Lecture 10 & 11

Segmentation and clustering

Administrative

A2 is out

- Was Due April 28th
- Date moved back

A3 is out

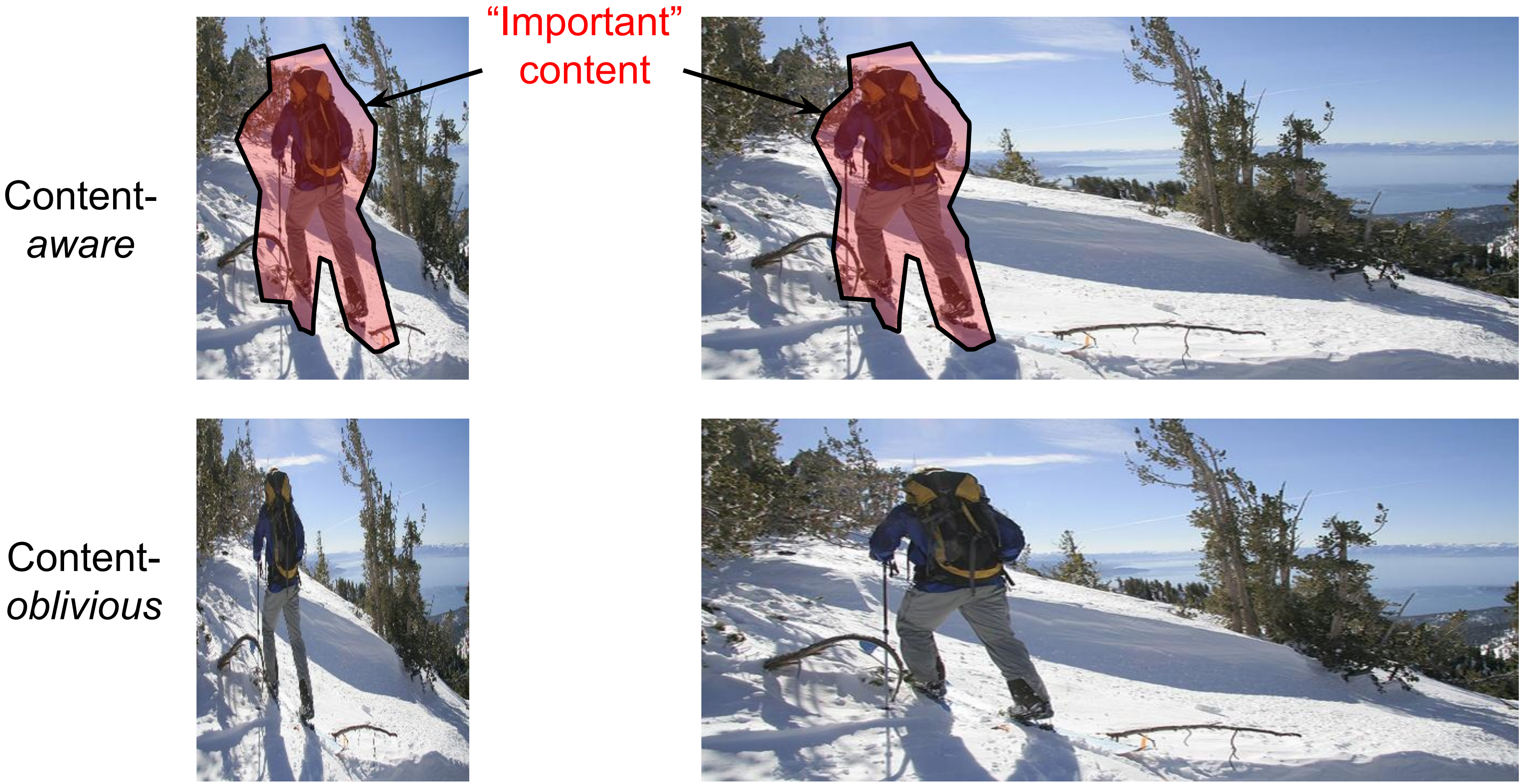
- Due May 9th

Administrative

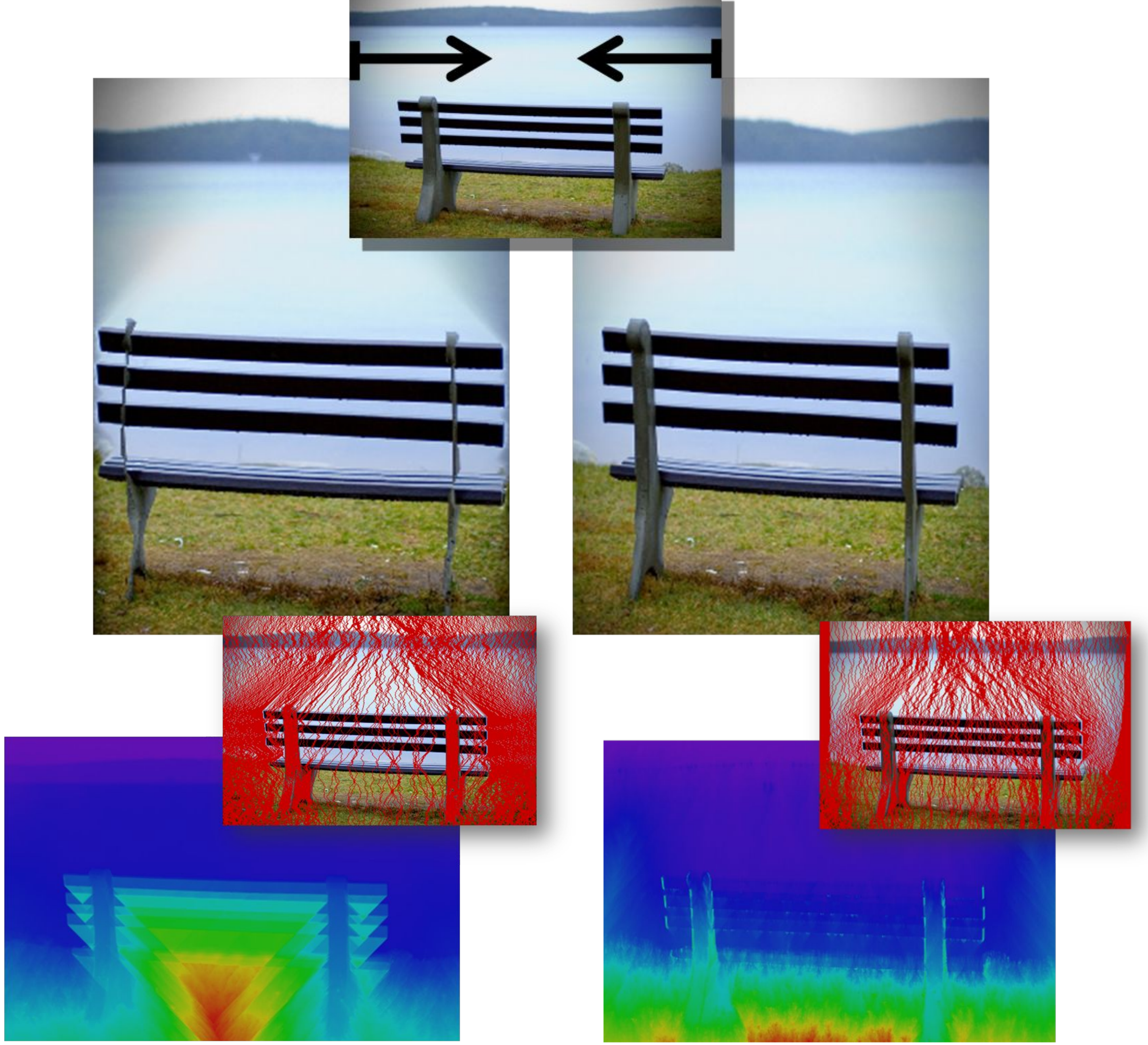
Recitation

- Xiaojuan Wang
- Panorama (part of your A2)
- detector, descriptor, RANSAC recap

Content-aware Retargeting Operators

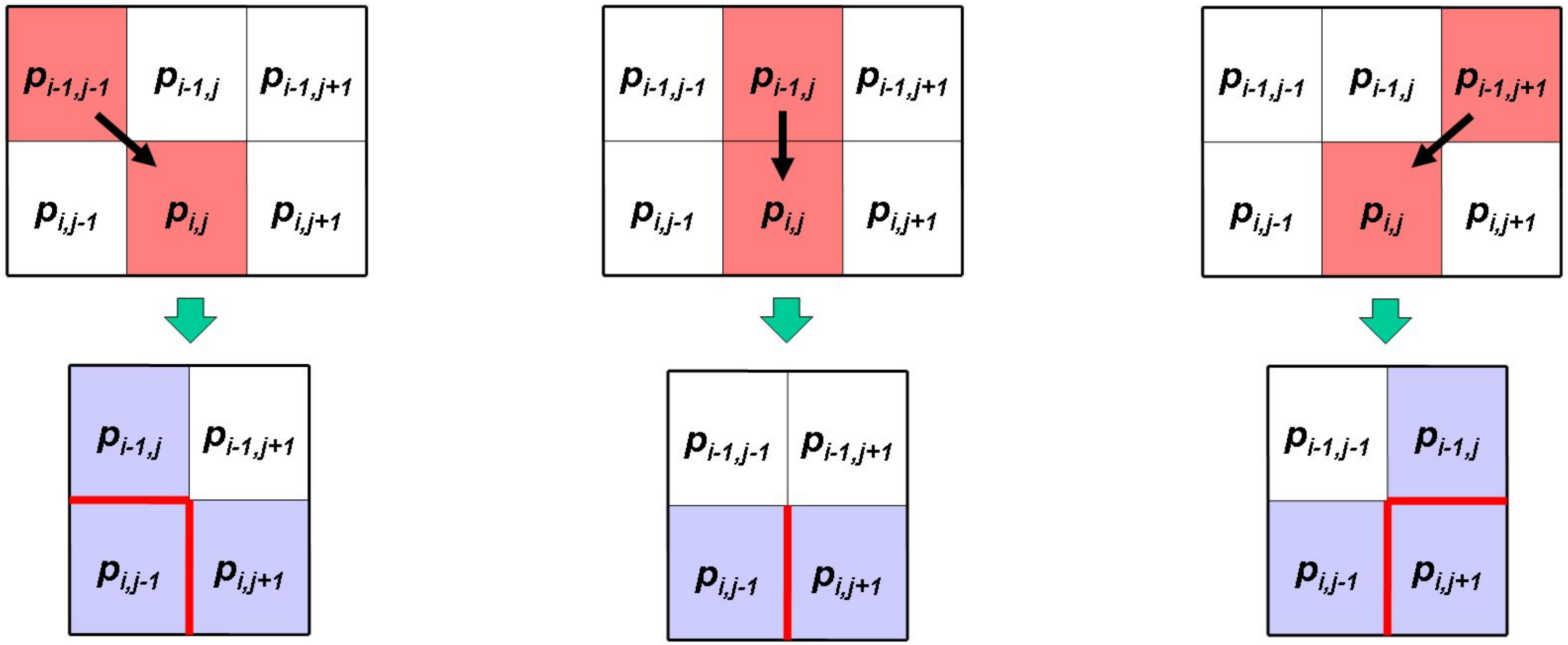


So far



So far: Seam carving with pixel energies

$$M(i,j) = E(i,j) + min \begin{cases} M(i-1,j-1) + C_L(i,j) \\ M(i-1,j) + C_V(i,j) \\ M(i-1,j+1) + C_R(i,j) \end{cases}$$



Retargeting in Both Dimensions

- Let T(r,c) denote a new cost matrix of obtaining an image of size (n-r)x(m-c).

$$\begin{aligned}\mathbf{T}(r,c) \quad &= \quad min(\mathbf{T}(r-1,c) + E(\mathbf{s^x}(\mathbf{I_{n-r-1\times m-c}})), \\ &\qquad \mathbf{T}(r,c-1) + E(\mathbf{s^y}(\mathbf{I_{n-r\times m-c-1}})))\end{aligned}$$

where $E(\mathbf{s^x}(\mathbf{I_{n-r-1\times m-c}})$ is the cost of removing a horizontal

seam from the image $\mathbf{I_{n-r-1\times m-c}}$

Today’s agenda

- Introduction to segmentation and clustering
- Gestalt theory for perceptual grouping
- Graph-based oversegmentation
- Agglomerative clustering
- K-means
- Mean-shift

Reading:
Szeliski, 2nd edition, Chapter 7.5

Today's agenda

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Q. What do you see?

