \times [-inf,inf] \rightarrow [0,255]$
- we assume that f[n, m] = 0 outside of the image rectangle

Images as functions

- **An Image** as a function *f* from R^2 to R^C:
  - f [n, m] gives the intensity at position [n, m]
  - Defined over a rectangle, with a finite range:

$$f: \underbrace{[a,b] \times [c,d]}_{\text{Domain support}} \rightarrow \underbrace{[0,255]}_{\text{range}}$$

Histograms are also a type of function



Today's agenda

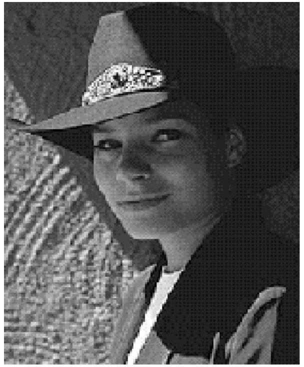
- Color spaces
- Image sampling and quantization
- Image histograms
- Images as functions
- Filters
- Properties of systems

Some background reading:
Forsyth and Ponce, Computer Vision, Chapter 7

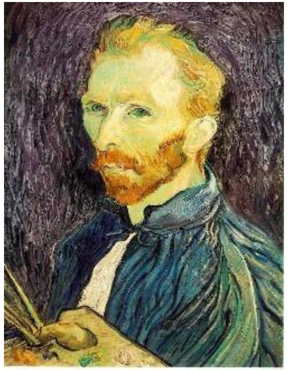
Applications of filters

De-noising

Salt and pepper noise



Super-resolution



In-painting

Bertamio et al

Systems and Filters

Filering:

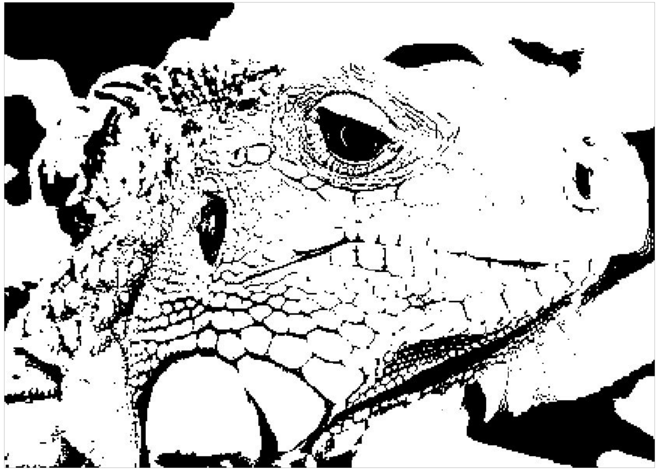
– Forming a new image whose pixel values are transformed from original pixel values

Goals of filters:

- Goal is to extract useful information from images, or transform images into another domain where we can modify/enhance image properties
  - Features (edges, corners, blobs…)
  - super-resolution; in-painting; de-noising

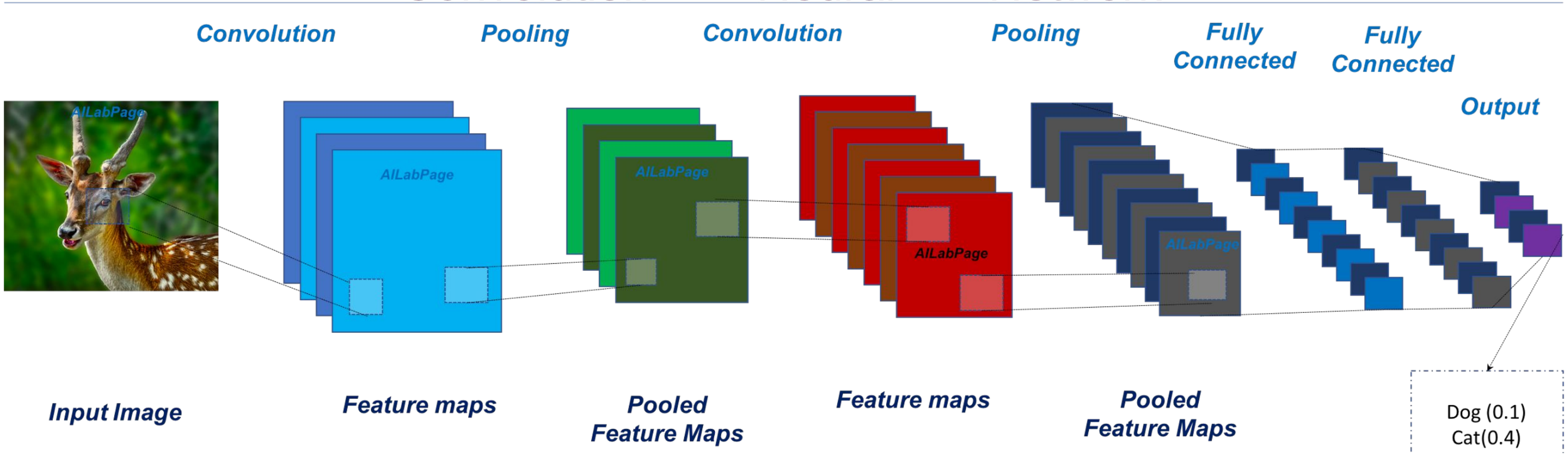
Intuition behind systems

- We will view systems as a sequence of filters applied to an image
- For example, multiplying by a constant leaves the semantic content intact
  - but can reveal interesting patterns



As an aside - we will go into detail later in the course:

- Neural networks and specifically **convolutional** neural networks are a sequence of filters (except they are a non-linear system) that contains multiple individual linear sub-systems.



Systems use Filters

- we define a system as a unit that converts an input function f[n,m] into an output (or response) function g[n,m]
  - where (n,m) index into the function
  - In the case for images, (n,m) represents the **spatial position in the image.**

$$f[n, m] \rightarrow \boxed{\text{System } \mathcal{S}} \rightarrow g[n, m]$$

Images produce a 2D matrix with pixel intensities at every location

$$f[n,m] = \begin{bmatrix} \ddots & & \vdots & & \\ & f[-1,-1] & f[0,-1] & f[1,-1] & \\ \dots & f[-1,0] & \underline{f[0,0]} & f[1,0] & \dots \\ & f[-1,1] & f[0,1] & f[1,1] & \\ & & \vdots & & \ddots \end{bmatrix}$$

Notation for discrete functions

2D discrete system
(system is a sequence of filters)

S is the **system operator**, defined as a mapping or assignment of possible inputs f[n,m] to some possible outputs g[n,m].

$$f[n,m] \rightarrow \boxed{\text{System } \mathcal{S}} \rightarrow g[n,m]$$

2D discrete system

S is the **system operator**, defined as a mapping or assignment of possible inputs f[n,m] to some possible outputs g[n,m].

$$f[n,m] \rightarrow \boxed{\text{System } \mathcal{S}} \rightarrow g[n,m]$$

Other notations:

$$g = \mathcal{S}[f], \quad g[n,m] = \mathcal{S}\{f[n,m]\}$$

$$f[n,m] \xrightarrow{\mathcal{S}} g[n,m]$$

Filter example #1: Moving Average

Original image

Q. What do you think will happen to the photo if we use a moving average filter?

Assume that the moving average replaces each pixel with an average value of itself and all its neighboring pixels.

