Computer Vision

CSE 455
Corner Detection

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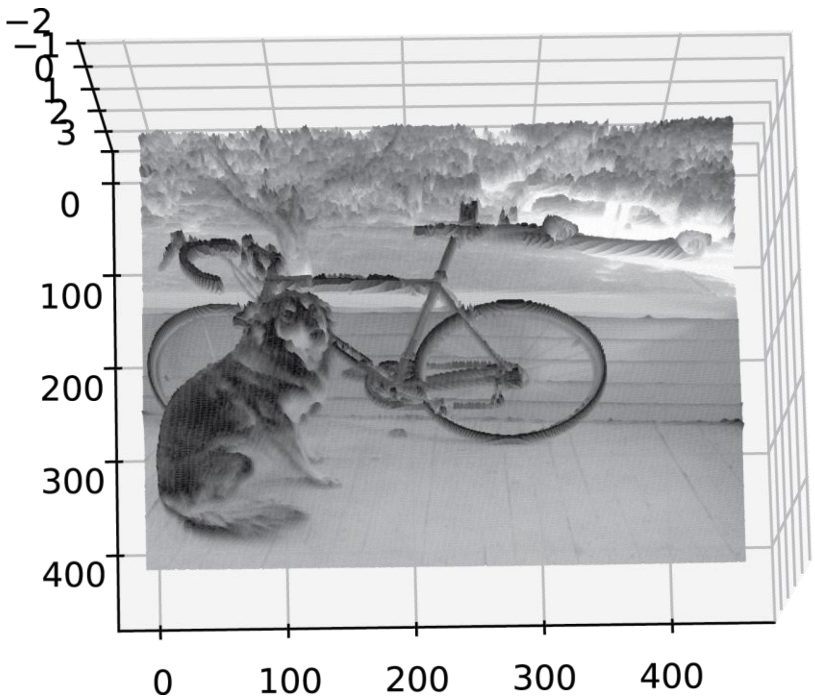
Review: What’s an edge?

- Image is a function
- Edges are rapid changes in this function



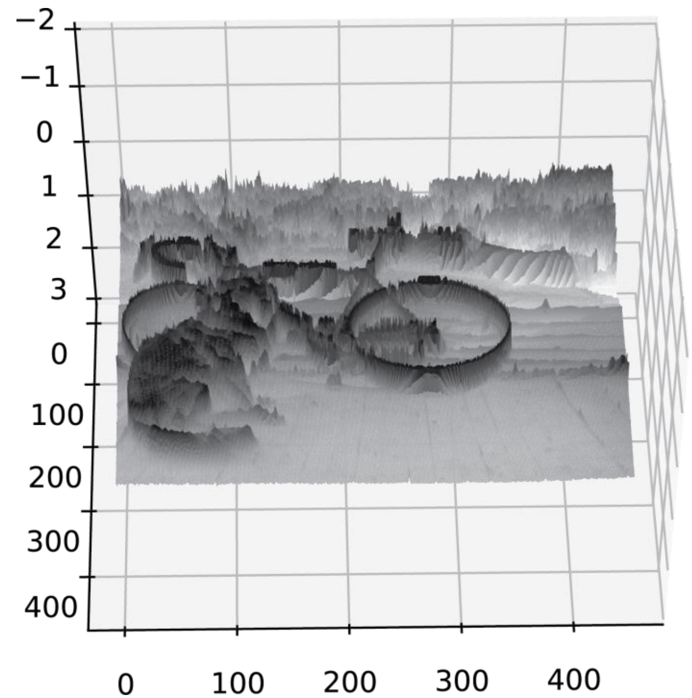
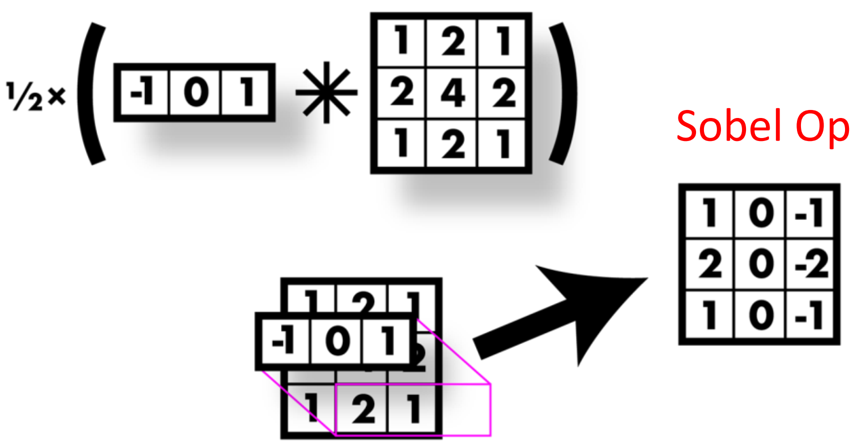
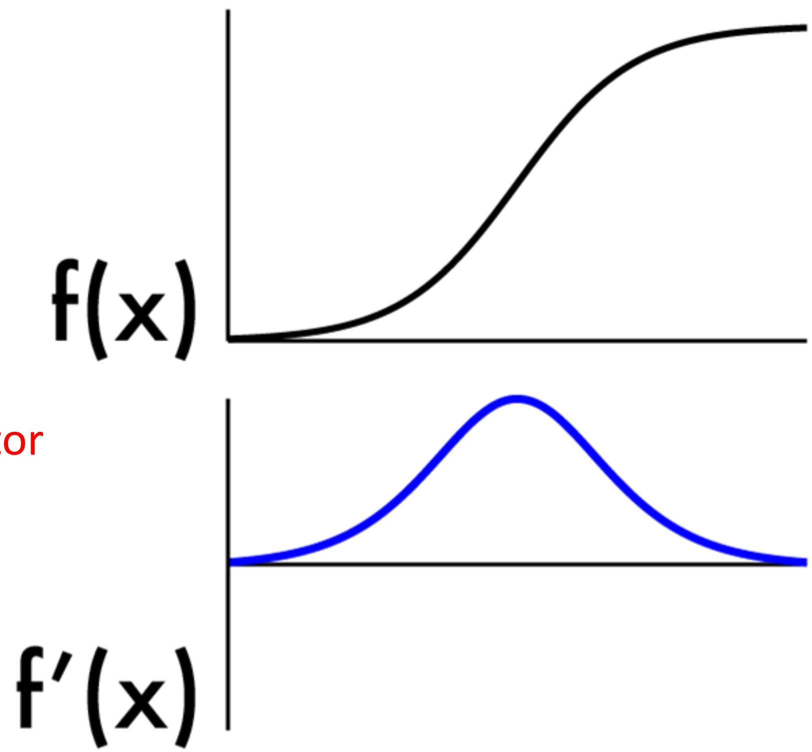


Image derivatives

- Recall:
  - $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$
- Want smoothing too!



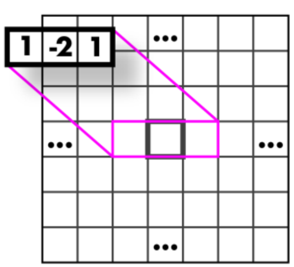


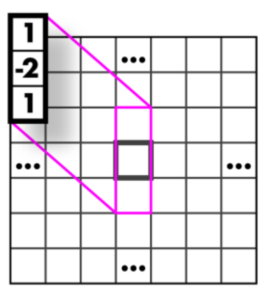
Laplacian (2nd derivative)!

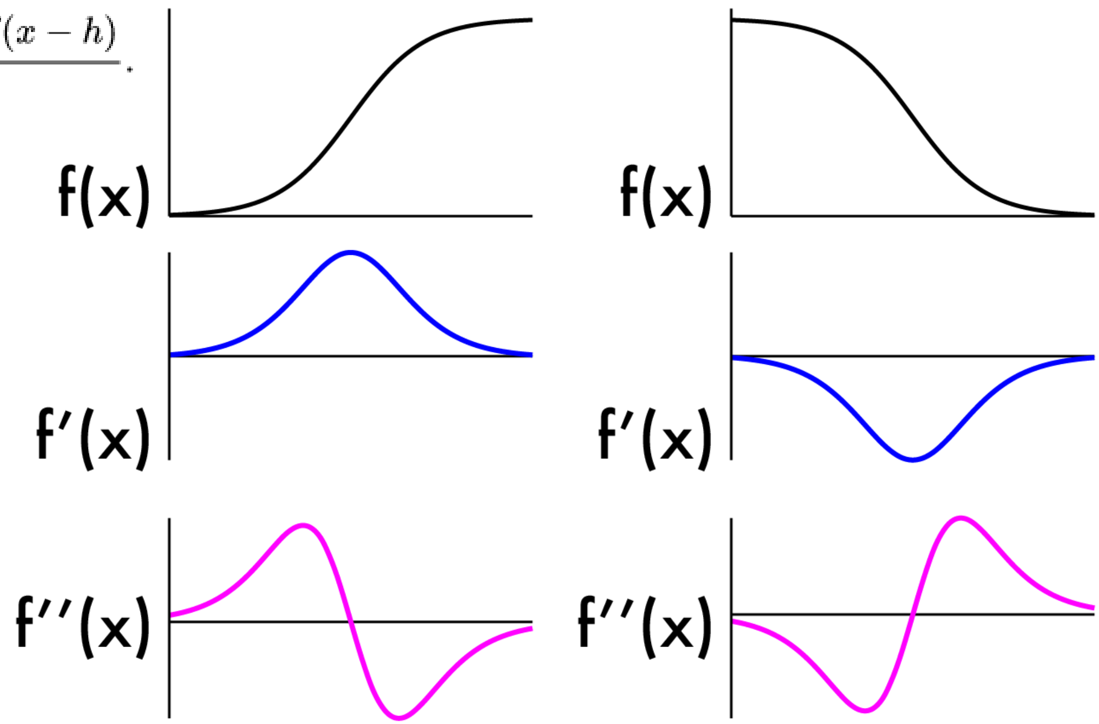
Laplacian Operator

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

- Crosses zero at extrema
- Recall:
  - $f''(x) = \lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}.$
- Laplacian:
  - $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$
- Again, have to estimate f''(x):

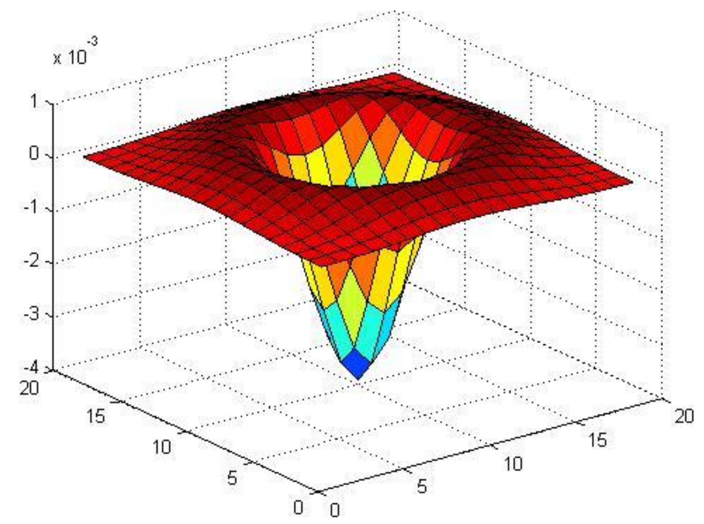






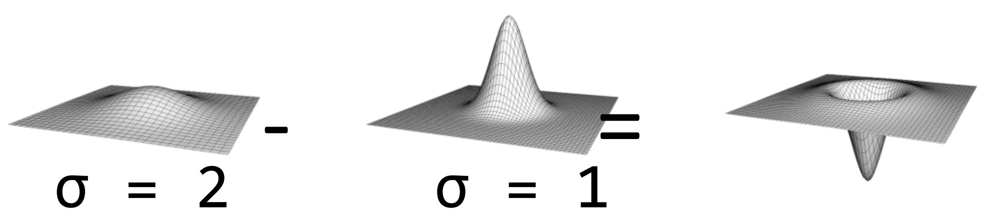
Laplacians also sensitive to noise

- Again, use gaussian smoothing
- Can just use one kernel since convs commute
- Laplacian of Gaussian, LoG
- Can get good approx. with
  5x5 - 9x9 kernels

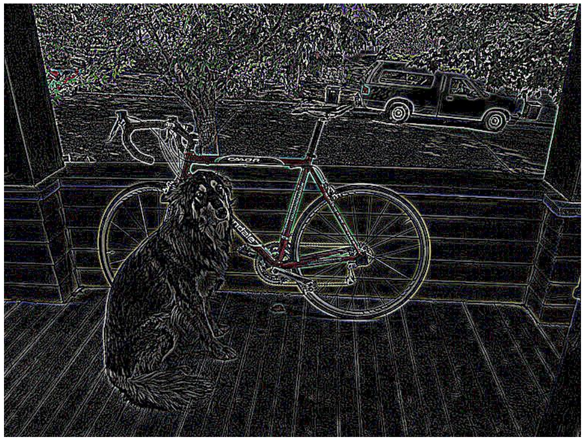


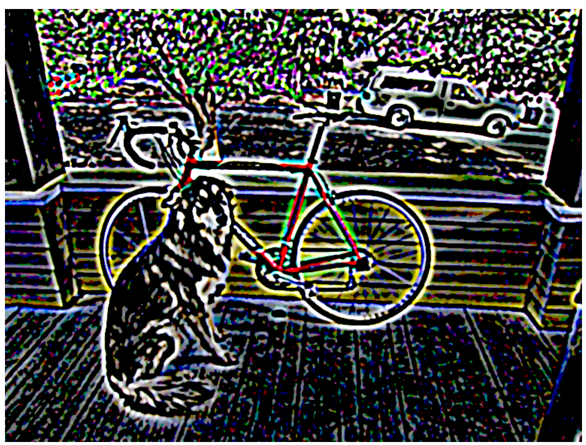
Difference of Gaussian (DoG)

- Gaussian is a low pass filter
- Strongly reduce components with frequency $f < \sigma$
- $(g*I)$ low frequency components
- $I - (g*I)$ high frequency components
- $g(\sigma 1)*I - g(\sigma 2)*I$
  - Components in between these frequencies
- $g(\sigma 1)*I - g(\sigma 2)*I = [g(\sigma 1) - g(\sigma 2)]*I$



DoGs






Canny Edge Detection

- Your first image processing pipeline!
  - Old-school CV is all about pipelines

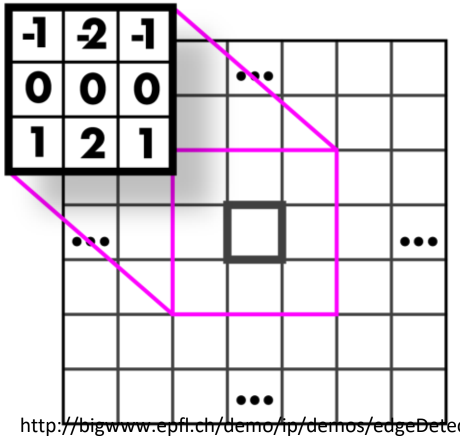
Algorithm:

- Smooth image (only want “real” edges, not noise)
- Calculate gradient direction and magnitude
- Non-maximum suppression perpendicular to edge
- Threshold into strong, weak, no edge
- Connect together components

http://bigwww.epfl.ch/demo/ip/demos/edgeDetector/

Gradient magnitude and direction

- Sobel filter



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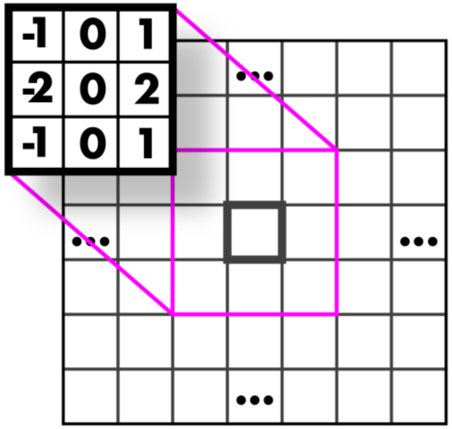
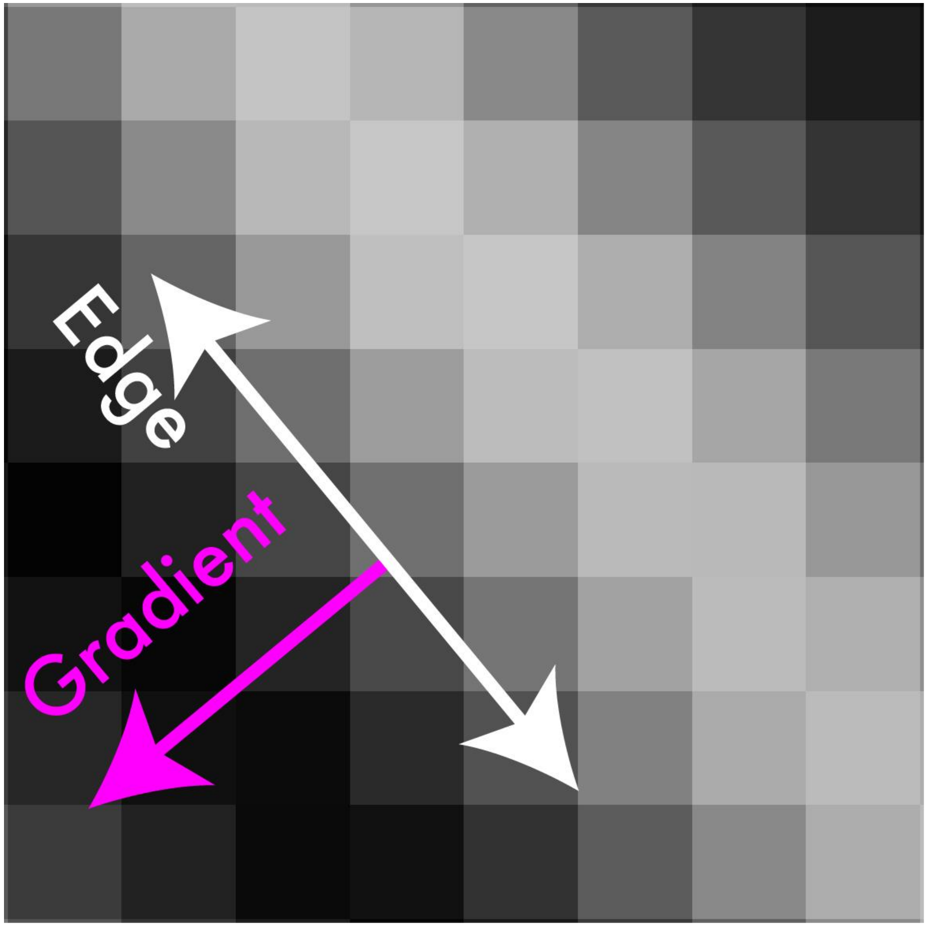
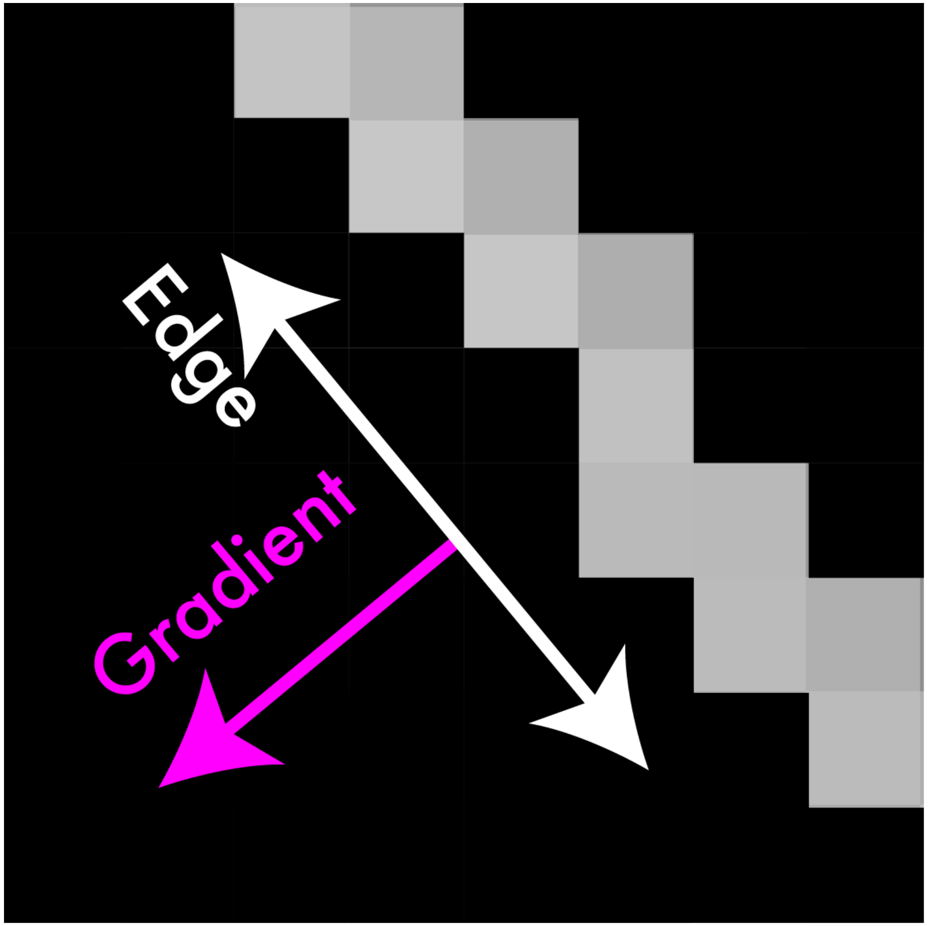


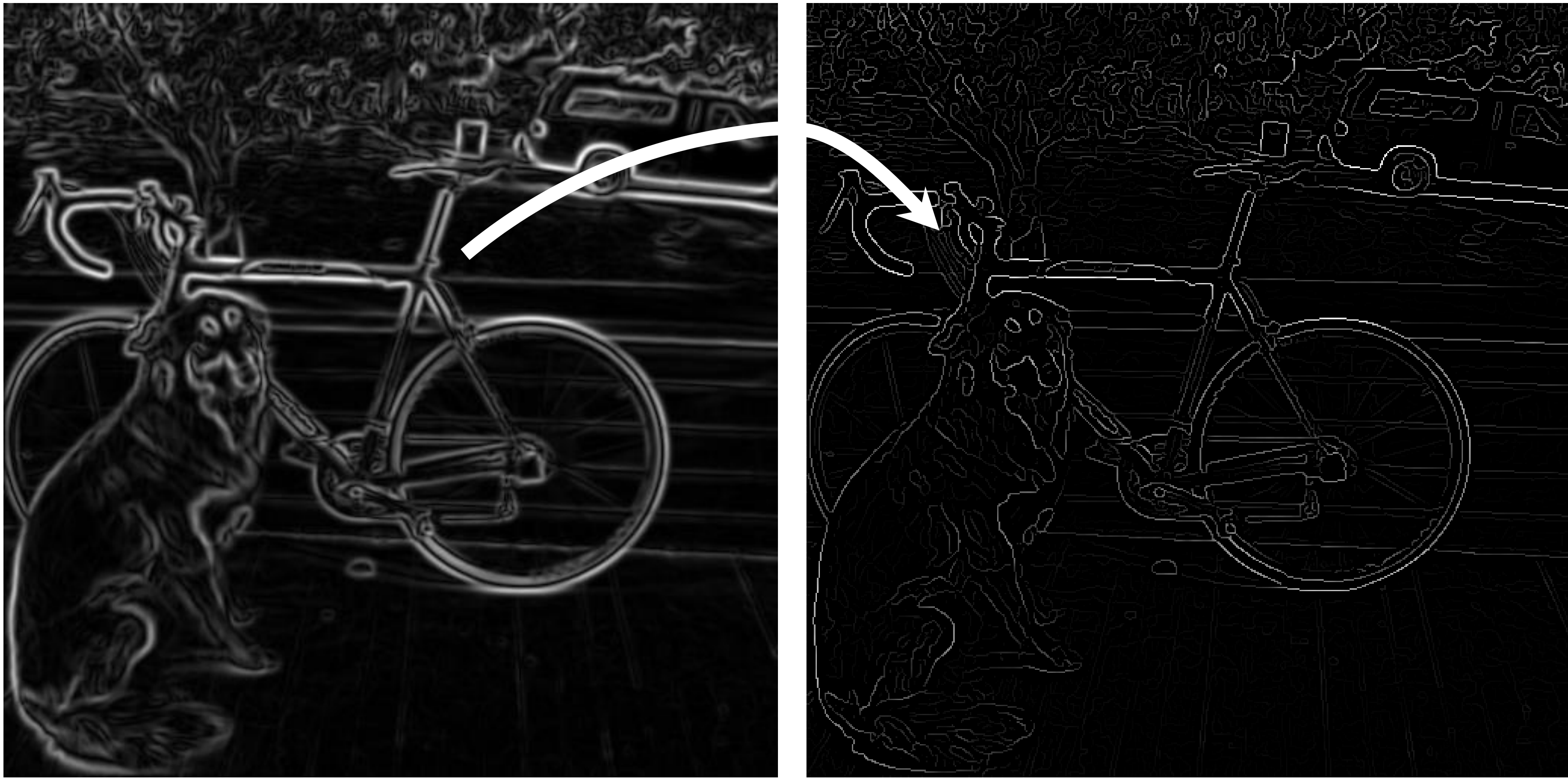
Non-maximum suppression



Non-maximum suppression



Non-maximum suppression



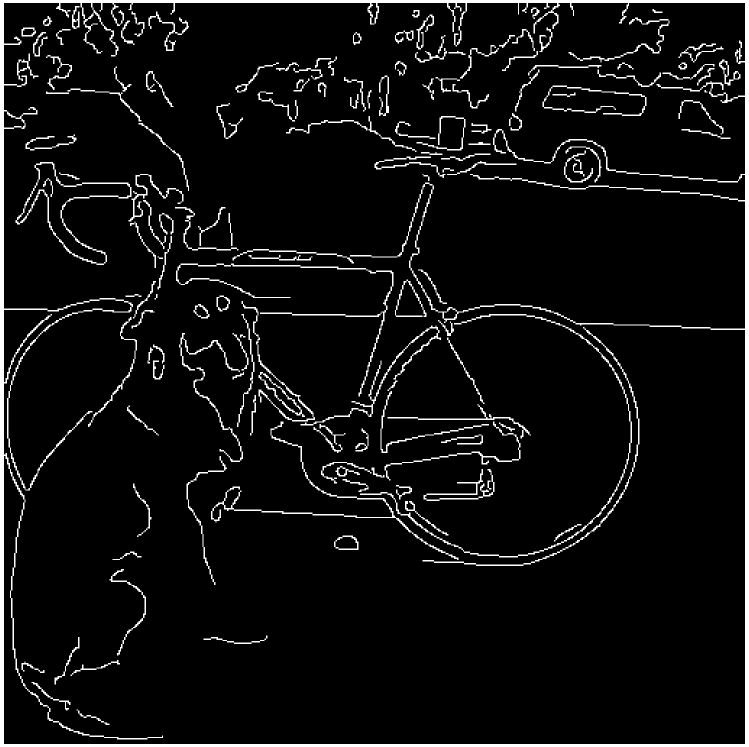
Threshold edges

- Still some noise
- Only want strong edges
- 2 thresholds, 3 cases
    - R > T: strong edge
    - R < T but R > t: weak edge
    - R < t: no edge
- Why two thresholds?

http://bigwww.epfl.ch/demo/ip/demos/edgeDetector/

Connect 'em up!

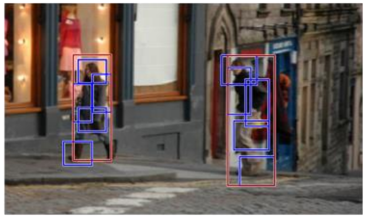
- Strong edges are edges!
- Weak edges are edges
  iff they connect to strong
- Look in some neighborhood
  (usually 8 closest)

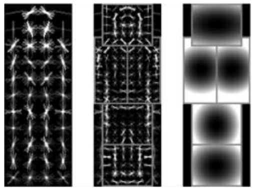


http://bigwww.epfl.ch/demo/ip/demos/edgeDetector/

Features!

- Highly descriptive local regions
- Ways to describe those regions
- Useful for:
  - Matching
  - Recognition
  - Detection





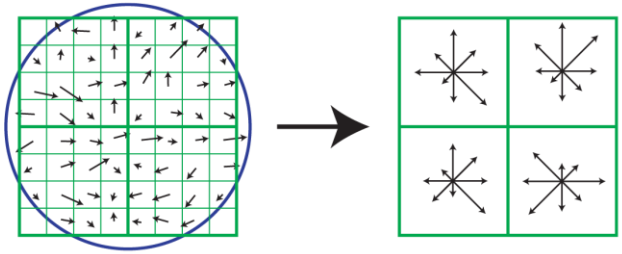
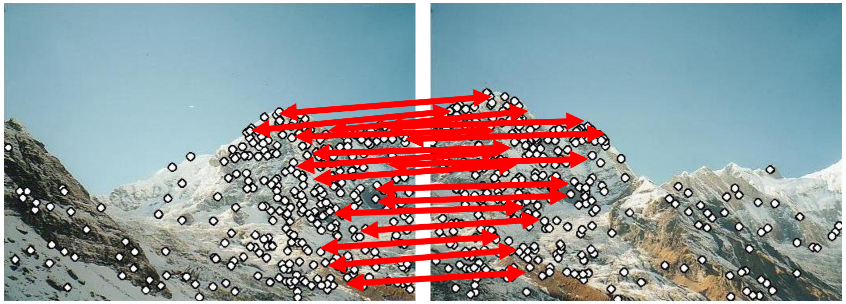
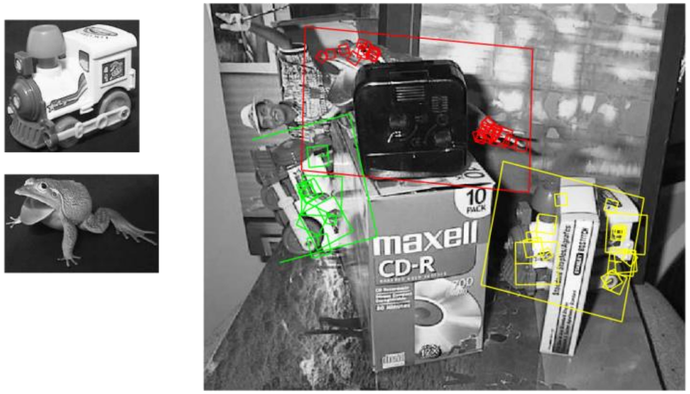


Image gradients

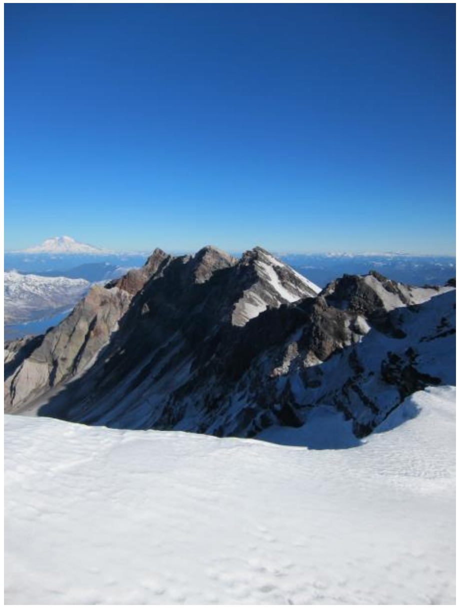
Keypoint descriptor

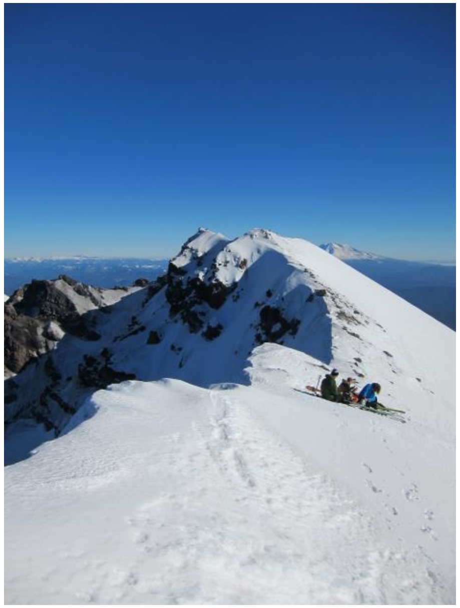




How to create a panorama

- Say we are stitching a panorama
- Want patches in image to match to other image
- Hopefully only match one spot



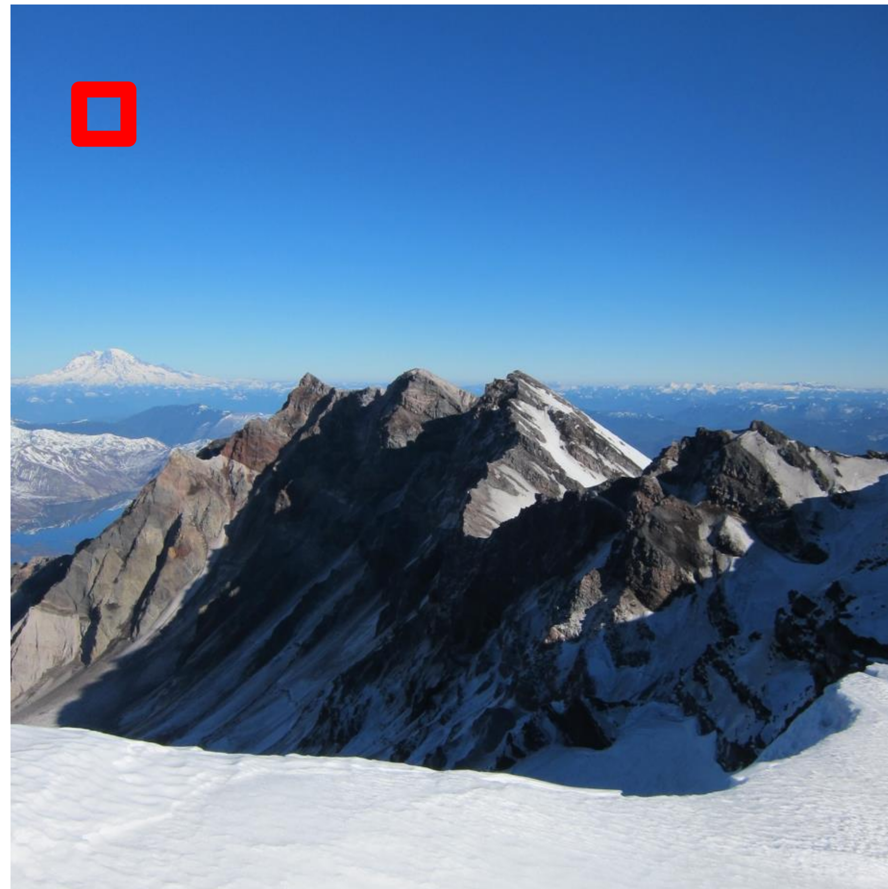


Q1: How close are two patches?