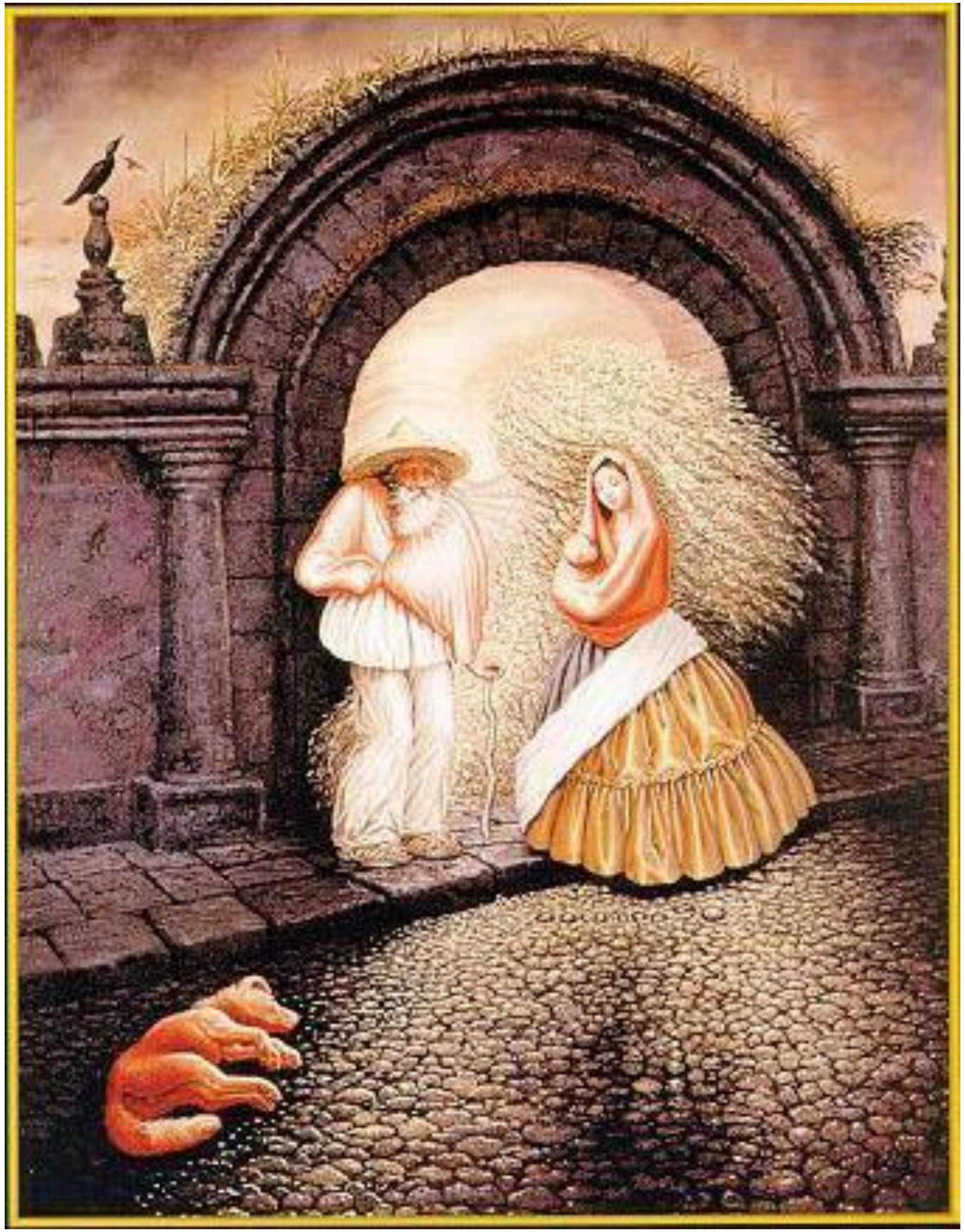
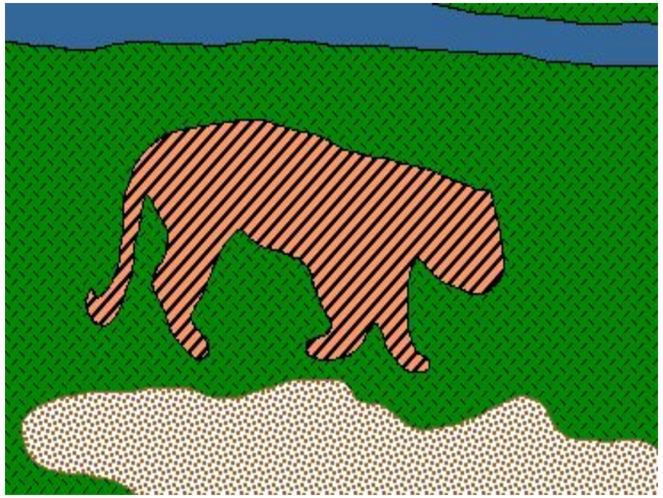


Image Segmentation

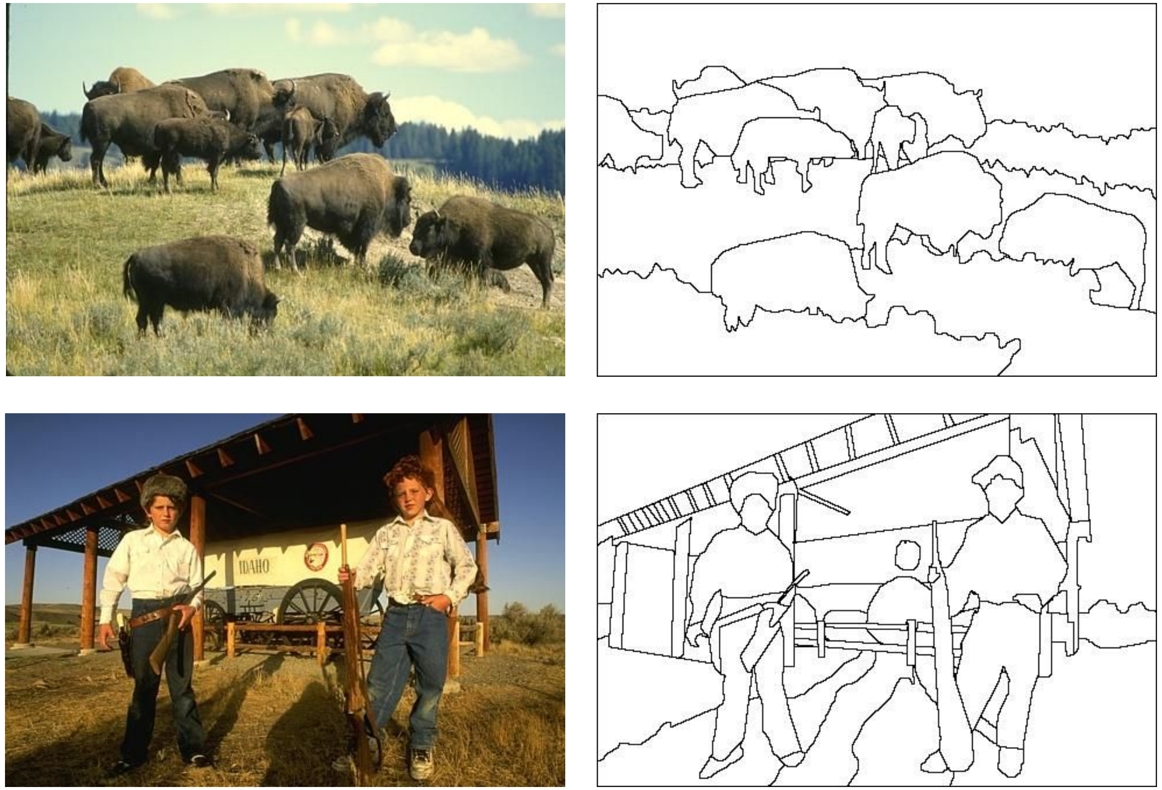
- Goal: identify groups of pixels that go together



The Goals of Segmentation

- Separate image into coherent "objects"

Image | Human segmentation

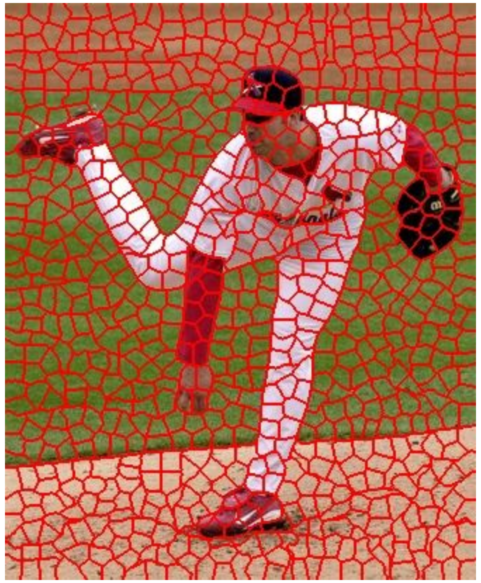


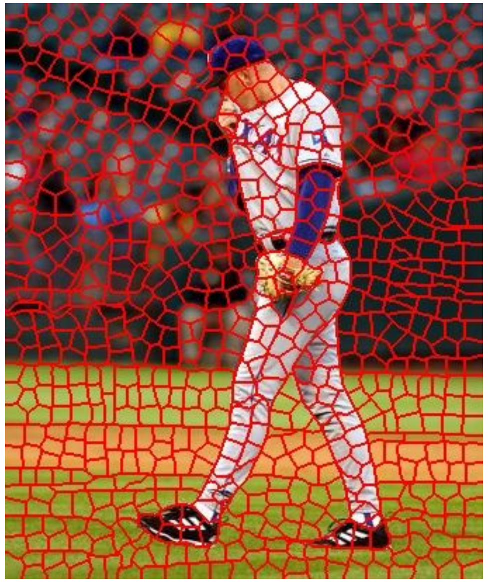
Slide credit: Svetlana Lazebnik

The Goals of Segmentation

- Separate image into coherent “objects”
- Group together similar-looking pixels for efficiency of further processing

“superpixels”





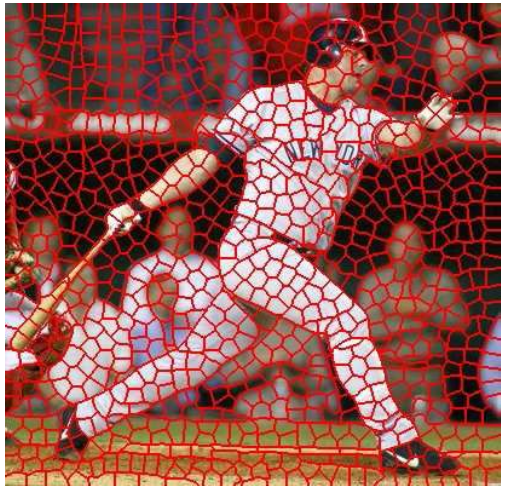
X. Ren and J. Malik. Learning a classification model for segmentation. ICCV 2003.

Segmentation for feature support

Slide: Derek Hoiem

Segmentation for efficiency

[Felzenszwalb and Huttenlocher 2004]



[Hoiem et al. 2005, Mori 2005]

[Shi and Malik 2001]

Slide: Derek Hoiem

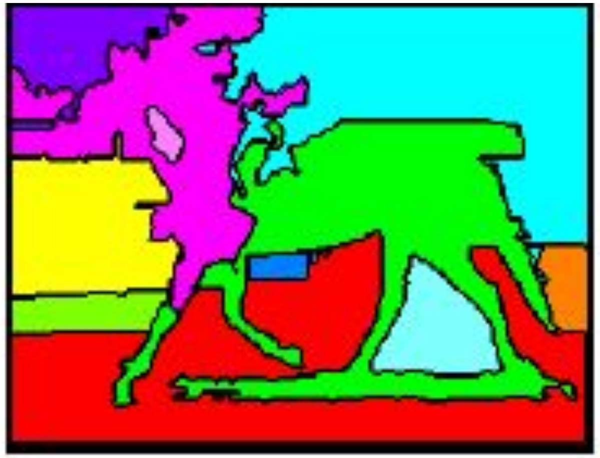
Segmentation is used in Adobe photoshop to remove background

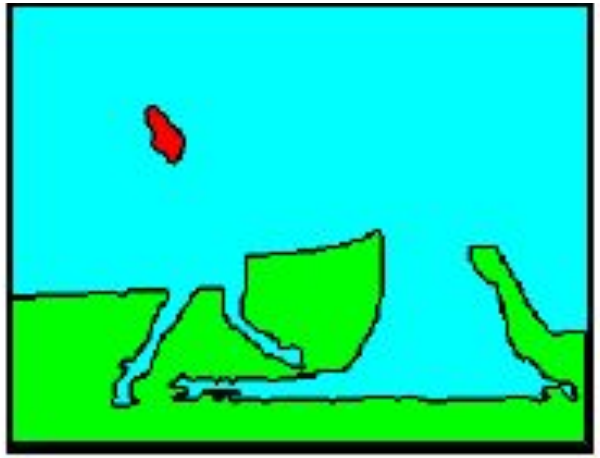
Rother et al. 2004

[Segment Anything](#) [2023]

Levels of segmentations

Over-segmentation





Under-segmentation

One way to think about “segmentation” is clustering

Clustering: group together similar data points and represent them with a single token

Key Challenges:

1) What makes two points/images/patches similar?
2) How do we compute an overall grouping from pairwise similarities?

Slide: Derek Hoiem

Why do we cluster?

- **Summarizing data**
  - Look at large amounts of data
  - Find clusters of pixels
  - Represent each cluster of pixels with a HoG feature
- **Counting**
  - Histograms of texture, color, SIFT vectors
- **Foreground-background separation**
  - Separate the image into different regions
- **Prediction**
  - Images in the same cluster may have the same labels

Slide: Derek Hoiem

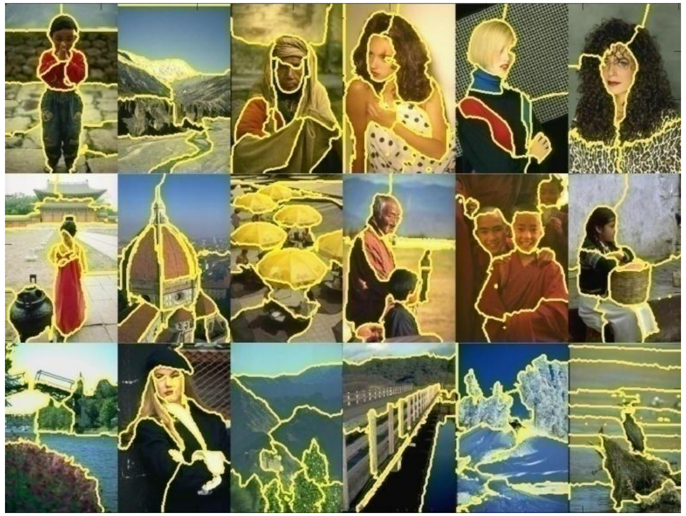
How do we cluster?