Filter example #1: Moving Average

Original image

Smoothed image

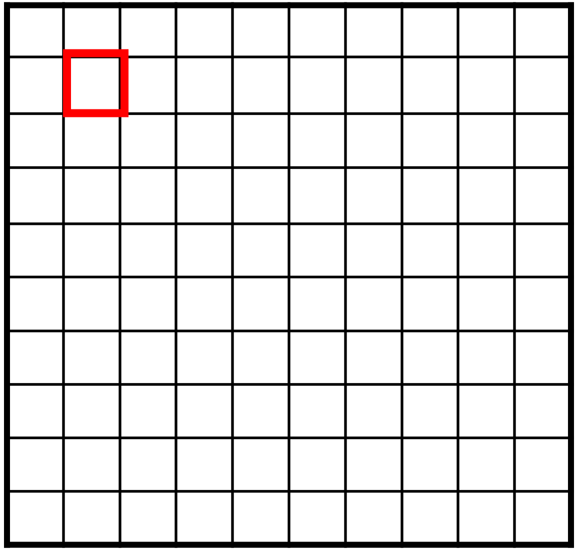
Visualizing what happens with a moving average filter

The red box is
the **h** matrix

$\mathrm{f}[n, m]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$\mathrm{g}[n, m]$



Courtesy of S. Seitz

Visualizing what happens with a moving average filter

$f[n, m]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[n, m]$

	0								

Visualizing what happens with a moving average filter

$f[n,m]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[n,m]$

	0	10							

Visualizing what happens with a moving average filter

$f[n,m]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[n,m]$

	0	10	20						

Visualizing what happens with a moving average filter

$f[n, m]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[n, m]$

	0	10	20	30					

Visualizing what happens with a moving average filter

$f[n,m]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

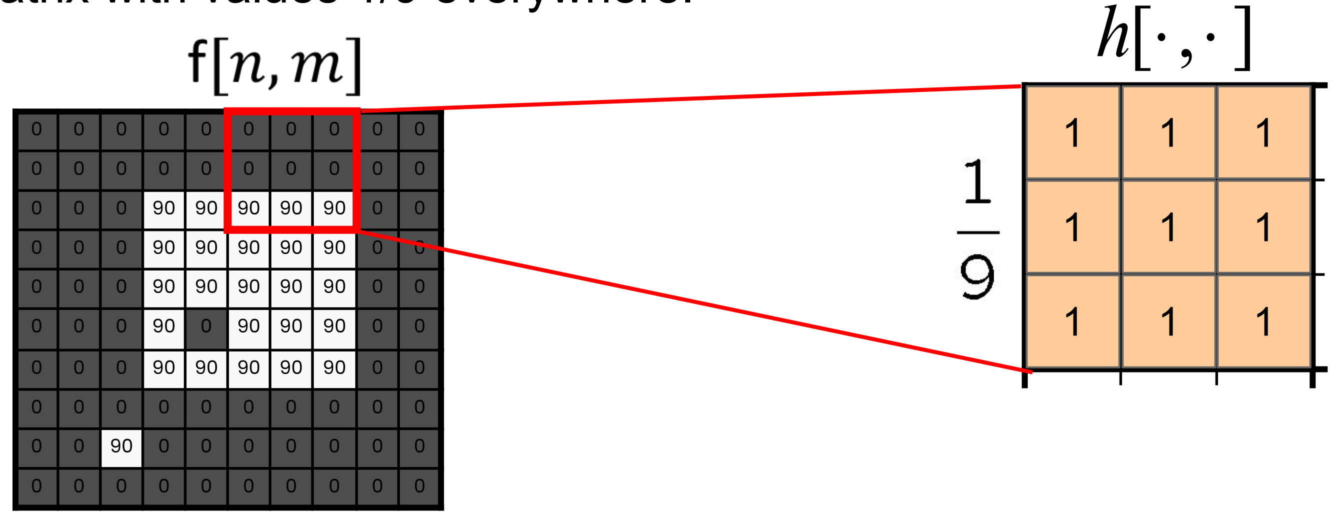
$g[n,m]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

Visual interpretation of moving average

A moving average over a 3 × 3 neighborhood window

h is a 3x3 matrix with values 1/9 everywhere.



Visual interpretation of moving average

A moving average over a 3 × 3 neighborhood window

h is a 3x3 matrix with values 1/9 everywhere.

Q. Why are the values 1/9?

$$h[\cdot,\cdot] = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Filter example #1: Moving Average

In summary:

- This filter “Replaces” each pixel with an average of its neighborhood.

- Achieve smoothing effect (remove sharp features)

$$h[\cdot,\cdot] = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Mathematical interpretation of moving average

How do we represent applying this filter mathematically?

$$f[n,m] \rightarrow \boxed{\text{System } \mathcal{S}} \rightarrow g[n,m]$$

$$h[\cdot,\cdot] = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Mathematical interpretation of moving average

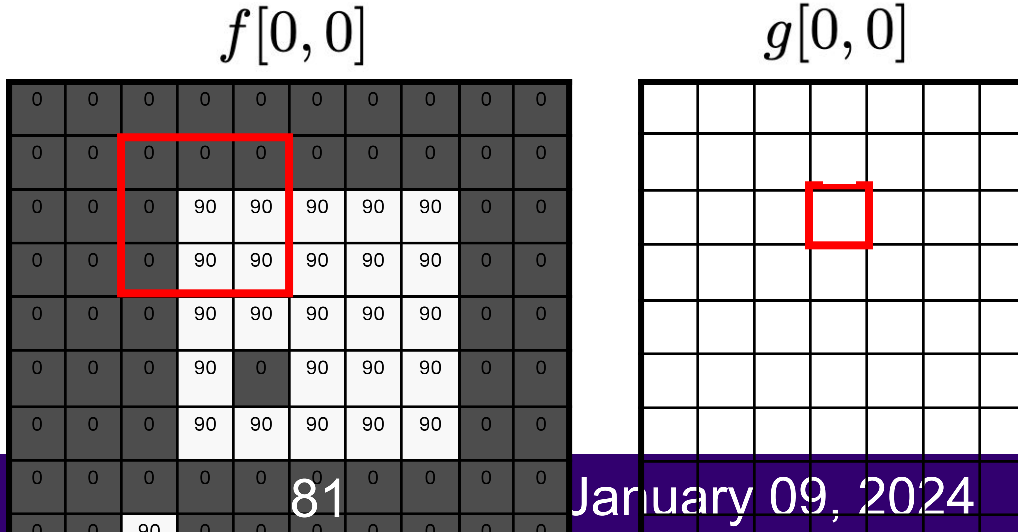
How do we represent applying this filter mathematically?

$$f[n,m] \rightarrow \boxed{\text{System } \mathcal{S}} \rightarrow g[n,m]$$

$$h[\cdot,\cdot] = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Mathematical interpretation of moving average

