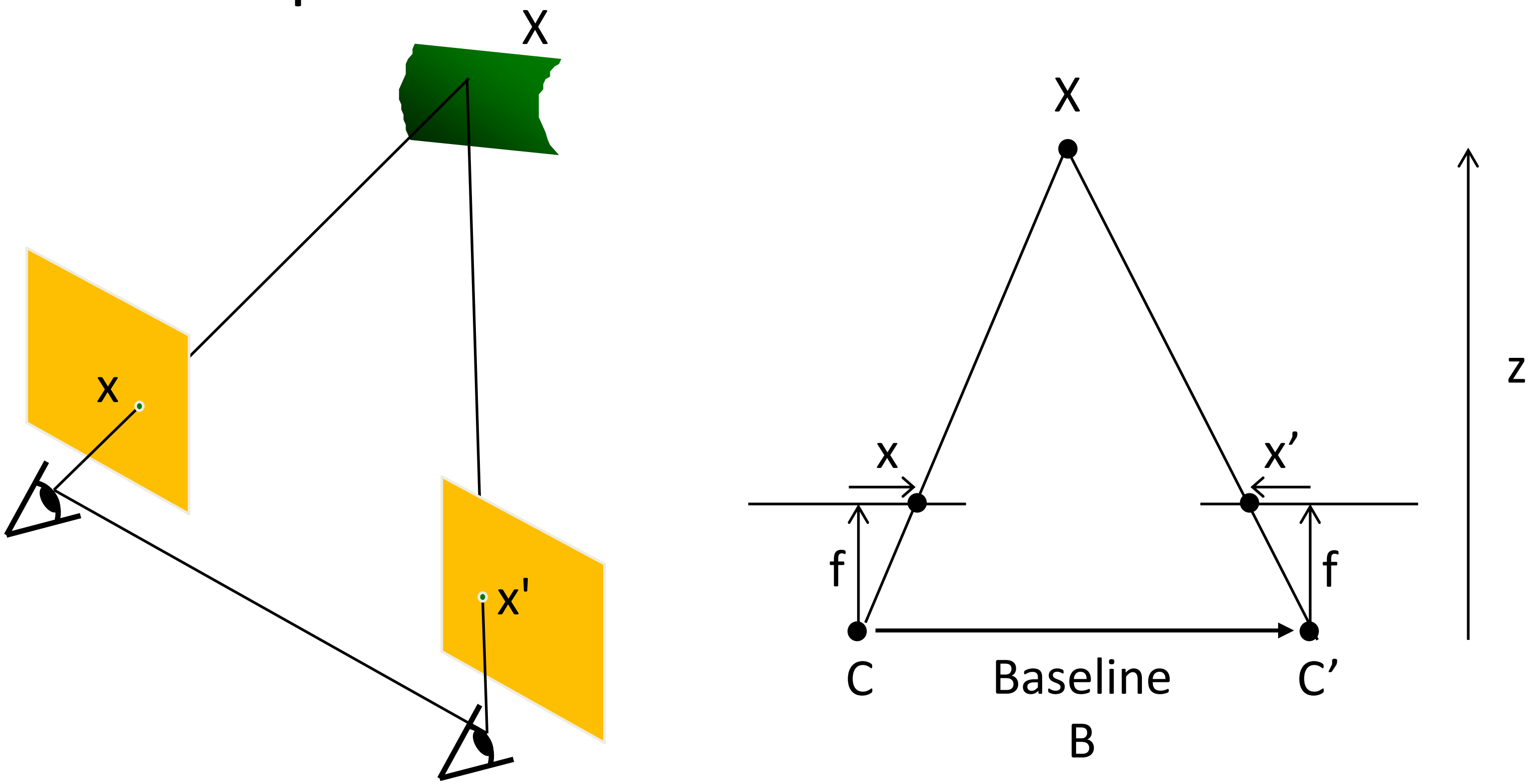
Stereo

Amount of horizontal movement is …

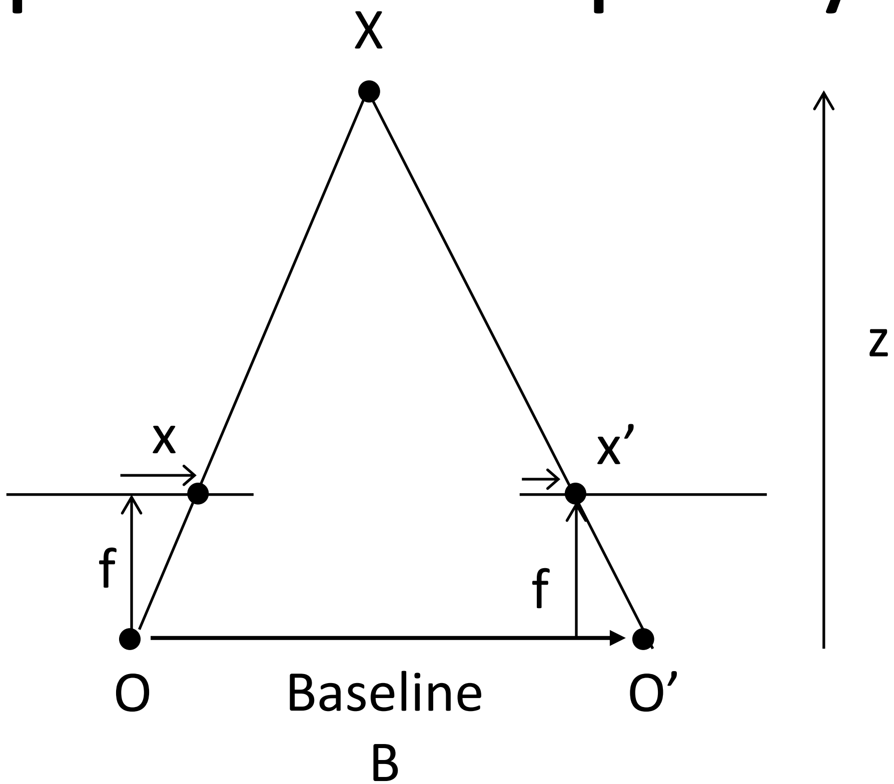
…inversely proportional to the distance from the camera

Depth from Stereo

- Goal: recover depth by finding image coordinate x' that corresponds to x



Depth from disparity



$$\frac{x - x'}{O - O'} = \frac{f}{z}$$

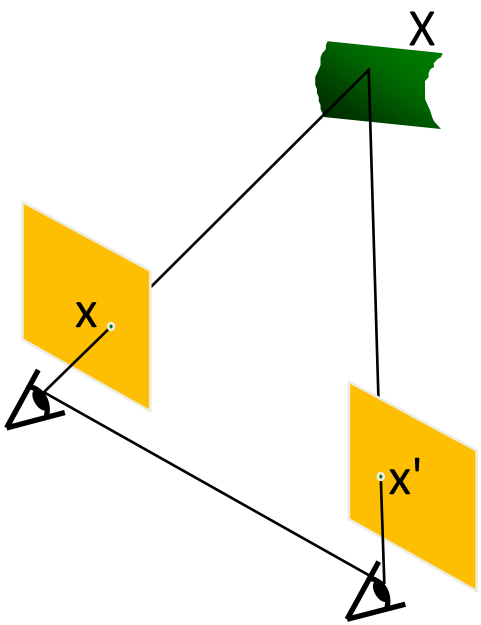
See Chapter 12 of Shapiro and Stockman Text.

$$disparity = x - x' = \frac{B \cdot f}{z}$$

Disparity is inversely proportional to depth.