- **Agglomerative clustering**
  - Start with each point as its own cluster and iteratively merge the closest clusters
- **K-means**
  - Iteratively re-assign points to the nearest cluster center
- **Mean-shift clustering**
  - Estimate modes of pdf

General ideas

- Tokens
  - Things that can be grouped together
  - (e.g. pixels, points, surface elements, etc., etc.)
- Bottom up clustering
  - tokens belong together because they are locally coherent
- Top down clustering
  - tokens belong together because they lie on the same visual entity (object, scene…)
- > These two are not mutually exclusive

Examples of Grouping in Vision

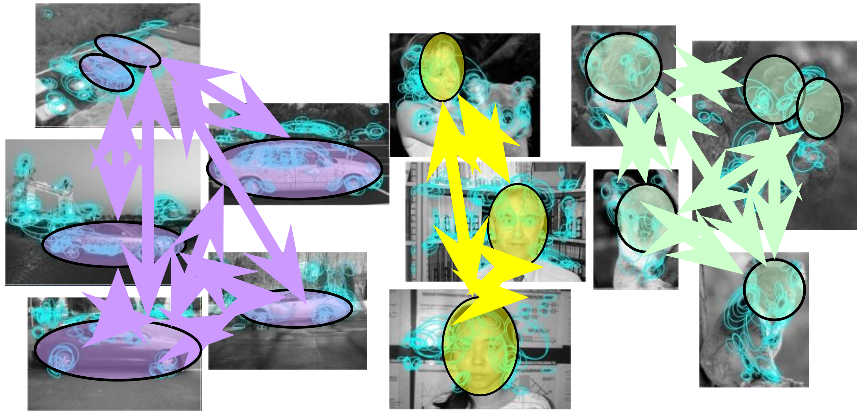


Determining image regions

Grouping video frames into shots

What things should be grouped?

What cues indicate groups?



Object-level grouping

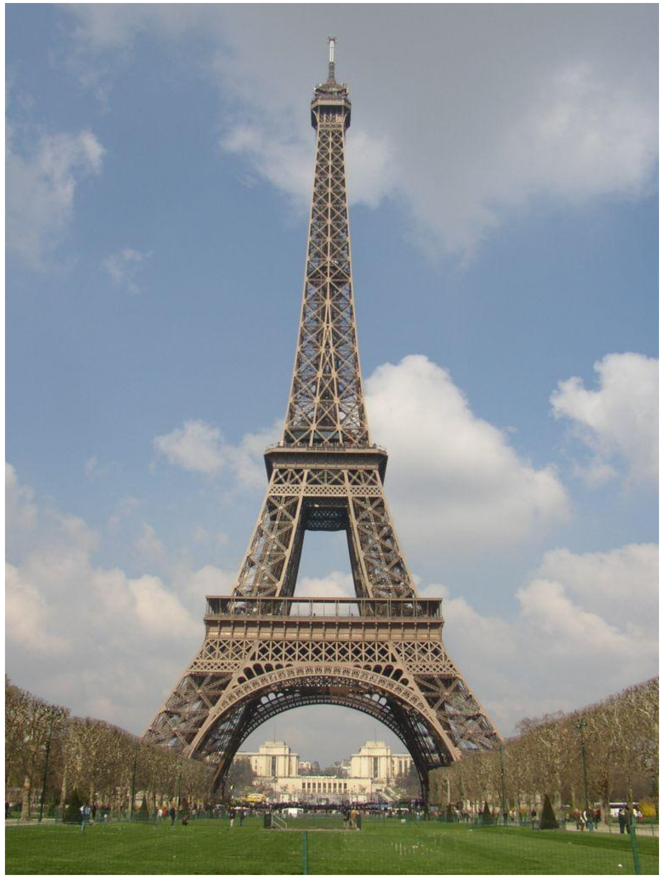
Figure-ground

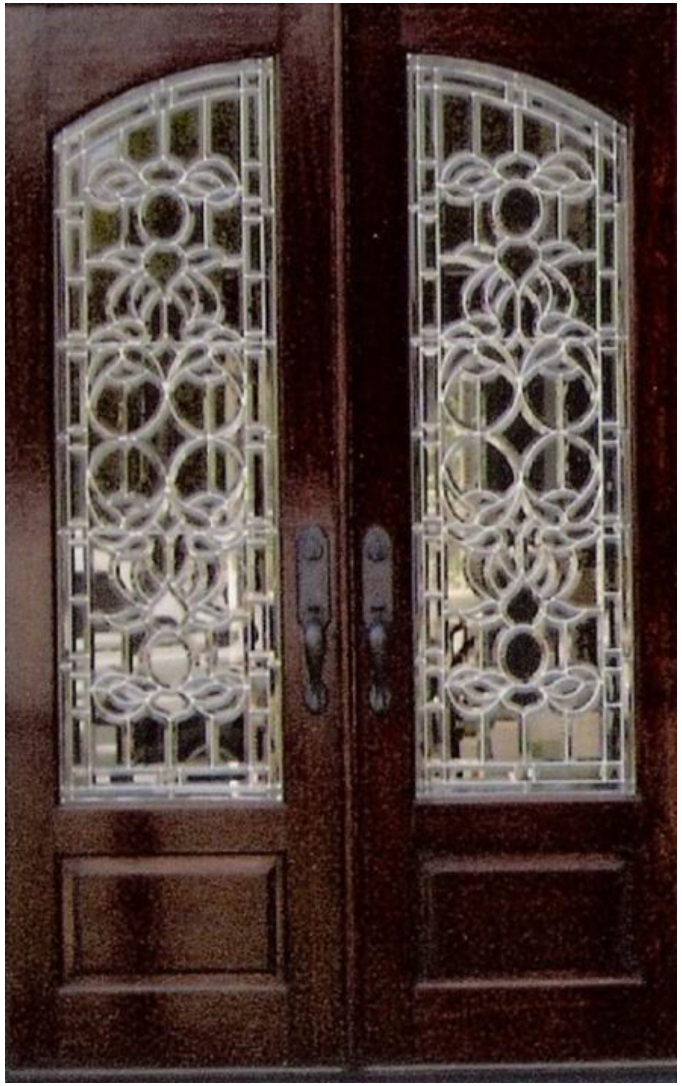
Slide credit: Kristen Grauman

Similarity

Slide credit: Kristen Grauman

Symmetry





Slide credit: Kristen Grauman

Common Fate

Image credit: Arthus-Bertrand (via F. Durand)

Slide credit: Kristen Grauman

Proximity

What will we learn today?

- Introduction to segmentation and clustering
- Gestalt theory for perceptual grouping
- Graph-based oversegmentation
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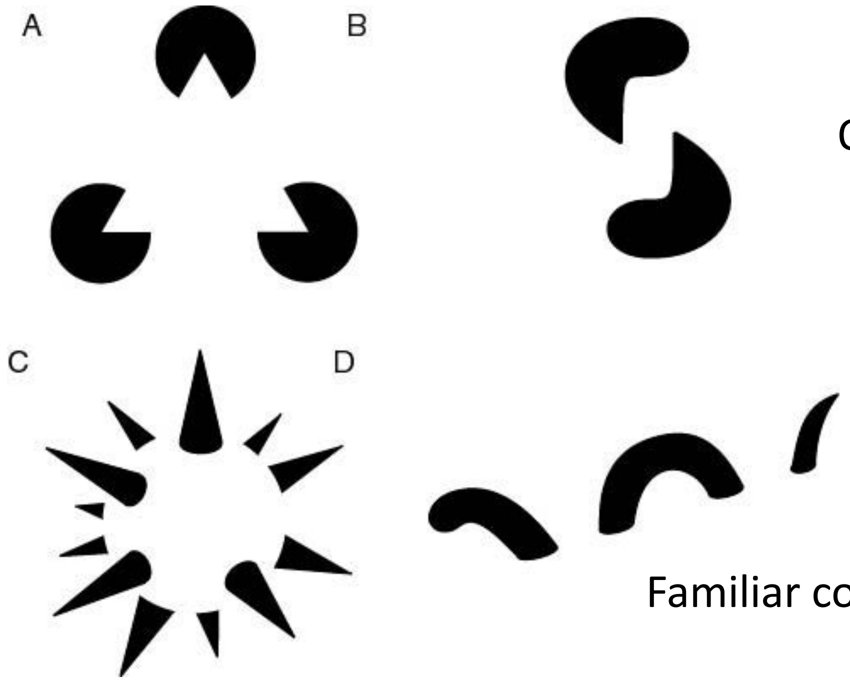
The Gestalt School

- Grouping is key to visual perception
- Elements in a collection can have properties that result from different **relationships (space, affordance, etc.)**
  - “The whole is greater than the sum of its parts”

Illusory/subjective contours



Occlusion

Familiar configuration

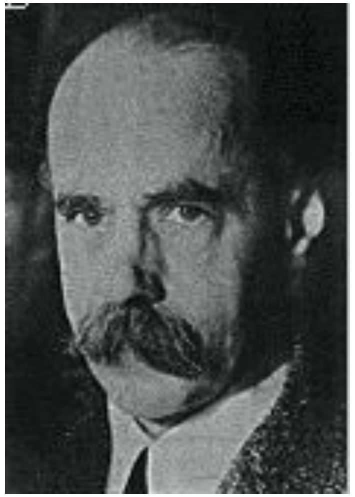
http://en.wikipedia.org/wiki/Gestalt_psychology

Gestalt Theory

- Gestalt: whole or group
  - Whole is greater than sum of its parts
  - Relationships among parts can yield new properties/features
- Psychologists identified series of factors that predispose set of elements to be grouped (by human visual system)

"I stand at the window and see a house, trees, sky. Theoretically I might say there were 327 brightnesses and nuances of colour. Do I have "327"? No. I have sky, house, and trees."

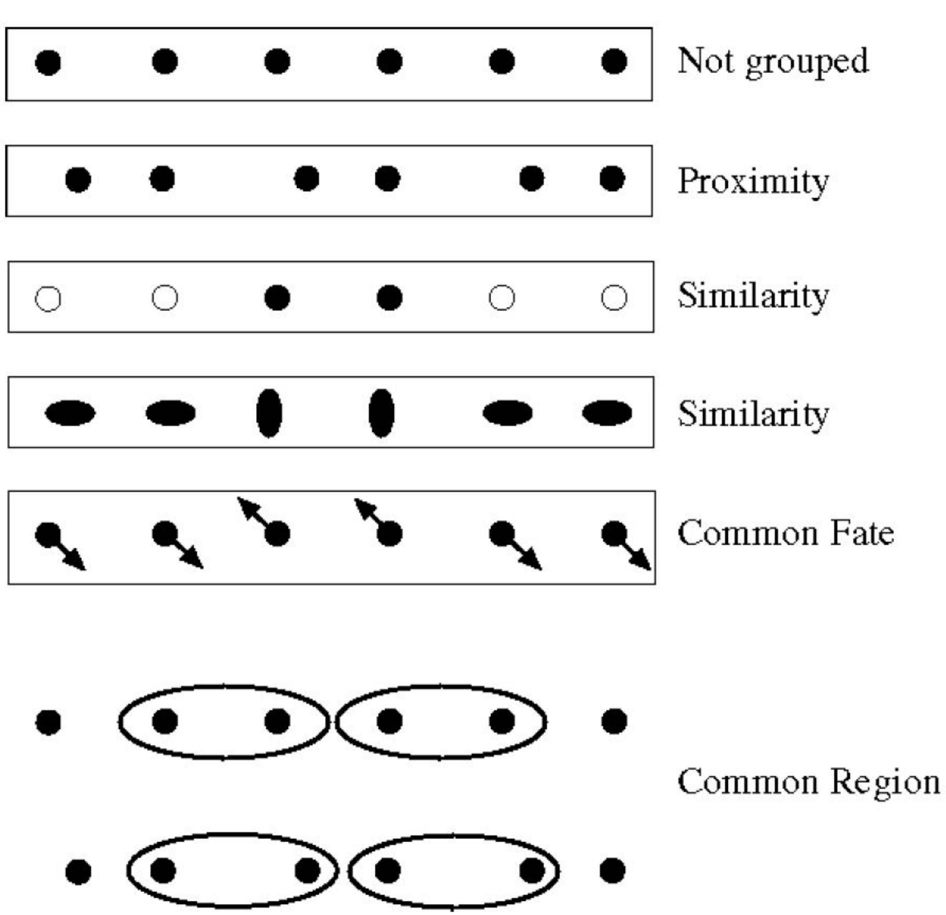
Max Wertheimer
(1880-1943)

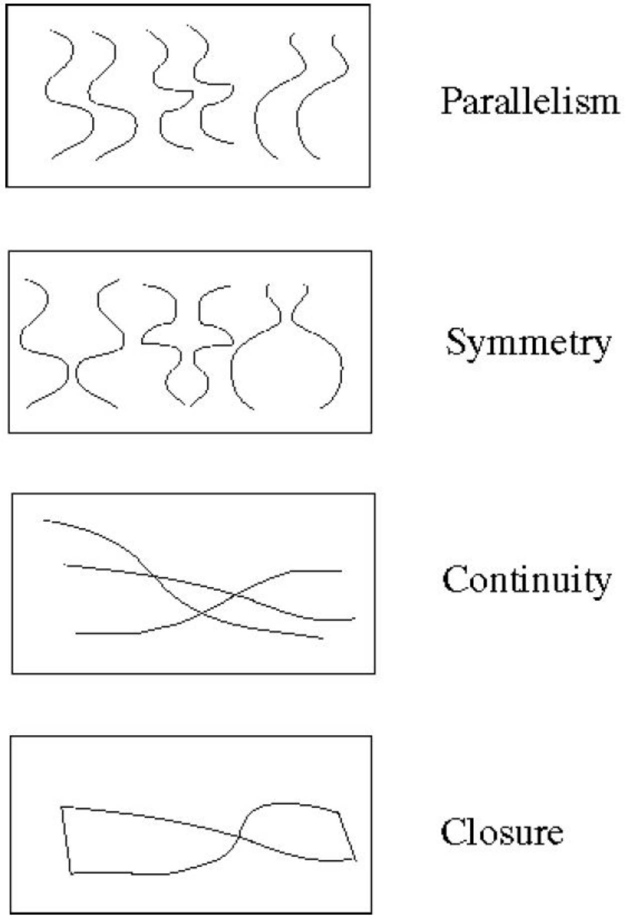


Untersuchungen zur Lehre von der Gestalt,
Psychologische Forschung, Vol. 4, pp. 301-350, 1923
http://psy.ed.asu.edu/~classics/Wertheimer/Forms/forms.htm