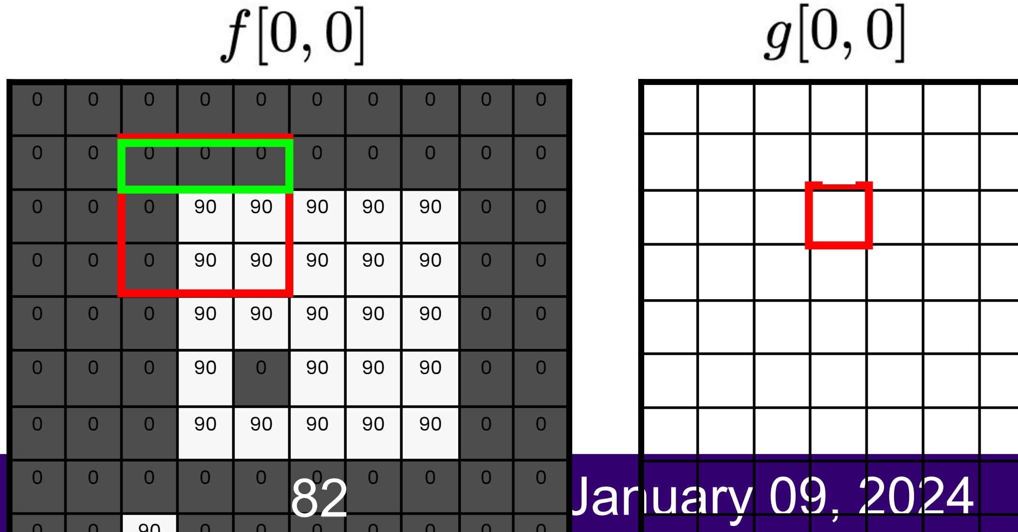
$$f[n,m] \rightarrow \boxed{\text{System } \mathcal{S}} \rightarrow g[n,m]$$



Mathematical interpretation of moving average

$$f[n,m] \rightarrow \boxed{\text{System } \mathcal{S}} \rightarrow g[n,m]$$

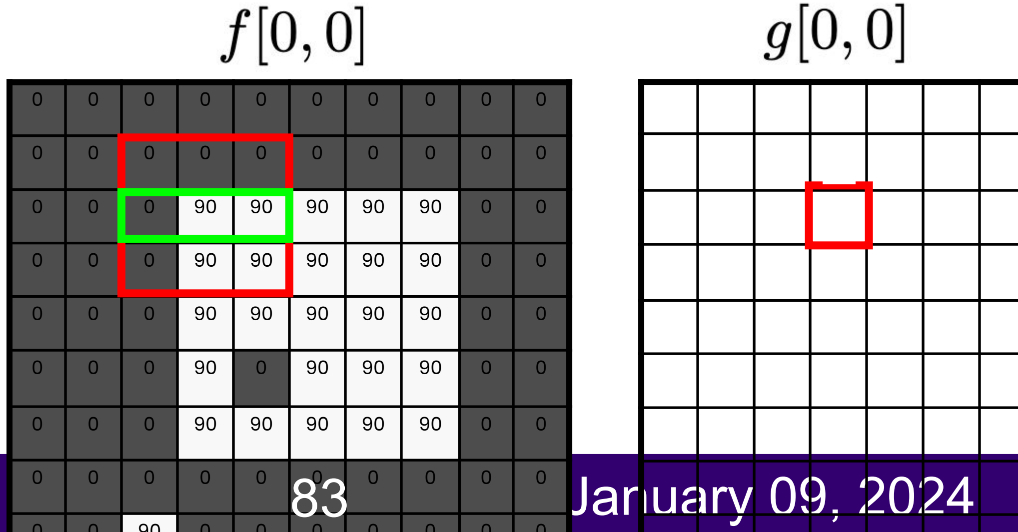
$$g[0,0] = f[-1,-1] + f[-1,0] + f[-1,1]$$
$$+ \ \dots$$



Mathematical interpretation of moving average

$$f[n,m] \rightarrow \boxed{\text{System } \mathcal{S}} \rightarrow g[n,m]$$

$$\begin{aligned} g[0,0] = &\ f[-1,-1] + f[-1,0] + f[-1,1] \\ &+ \boxed{f[0,-1] + f[0,0] + f[0,1]} \\ &+ \ \dots \end{aligned}$$

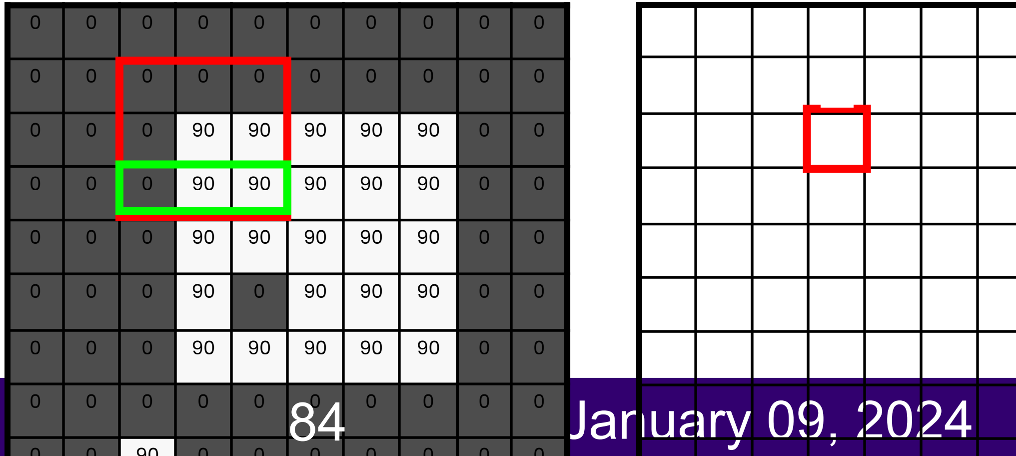


Mathematical interpretation of moving average

$$f[n,m] \rightarrow \boxed{\text{System } \mathcal{S}} \rightarrow g[n,m]$$

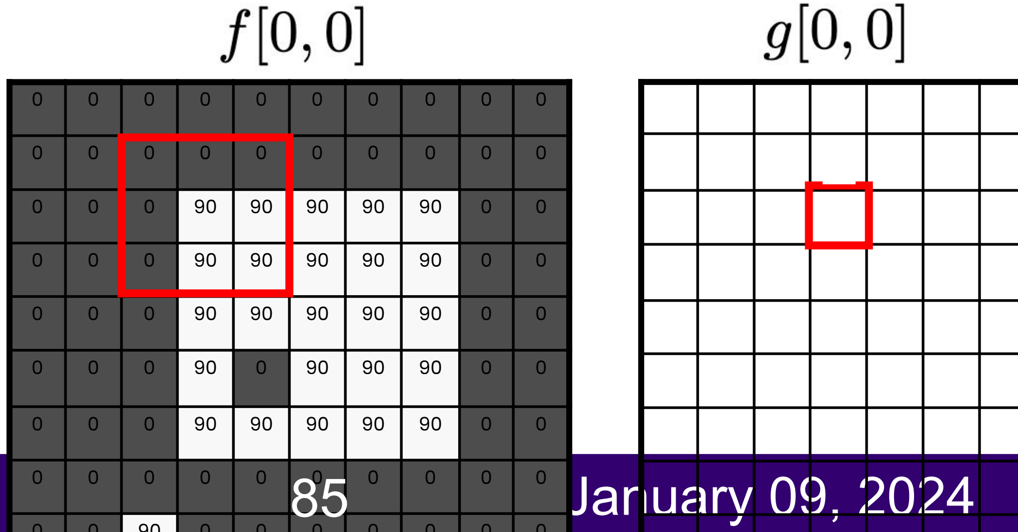
$$\begin{aligned} g[0,0] = &\ f[-1,-1] + f[-1,0] + f[-1,1] \\ &+ f[0,-1] + f[0,0] + f[0,1] \\ &+ f[1,-1] + f[1,0] + f[1,1] \end{aligned}$$