Depth from Stereo

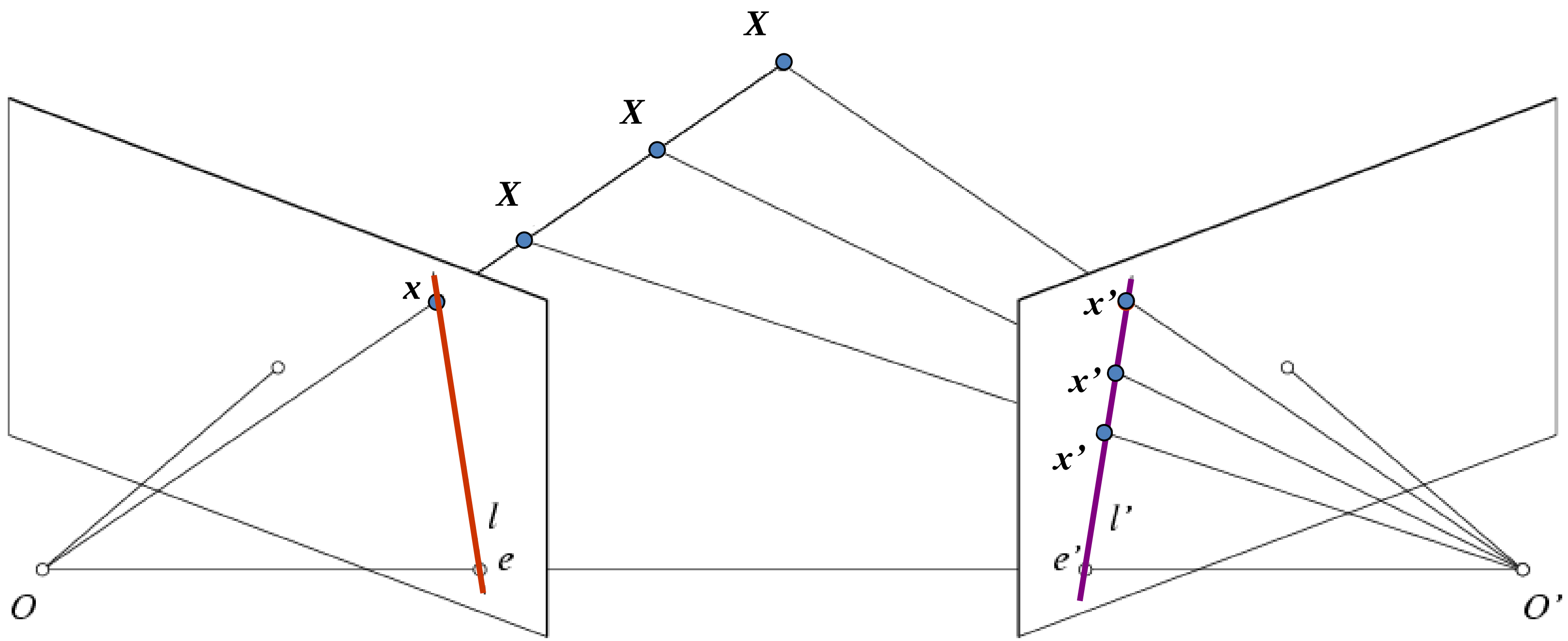
- Goal: recover depth by finding image coordinate x' that corresponds to x
- Sub-Problems
  1. Calibration: How do we recover the relation of the cameras (if not already known)?
  2. Correspondence: How do we search for the matching point x'?



Correspondence Problem

- We have two images taken from cameras with different intrinsic and extrinsic parameters
- How do we match a point in the first image to a point in the second? How can we constrain our search?

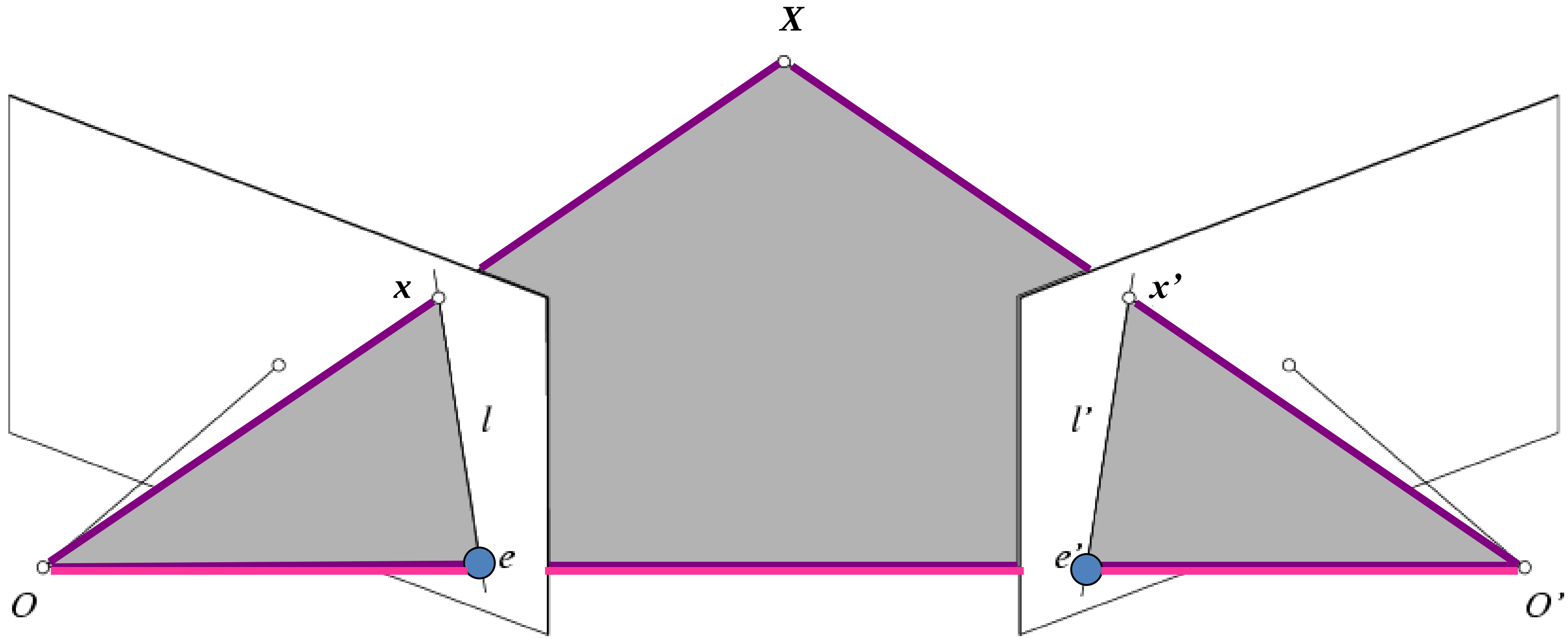
Key idea: Epipolar constraint



Potential matches for *x* have to lie on the corresponding line *l'*.

Potential matches for *x'* have to lie on the corresponding line *l*.

Epipolar geometry: notation



- **Baseline** – line connecting the two camera centers

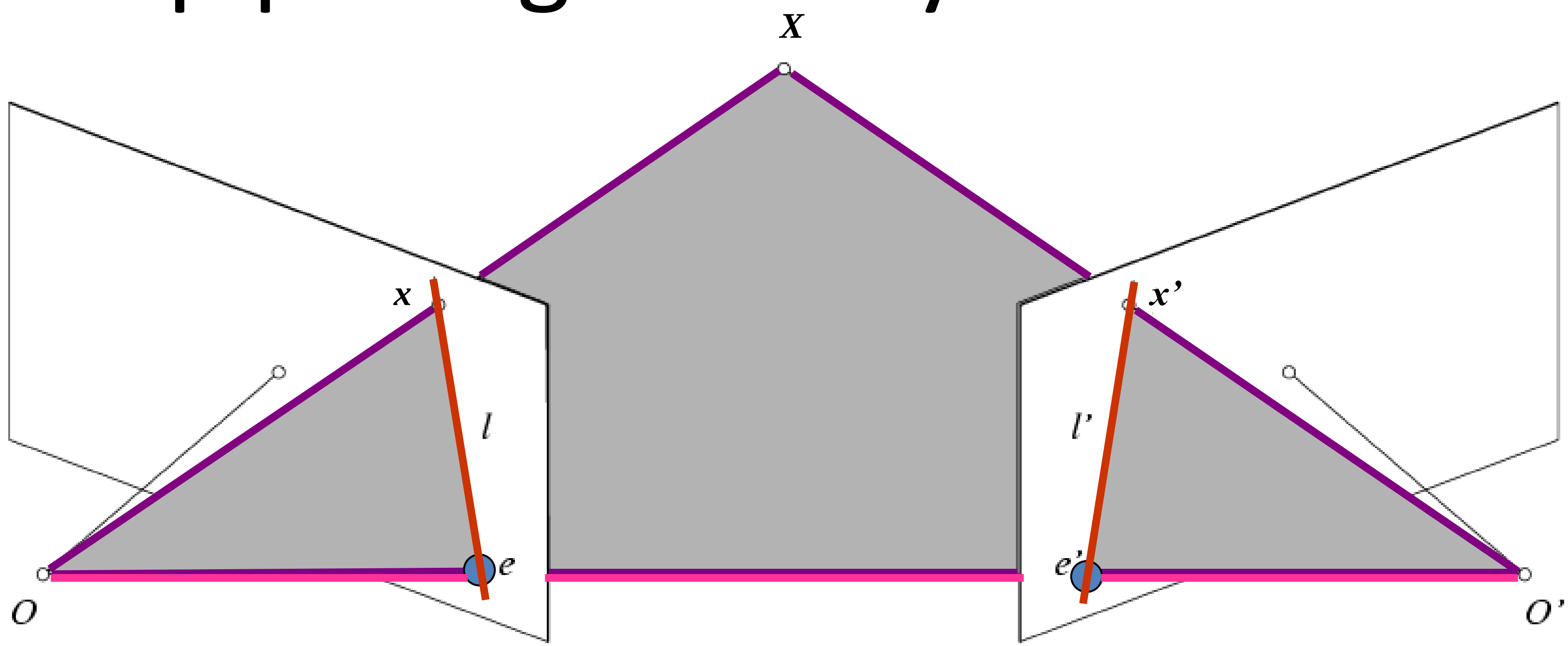
- **Epipoles**

= intersections of baseline with image planes

= projections of the other camera center

- **Epipolar Plane** – plane containing baseline (1D family)