- Sum squared difference
- Images I, J
- $\Sigma_{x,y} (I(x,y) - J(x,y))^2$

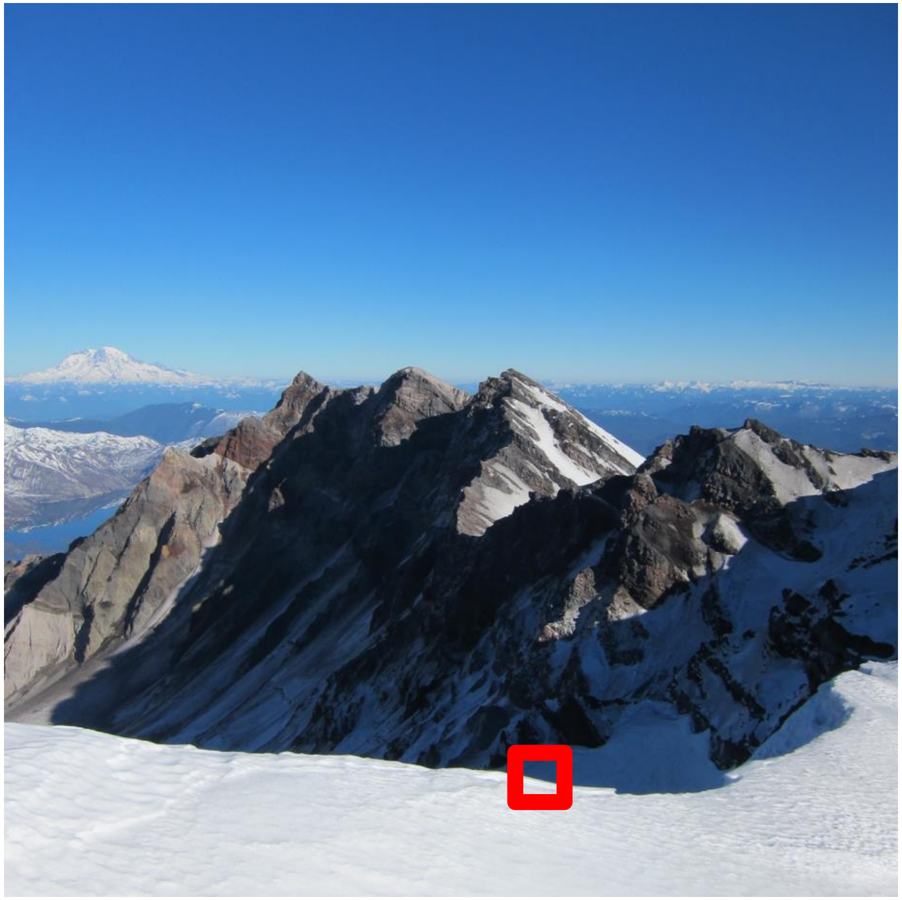
Q2: How can we find unique patches?

- Sky: bad
  - Very little variation
  - Could match any other sky



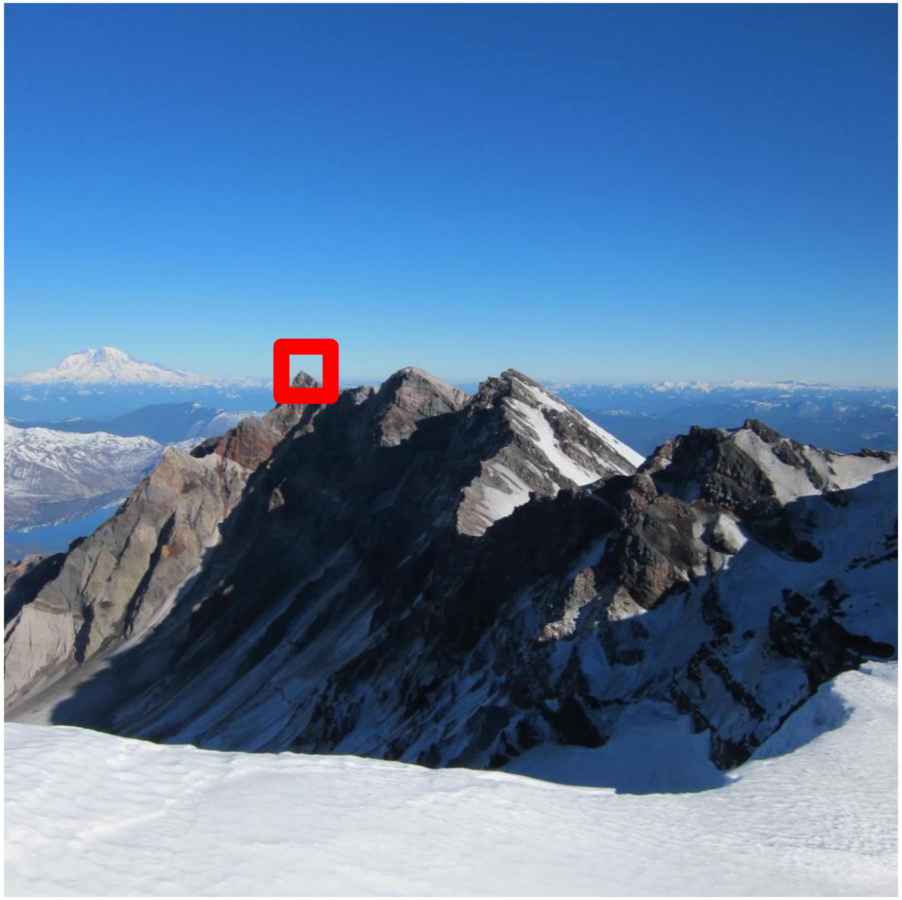
How can we find unique patches?

- Sky: bad
  - Very little variation
  - Could match any other sky
- Edge: ok
  - Variation in one direction
  - Could match other patches along same edge



How can we find unique patches?

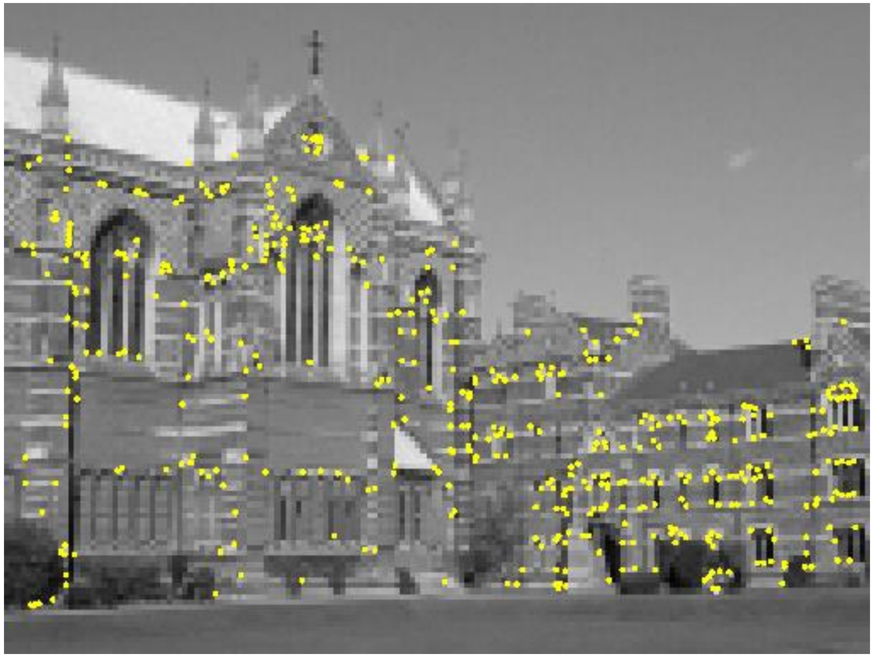
- Sky: bad
  - Very little variation
  - Could match any other sky
- Edge: ok
  - Variation in one direction
  - Could match other patches along same edge
- Corners: good!
  - Only one alignment matches

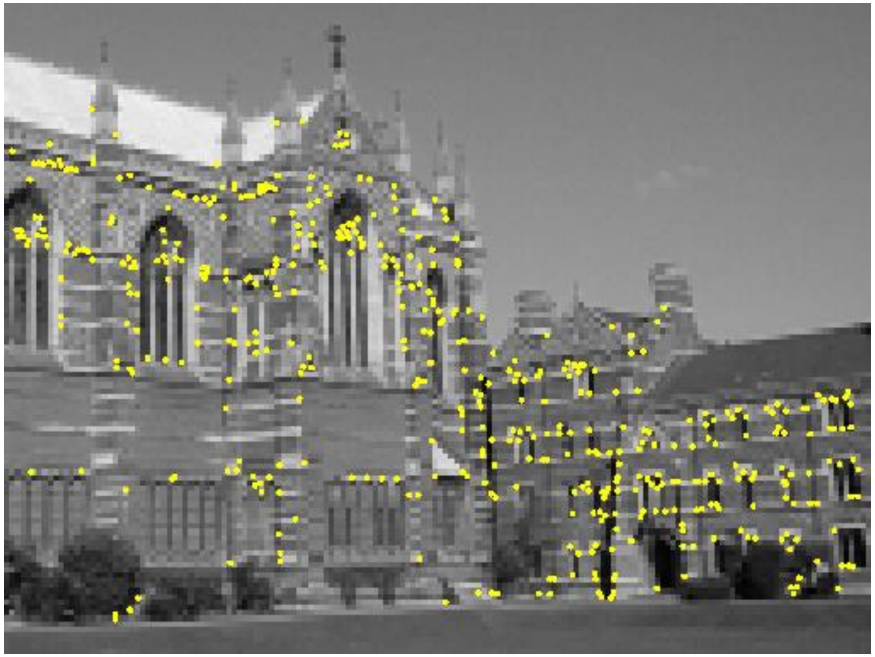


What are we going to do?

- We are going to build a panorama from two (or more) images.
- We need to learn about
  - Finding interest points
  - Describing small patches about such points
  - Finding matches between pairs of such points on two images, using the descriptors
  - Selecting the best set of matches and saving them
  - Constructing homographies (transformations) from one image to the other and picking the best one
  - Stitching the images together to make the panorama

Preview: Harris detector

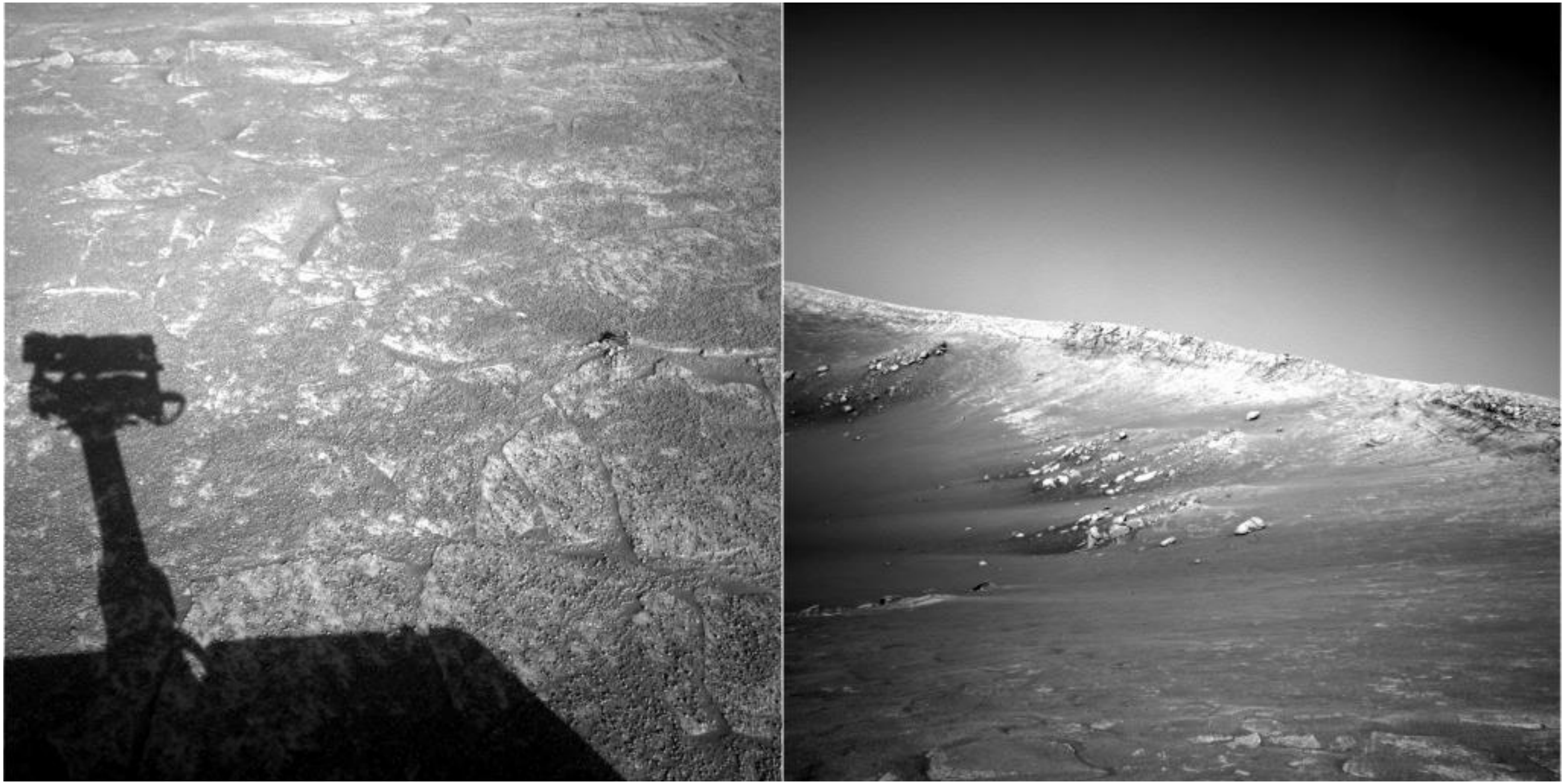




Interest points extracted with Harris (~ 500 points)

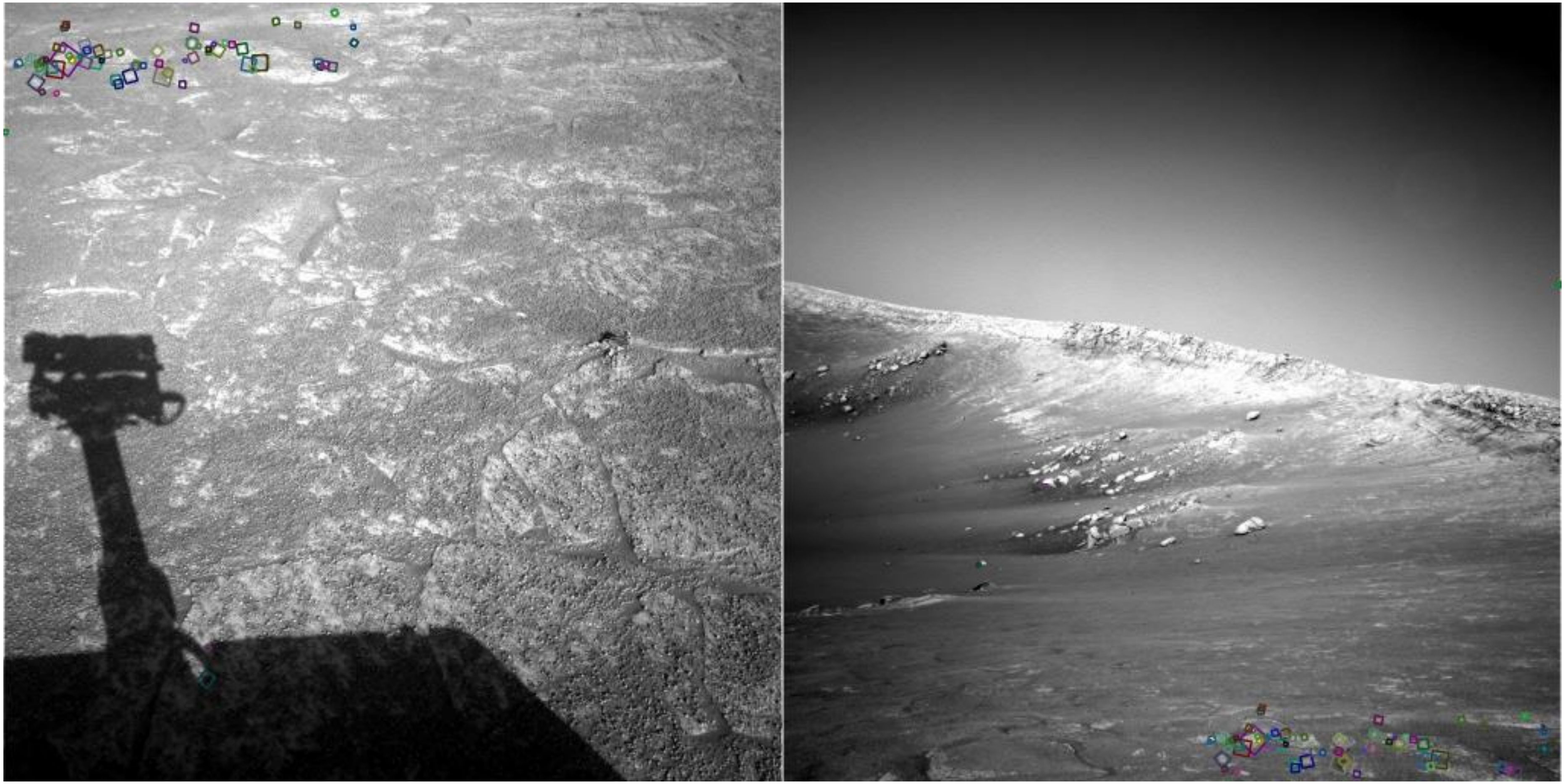
How can we find corresponding points?