$f[0,0]$ $g[0,0]$

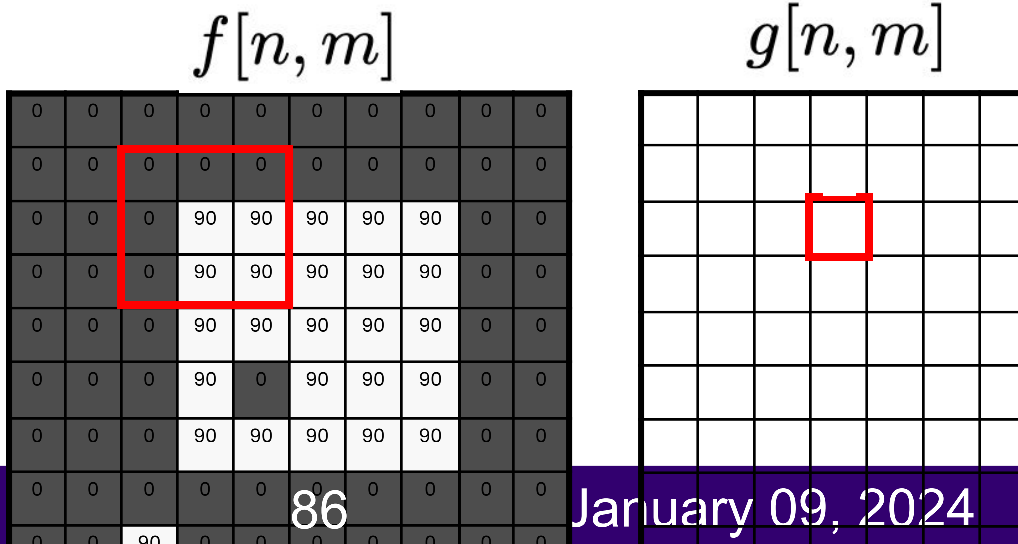


Lastly, divide by 1/9

$$g[0,0] = \frac{1}{9}[f[-1,-1] + f[-1,0] + f[-1,1] + f[0,-1] + f[0,0] + f[0,1] + f[1,-1] + f[1,0] + f[1,1]]$$

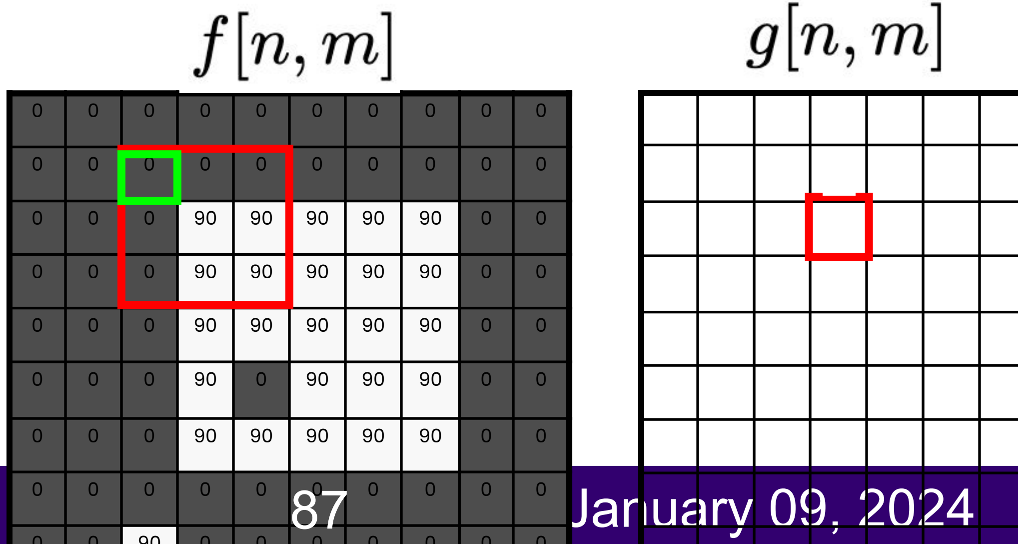


Now, instead of [0, 0], let’s do [n, m]



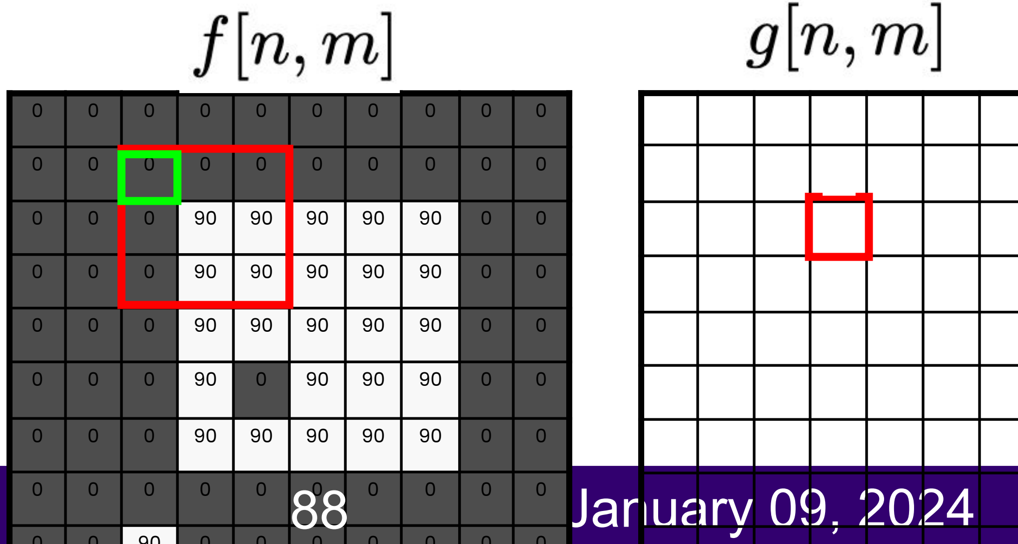
Now, instead of [0, 0], let's do [n, m]

$$g[n, m] = \; ...$$



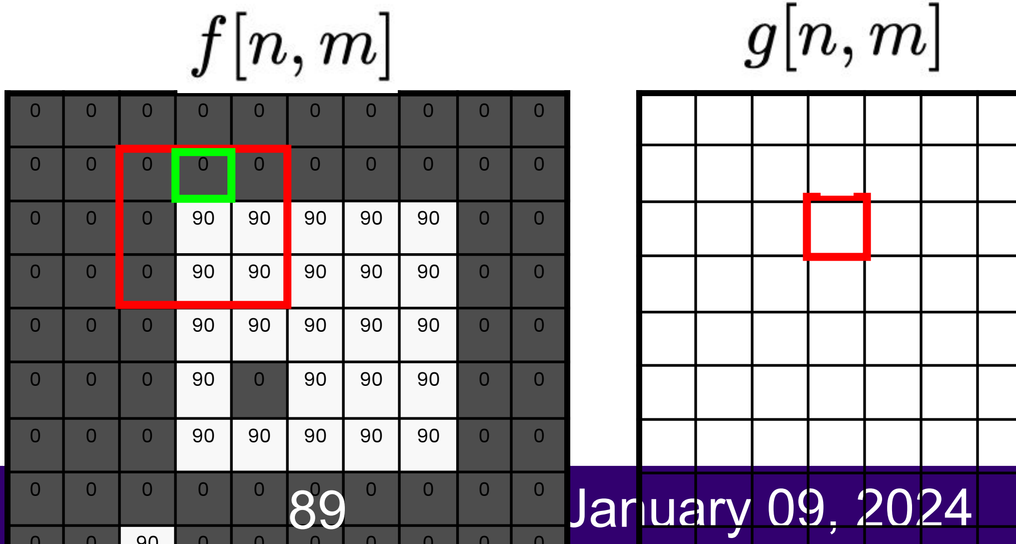
Now, instead of [0, 0], let's do [n, m]