Epipolar geometry: notation

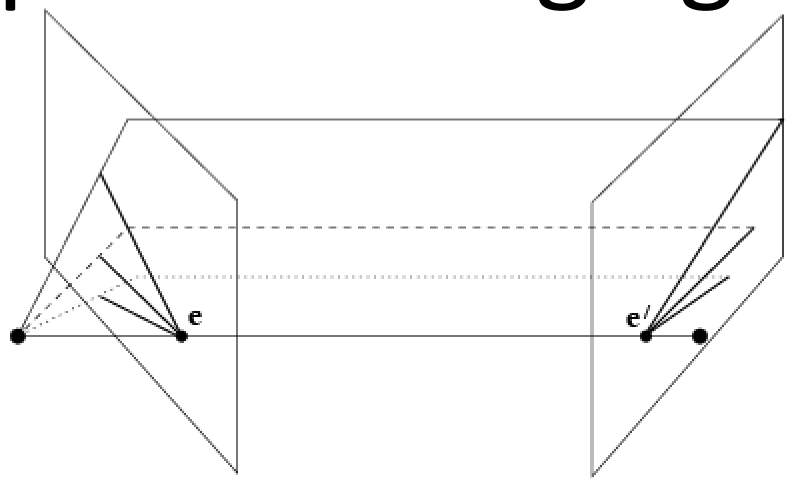


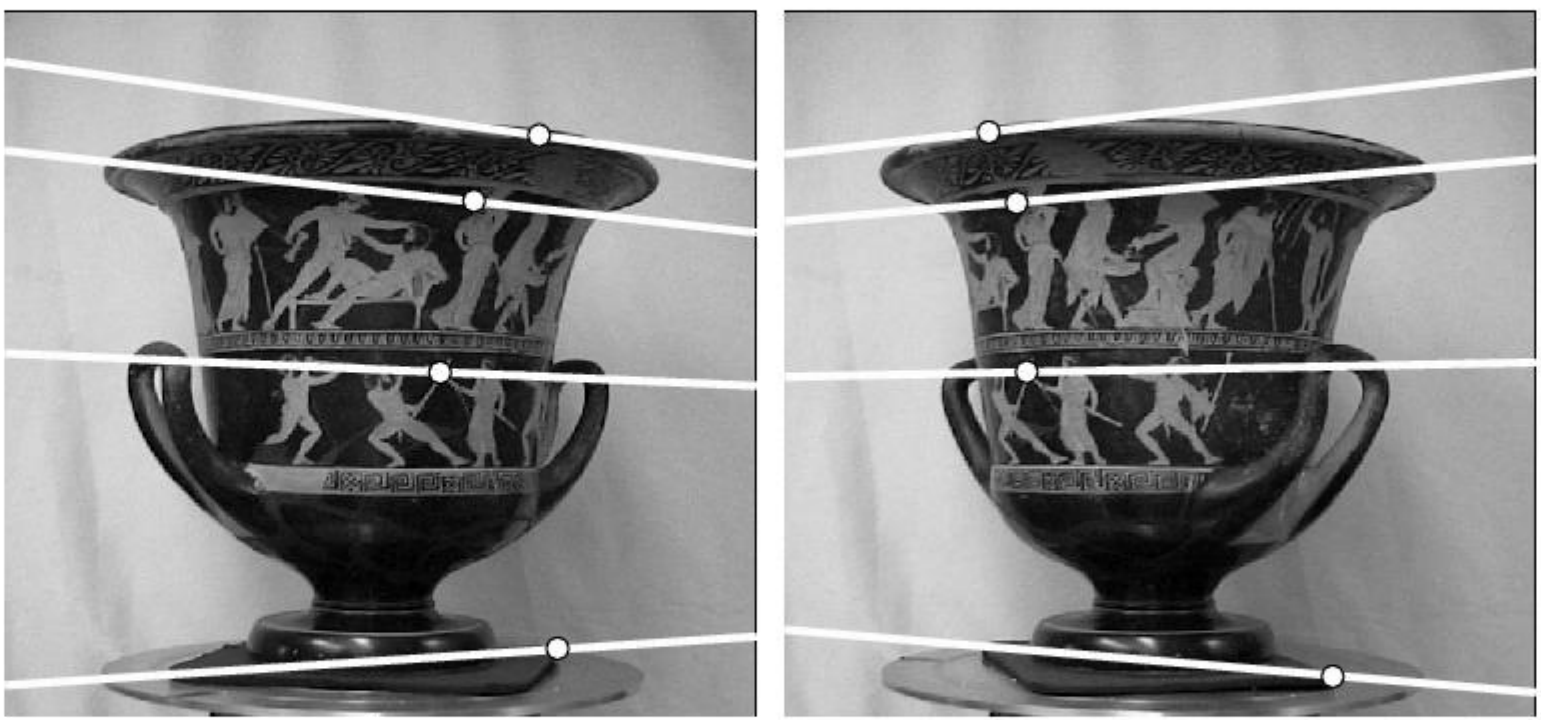
- **Baseline** – line connecting the two camera centers
- **Epipoles**

= intersections of baseline with image planes
= projections of the other camera center

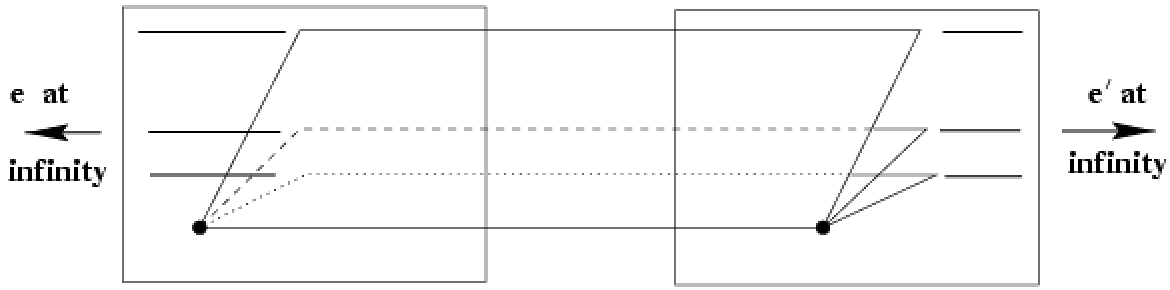
- **Epipolar Plane** – plane containing baseline (1D family)
- **Epipolar Lines** - intersections of epipolar plane with image planes (always come in corresponding pairs)