Not always easy



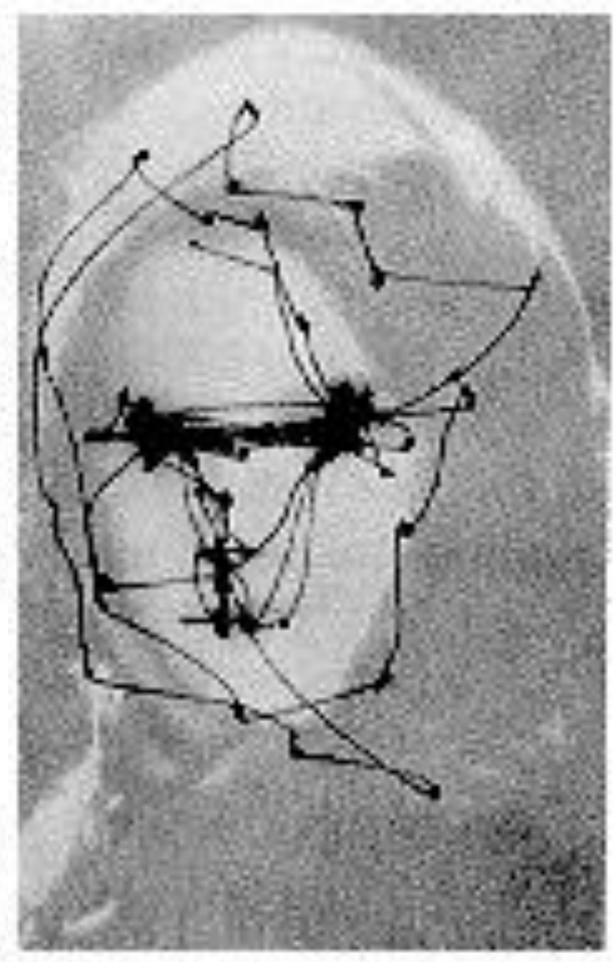
NASA Mars Rover images

Answer below (look for tiny colored squares…)



NASA Mars Rover images
with SIFT feature matches
Figure by Noah Snavely

Human eye movements

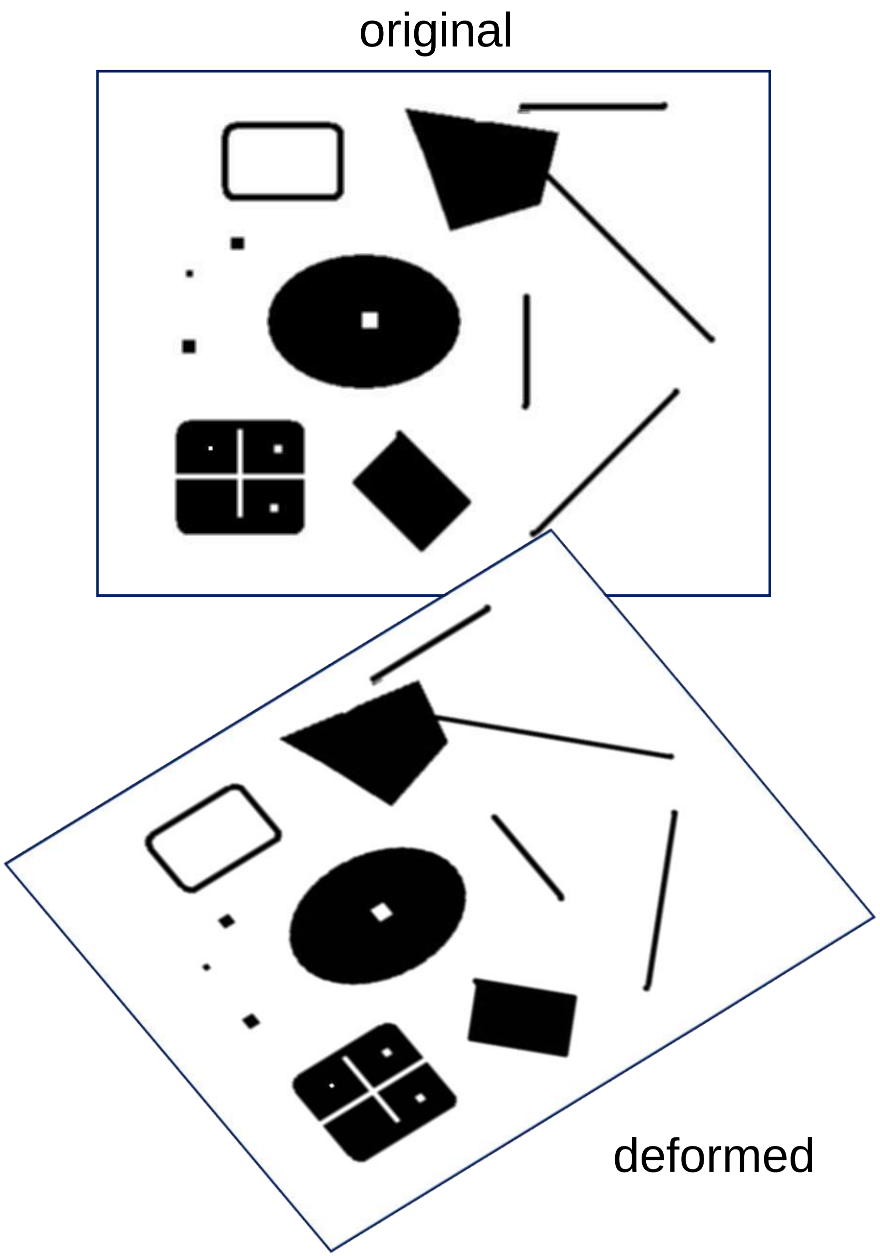


Yarbus eye tracking

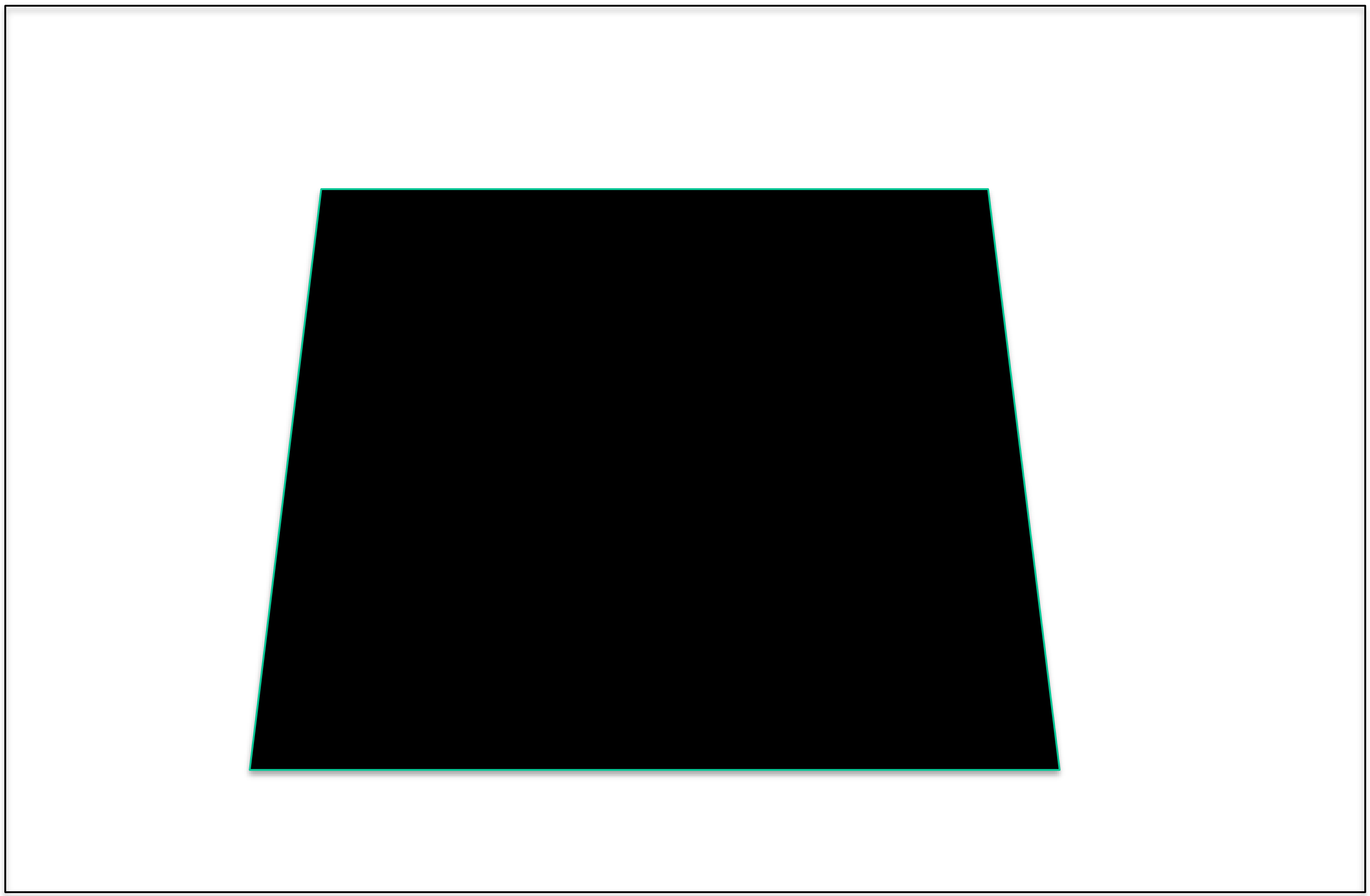
What catches your interest?

Interest points

- Suppose you have to click on some point, go away and come back after I deform the image, and click on the same points again.
  - Which points would you choose?

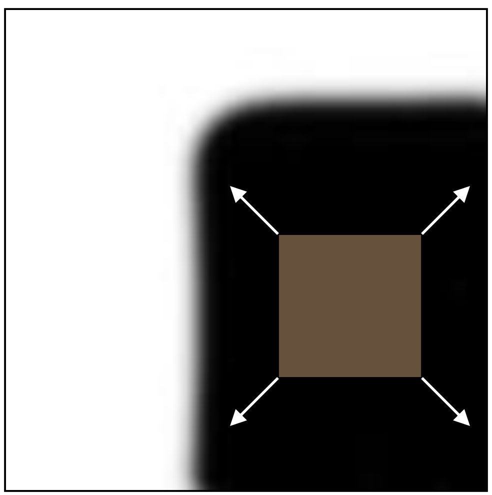


Intuition

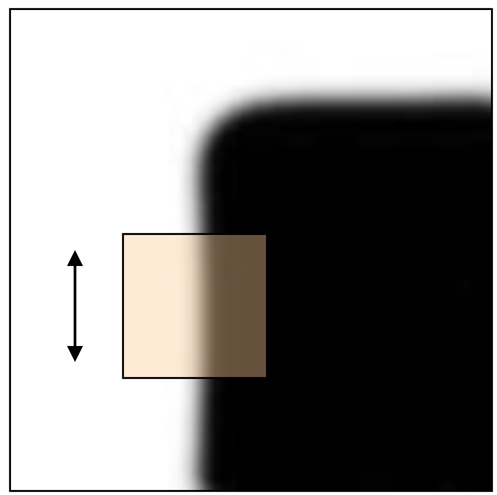


Corners

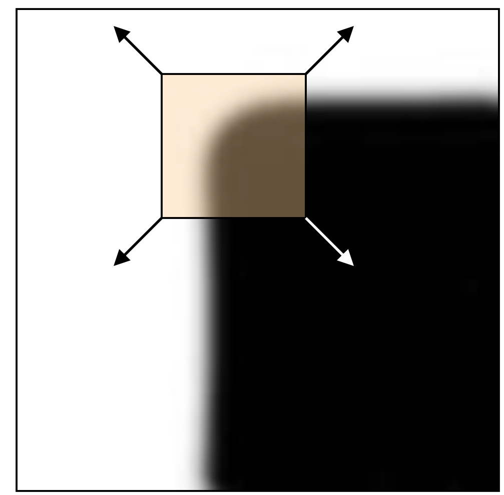
- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



"flat" region:
no change in
all directions



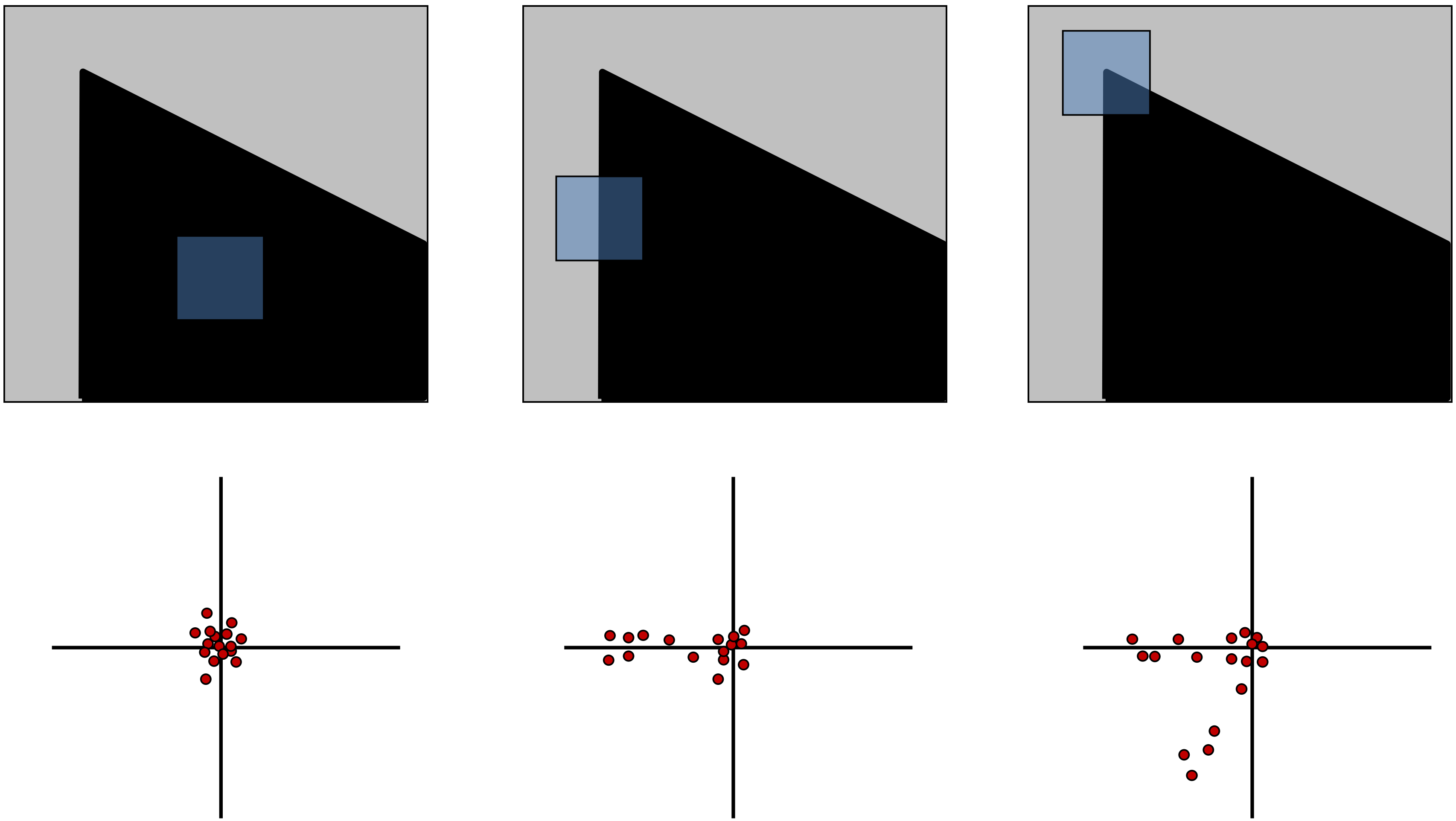
"edge":
no change along
the edge
direction



"corner":
significant
change in all
directions

Source: A. Efros

Let's look at the gradient distributions



Principal Component Analysis

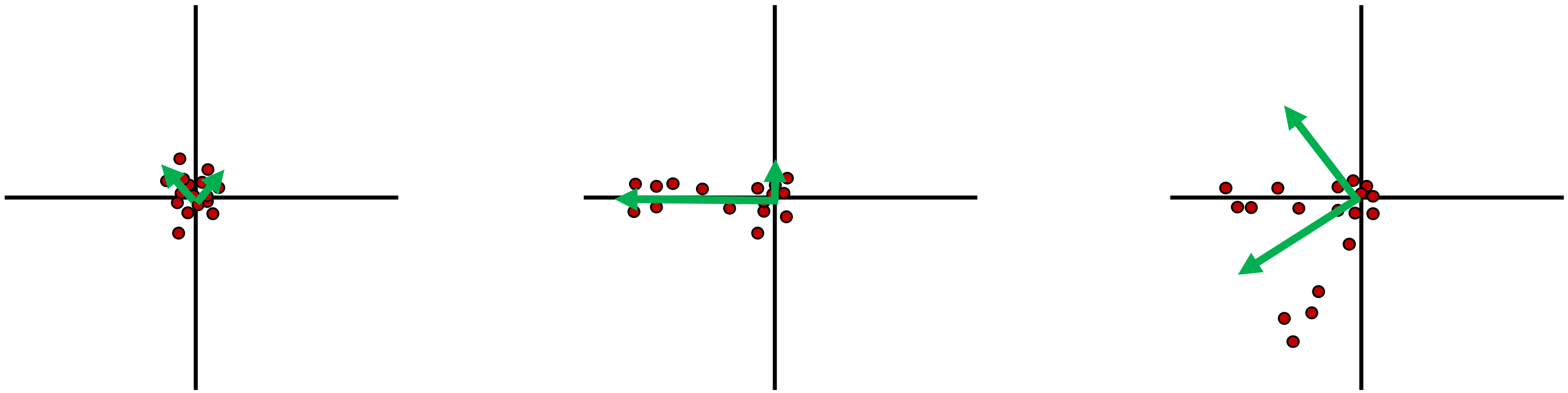
Principal component is the direction of highest variance.