$$g[n,m] = f[n-1,m-1] + \ \dots$$



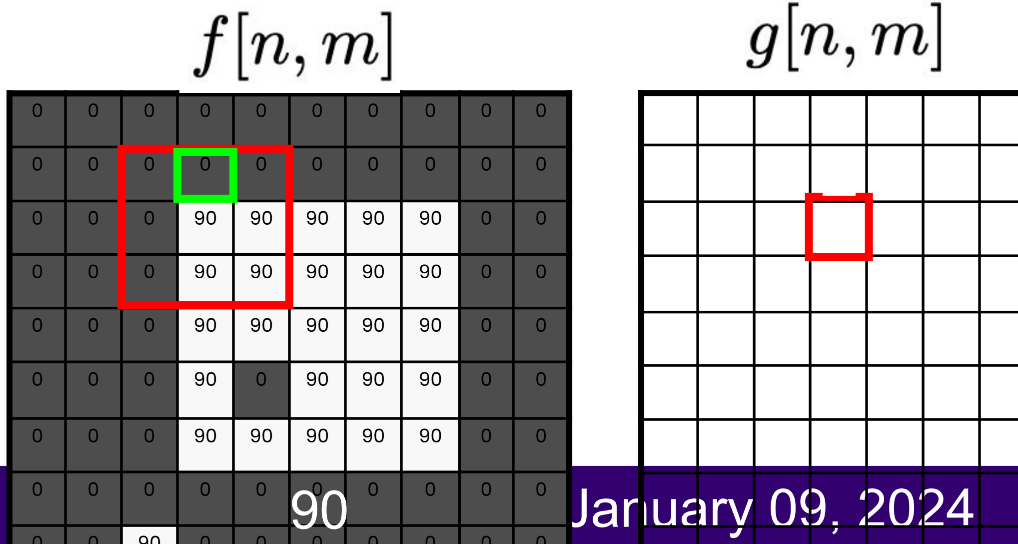
Now, instead of [0, 0], let's do [n, m]

$$g[n, m] = f[n-1, m-1] + \dots$$



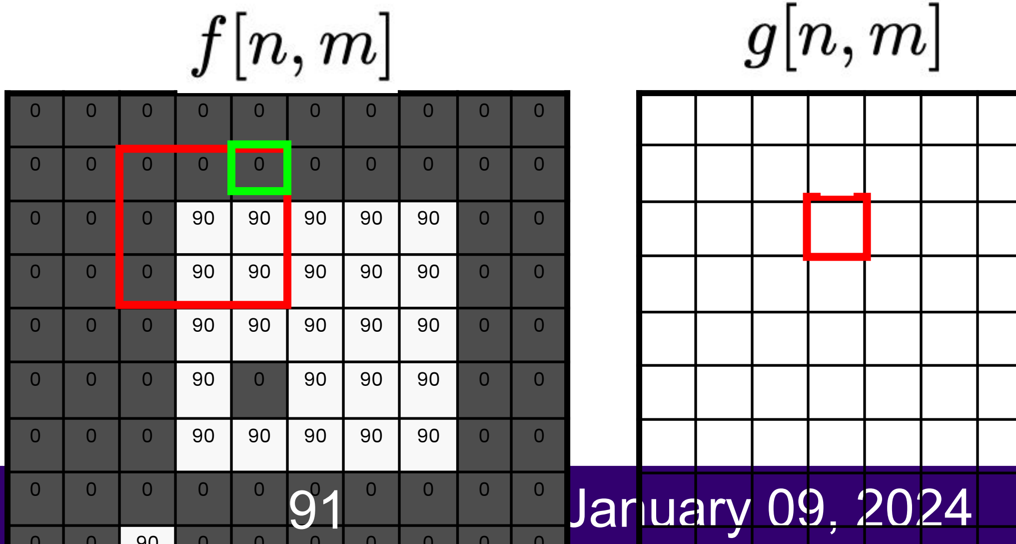
Now, instead of [0, 0], let's do [n, m]

$$g[n,m] = f[n-1,m-1] + \boxed{f[n-1,m]} + \ \dots$$



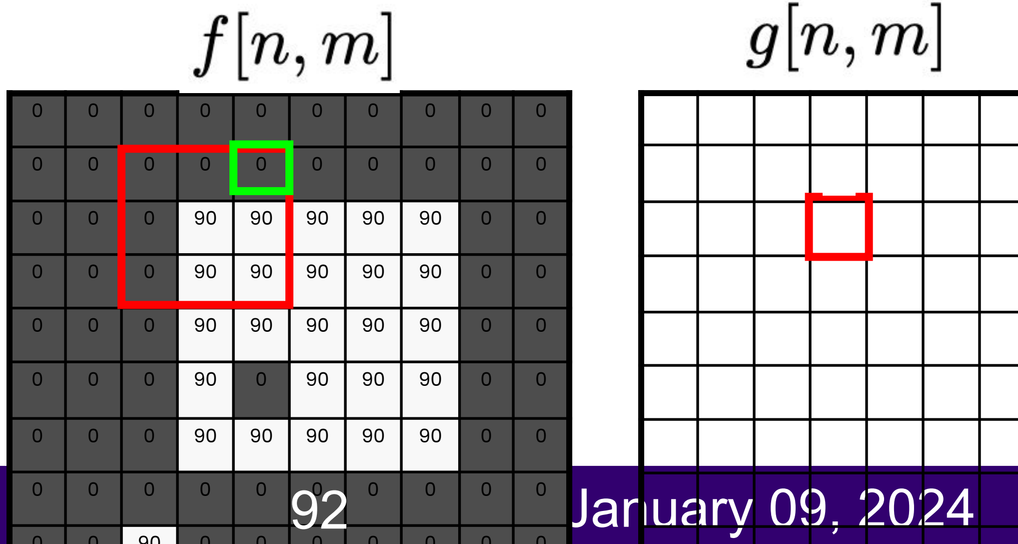
Now, instead of [0, 0], let's do [n, m]