Example: Converging cameras

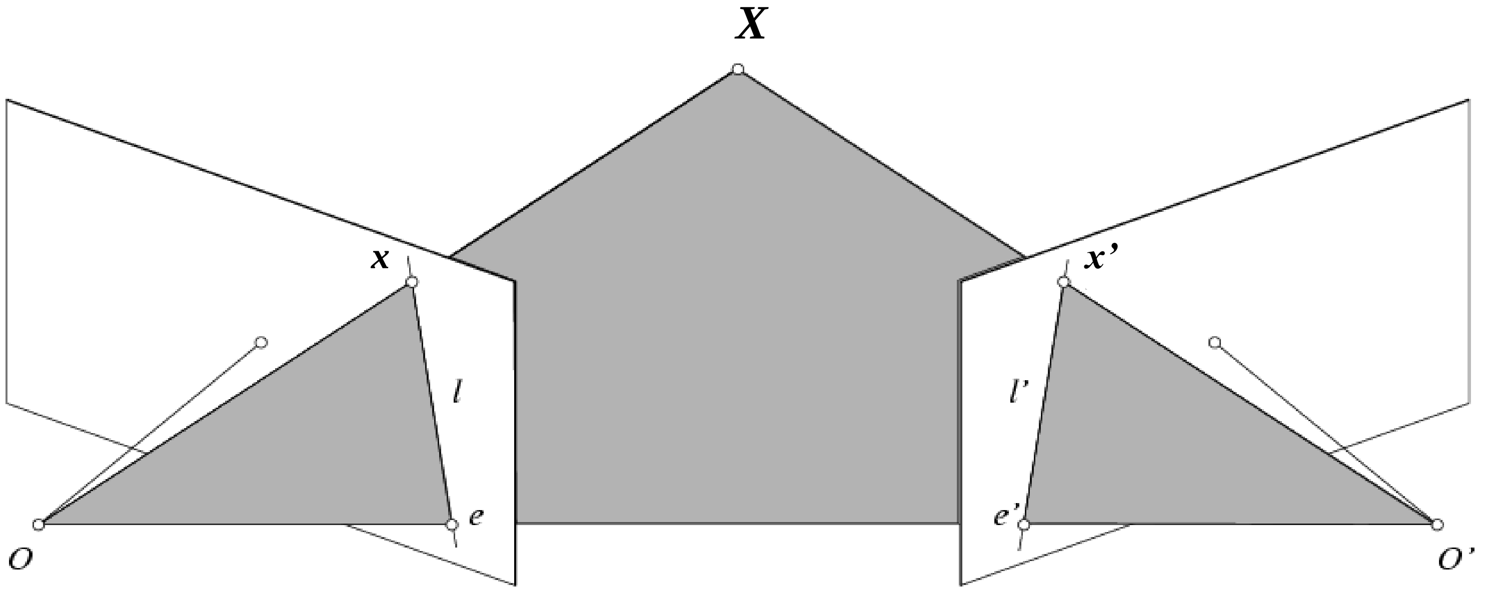




Example: Motion parallel to image plane



Epipolar constraint: Calibrated case



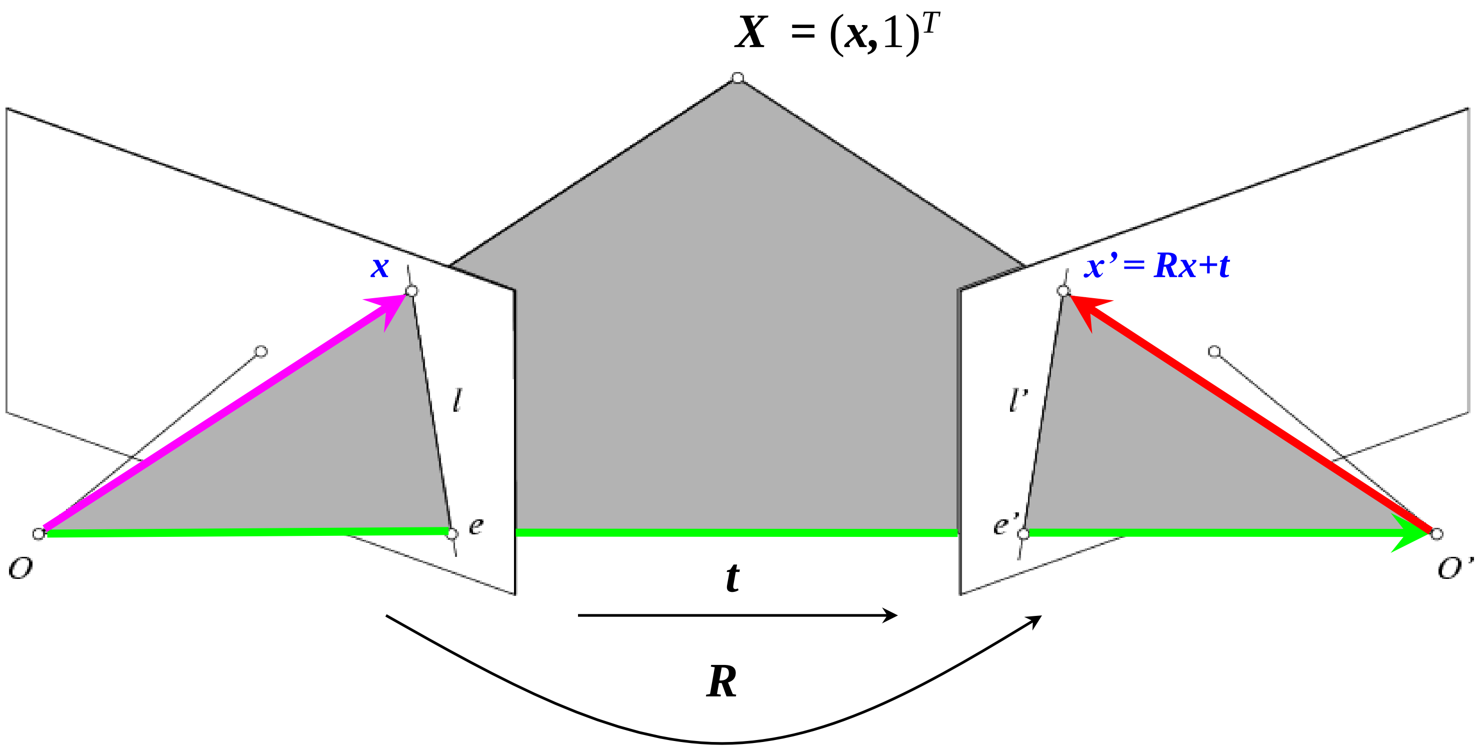
- Assume that the intrinsic and extrinsic parameters of the cameras are known
- We can multiply the projection matrix of each camera (and the image points) by the inverse of the calibration matrix to get *normalized image coordinates*
- We can also set the global coordinate system to the coordinate system of the first camera. Then the projection matrices of the two cameras can be written as **[I | 0]** and **[R | t]**

Simplified Matrices for the 2 Cameras

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \left(\, \mathbf{I} \mid \mathbf{0} \, \right)$$

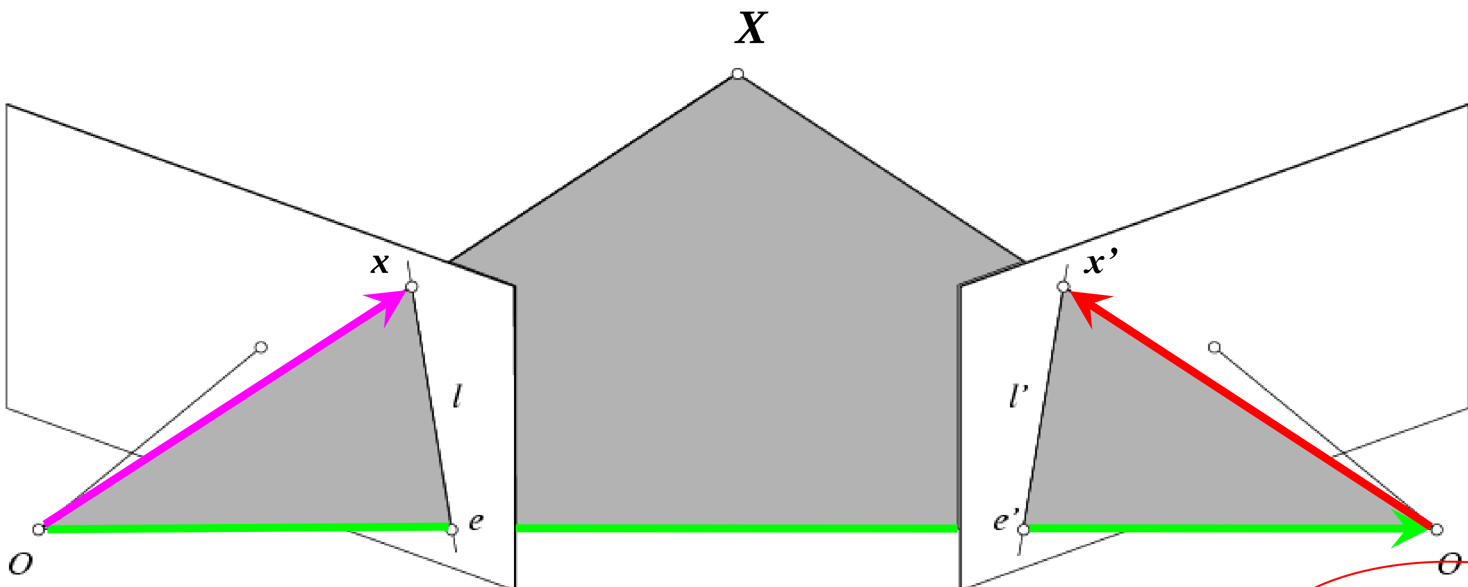
$$\left(\begin{array}{c|c} \mathbf{R} & \mathbf{t} \\ \hline \mathbf{0} & 1 \end{array} \right) = (R \mid T)$$

Epipolar constraint: Calibrated case



The vectors ***Rx***, ***t***, and ***x'*** are coplanar

Epipolar constraint: Calibrated case

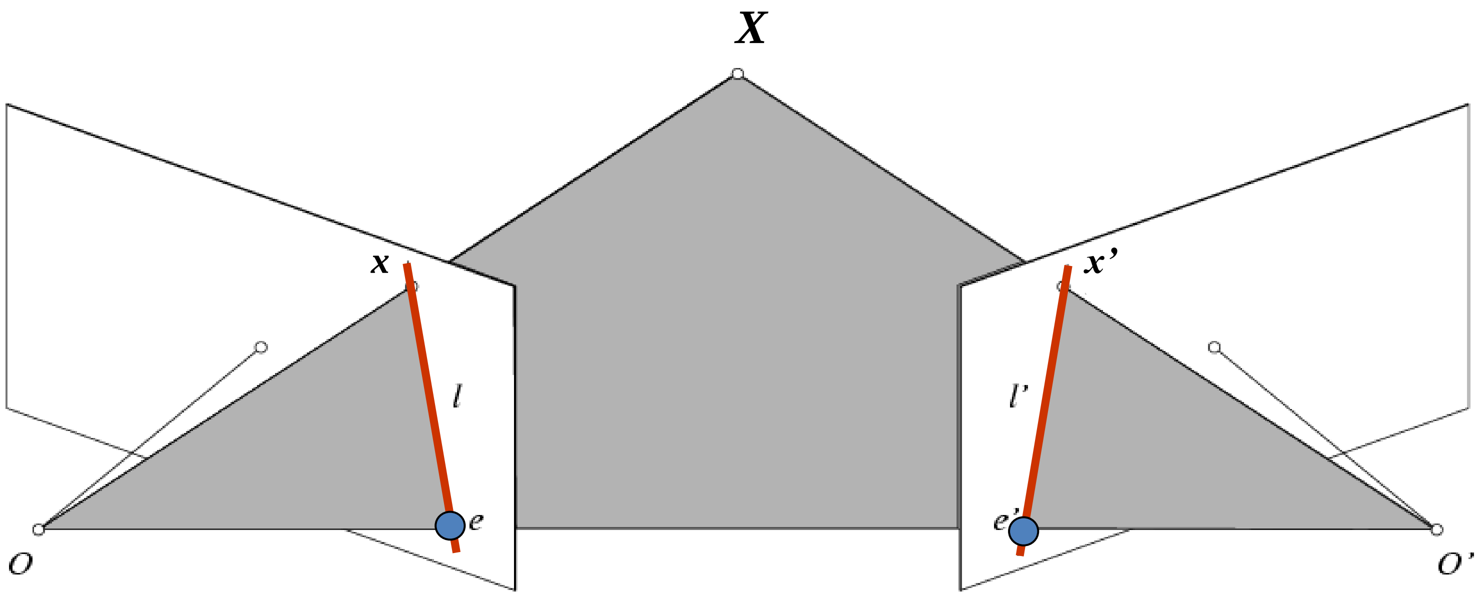


$$\boldsymbol{x}' \cdot [\boldsymbol{t} \times (\boldsymbol{R}\boldsymbol{x})] = 0 \quad \Rightarrow \quad \boldsymbol{x}'^T \boldsymbol{E}\, \boldsymbol{x} = 0 \quad \text{with} \quad \boldsymbol{E} = [\boldsymbol{t}_\times]\boldsymbol{R}$$