Next, highest component is the direction with highest variance *orthogonal* to the previous components.

How to compute PCA components:

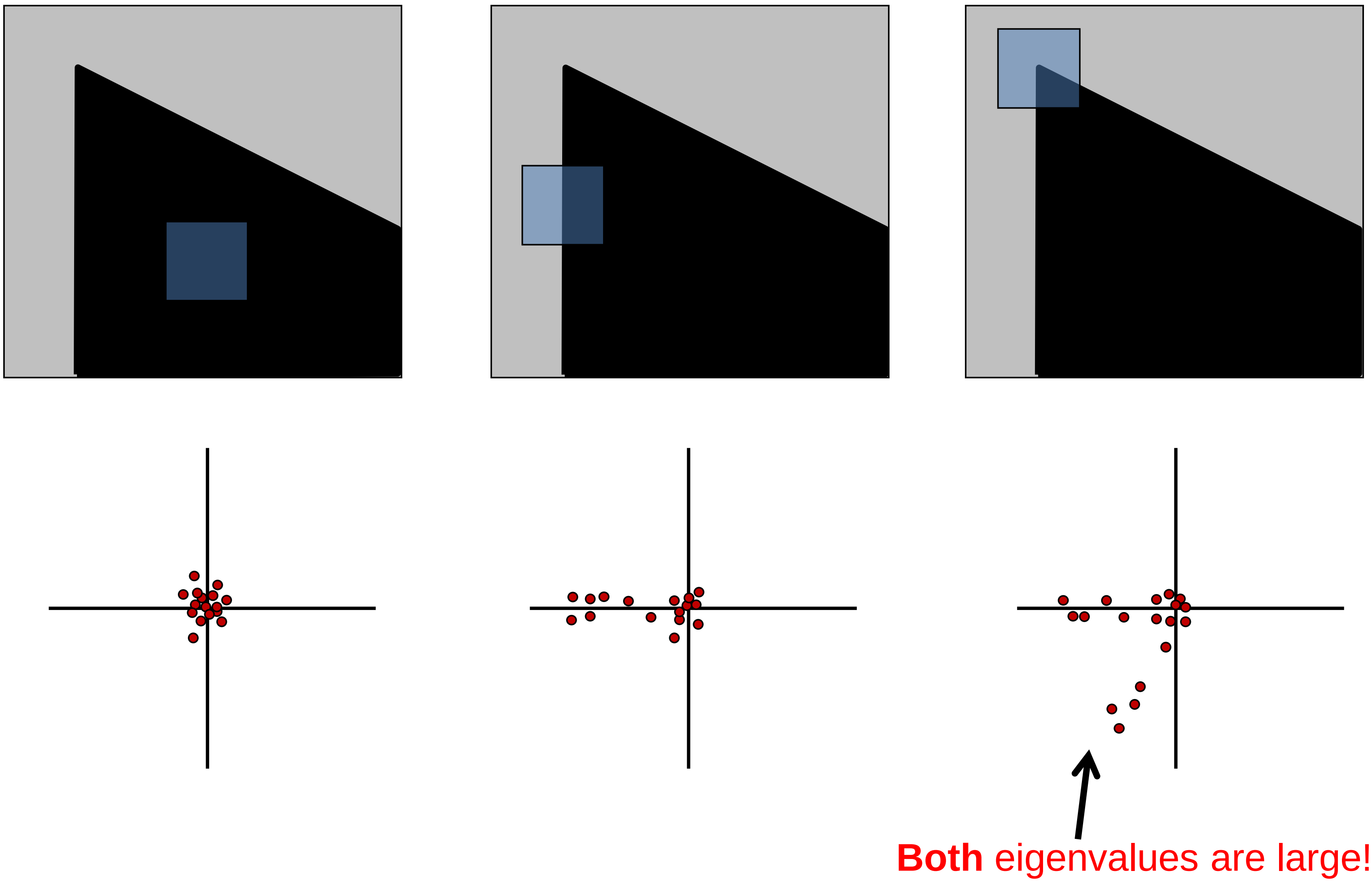
1. Subtract off the mean for each data point.
2. Compute the covariance matrix.
3. Compute eigenvectors and eigenvalues.
4. The components are the eigenvectors ranked by the eigenvalues.

$$Hx = \lambda x$$



Definition: A scalar λ is called an eigenvalue of the n×n matrix A if there is a nontrivial solution x of Ax=λx. Such x is called an eigenvector corresponding to the eigenvalue λ.

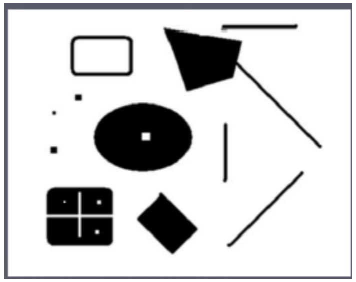
Corners have …

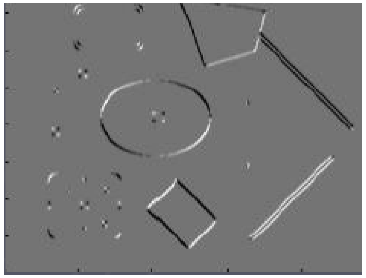


Second Moment Matrix or Harris Matrix

$$H = \begin{vmatrix} \Sigma_i w_i\, I_x\, I_x & \Sigma_i w_i\, I_x\, I_y \\ \Sigma_i w_i\, I_x\, I_y & \Sigma_i w_i\, I_y\, I_y \end{vmatrix}$$

2 x 2 matrix of image derivatives smoothed by Gaussian weights.



Notation: $I_x \Leftrightarrow \dfrac{\partial I}{\partial x}$ $\quad I_y \Leftrightarrow \dfrac{\partial I}{\partial y}$ $\quad I_x I_y \Leftrightarrow \dfrac{\partial I}{\partial x}\dfrac{\partial I}{\partial y}$

- First compute I_x, I_y, and $I_x\ I_x$, $I_y\ I_y$, I_xI_y; then apply Gaussian to each.

From HW3: Structure matrix

- Weighted sum of gradient information
  - | $\Sigma_i w_i I_x(i) I_x(i)$ $\Sigma_i w_i I_x(i) I_y(i)$ |
  - | $\Sigma_i w_i I_x(i) I_y(i)$ $\Sigma_i w_i I_y(i) I_y(i)$ |

We'll tell you how to store this!

- Use Gaussian weighting
- Eigen vectors/values of this matrix summarize the distribution of the gradients nearby
- λ_1 and λ_2 are eigenvalues
  - λ_1 and λ_2 both small: no gradient
  - $\lambda_1 >> \lambda_2$: gradient in one direction
  - λ_1 and λ_2 similar: multiple gradient directions, corner

Estimating Response

$$H = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \lambda_{\pm} = \frac{1}{2}\left((a+d) \pm \sqrt{4bc + (a-d)^2}\right)$$

- A few methods we use to estimate:
- Calculate these directly from the 2x2 matrix
  - $\det(S) = ad - bc = \lambda_1 * \lambda_2$
  - $\mathrm{trace}(S) = a + d = \lambda_1 + \lambda_2$
- Estimate formula 1: $R = \det(S) - \alpha\, \mathrm{trace}(S)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1+\lambda_2)^2$
- Estimate formula 2: $R = \det(S) / \mathrm{trace}(S) = \lambda_1\lambda_2/(\lambda_1+\lambda_2)$
- If these estimates are large, we call it a corner

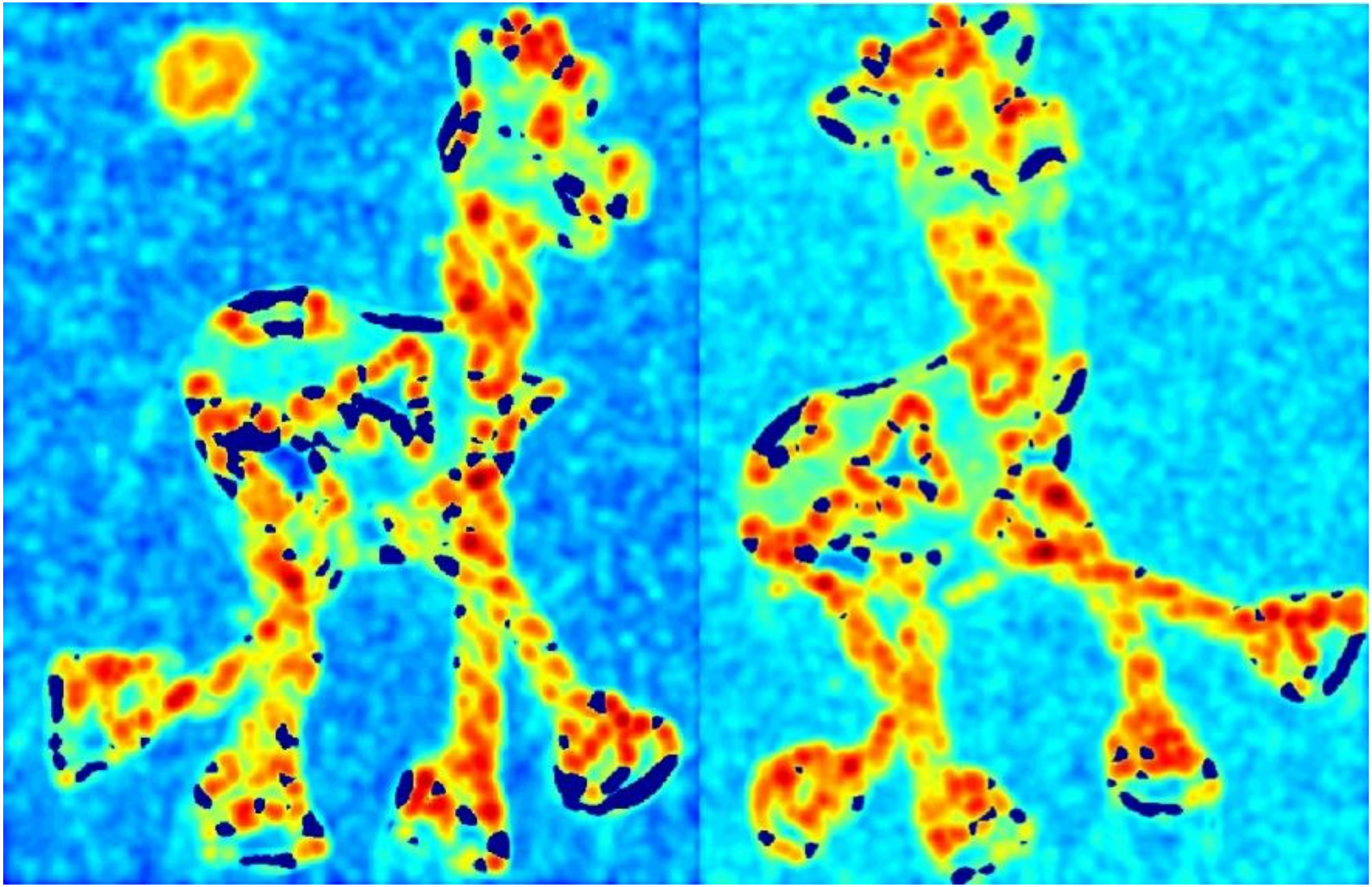
Harris Corner Detector

- Calculate derivatives I_x and I_y
- Calculate 3 measures I_xI_x, I_yI_y, I_xI_y
- Calculate weighted sums
  - Want a weighted sum of nearby pixels, guess what this is?
  - Gaussian!
- Estimate response
- Non-max suppression!