$$g[n, m] = f[n-1, m-1] + f[n-1, m] + \ \ldots$$



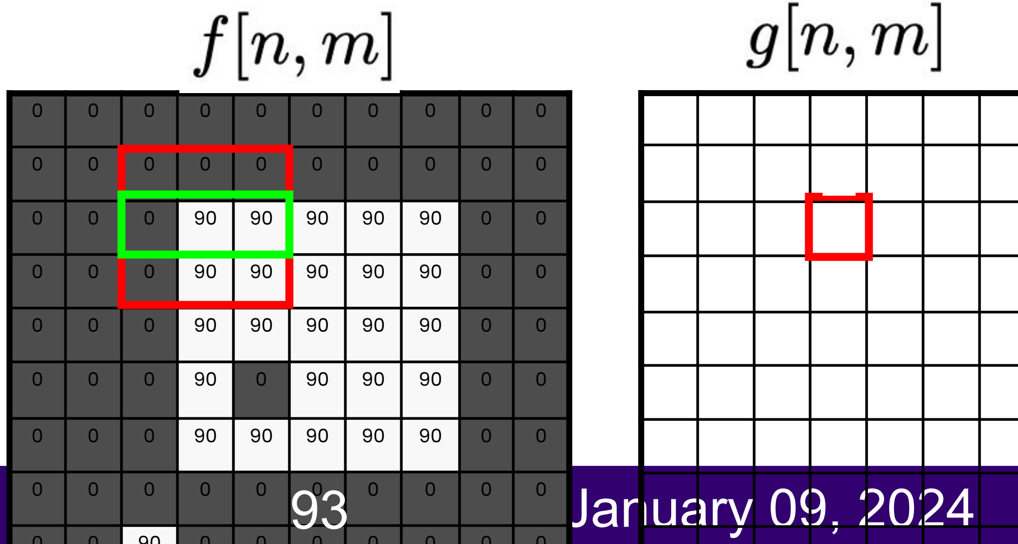
Now, instead of [0, 0], let's do [n, m]

$$g[n,m] = f[n-1,m-1] + f[n-1,m] + \boxed{f[n-1,m+1]}$$



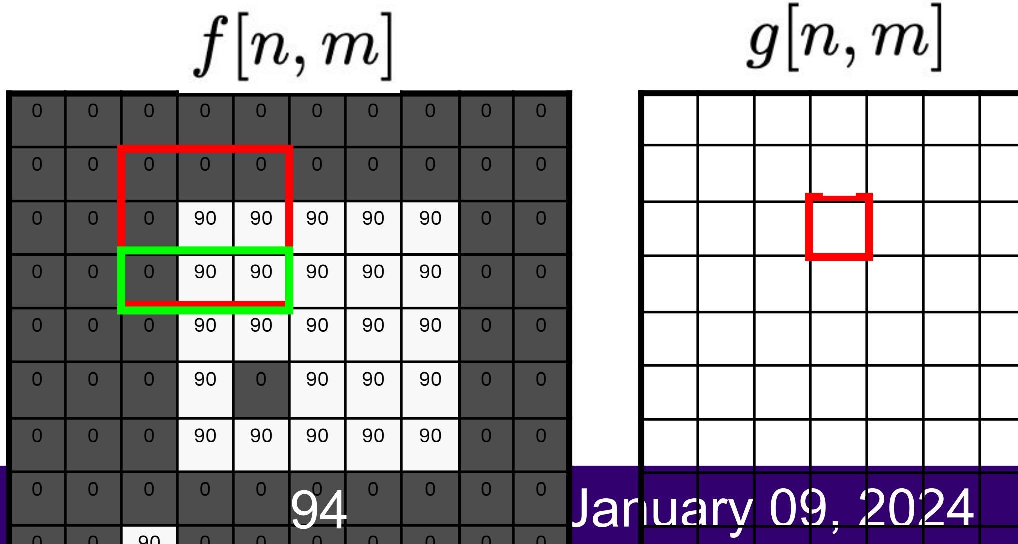
Now, instead of [0, 0], let's do [n, m]

$$g[n,m] = f[n-1,m-1] + f[n-1,m] + f[n-1,m+1]$$
$$+ f[n,m-1] + f[n,m] + f[n,m+1]$$



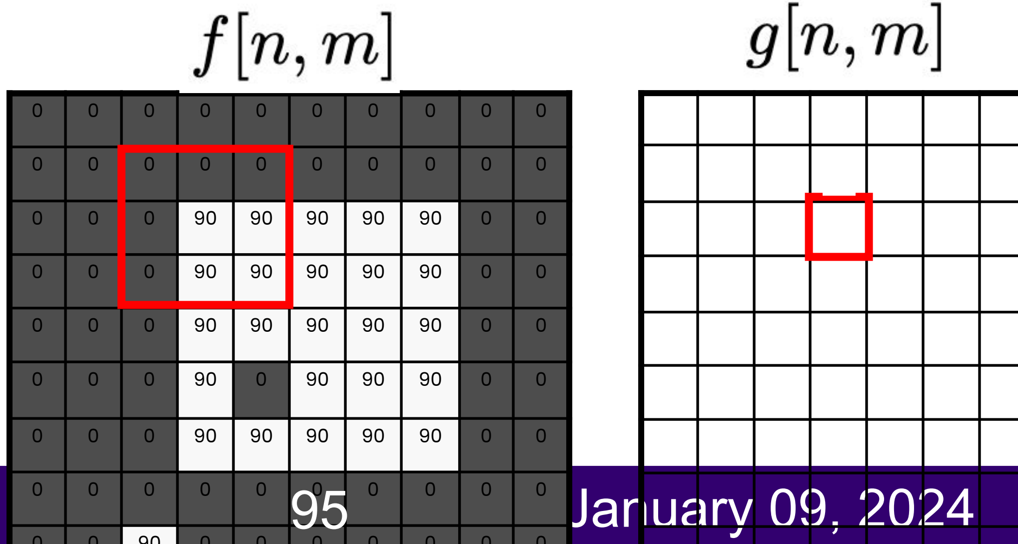
Now, instead of [0, 0], let's do [n, m]

$$g[n,m] = f[n-1,m-1] + f[n-1,m] + f[n-1,m+1]$$
$$+ f[n,m-1] + f[n,m] + f[n,m+1]$$
$$+ f[n+1,m-1] + f[n+1,m] + f[n+1,m+1]$$



Lastly, divide by 1/9

$$g[n,m] = \frac{1}{9}[f[n-1,m-1] + f[n-1,m] + f[n-1,m+1] + f[n,m-1] + f[n,m] + f[n,m+1] + f[n+1,m-1] + f[n+1,m] + f[n+1,m+1]]$$



Mathematical interpretation of moving average

We can re-write the equation using summations

$$g[n,m] = \frac{1}{9} \sum_{k=??}^{??} \sum_{l=??}^{??} f[k,l]$$

$$h[\cdot,\cdot] = \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Q. What values will **k** take?

Mathematical interpretation of moving average

How do we represent applying this filter mathematically?

$$g[n,m] = \frac{1}{9} \sum_{k=n-1}^{n+1} \sum_{l=??}^{??} f[k,l]$$

$$h[\cdot,\cdot] = \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

k goes from n-1 to n+1

Mathematical interpretation of moving average

How do we represent applying this filter mathematically?

$$g[n,m] = \frac{1}{9} \sum_{k=n-1}^{n+1} \sum_{l=??}^{??} f[k,l]$$

$$h[\cdot,\cdot] = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Q. What values will **l** take?