Essential Matrix E
(Longuet-Higgins, 1981)

The vectors $\boldsymbol{Rx}$, $\boldsymbol{t}$, and $\boldsymbol{x}'$ are coplanar

Epipolar constraint: Calibrated case



$$\mathbf{x}' \cdot [\boldsymbol{t} \times (\boldsymbol{R}\,\boldsymbol{x})] = 0 \quad \Rightarrow \quad \mathbf{x}'^T \boldsymbol{E}\,\mathbf{x} = 0 \quad \text{with} \quad \boldsymbol{E} = [\boldsymbol{t}_\times]\boldsymbol{R}$$

- $\boldsymbol{E}\,\boldsymbol{x}$ is the epipolar line associated with $\boldsymbol{x}$ ($\boldsymbol{l}' = \boldsymbol{E}\,\boldsymbol{x}$)
- $\boldsymbol{E}^T\boldsymbol{x}'$ is the epipolar line associated with $\boldsymbol{x}'$ ($\boldsymbol{l} = \boldsymbol{E}^T\boldsymbol{x}'$)
- $\boldsymbol{E}\,\boldsymbol{e} = 0$ and $\boldsymbol{E}^T\boldsymbol{e}' = 0$
- $\boldsymbol{E}$ is singular (rank two)
- $\boldsymbol{E}$ has five degrees of freedom

Moving on to stereo…

Fuse a calibrated binocular stereo pair to produce a depth image

image 1

image 2

Dense depth map

Many of these slides adapted from Steve Seitz and Lana Lazebnik

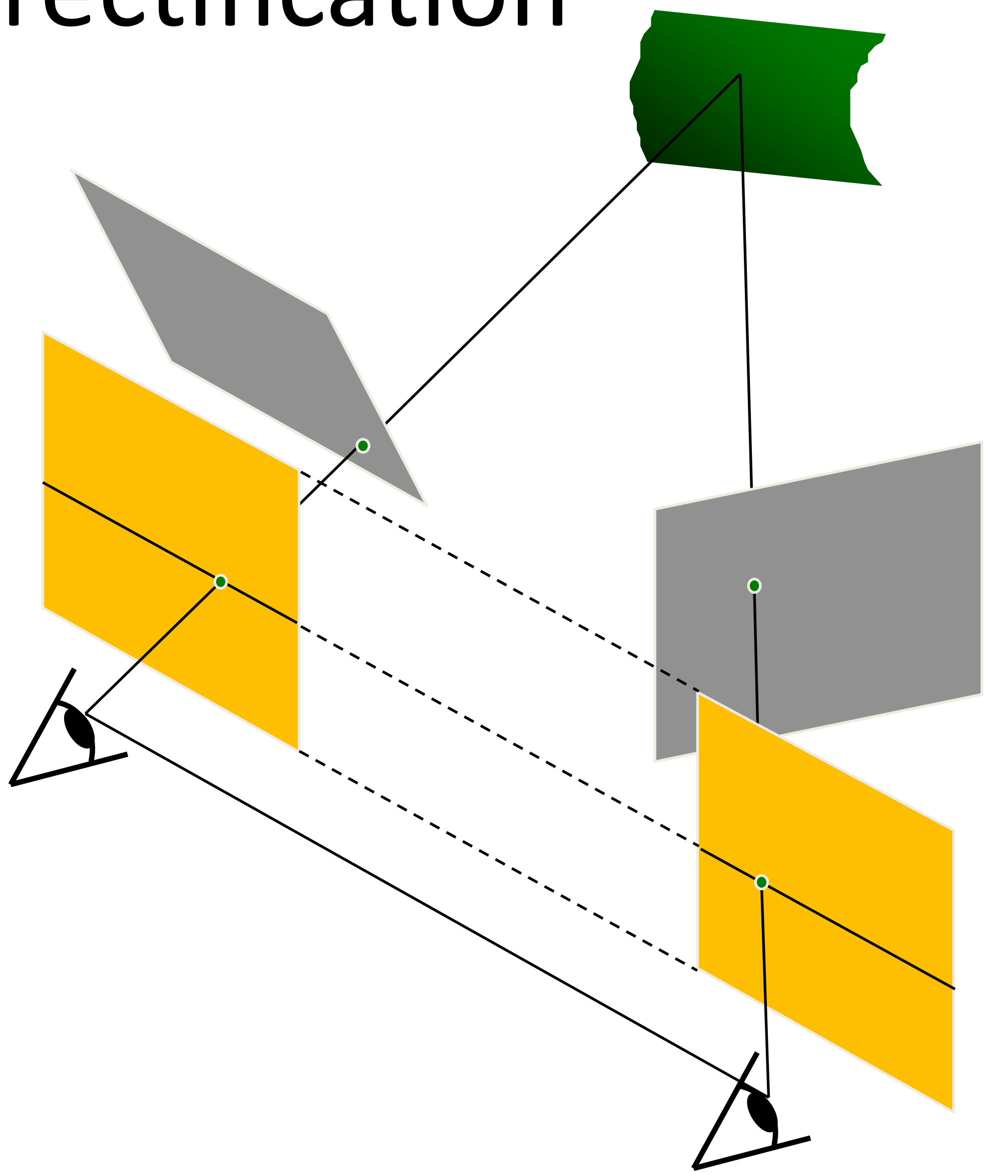
Stereo image rectification

- Reproject image planes onto a common plane parallel to the line between camera centers

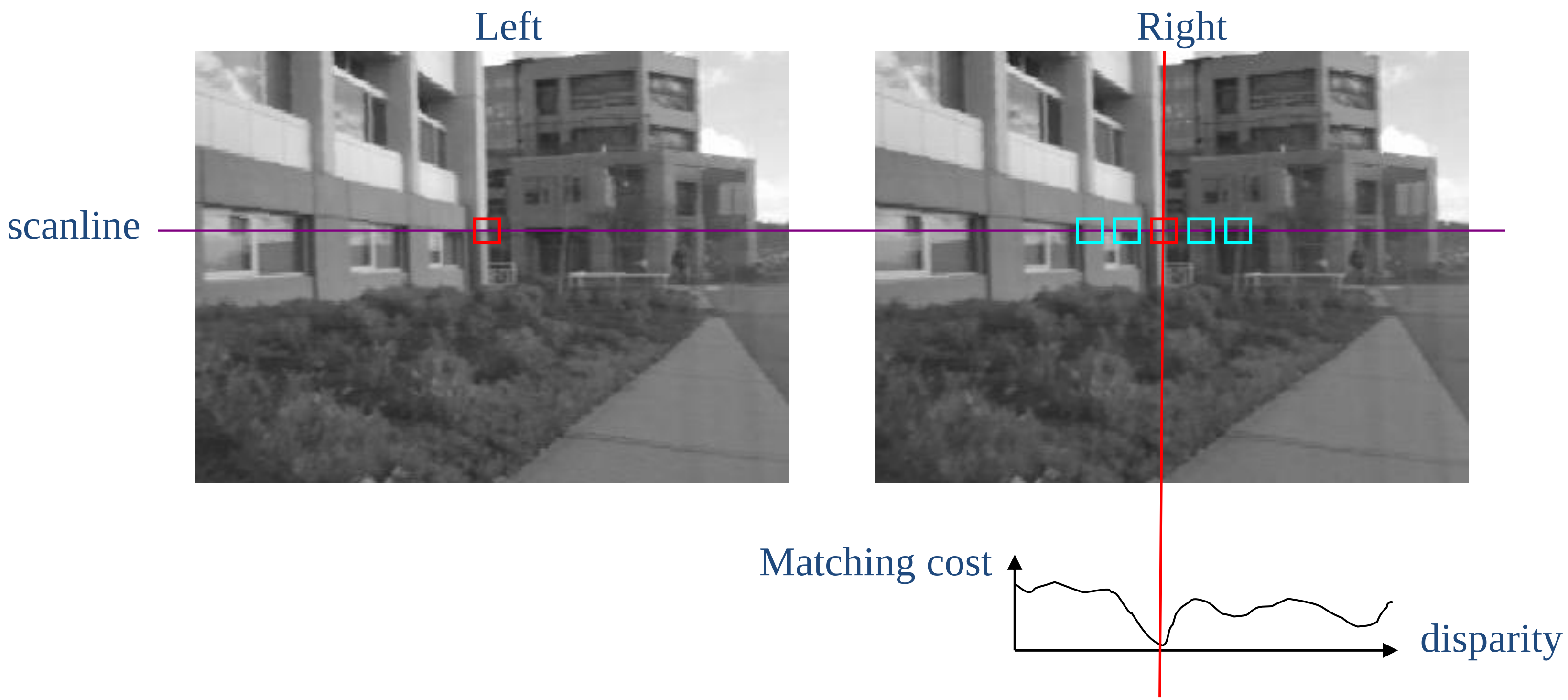
- Pixel motion is horizontal after this transformation

- Two homographies (3x3 transform), one for each input image reprojection

➢ C. Loop and Z. Zhang. [Computing Rectifying Homographies for Stereo Vision](). IEEE Conf. Computer Vision and Pattern Recognition, 1999.