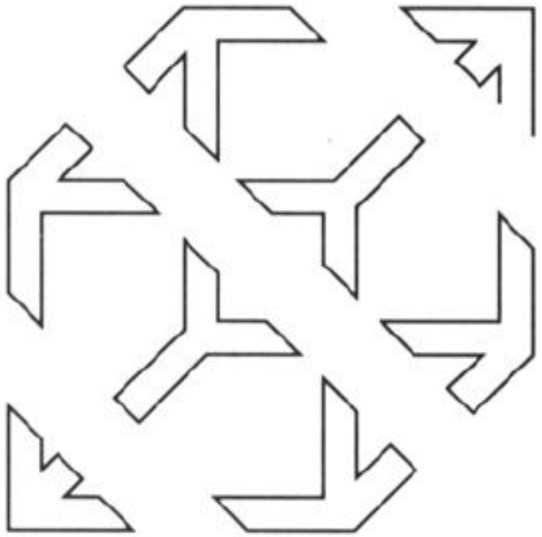
Gestalt Factors

These factors make intuitive sense, but are very difficult to translate into algorithms.

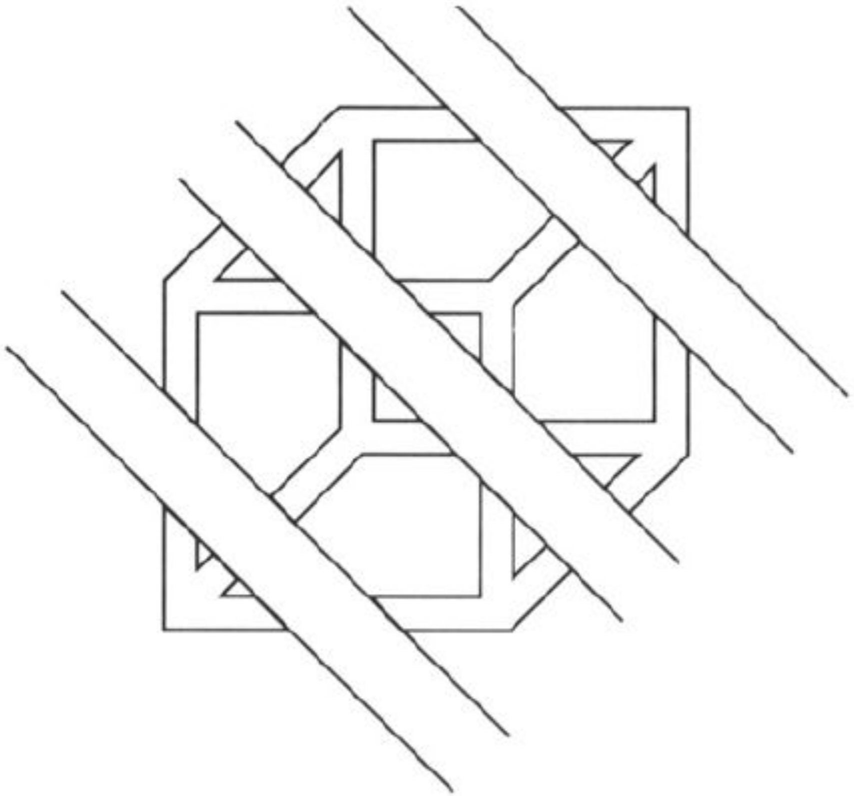




Continuity through Occlusion Cues

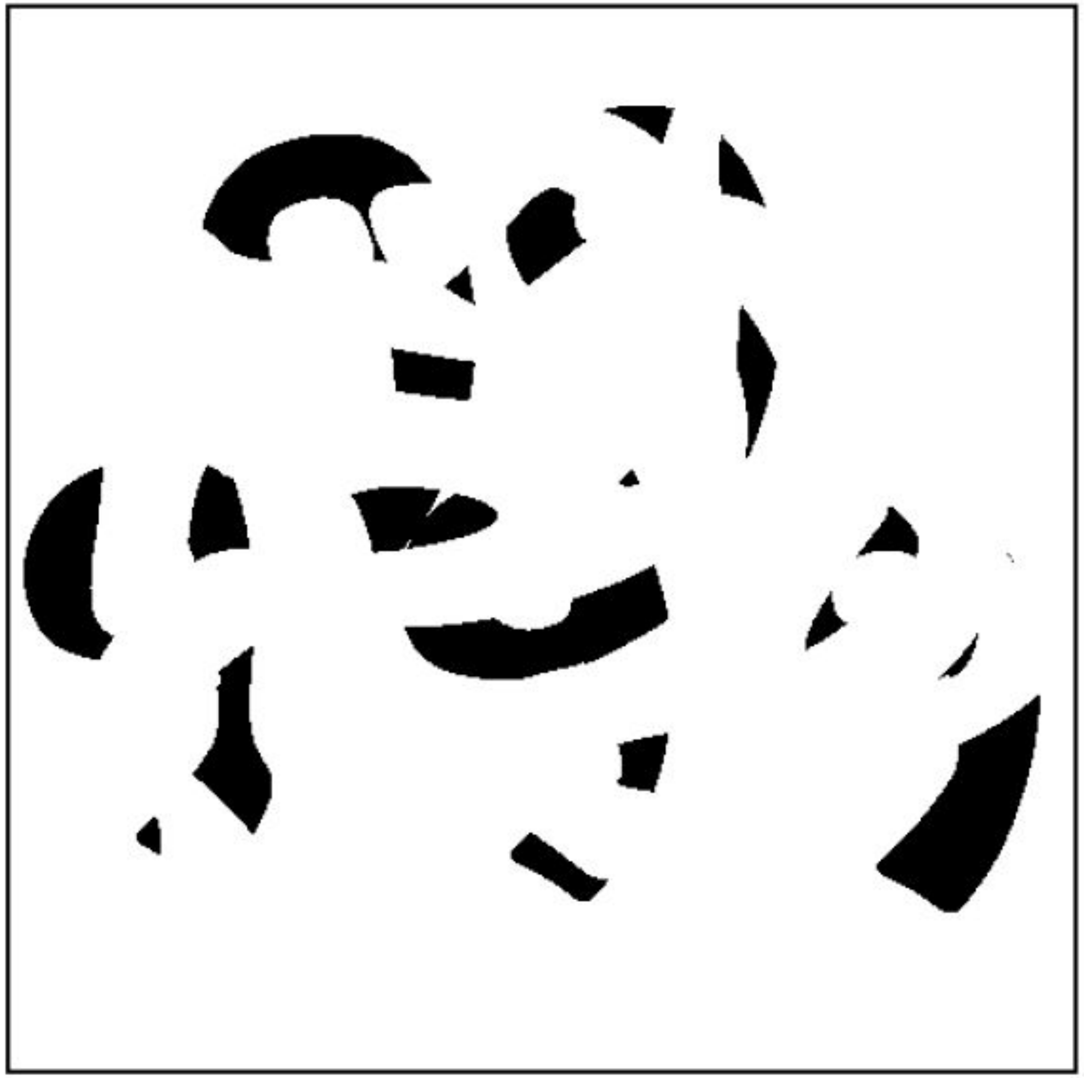


Continuity through Occlusion Cues



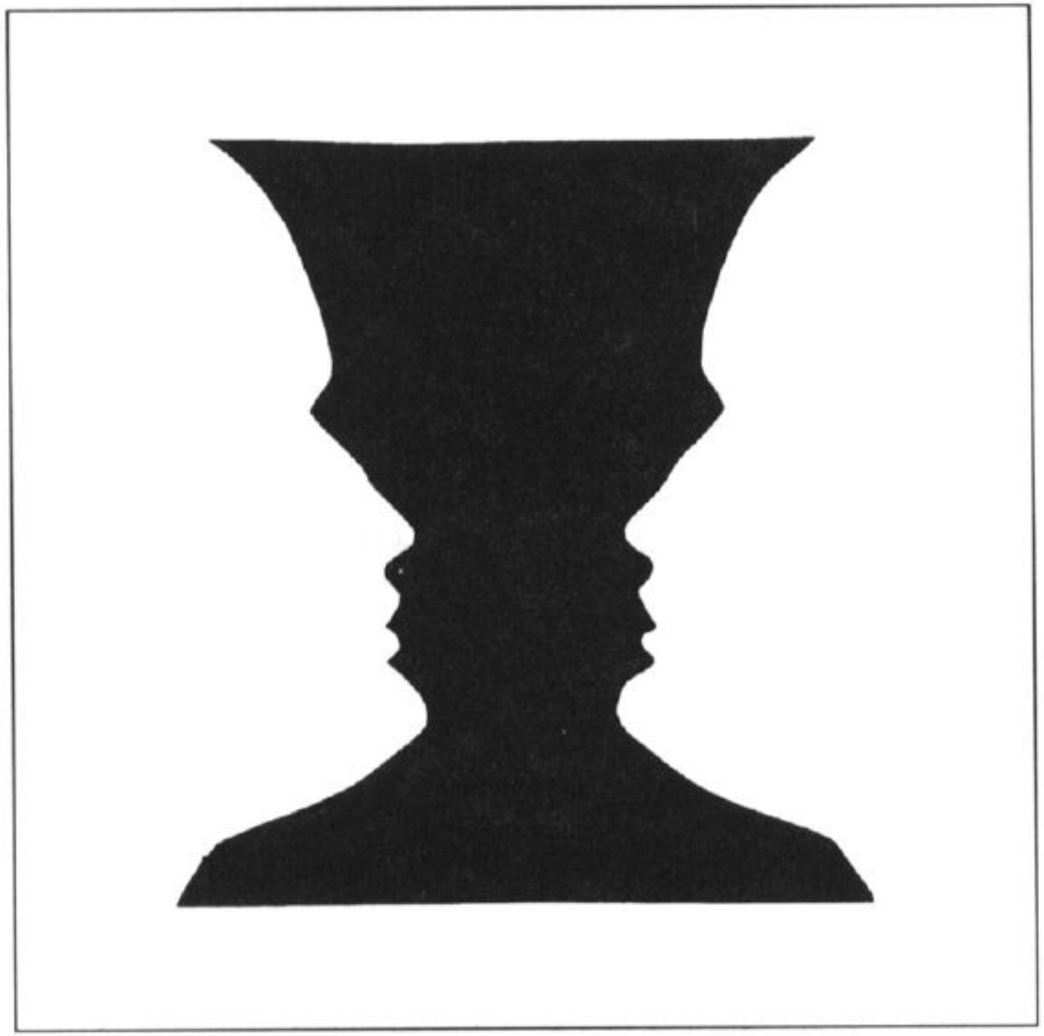
Continuity, explanation by occlusion

Continuity through Occlusion Cues



Continuity through Occlusion Cues

Figure-Ground Discrimination



The Ultimate Gestalt?

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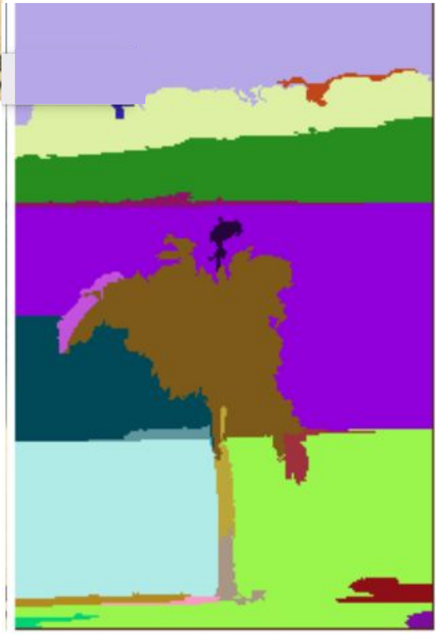
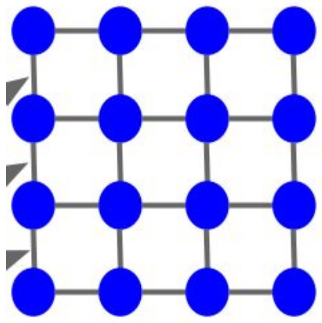


Over-segmenting images

- Graph-based clustering for Image Segmentation
  - Introduced by *Felzenszwalb and Huttenlocher* in the paper titled *Efficient Graph-Based Image Segmentation.*

Image as a Graph - Features and weights

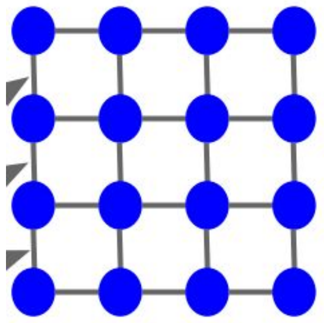
- Every pixel is connected to its 8 neighboring pixels
- The edges between neighbors have weights that are determined by the distance between them.
- Edge weights between pixels are determined using dist(x, x') distance in feature space.
  - where p and p' are two neighboring pixels



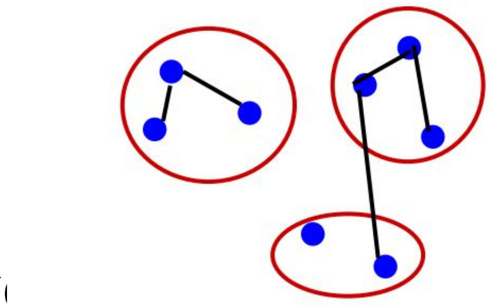
- Q. What is a good feature space?