Mathematical interpretation of moving average

How do we represent applying this filter mathematically?

$$g[n,m] = \frac{1}{9} \sum_{k=n-1}^{n+1} \sum_{l=m-1}^{m+1} f[k,l]$$

$$h[\cdot,\cdot] = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

l goes from m-1 to m+1

Math formula for the moving average filter

A moving average over a 3 × 3 neighborhood window

We can write this operation mathematically:

$$g[n,m] = \frac{1}{9} \sum_{k=n-1}^{n+1} \sum_{l=m-1}^{m+1} f[k,l]$$

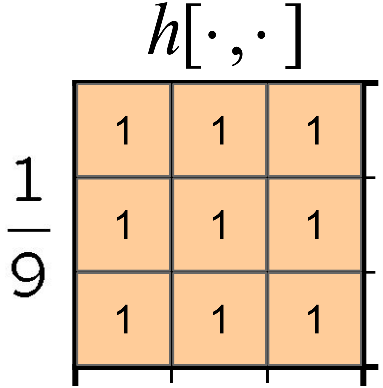
$$h[\cdot,\cdot] = \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Rewriting this formula

We are almost done. Let's rewrite this formula a little bit

Let $k' = n - k$

$$g[n,m] = \frac{1}{9} \sum_{k=n-1}^{n+1} \sum_{l=m-1}^{m+1} f[k,l]$$



Rewriting this formula

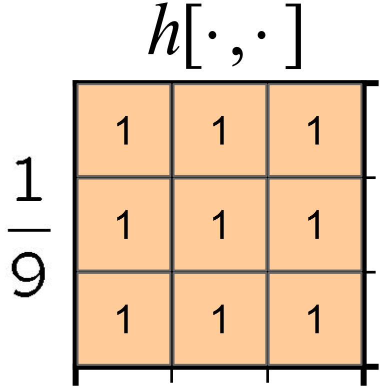
We are almost done. Let's rewrite this formula a little bit

Let $k' = n - k$

therefore, $k = n - k'$

$$g[n,m] = \frac{1}{9} \sum_{k=n-1}^{n+1} \sum_{l=m-1}^{m+1} f[k,l]$$

Now we can replace k in the equation above



Rewriting this formula

We are almost done. Let's rewrite this formula a little bit

Let $k' = n - k$

therefore, $k = n - k'$

$$g[n,m] = \frac{1}{9} \sum_{k=n-1}^{n+1} \sum_{l=m-1}^{m+1} f[k,l]$$

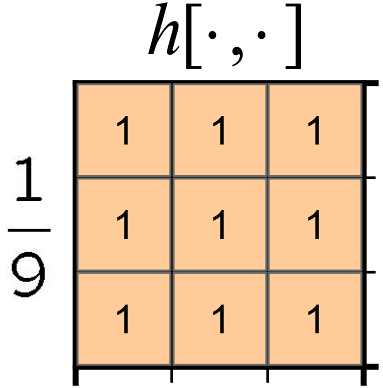
$$g[n,m] = \frac{1}{9} \sum_{n-k'=n-1}^{n-k'=n+1} \sum_{l=m-1}^{m+1} f[n-k',l]$$

$$h[\cdot,\cdot] \qquad \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Rewriting this formula

So now we have this:

$$g[n,m] = \frac{1}{9} \sum_{n-k'=n-1}^{n-k'=n+1} \sum_{l=m-1}^{m+1} f[n-k', l]$$



Rewriting this formula

So now we have this:

$$g[n,m] = \frac{1}{9} \sum_{n-k'=n-1}^{n-k'=n+1} \sum_{l=m-1}^{m+1} f[n-k',l]$$

We can simplify the equations in red:

$$g[n,m] = \frac{1}{9} \sum_{k'=1}^{k'=-1} \sum_{l=m-1}^{m+1} f[n-k',l]$$

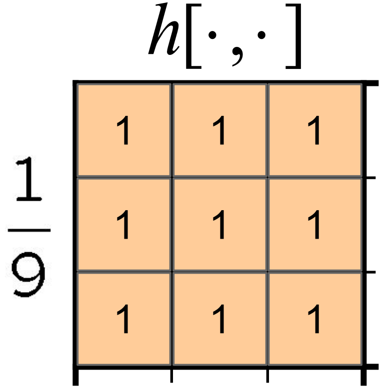
$$h[\cdot,\cdot] \quad \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Rewriting this formula

So now we have this:

$$g[n,m] = \frac{1}{9} \sum_{k'=1}^{k'=-1} \sum_{l=m-1}^{m+1} f[n-k', l]$$

Remember that summations are just for-loops!!



Rewriting this formula

So now we have this:

$$g[n,m] = \frac{1}{9} \sum_{k'=1}^{k'=-1} \sum_{l=m-1}^{m+1} f[n-k', l]$$

Remember that summations are just for-loops!!

$$g[n,m] = \frac{1}{9} \sum_{k'=-1}^{1} \sum_{l=m-1}^{m+1} f[n-k', l]$$

$h[\cdot,\cdot]$

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

Rewriting this formula

One last change: since there are no more k and only k',
let's just write k' as k

$$g[n,m] = \frac{1}{9} \sum_{k'=-1}^{1} \sum_{l=m-1}^{m+1} f[n-k', l]$$

$$g[n,m] = \frac{1}{9} \sum_{k=-1}^{1} \sum_{l=m-1}^{m+1} f[n-k, l]$$

$$h[\cdot,\cdot] = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Mathematical interpretation of moving average

Let’s repeat for l, just like we did for k

$$g[n,m] = \frac{1}{9} \sum_{k=n-1}^{n+1} \sum_{l=m-1}^{m+1} f[k,l]$$

$$= \frac{1}{9} \sum_{k=-1}^{1} \sum_{l=-1}^{1} f[n-k, m-l]$$

$$h[\cdot,\cdot] = \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Filter example #1: Moving Average

Original image

Smoothed image

Filter example #2: Image Segmentation

Q. How would you use pixel values to design a filter to segment an image so that you only keep around the edges?

Filter example #2: Image Segmentation

- Use a simple pixel threshold:

$$g[n,m] = \begin{cases} 255, & f[n,m] > 100 \\ 0, & \text{otherwise.} \end{cases}$$

Summary so far

- Discrete systems convert input discrete signals and convert them into something more meaningful.
- There are an infinite number of possible filters we can design.
- What are ways we can category the space of possible systems?

Today's agenda

- Color spaces
- Image sampling and quantization
- Image histograms
- Images as functions
- Filters
- Properties of systems

Properties of systems

- Amplitude properties:
  - Additivity

$$\mathcal{S}[f_i[n,m] + f_j[n,m]] = \mathcal{S}[f_i[n,m]] + \mathcal{S}[f_j[n,m]]$$

Example question:

Q. Is the moving average filter additive?

$$\mathcal{S}[f_i[n,m] + f_j[n,m]] = \mathcal{S}[f_i[n,m]] + \mathcal{S}[f_j[n,m]]$$

$$h[\cdot,\cdot] = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

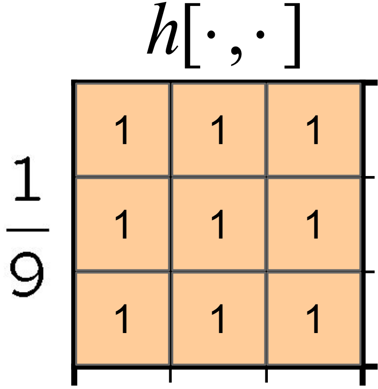
How would you prove it?

$$g[n,m] = \frac{1}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} f[n-k, m-l]$$