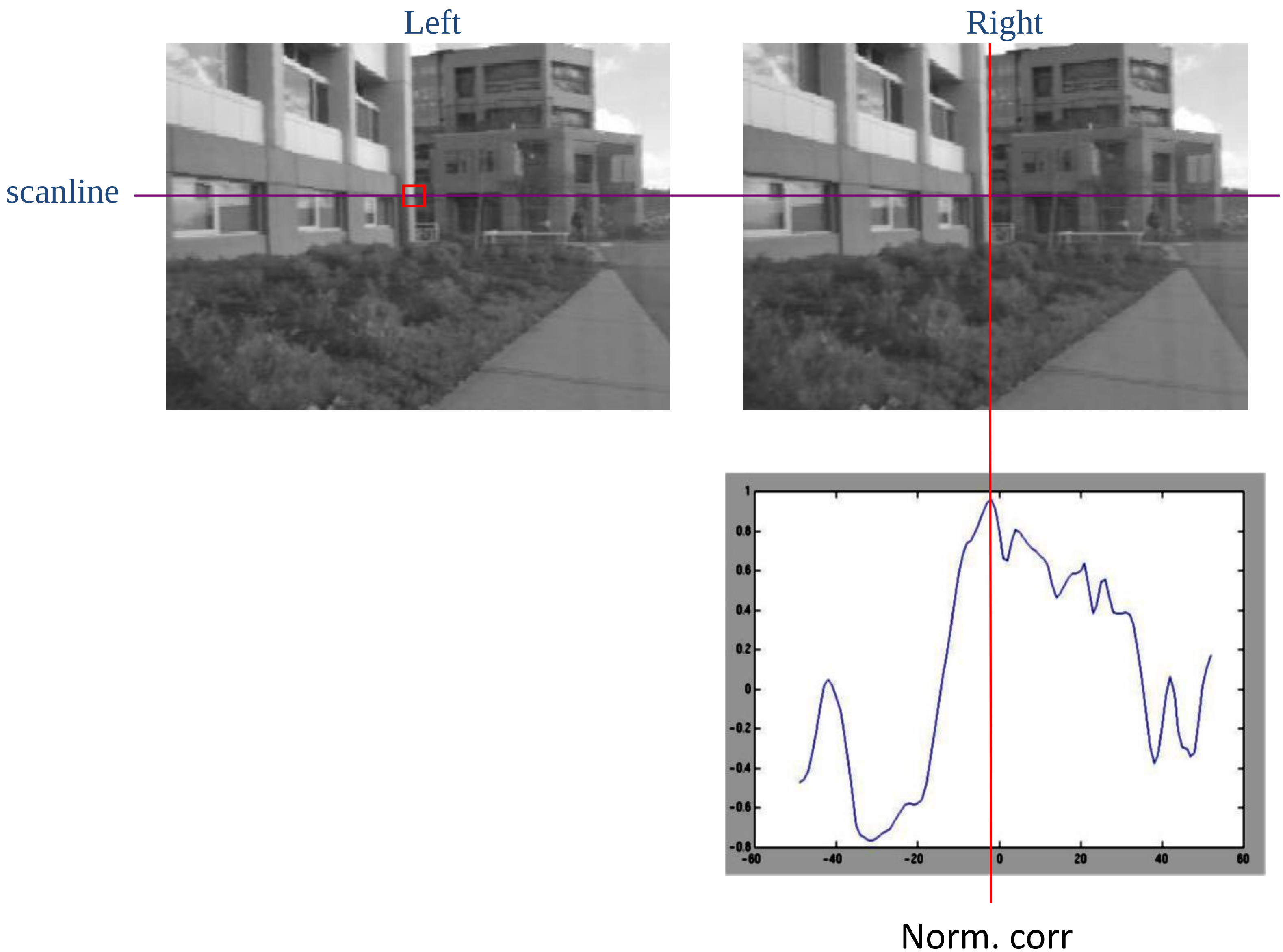


Example



- Slide a window along the right scanline and compare contents of that window with the reference window in the left image
- Matching cost: SSD, SAD, or normalized correlation