What are good pixel features?

- Use RGB values?
  - v = [r, g, b]
  - It is 3-dimensional
- Use location?
  - v = [x, y]
  - 2-dim
- Use RGB + location?
  - v = [x, y, r, g, b]
  - 5-dim
- Use gradient magnitude?
  - v = [df/dx, df/dy]
  - 2-d



Problem Formulation



- Graph $G = (V, E)$
- V is set of nodes (i.e. pixels)
- E is a set of undirected edges between pairs of pix
- $dist(v_i, v_j)$ is the weight/distance of the edge between nodes v_i and v_j.

- S is a segmentation of a graph G such that $G' = (V, E')$ where $E' \subset E$.
  - That is, **we keep all vertices**, but **select a subset E'** from all initial edges E.
- S divides G into G' such that it contains distinct clusters C.

Weights of edges: distance measure

Clustering is an unsupervised learning method. Given items $v_1, v_2, \ldots, v_n \in \mathcal{R}^D$, the goal is to group them into clusters.

We need a pairwise distance/similarity function between items, and sometimes the desired number of clusters.

When data (e.g. images, objects, documents) are represented by feature vectors, commonly used measures are:

- *Euclidean distance.*
- *Cosine similarity.*

Defining Distance Measures

Let x and x' be two objects from the universe of possible objects.
The distance (or similarity) between x and x' is a real number:

- The Euclidean distance is defined as $$dist(v_1, v_2) = \sqrt{\sum_i (v_{1i} - v_{2i})^2}$$

- In contrast, the cosine similarity measure would be

$$\begin{aligned} dist(v_1, v_2) &= 1 - cos(v_1, v_2) \\ &= 1 - \frac{v_1^T v_2}{||v_1|| \cdot ||v_2||} \end{aligned}$$

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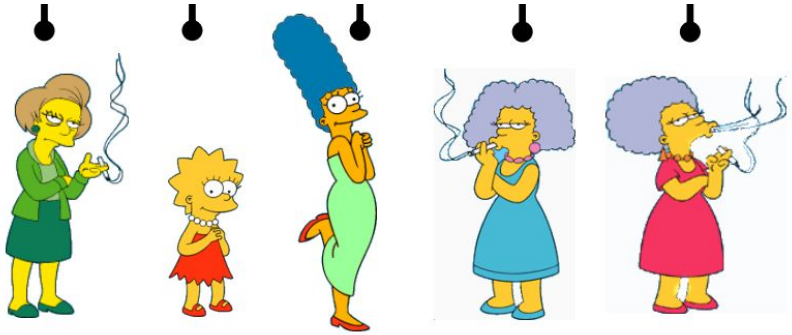
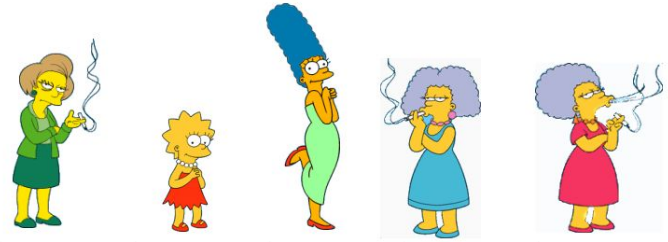
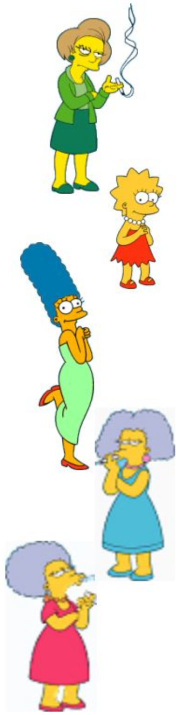
Animated example



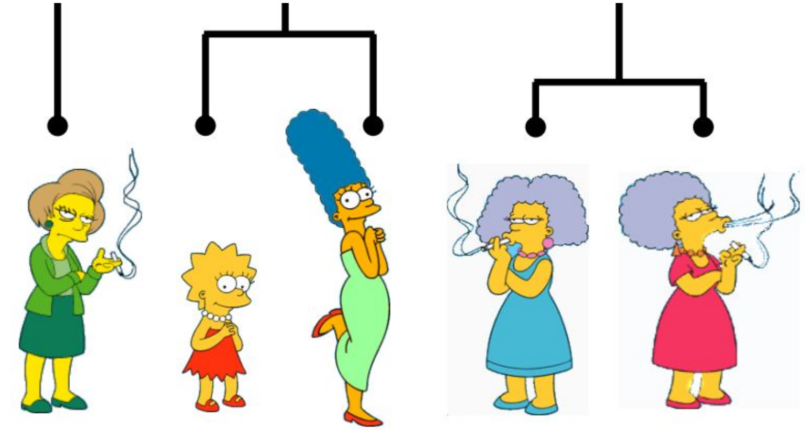


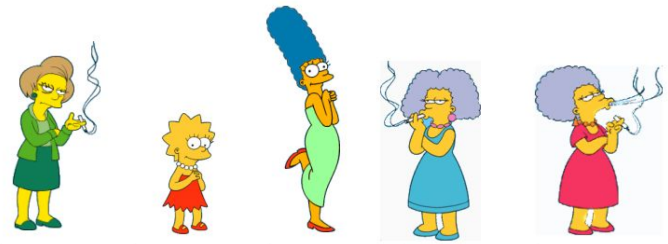


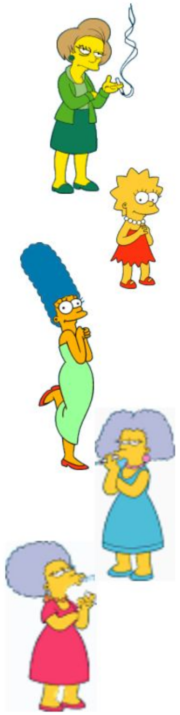
0	8	8	7	7
	0	2	4	4
		0	3	3
			0	1
				0

source

Animated example



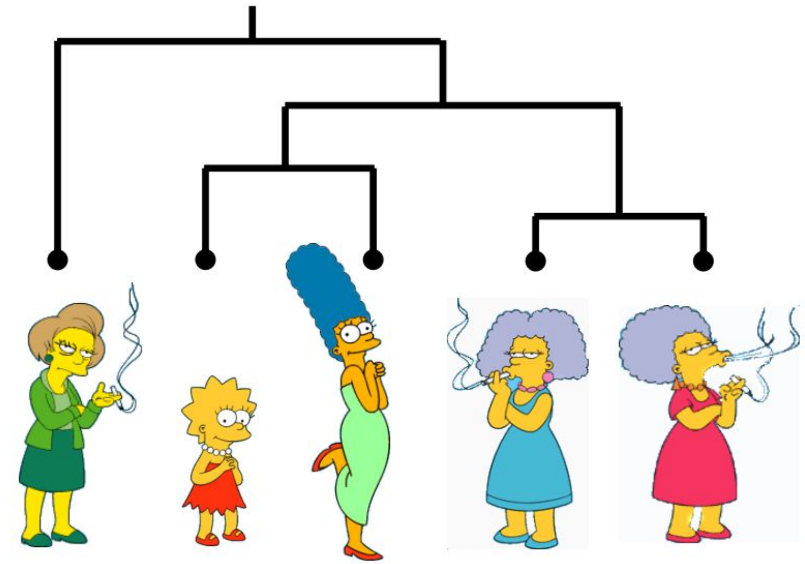


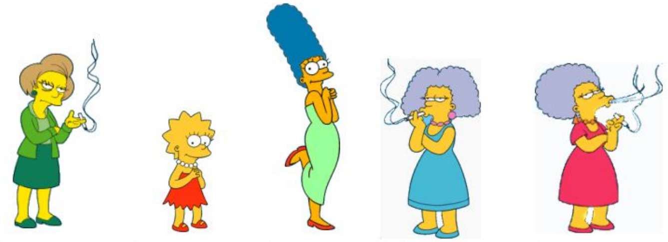


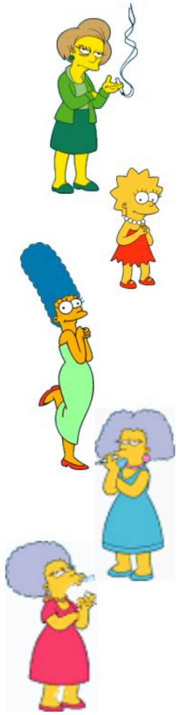
0	8	8	7	7
	0	2	4	4
		0	3	3
			0	1
				0

source

Animated example



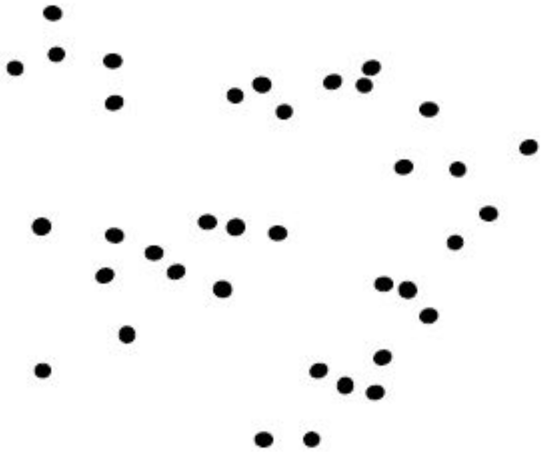




0	8	8	7	7
	0	2	4	4
		0	3	3
			0	1
				0

source

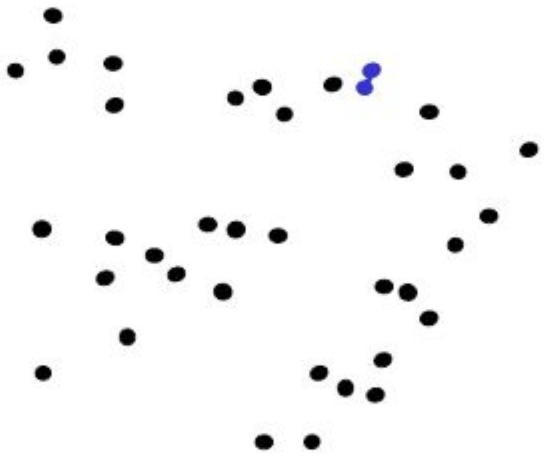
Agglomerative clustering



1. Say "Every point is its own cluster"

Slide credit: Andrew Moore

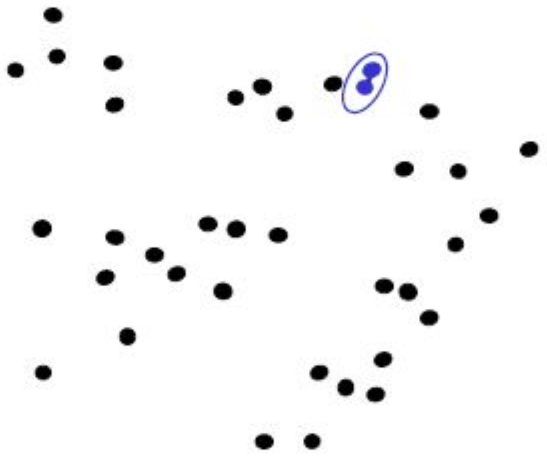
Agglomerative clustering



1. Say "Every point is its own cluster"
2. Find "most similar" pair of clusters

Slide credit: Andrew Moore

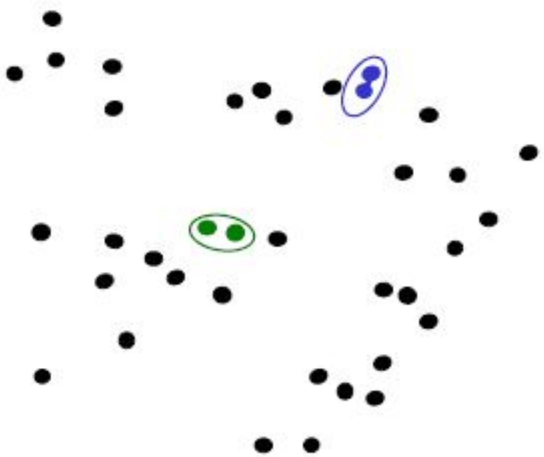
Agglomerative clustering



1. Say "Every point is its own cluster"
2. Find "most similar" pair of clusters
3. Merge it into a parent cluster

Slide credit: Andrew Moore

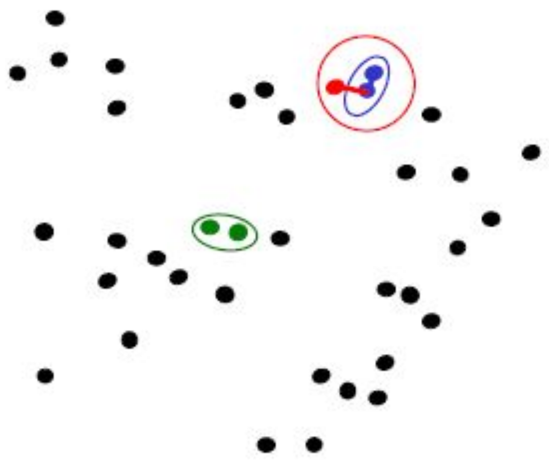
Agglomerative clustering



1. Say "Every point is its own cluster"
2. Find "most similar" pair of clusters
3. Merge it into a parent cluster
4. Repeat