Example question:

$$\mathcal{S}[f_i[n,m] + f_j[n,m]] = \mathcal{S}[f_i[n,m]] + \mathcal{S}[f_j[n,m]]$$

Let $f'[n,m] = f_i[n,m] + f_j[n,m]$



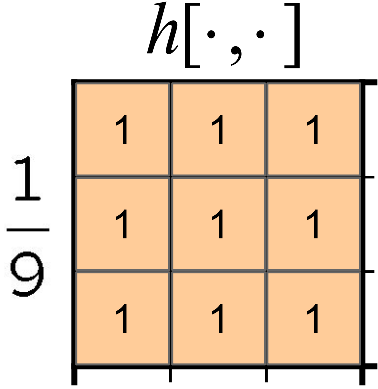
$$g[n,m] = \frac{1}{9} \sum_{k=-1}^{1} \sum_{l=-1}^{1} f[n-k, m-l]$$

Example question:

$$\mathcal{S}[f_i[n,m] + f_j[n,m]] = \mathcal{S}[f_i[n,m]] + \mathcal{S}[f_j[n,m]]$$

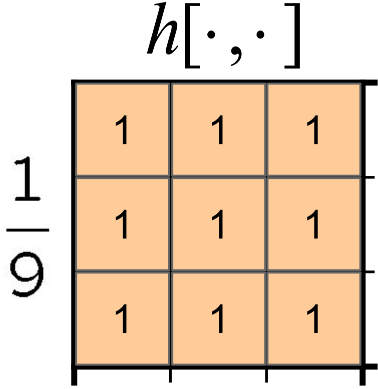
Let $f'[n,m] = f_i[n,m] + f_j[n,m]$

$$\mathcal{S}[f_i[n,m] + f_j[n,m]] = \mathcal{S}[f'[n,m]]$$



$$g[n,m] = \frac{1}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} f[n-k, m-l]$$

Example question:



$$g[n,m] = \frac{1}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} f[n-k,m-l]$$

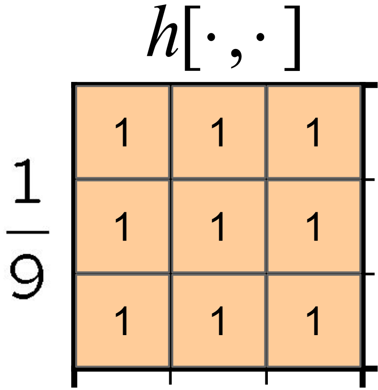
$$\mathcal{S}[f_i[n,m] + f_j[n,m]] = \mathcal{S}[f_i[n,m]] + \mathcal{S}[f_j[n,m]]$$

Let $f'[n,m] = f_i[n,m] + f_j[n,m]$

$$\mathcal{S}[f_i[n,m] + f_j[n,m]] = \mathcal{S}[f'[n,m]]$$

$$= \frac{1}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} f'[n-k,m-l]$$

Example question:

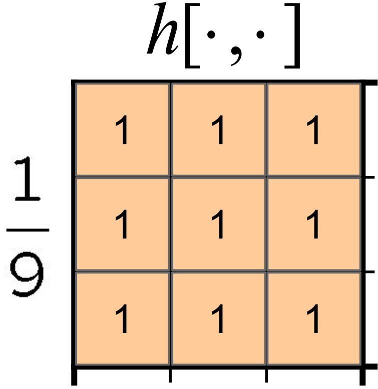


$$\mathcal{S}[f_i[n,m] + f_j[n,m]] = \mathcal{S}[f_i[n,m]] + \mathcal{S}[f_j[n,m]]$$

Let $f'[n,m] = f_i[n,m] + f_j[n,m]$

$$g[n,m] = \frac{1}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} f[n-k, m-l]$$

$$\begin{aligned}\mathcal{S}[f_i[n,m] + f_j[n,m]] &= \mathcal{S}[f'[n,m]] \\ &= \frac{1}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} f'[n-k, m-l] \\ &= \frac{1}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} [f_i[n-k, m-l] + f_j[n-k, m-l]]\end{aligned}$$



Example question:

$$\mathcal{S}[f_i[n,m] + f_j[n,m]] = \mathcal{S}[f_i[n,m]] + \mathcal{S}[f_j[n,m]]$$

Let $f'[n,m] = f_i[n,m] + f_j[n,m]$

$$g[n,m] = \frac{1}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} f[n-k, m-l]$$

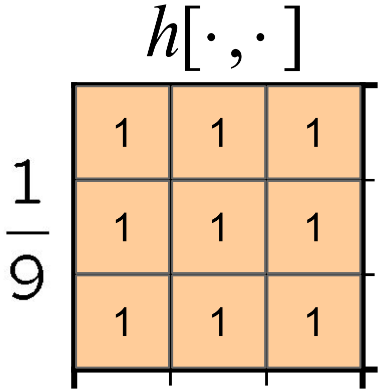
$$\begin{aligned}
\mathcal{S}[f_i[n,m] + f_j[n,m]] &= \mathcal{S}[f'[n,m]] \\
&= \frac{1}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} f'[n-k, m-l] \\
&= \frac{1}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} [f_i[n-k, m-l] + f_j[n-k, m-l]] \\
&= \frac{1}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} f_i[n-k, m-l] + \frac{1}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} f_j[n-k, m-l]]
\end{aligned}$$

Example question:

$$\mathcal{S}[f_i[n,m] + f_j[n,m]] = \mathcal{S}[f_i[n,m]] + \mathcal{S}[f_j[n,m]]$$

Let $f'[n,m] = f_i[n,m] + f_j[n,m]$

$$\begin{aligned}
\mathcal{S}[f_i[n,m] + f_j[n,m]] &= \mathcal{S}[f'[n,m]] \\
&= \frac{1}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} f'[n-k, m-l] \\
&= \frac{1}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} [f_i[n-k, m-l] + f_j[n-k, m-l]] \\
&= \frac{1}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} f_i[n-k, m-l] + \frac{1}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} f_j[n-k, m-l]] \\
&= \mathcal{S}[f_i[n,m]] + \mathcal{S}[f_j[n,m]]
\end{aligned}$$



$$g[n,m] = \frac{1}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} f[n-k, m-l]$$

Properties of systems

- Amplitude properties:
  - Additivity

$$\mathcal{S}[f_i[n,m] + f_j[n,m]] = \mathcal{S}[f_i[n,m]] + \mathcal{S}[f_j[n,m]]$$

Properties of systems

- Amplitude properties:
  - Additivity

$$\mathcal{S}[f_i[n,m] + f_j[n,m]] = \mathcal{S}[f_i[n,m]] + \mathcal{S}[f_j[n,m]]$$

  - Homogeneity

$$\mathcal{S}[\alpha f[n,m]] = \alpha \mathcal{S}[f[n,m]]$$

Another question:

Q. Is the moving average filter homogeneous?

$$\mathcal{S}[\alpha f[n,m]] = \alpha \mathcal{S}[f[n,m]]$$

$h[\cdot,\cdot]$

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

Practice proving it at home using:

$$g[n,m] = \frac{1}{9} \sum_{k=-1}^{1} \sum_{l=-1}^{1} f[n-k, m-l]$$

What we covered today

- Color spaces
- Image sampling and quantization
- Image histograms
- Images as functions
- Filters
- Properties of systems

Next time:

Linear systems and convolutions