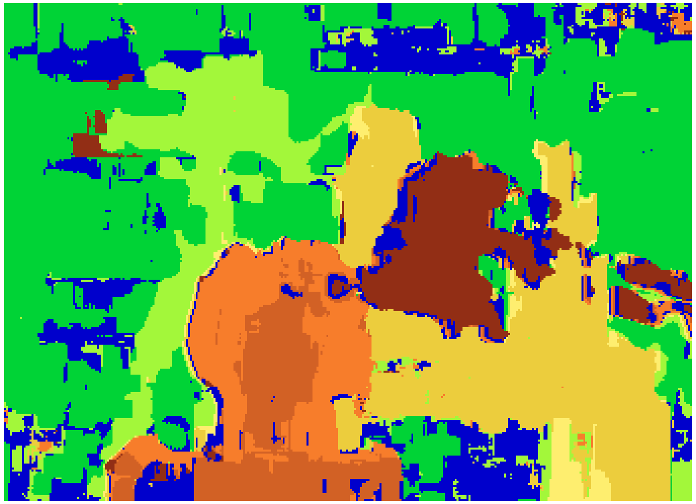
Correspondence search



Results with window search

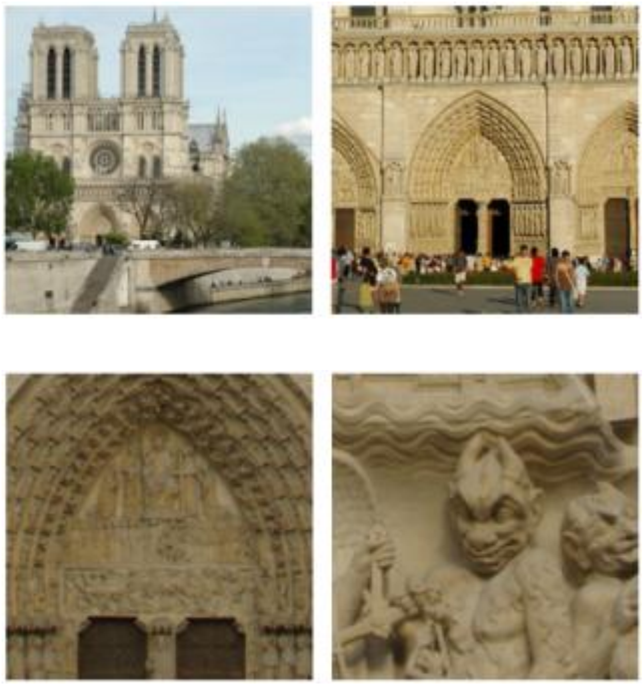
Data

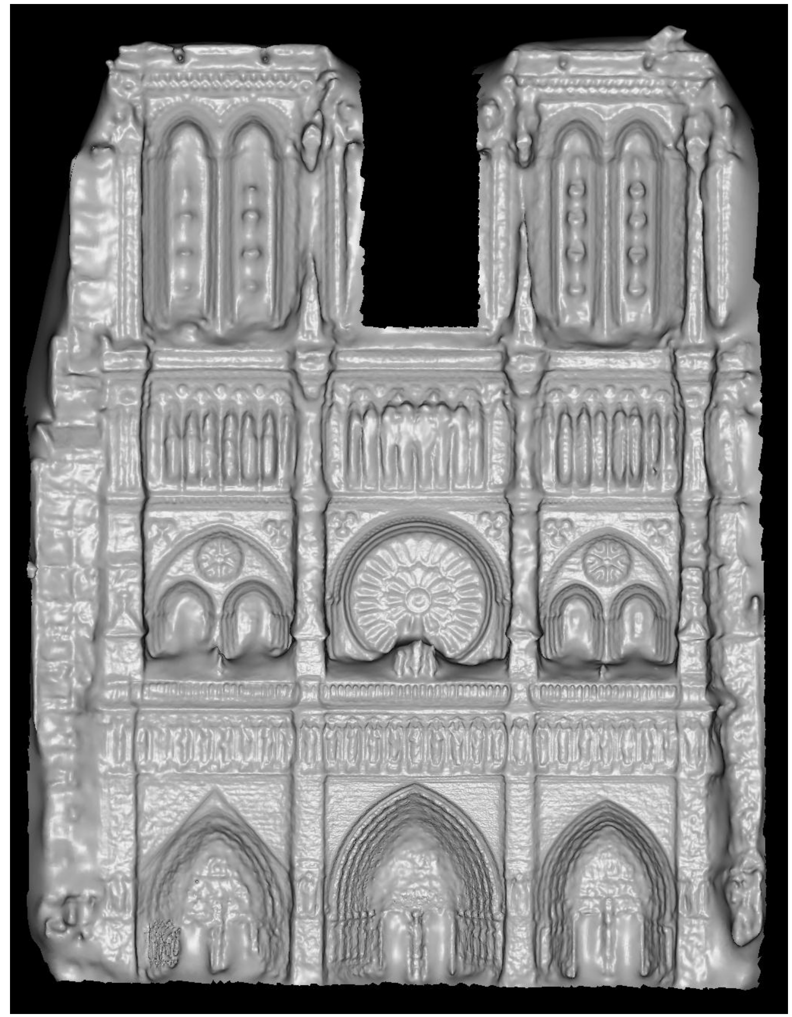
Window-based matching



Ground truth

Using more than two images

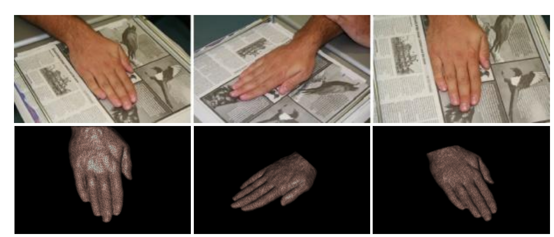




Multi-View Stereo for Community Photo Collections
M. Goesele, N. Snavely, B. Curless, H. Hoppe, S. Seitz
Proceedings of ICCV 2007,

3D model

- “Digital copy” of real object
- Allows us to
  - Inspect details of object
  - Measure properties
  - Reproduce in different material
- Many applications
  - Cultural heritage preservation
  - Computer games and movies
  - City modelling
  - E-commerce

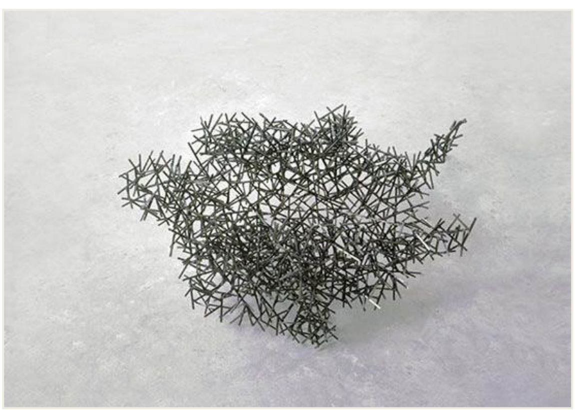


Applications: cultural heritage

SCULPTEUR European project

Applications: art

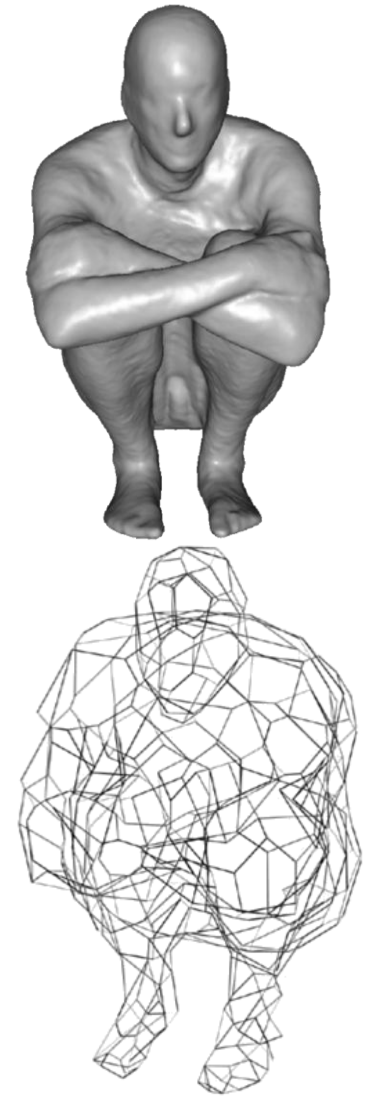
Block Works Precipitate III 2004
Mild steel blocks 80 x 46 x 66 cm



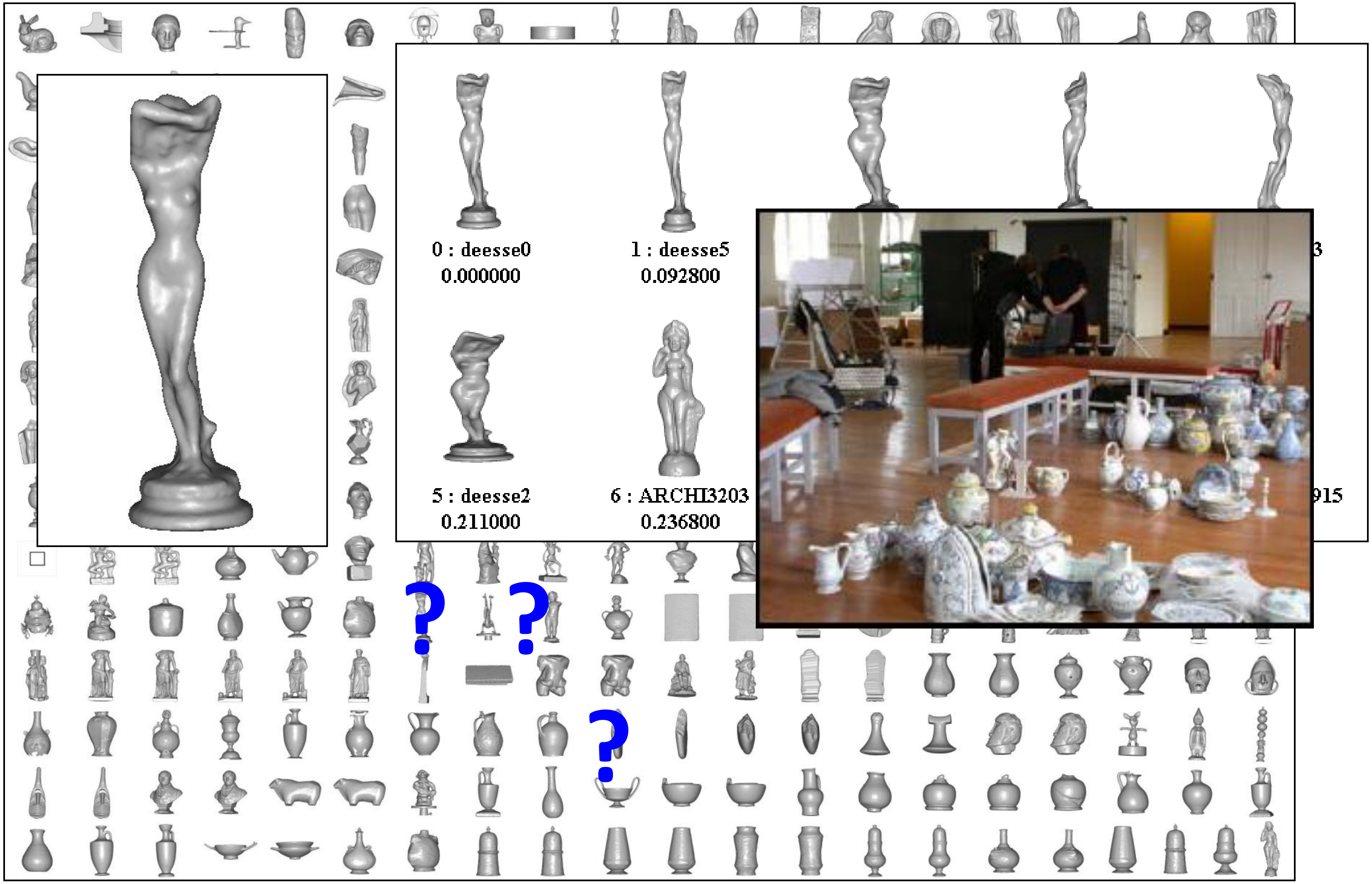
Domain Series Domain VIII Crouching
1999 *Mild steel bar* 81 x 59 x 63 cm