Slide credit: Andrew Moore

Agglomerative clustering



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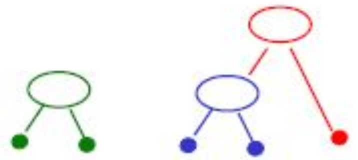
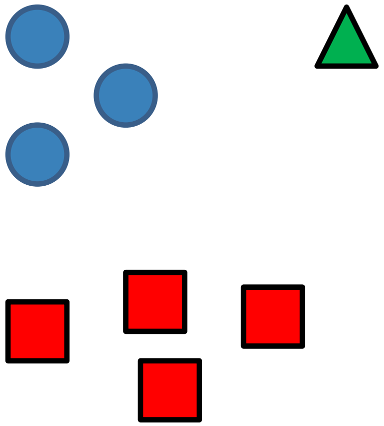


Slide credit: Andrew Moore

Agglomerative clustering

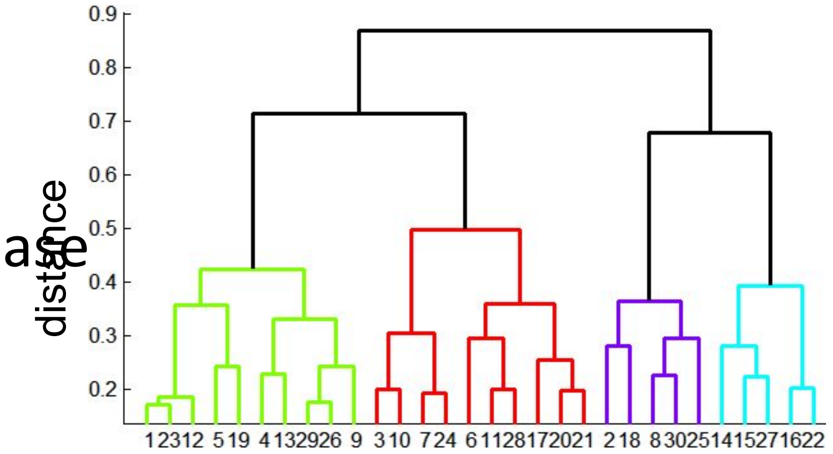
How to define cluster similarity?

- Average distance between all pixels between the two cluster?
- Maximum distance?
- Minimum distance?
- Distance between means?



How many clusters?

- Clustering creates a dendrogram (a tree)
- Threshold based on max number of clusters or based on distance between merges



Agglomerative Hierarchical Clustering - Algorithm

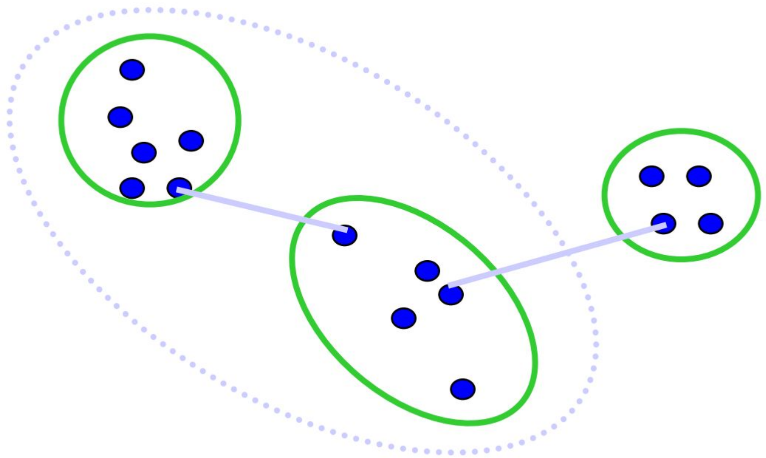
Inputs:
- An input image
- Feature representation for each pixel
- Distance metric dist(-,-)

- ➢ Initially, each pixel $v_1, \ldots, v_n$ is its own cluster $C_1, \ldots, C_n$
- ➢ While True:
  - ○ Find two nearest clusters according to dist(C_i, C_j)
  - ○ Merge C = (C_i, C_j)
  - ○ If only 1 cluster is left:
    - ■ break

Different measures of nearest clusters

Single Linkage: $dist(C_i, C_j) = \min_{v_i \in C_i, v_j \in C_j, (C_i, C_j) \in E} dist(v_i, v_j)$

Connects the clusters based on the distance of their closest pixels
It produces “long” clusters.

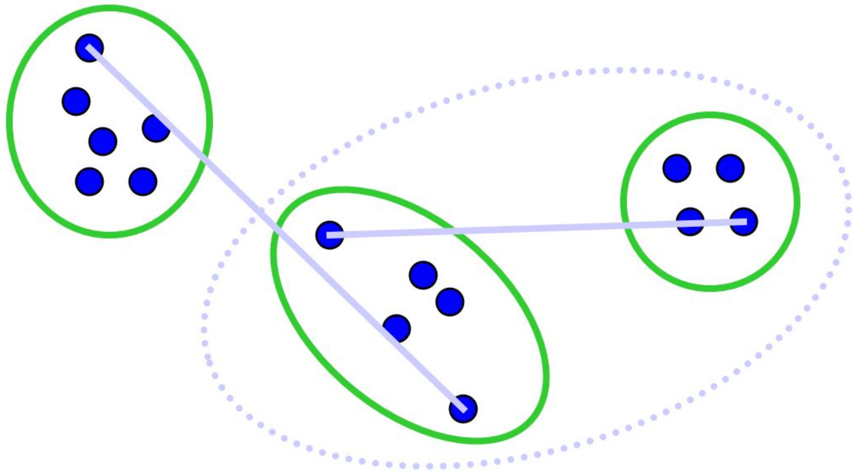


Long, skinny clusters

Different measures of nearest clusters

Complete Link: $$dist(C_i, C_j) = \max_{v_i \in C_i, v_j \in C_j, (C_i, C_j) \in E} dist(v_i, v_j)$$

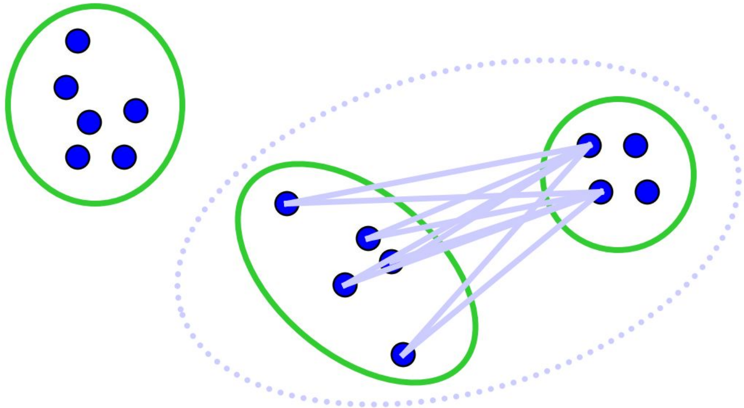
Produces compact clusters that are similar in diameter



Tight clusters

Different measures of nearest clusters

Average Link: $$dist(C_i, C_j) = \frac{\sum_{v_i \in C_i, v_j \in C_j, (C_i, C_j) \in E} dist(v_i, v_j)}{|C_i||C_j|}$$

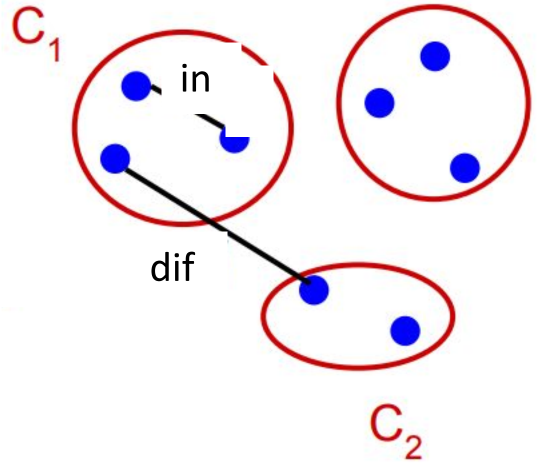


Robust against noise.

Different measures of nearest clusters

Inlier-outlier linkage:

$$Merge(C_1, C_2) = \begin{cases} True & if\ dif(C_1, C_2) < in(C_1, C_2) \\ False & otherwise \end{cases}$$



Where

- dif(C1 , C2) is the difference between two clusters.
- in(C1 , C2) is the internal difference in the clusters C1 and C2

Different measures of nearest clusters

Inlier-outlier linkage:

$$Merge(C_1, C_2) = \begin{cases} True & if\ dif(C_1, C_2) < in(C_1, C_2) \\ False & otherwise \end{cases}$$

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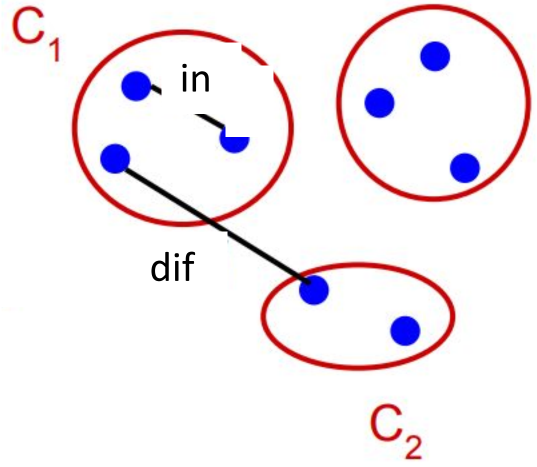

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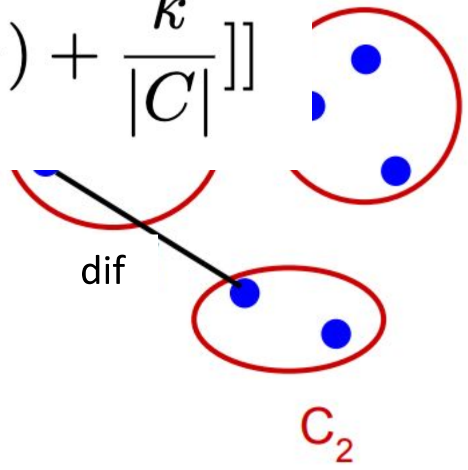
$$dif(C_i, C_j) = \min_{v_i \in C_i, v_j \in C_j, (C_i, C_j) \in E} dist(v_i, v_j)$$

$$in(C_i, C_j) = \min_{C \in \{C_i, C_j\}} [\max_{v_i, v_j \in C} [dist(v_i, v_j) + \frac{k}{|C|}]]$$

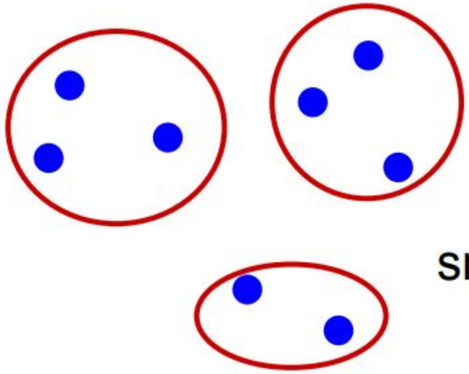


Where

- dif(C1 , C2) is the difference between two clusters.
- in(C1 , C2) is the internal difference in the clusters C1 and C2

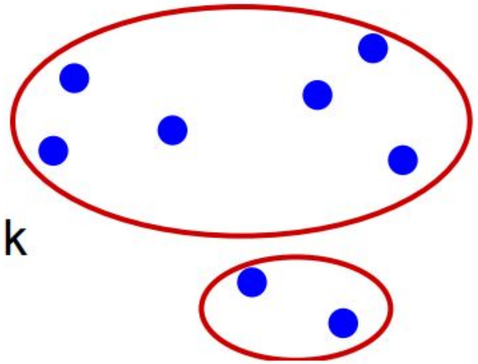
inlier-outlier linkage for Segmentation

- k/|C| sets the threshold by which the clusters need to be different from the internal pixels in a cluster.
- Effect of k:
  - **If k is large, it causes a preference for larger objects.**



small k

large k



Results

Conclusions: Agglomerative Clustering

Pros:

- Simple to implement, widespread application.
- Clusters have adaptive shapes.
- Provides a hierarchy of clusters.
- No need to specify number of clusters **in advance**.

Cons:

- May have imbalanced clusters.
- Still have to choose number of clusters eventually for an application
- Does not scale well. Runtime of $O(n^3)$.
- Can get stuck at a local optima.