Harris Detector: Steps

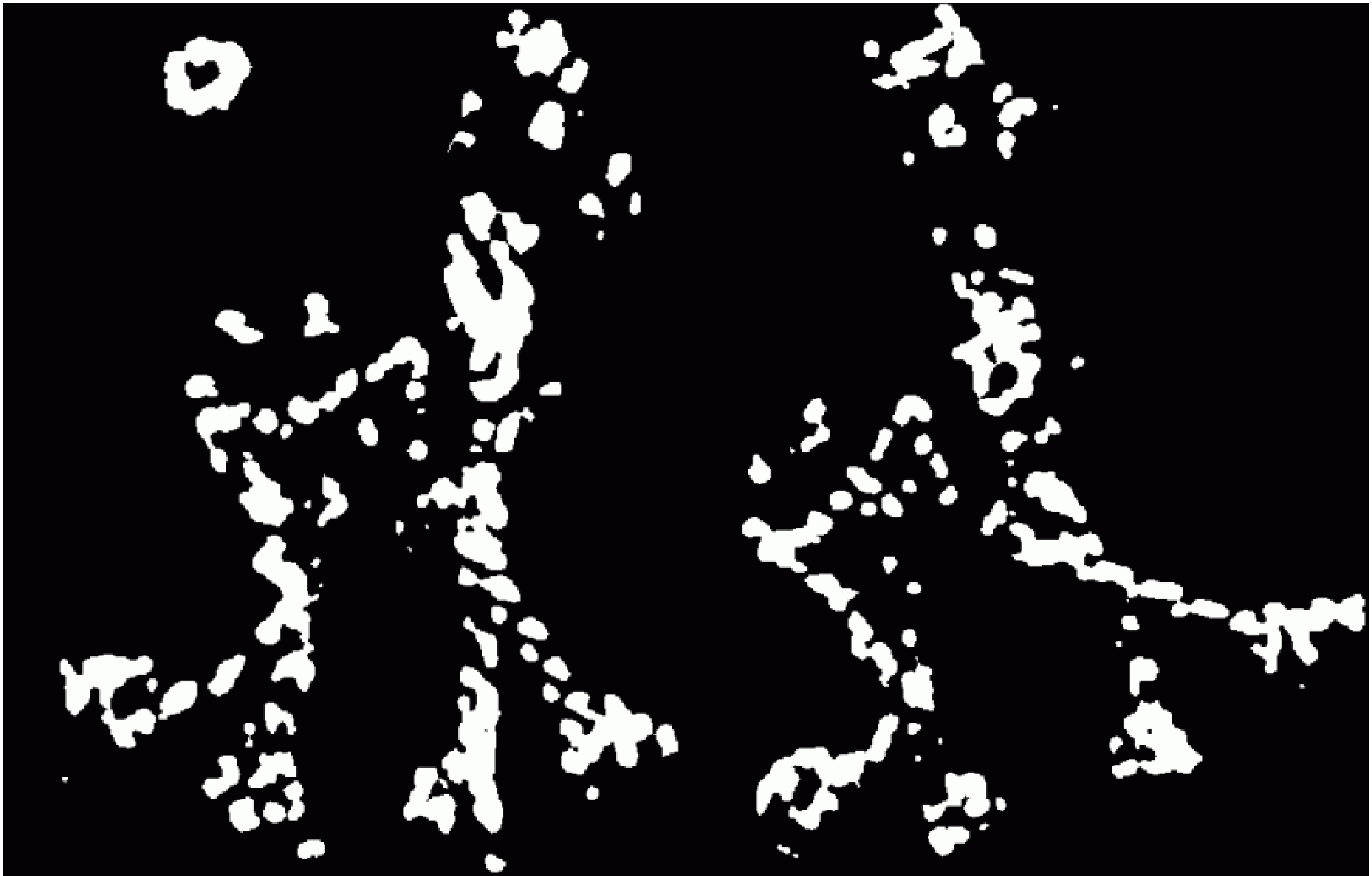
Harris Detector: Steps

Compute corner response R



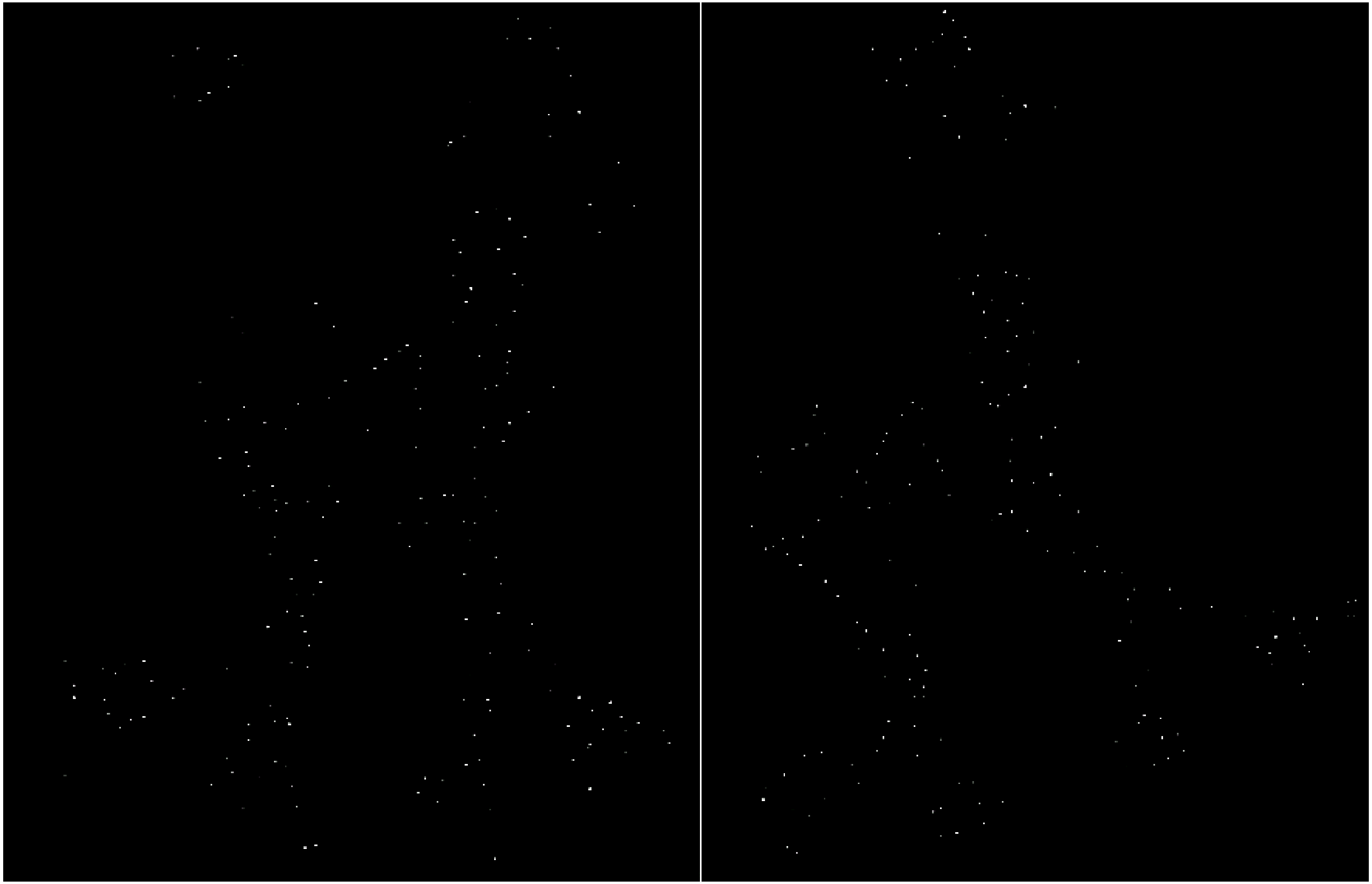
Harris Detector: Steps

Find points with large corner response: R > threshold



Harris Detector: Steps

Take only the points of local maxima of *R*

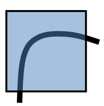


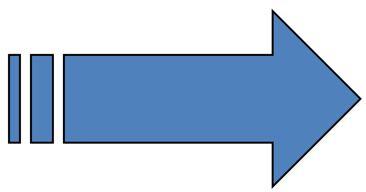
Harris Detector: Results

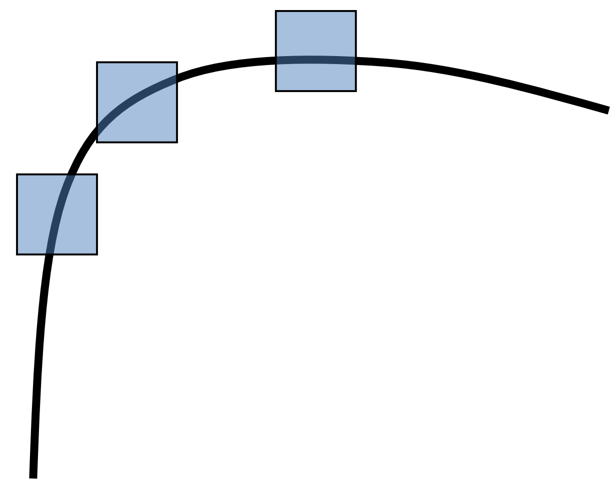
Properties of the Harris corner detector

- Translation invariant? Yes
- Rotation invariant? Yes
- Scale invariant? No

What's the problem?

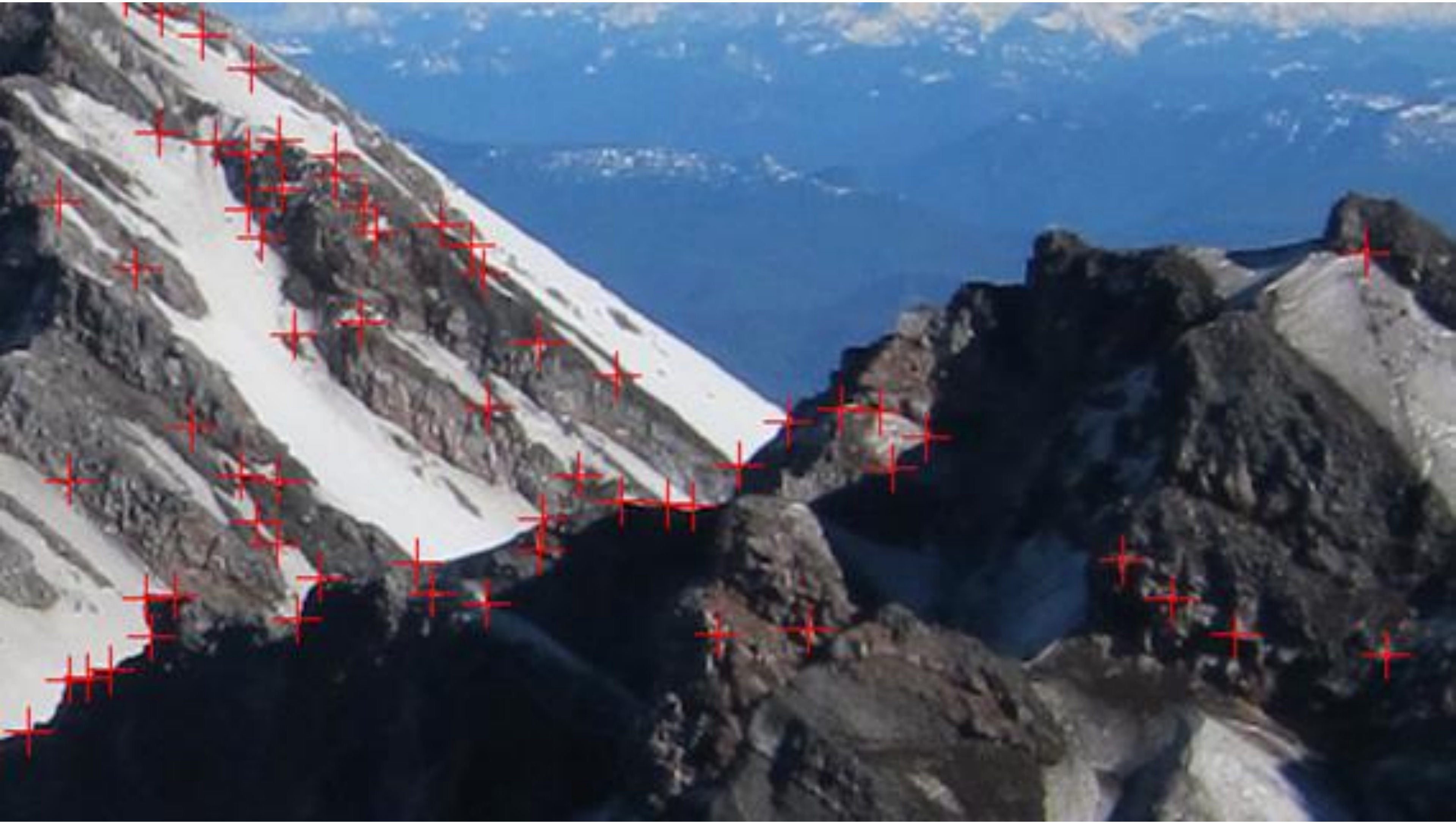






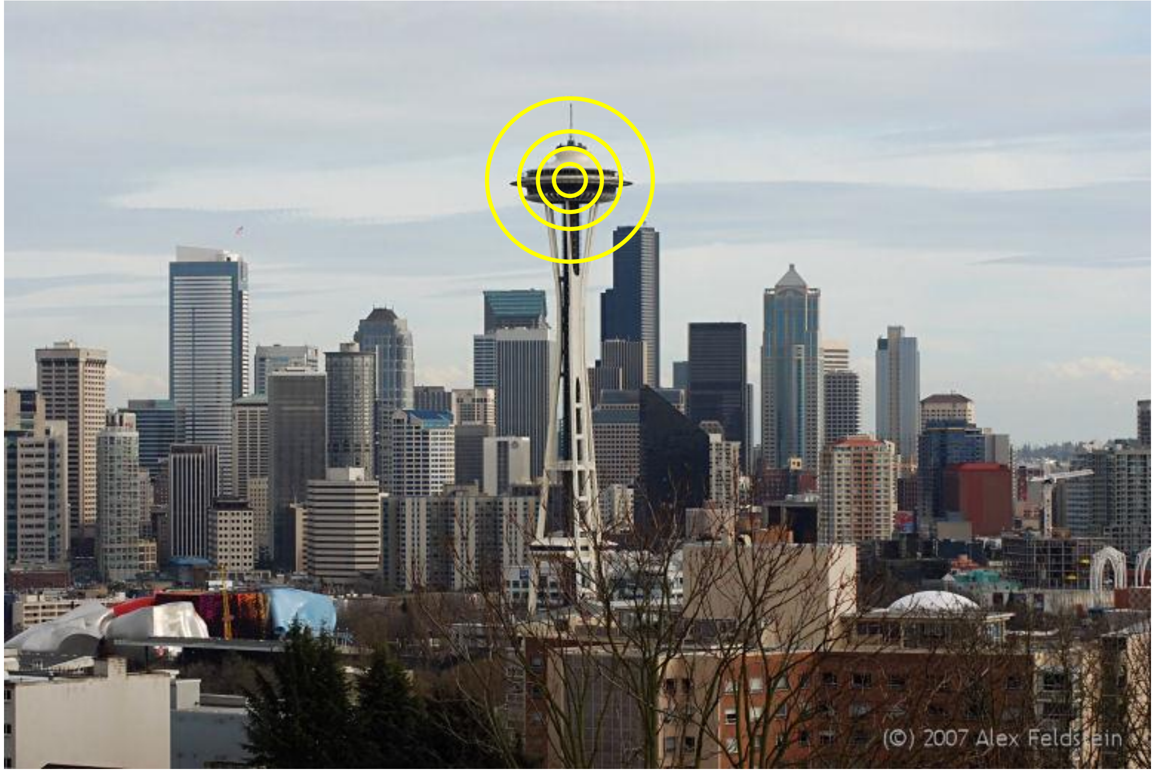
Corner !

All points will be classified as edges



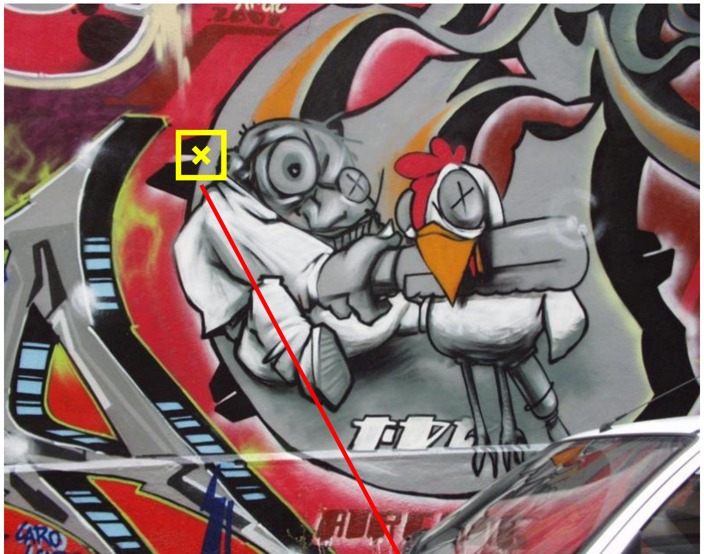
- Can we do better than Harris?
- The Harris corner detector is not widely used except in class assignments.
- The SIFT detector/descriptor is the standard.
- Let’s take a look.

Scale



What is the “best” scale?

Scale Invariance

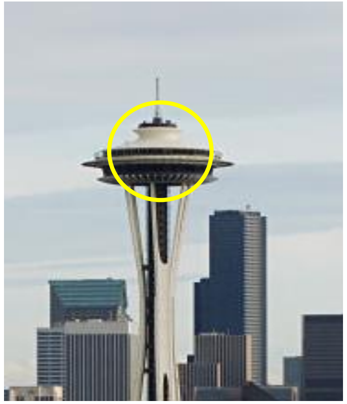


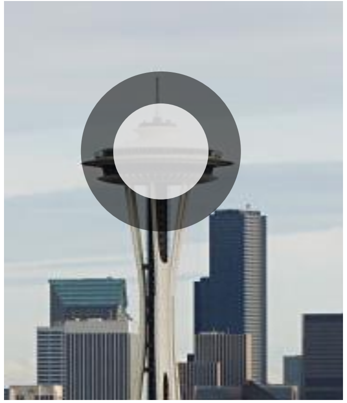
$$f(I_{i_1 \ldots i_m}(x, \sigma)) = f(I_{i_1 \ldots i_m}(x', \sigma'))$$

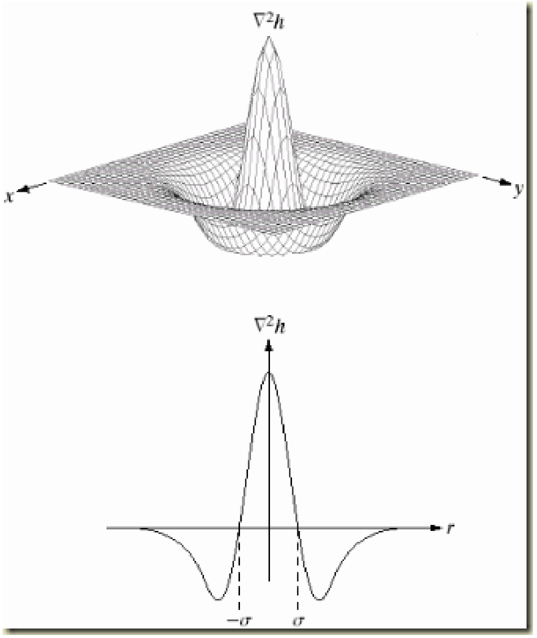
How can we independently select interest points in each image, such that the detections are repeatable across different scales?

K. Grauman, B. Leibe

Differences between Inside and Outside







1. We can use a Laplacian function

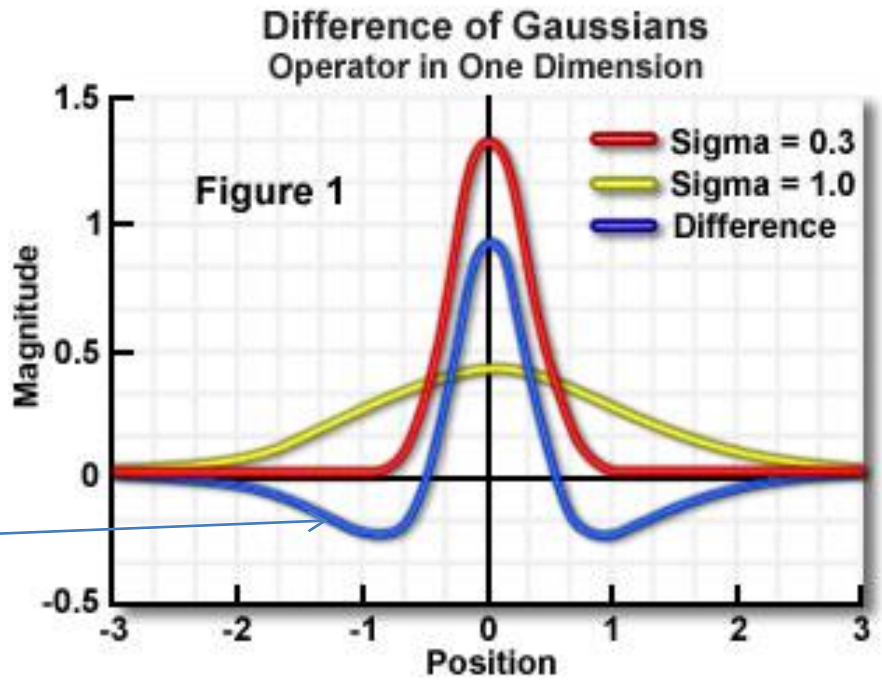
Scale

But we use a Gaussian.