Applications: structure engineering

BODY / SPACE / FRAME, Antony Gormley, Lelystad, Holland



Applications: 3D indexation

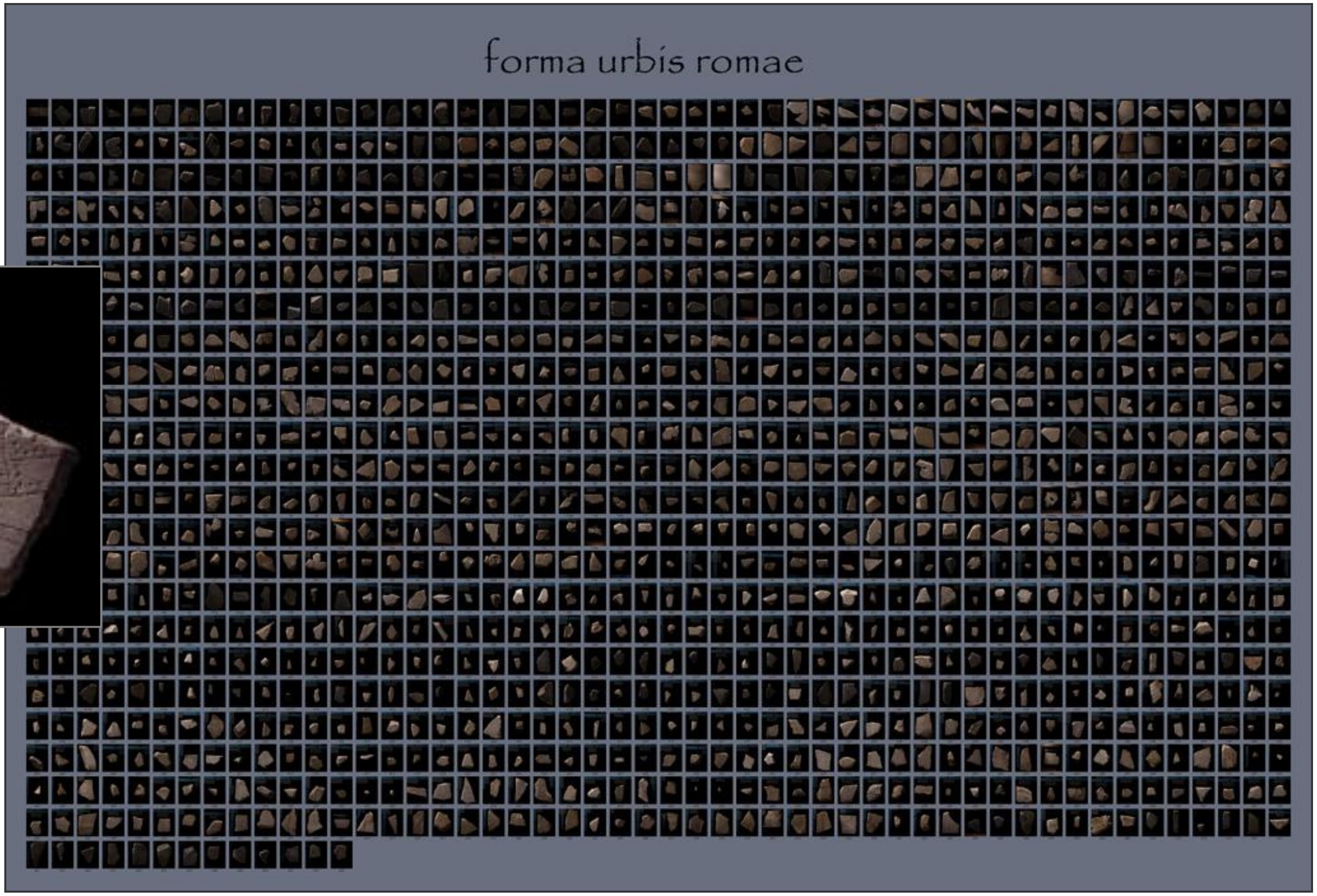


Applications: archaeology

- "forma urbis romae" project

Fragments of the City: Stanford's Digital Forma Urbis Romae Project
David Koller, Jennifer Trimble, Tina Najbjerg, Natasha Gelfand, Marc Levoy
Proc. Third Williams Symposium on Classical Architecture, Journal of Roman Archaeology supplement, 2006.

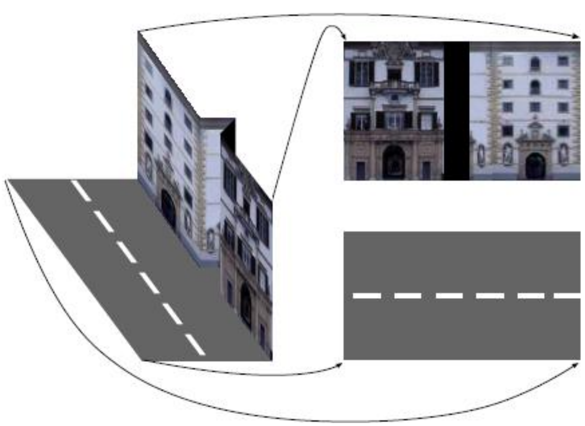
1186 fragments



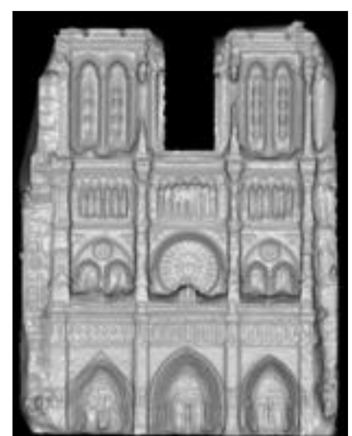
Applications: large scale modelling

[Furukawa10]

[Pollefeys08]

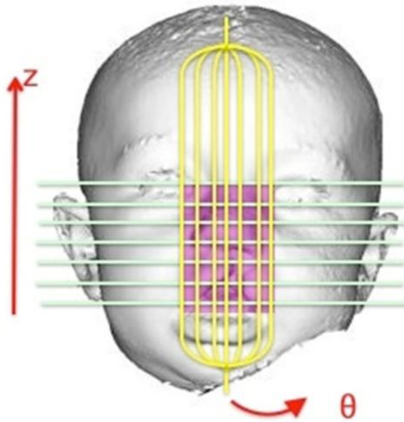


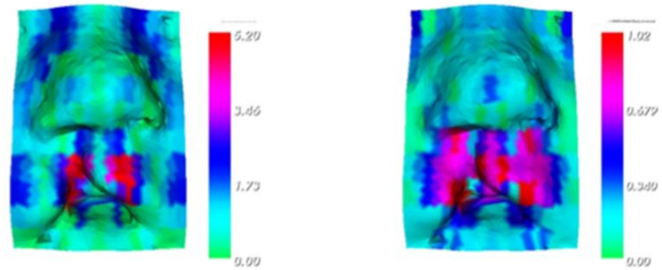
[Cornelis08]



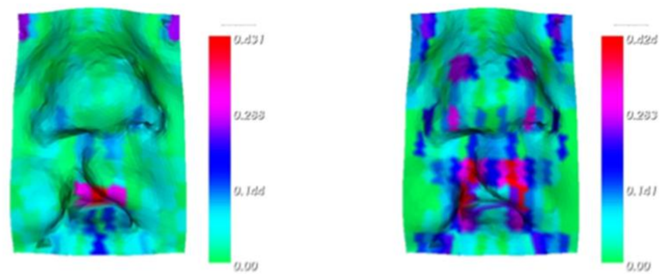
[Goesele07]

Applications: Medicine

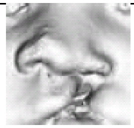
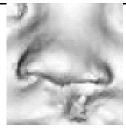
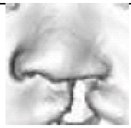
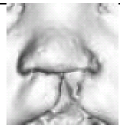




(a) Radius difference (b) Angle difference

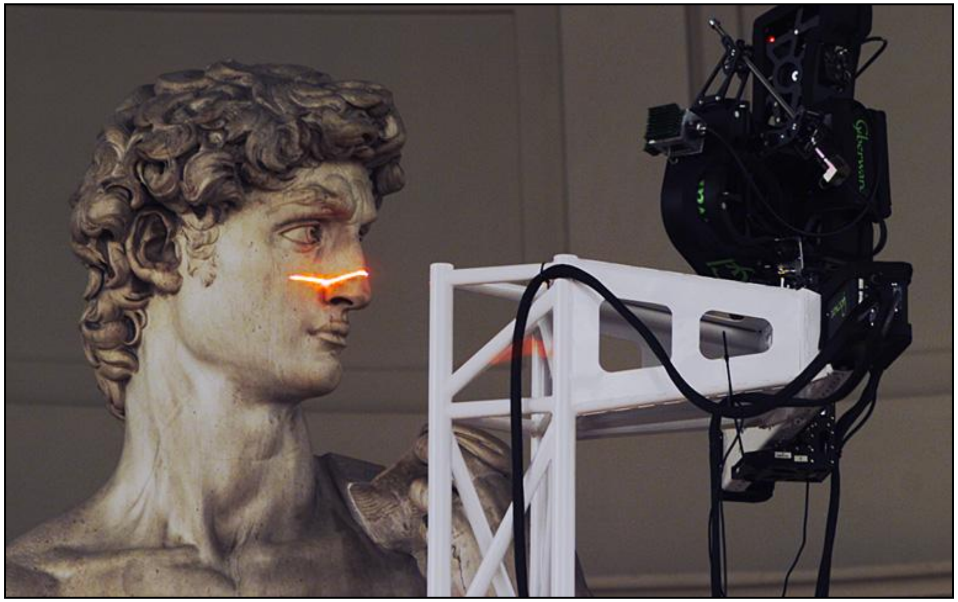


(c) Curvature difference (d) Edge difference

expert's order	1	2	3	4	5	6	7	8	9	10
images										
learning	1	3	2	4	5	6	8	9	7	10
a-lmk	1	2	3	5	6	4	8	7	9	10
mirror	1	2	4	8	5	6	9	3	7	10
m-lmk	1	2	3	4	5	6	9	7	10	8
plane	1	2	3	5	4	6	7	9	10	8

Scanning technologies

- Laser scanner, coordinate measuring machine
  - Very accurate
  - Very Expensive
  - Complicated to use



Minolta

Contura CMM

Medical Scanning System

The “Us” Data Set (subset)