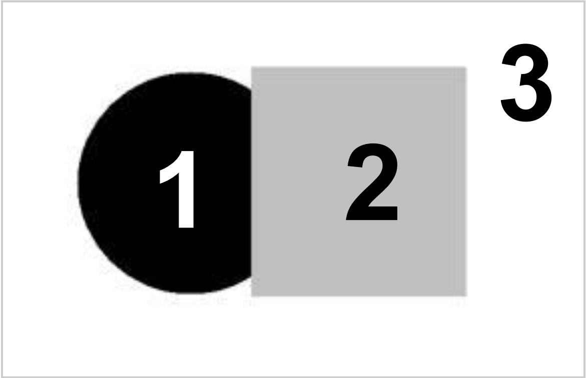
Today's agenda

- Introduction to segmentation and clustering
- Gestalt theory for perceptual grouping
- Graph-based oversegmentation
- Agglomerative clustering
- K-means clustering
- Mean-shift clustering

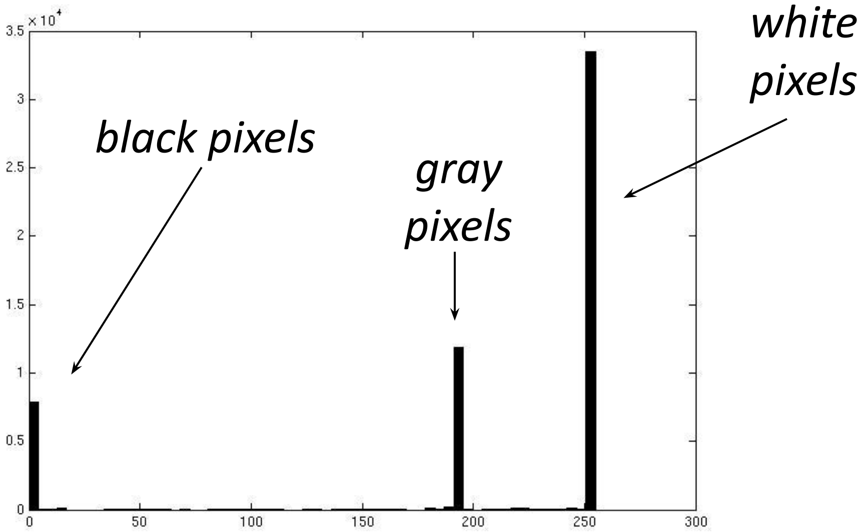
Reading: Szeliski Chapters: 5.2.2, 7.5.2

D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.

Image Segmentation: Binary image Example

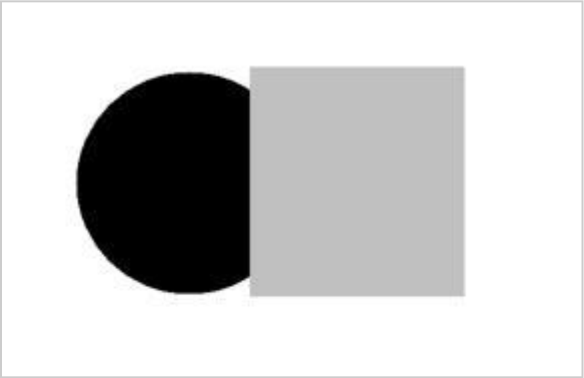


input image

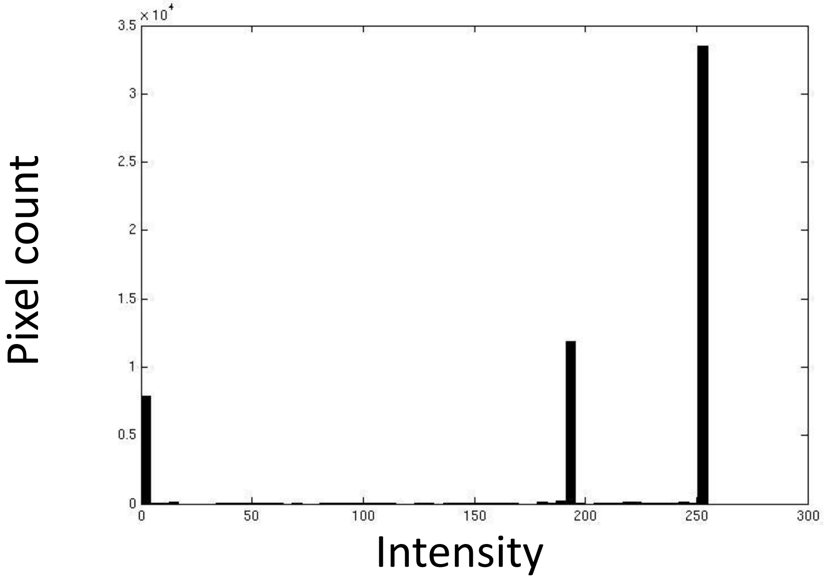


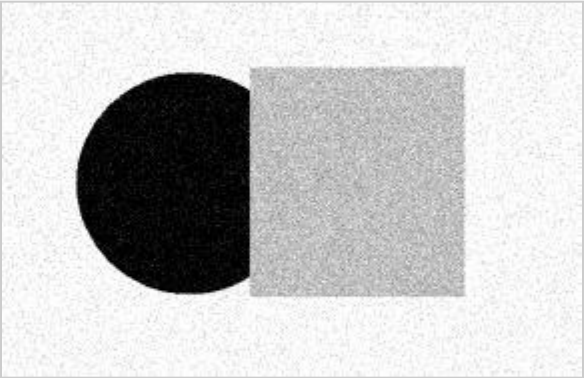
intensity

- These pixel values show that there are three things in the image.
- We could label every pixel in the image according to which of these primary intensities it is.
  - i.e., segment the image based on the intensity feature.
- What if the image isn't quite so simple?

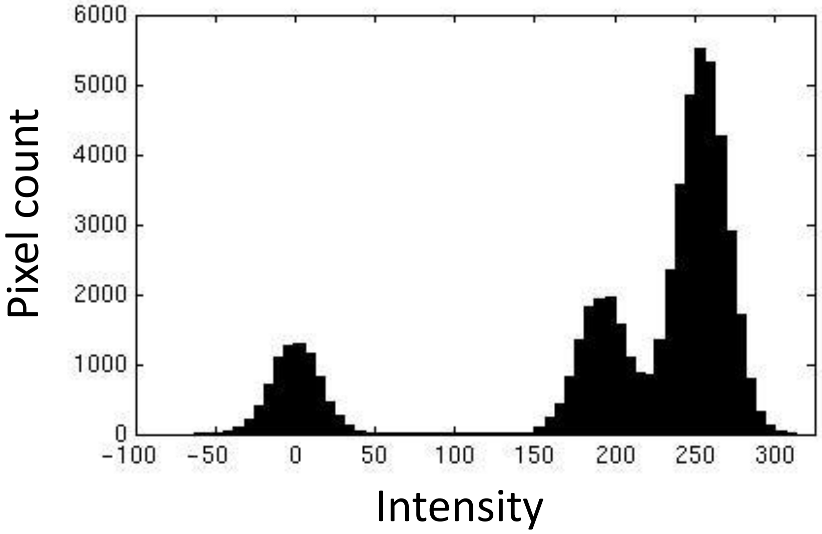


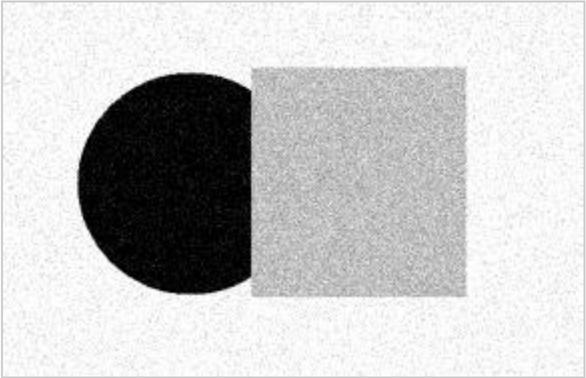
Input image



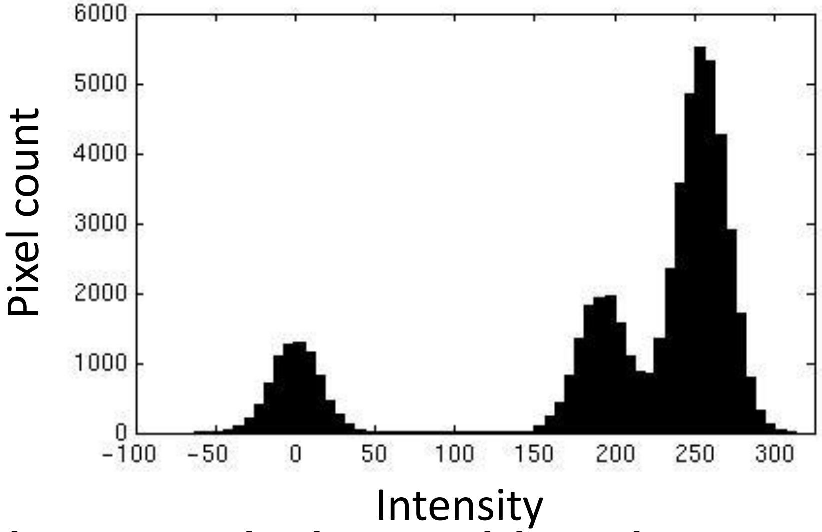


Input image

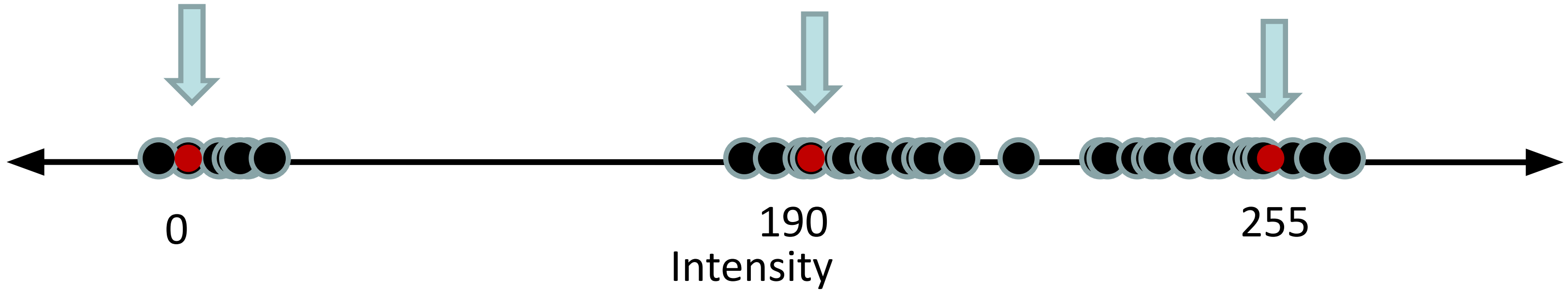


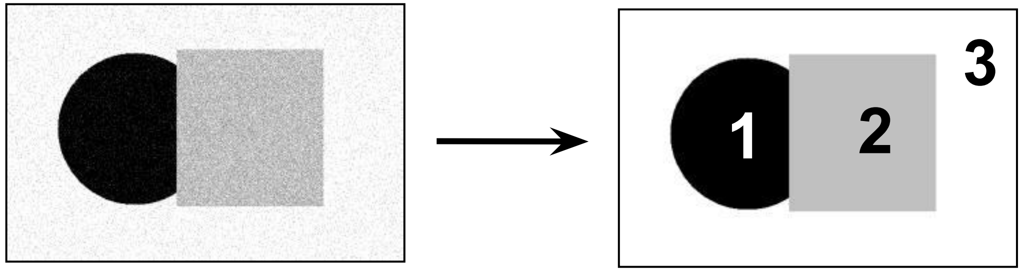


Input image



- How do we determine the three main intensities that define our groups?
- Each cluster has a cluster center
  - A mean cluster value.





- Goal: choose three "centers" as the representative intensities and label every pixel according to which of these centers it is nearest to.

- Best cluster centers are those that minimize **Sum of Square Distance** (SSD) between all points and their nearest cluster center c_i:

$$SSD = \sum_{C} \sum_{v \in C} (v - c_i)^2$$

Clustering

Goal: cluster to minimize distance of pixels to their cluster centers

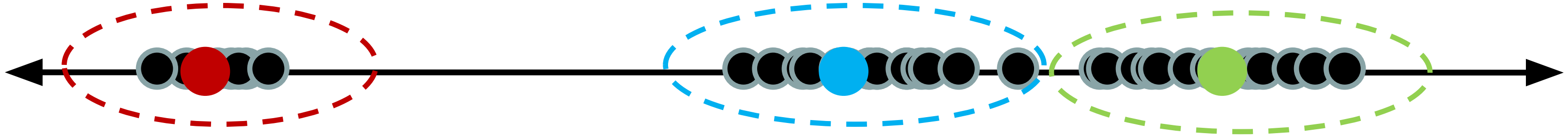
Cluster center Data

$$c^*, \delta^* = \arg\min_{c,\delta} \sum_j^N \sum_i^N \delta_{ij}(c_i - v_j)^2$$

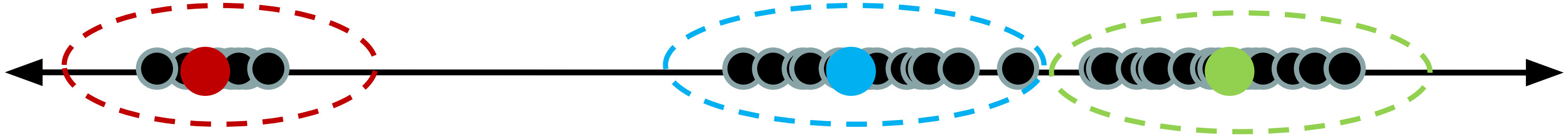
Whether v_j is assigned to c_i

Clustering

- With this objective, it is a "chicken and egg" problem:
  - If we knew the *cluster centers*, we could allocate points to groups by assigning each to its closest center.



  - If we knew the *group memberships*, we could get the centers by computing the mean per group.