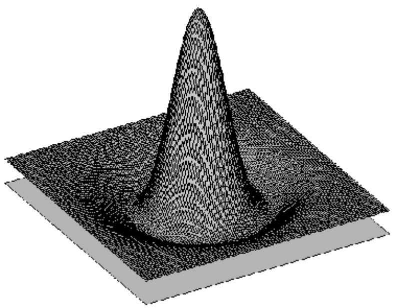
Why Gaussian?

It is invariant to scale change, i.e., $f * \mathcal{G}_\sigma * \mathcal{G}_{\sigma'} = f * \mathcal{G}_{\sigma''}$ and has several other nice properties. Lindeberg, 1994

In practice, the Laplacian is approximated using a Difference of Gaussian (DoG).



Difference-of-Gaussian (DoG)



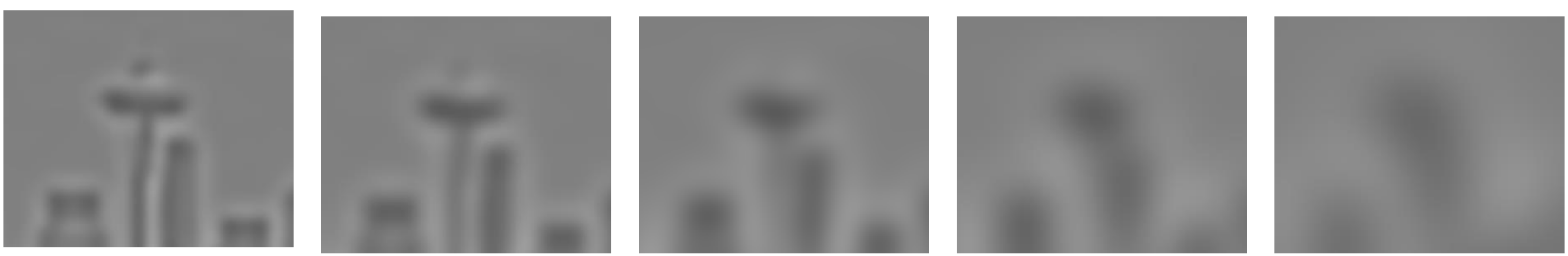
G1 - G2 = DoG

K. Grauman, B. Leibe

DoG example

Take Gaussians at multiple spreads and uses DoGs.

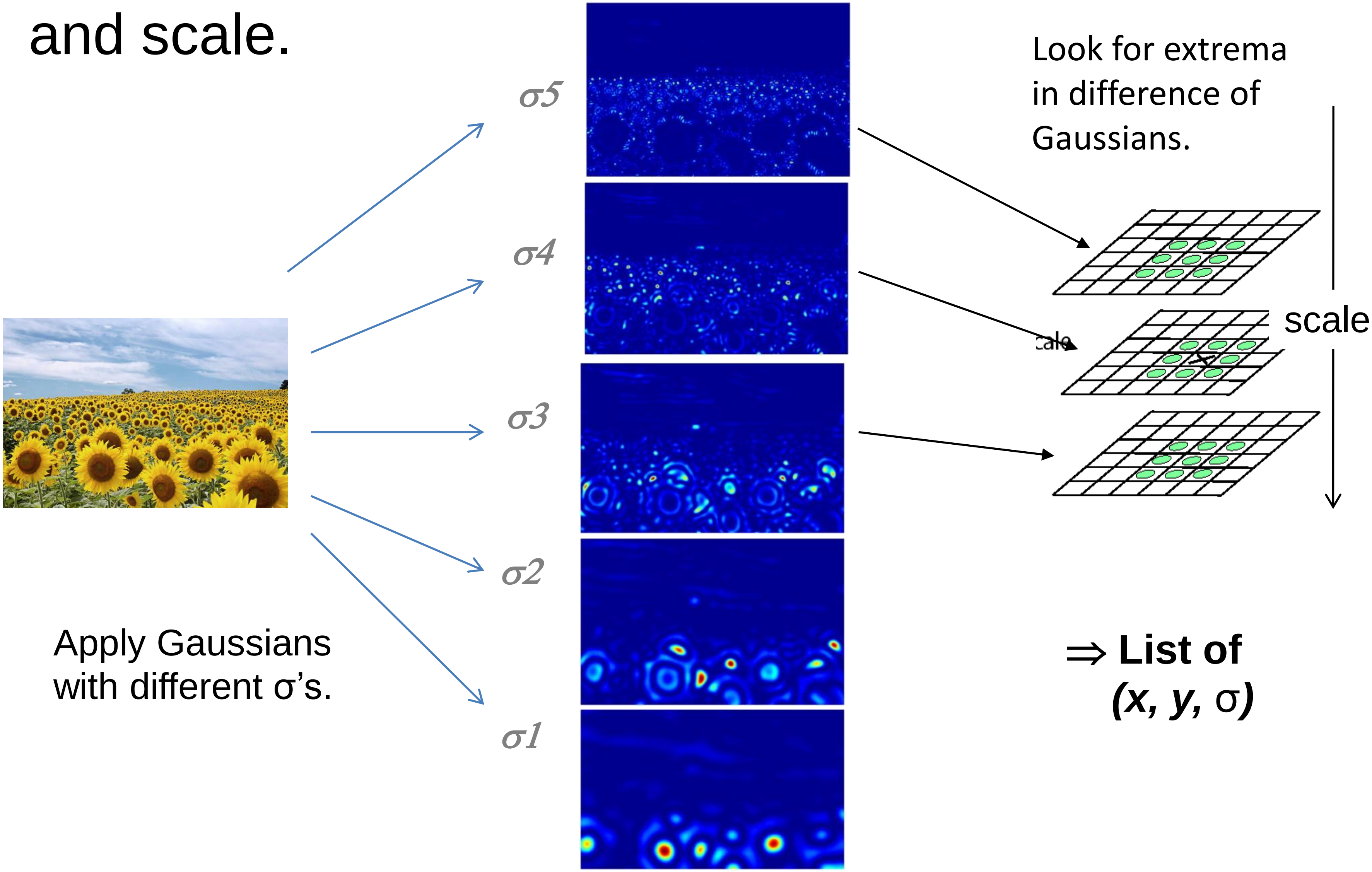
σ = 1



σ = 66

Scale invariant interest points

Interest points are local maxima in both position and scale.



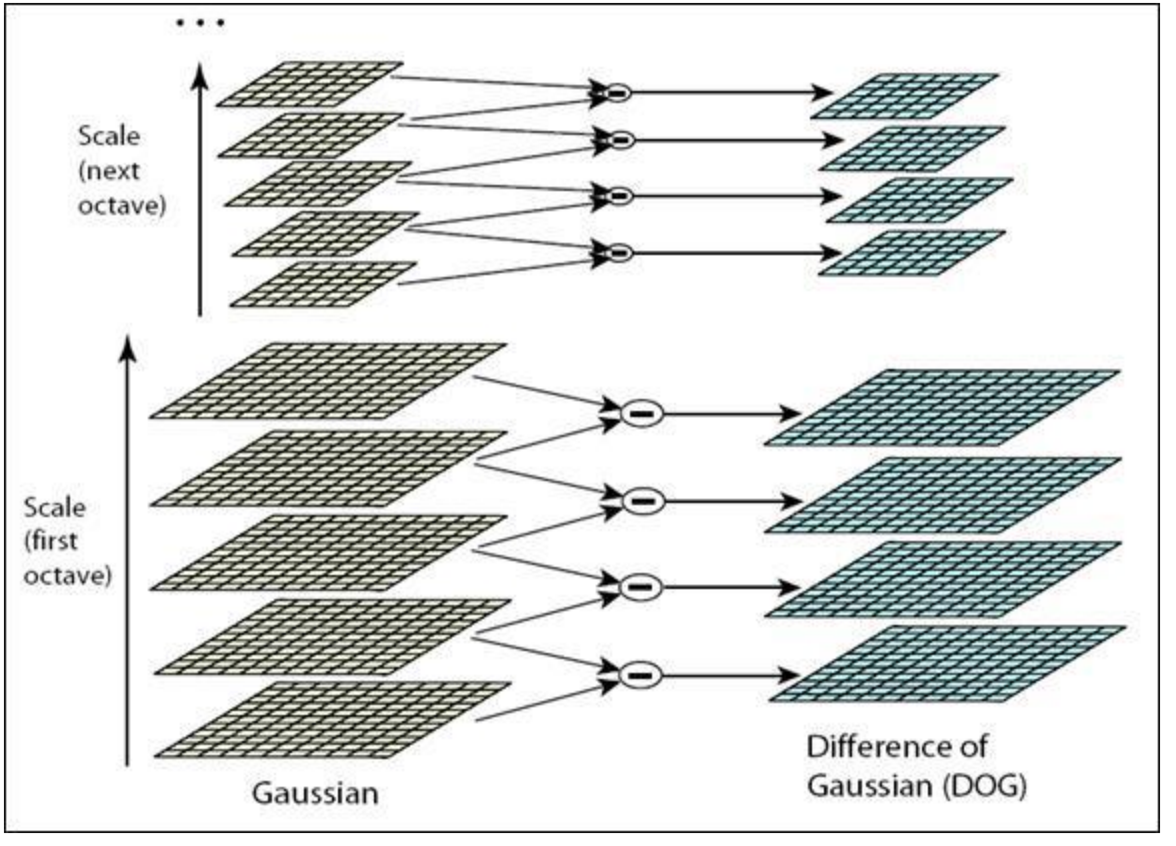
Apply Gaussians with different σ's.

Look for extrema in difference of Gaussians.

⇒ **List of *(x, y,* σ*)***

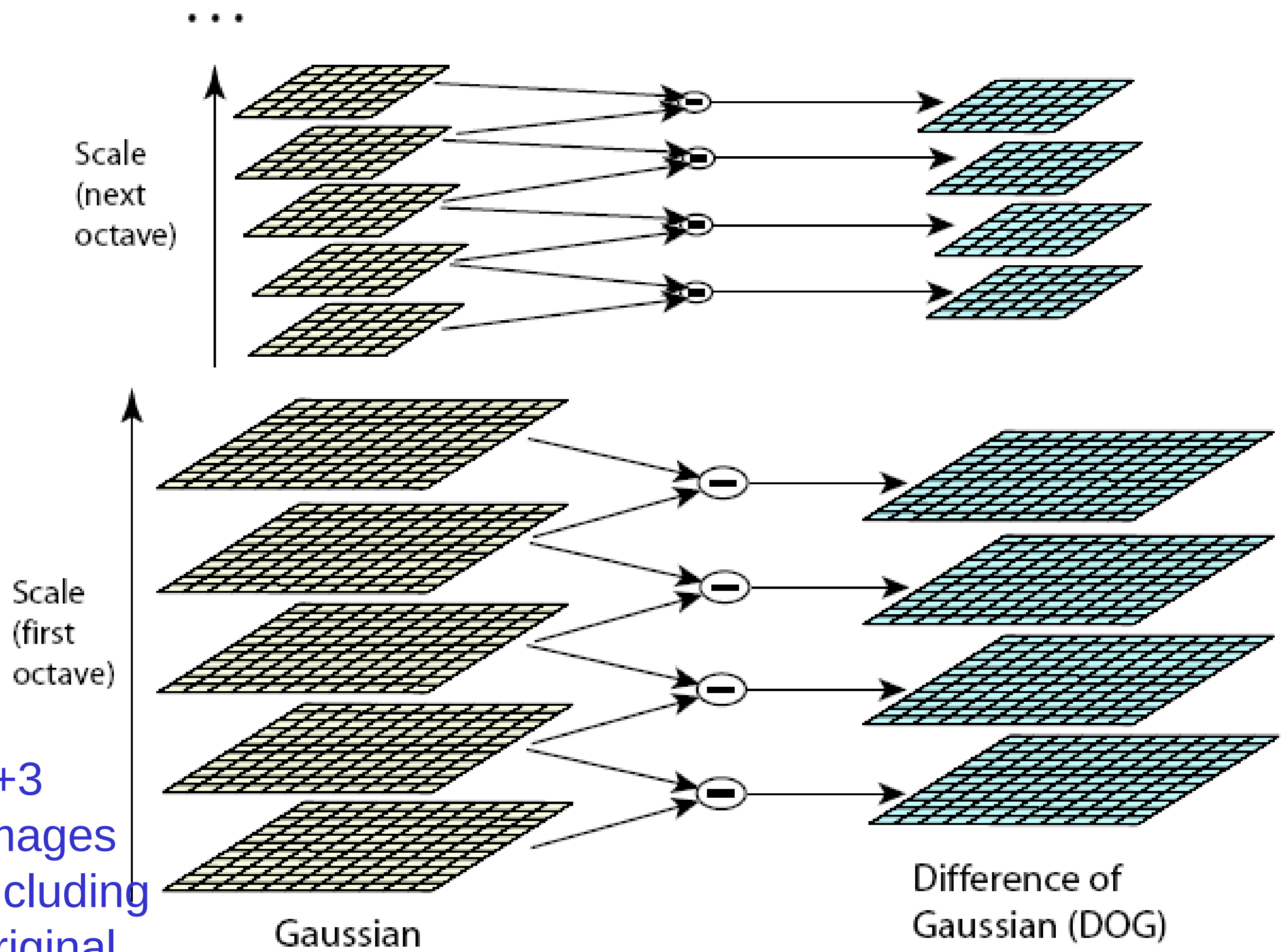
Scale (Lowe uses multiple scales)

In practice the image is downsampled for larger sigmas.



Lowe, 2004.

Lowe's Pyramid Scheme



s+2 filters

$\sigma_{s+1}=2^{(s+1)/s}\sigma_0$

$\sigma_s=2^{s/s}\sigma_0$

.

.

$\sigma_i=2^{i/s}\sigma_0$

.

.

$\sigma_2=2^{2/s}\sigma_0$

$\sigma_1=2^{1/s}\sigma_0$

σ_0

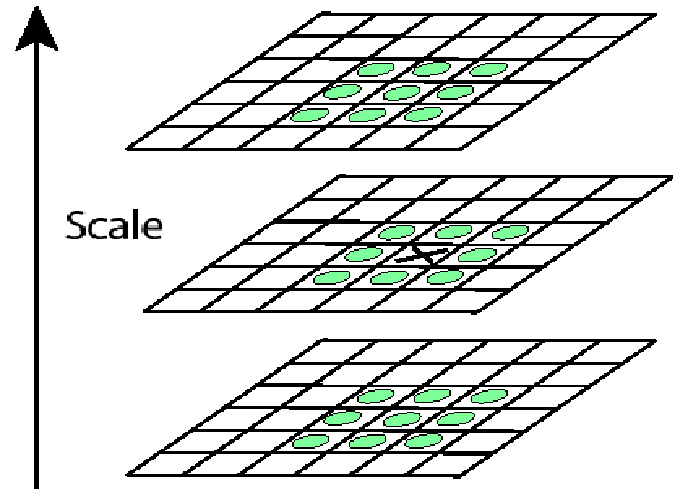
s+3 images including original

s+2 differ-ence images

The parameter **s** determines the number of images per octave.

Key point localization

s+2 difference images.
top and bottom ignored.
s planes searched.

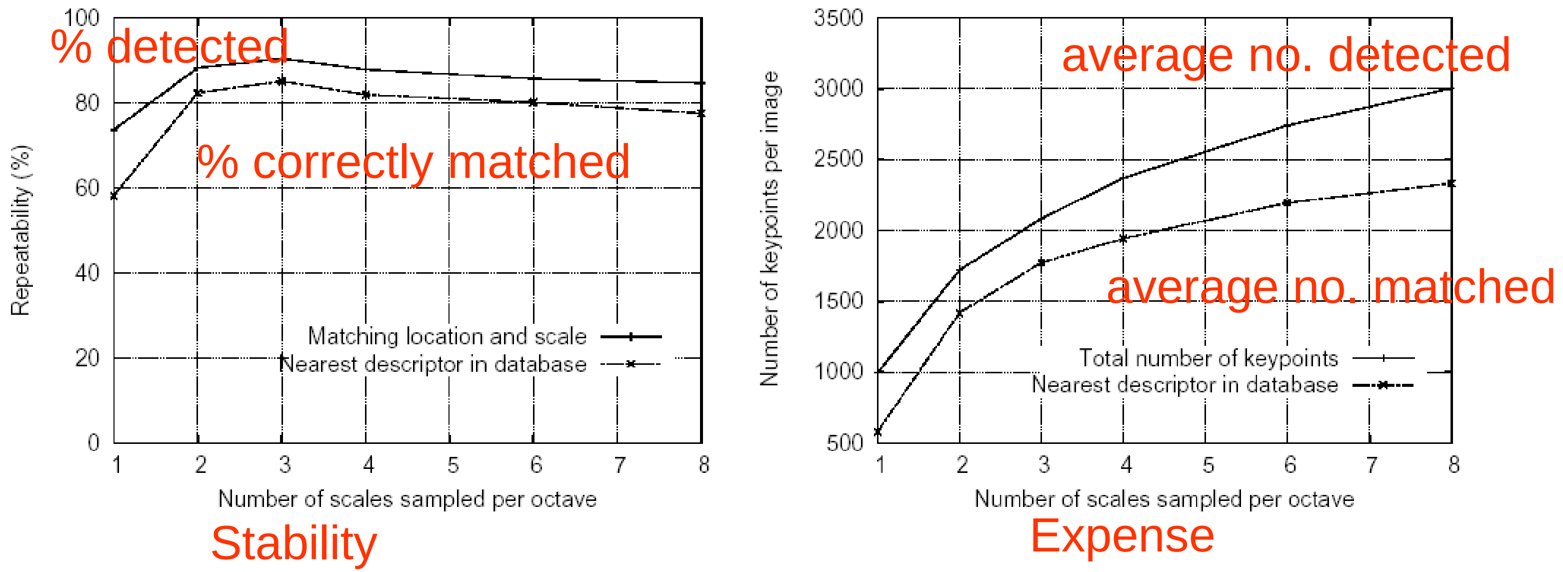


- Detect maxima and minima of difference-of-Gaussian in scale space

- Each point is compared to its 8 neighbors in the current image and 9 neighbors each in the scales above and below

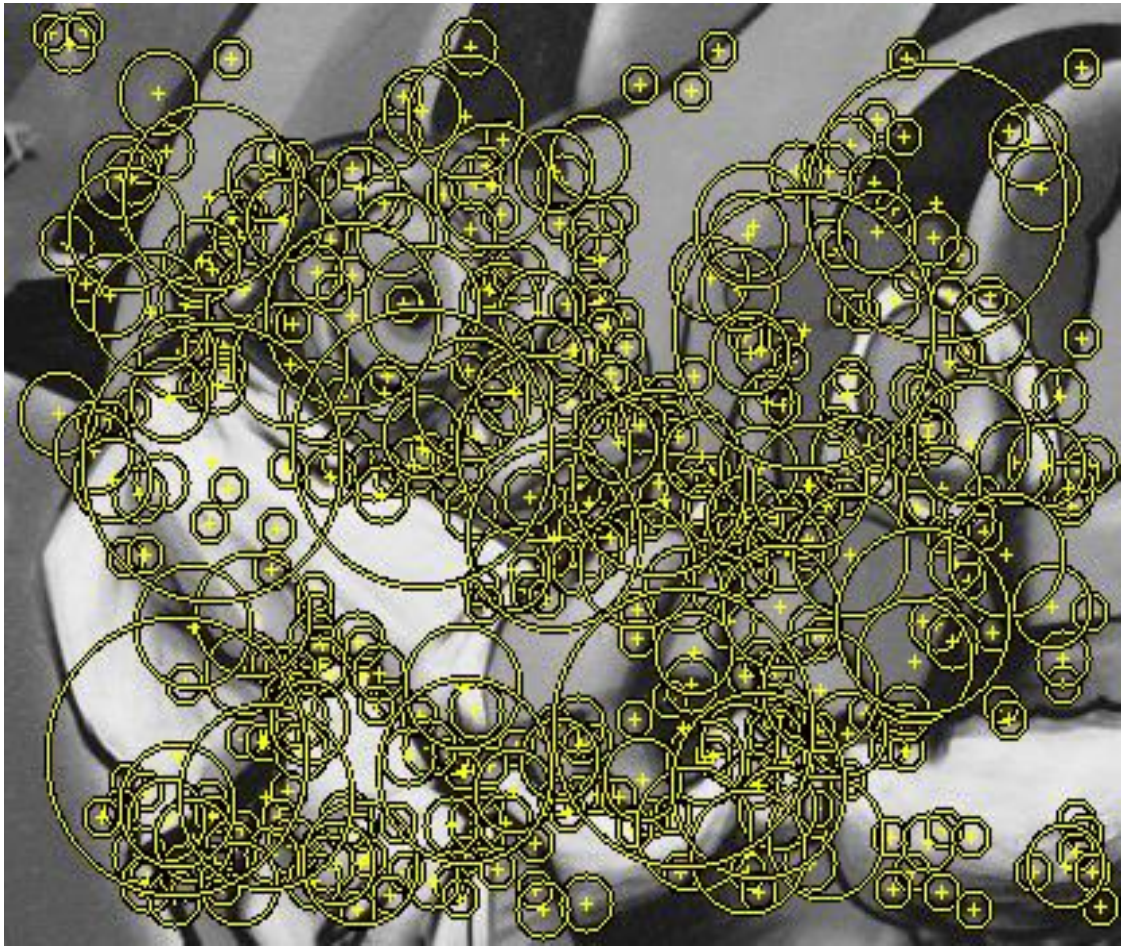
For each max or min found, output is the **location** and the **scale**.

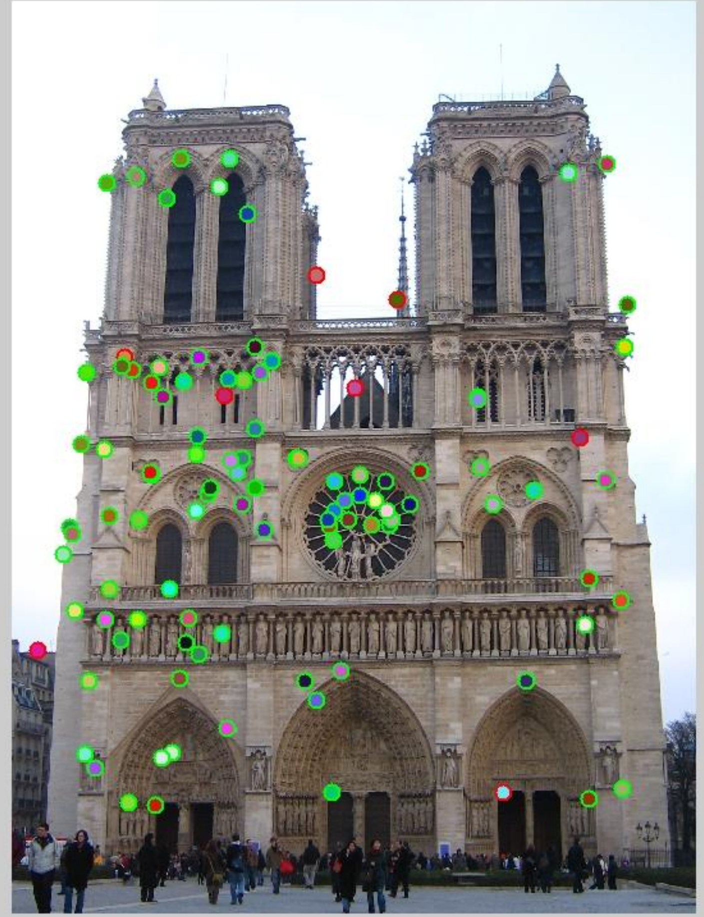
Scale-space extrema detection: experimental results over 32 images that were synthetically transformed and noise added.



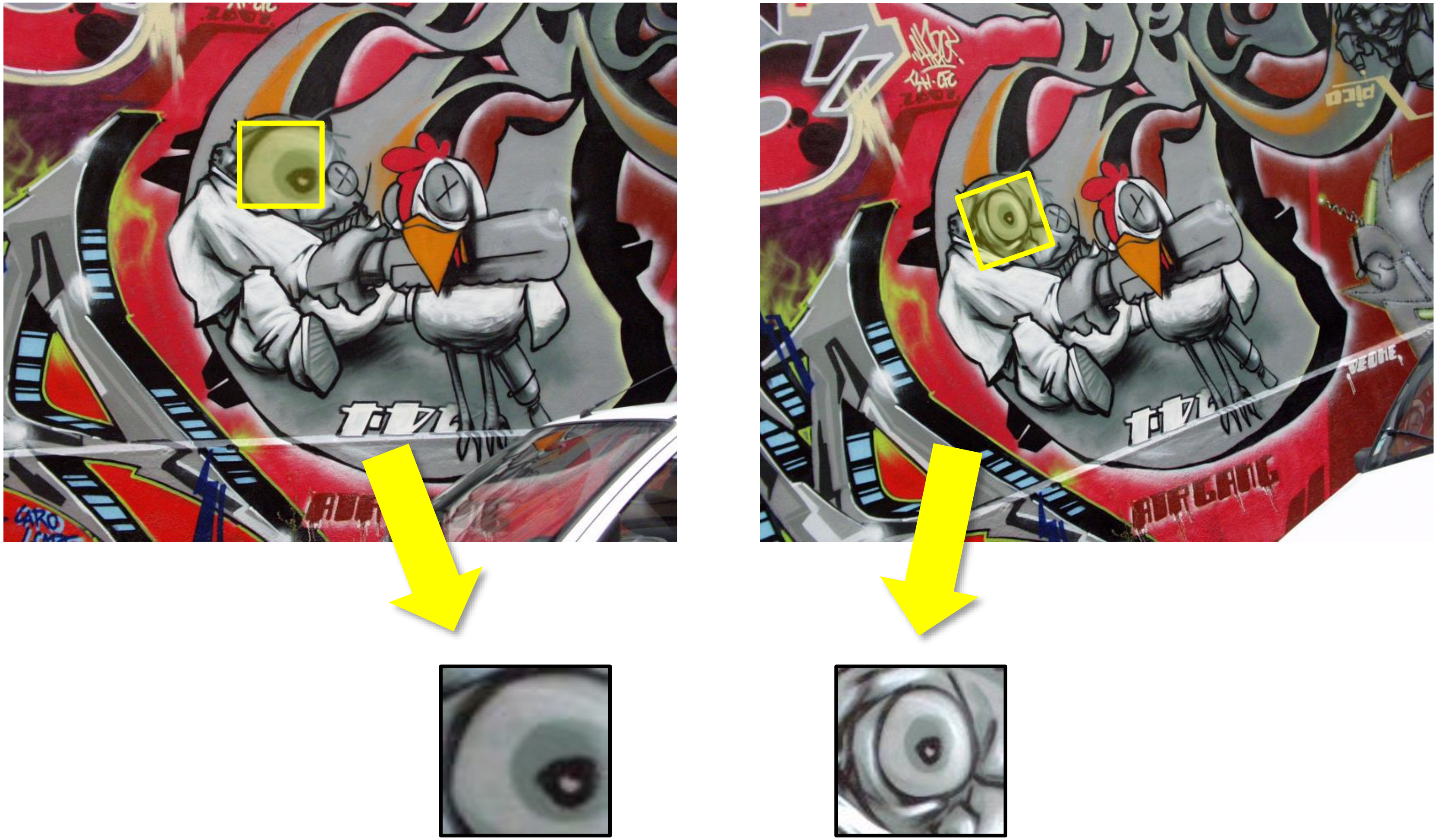
- Sampling in scale for efficiency
  - How many scales should be used per octave? S=?
    - More scales evaluated, more keypoints found
    - S < 3, stable keypoints increased too
    - S > 3, stable keypoints decreased
    - S = 3, maximum stable keypoints found

Results: Difference-of-Gaussian



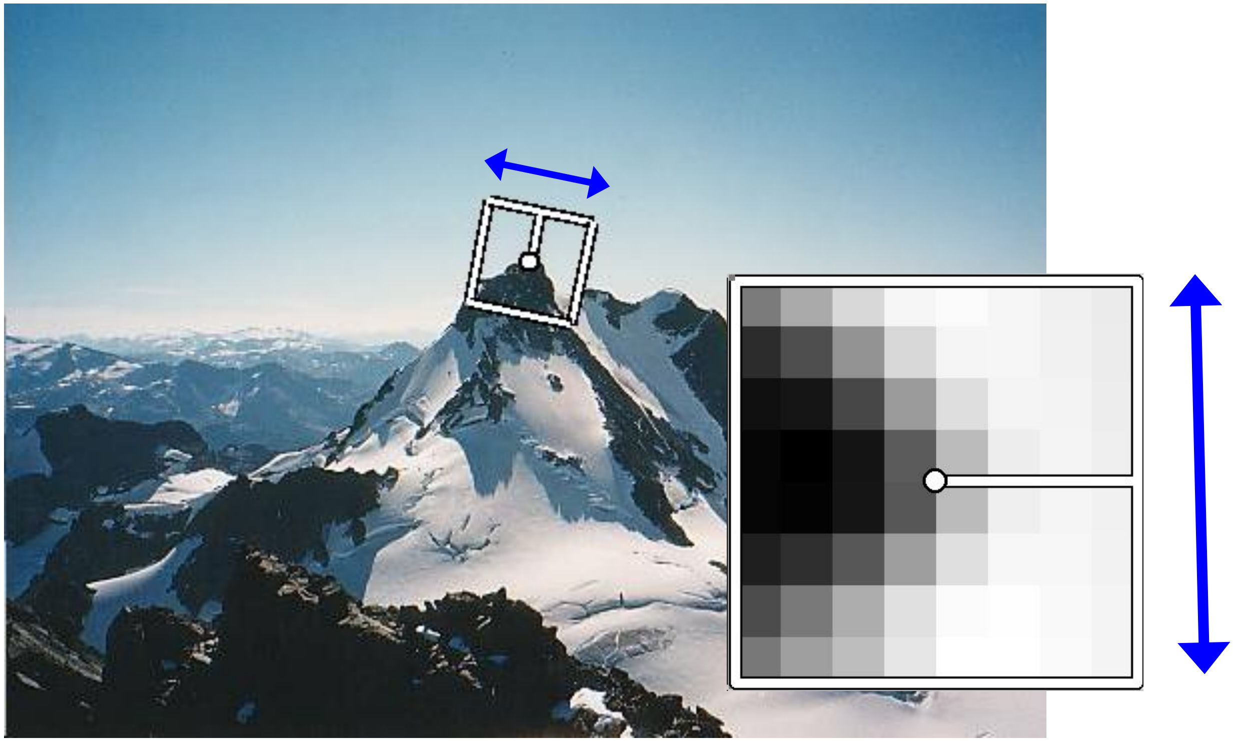


How can we find correspondences?



Similarity transform

Rotation invariance



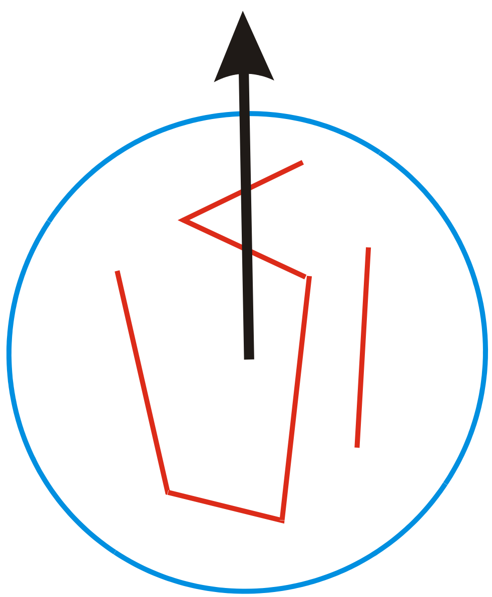
- Rotate patch according to its dominant gradient orientation
- This puts the patches into a canonical orientation.

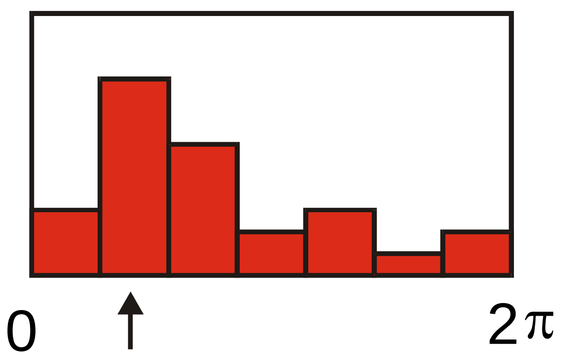
Image from Matthew Brown

Orientation Normalization

- Compute orientation histogram
- Select dominant orientation
- Normalize: rotate to fixed orientation

[Lowe, SIFT, 1999]

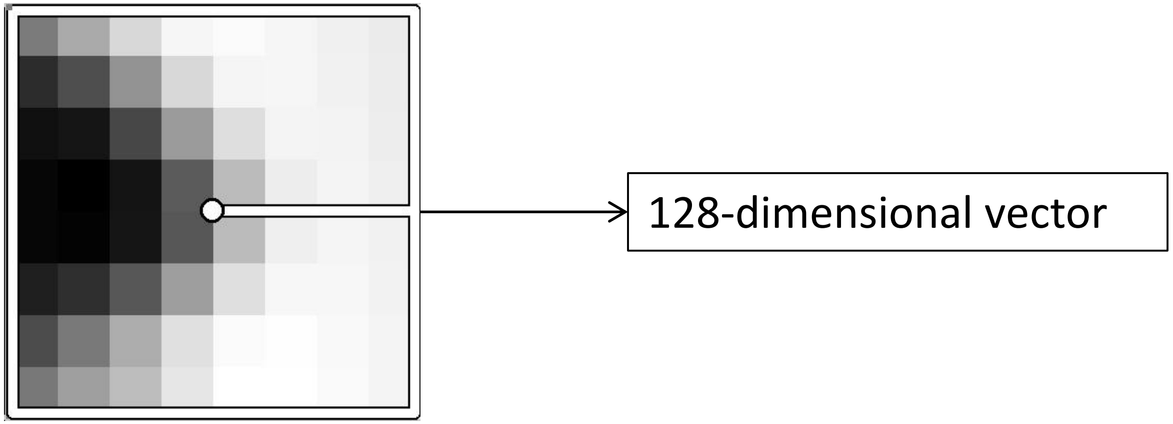




What’s next?

Once we have found the keypoints and a dominant orientation for each,

we need to describe the (rotated and scaled) neighborhood about each.



Important Point

- People just say "SIFT".
- But there are TWO parts to SIFT.

1. an interest point detector
2. a region descriptor

- They are independent. Many people use the region descriptor without looking for the points.