K-means clustering

1. Initialize ($t = 0$): cluster centers $c_1, ..., c_K$

K-means clustering

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2. Compute δ^t : assign each point to the closest center
   - δ^t denotes the set of assignment for each v_j to cluster c_i at iteration *t*

$$\delta^t = \arg\min_{\delta} \frac{1}{N} \sum_{j}^{N} \sum_{i}^{K} \delta_{ij}^{t-1} (c_i^{t-1} - v_j)^2$$

Slide: Derek Hoiem

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3. Computer c^t : update cluster centers as the mean of the points

$$c^t = \arg\min_{c} \frac{1}{N} \sum_{j}^{N} \sum_{i}^{K} \delta_{ij}^{t} (c_i^{t-1} - v_j)^2$$

Slide: Derek Hoiem

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4. Update $t = t + 1$, Repeat Step 2-3 till stopped

Slide: Derek Hoiem

K-means clustering

1. Initialize ($t=0$): cluster centers $c_1,...,c_K$

2. Compute δ^t : assign each point to the closest center
   - δ^t denotes the set of assignment for each v_j to cluster c_i at iteration *t*

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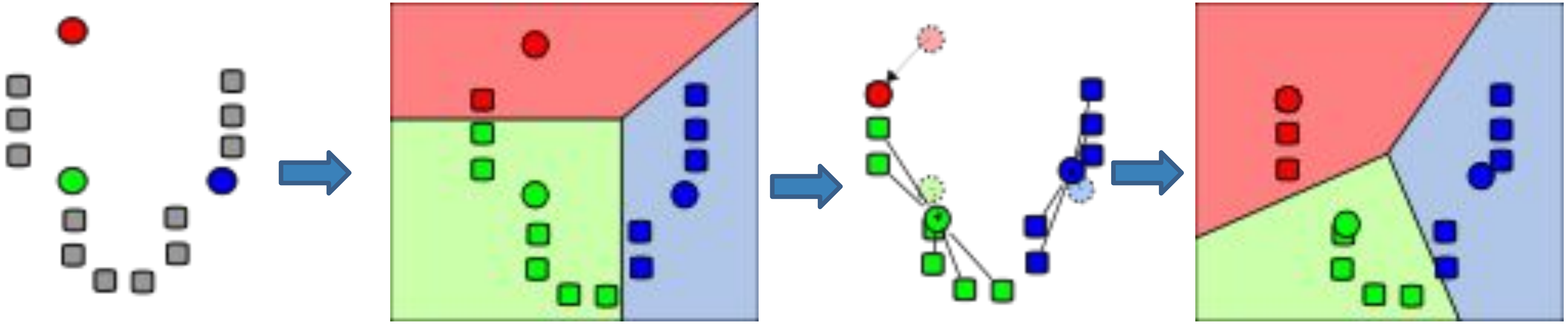
Slide: Derek Hoiem

K-means clustering



1. Initialize Cluster Centers
2. Assign Points to Clusters
3. Re-compute Means

Repeat (2) and (3)

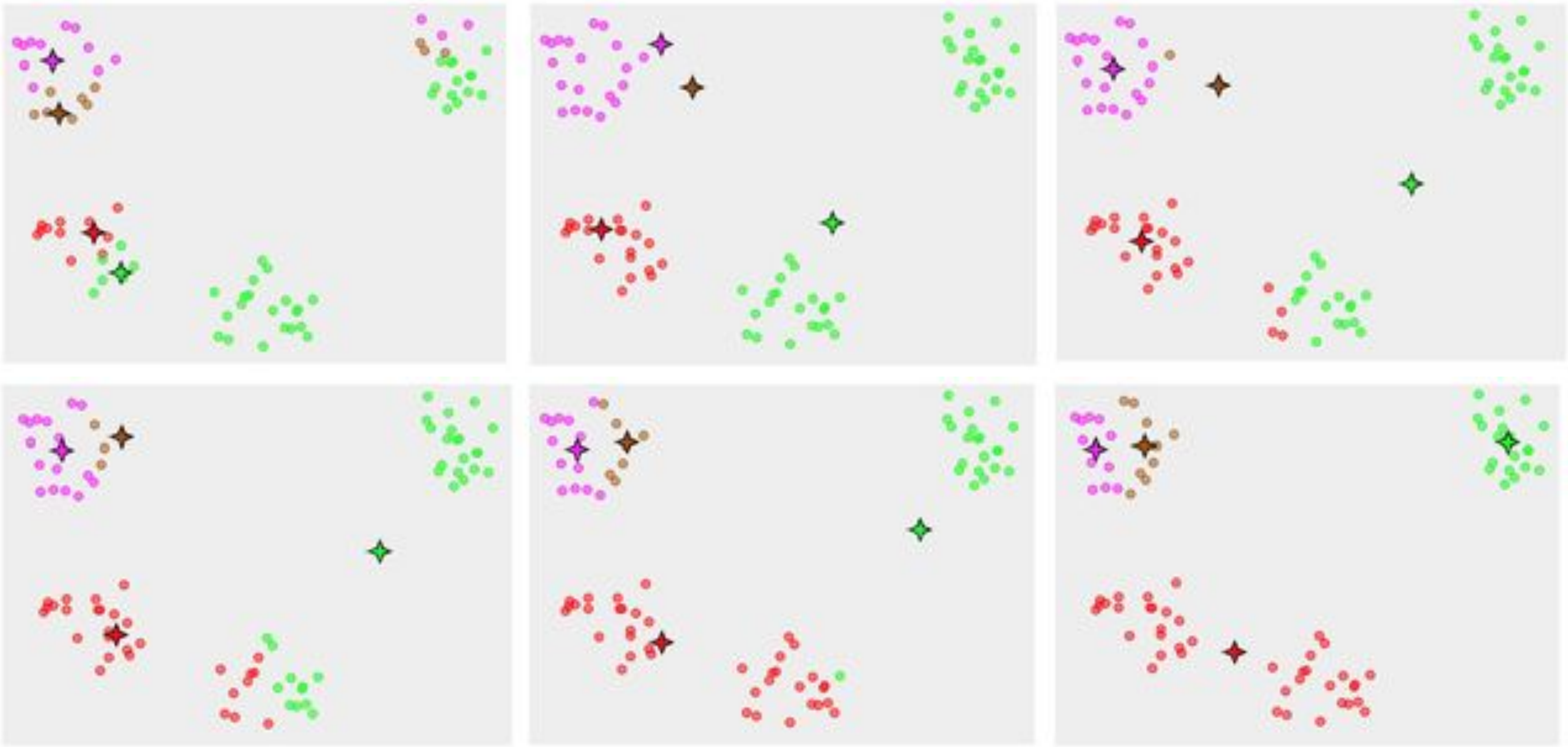
K-means clustering

Initial cluster centers are randomly initialized

- Can lead to bad initializations
- Can cause bad clusters

K-means Converges to a local minimum solution

Initialize multiple runs!



K-Means++

Tries to prevent arbitrarily bad local minima?

1. Randomly choose first center.
2. Pick new center with prob. proportional to $(c_i - v_j)^2$
    a. Basically we want to find as good of an initialization as possible
3. Repeat until *K* centers.

K-means clustering

Initial cluster centers are randomly initialized

- Can lead to bad initializations
- Can cause bad clusters

Different distance measures can change K-Means clusters

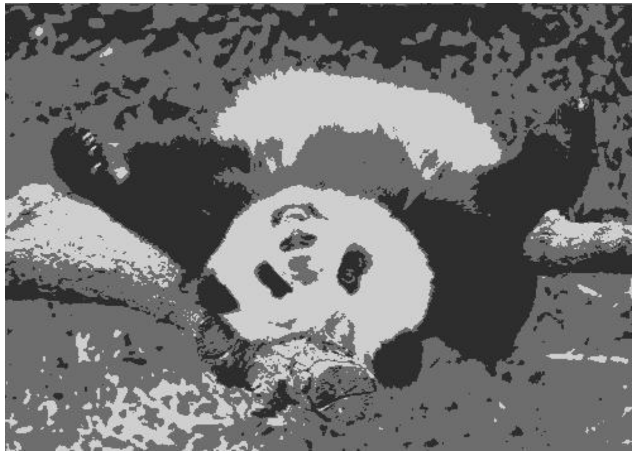
- Euclidean distance of cosine distance.

Different feature space can lead to different cluster

Segmentation as Clustering

Original image

2 clusters



3 clusters

Feature Space: pixel value

- Feature space: what measurements do we include in x_i?
- Depending on what we choose as the *feature space*, we can group pixels in different ways.

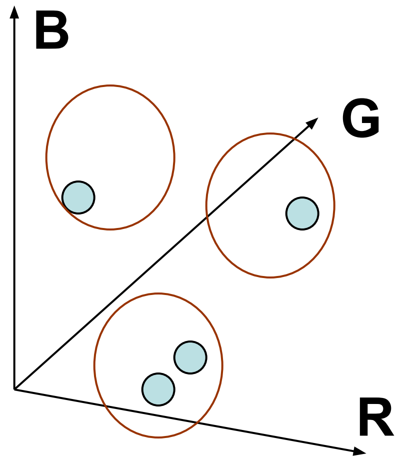
- Grouping pixels based on intensity similarity

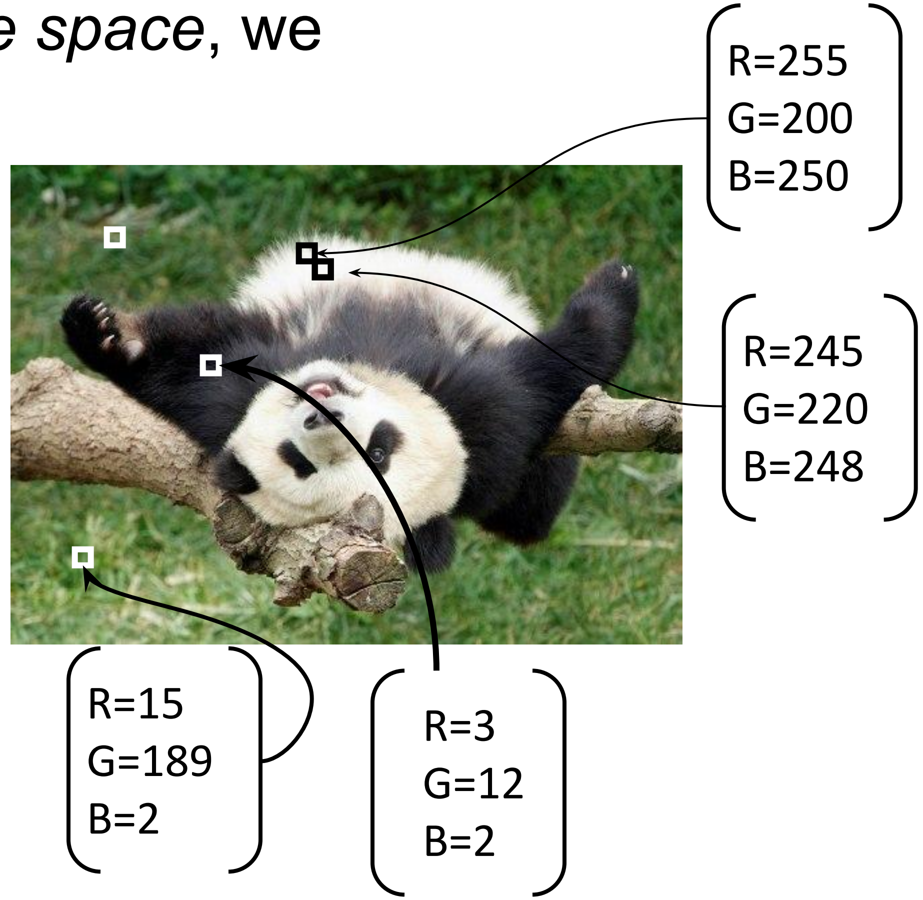
- Feature space: intensity value (1D)

Feature Space: RGB

- Depending on what we choose as the *feature space*, we can group pixels in different ways.

- Grouping pixels based on color similarity



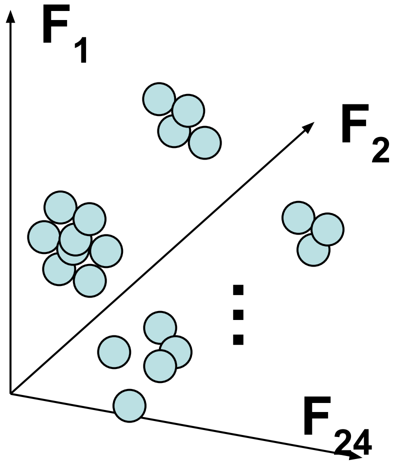


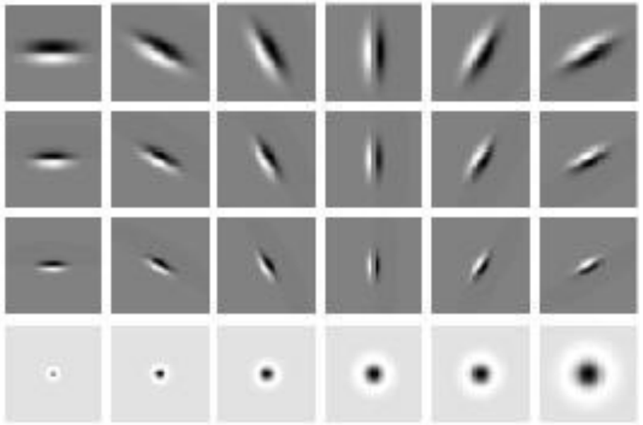
- Feature space: color value (3-dim)

Feature Space: edges and blobs

- Depending on what we choose as the *feature space*, we can group pixels in different ways.

- Grouping pixels based on oriented gradient similarity



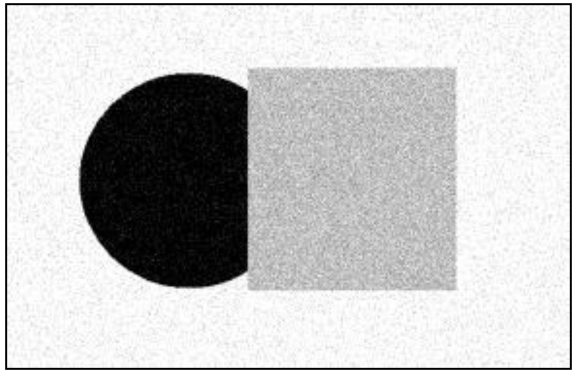


24 edge & blog filters

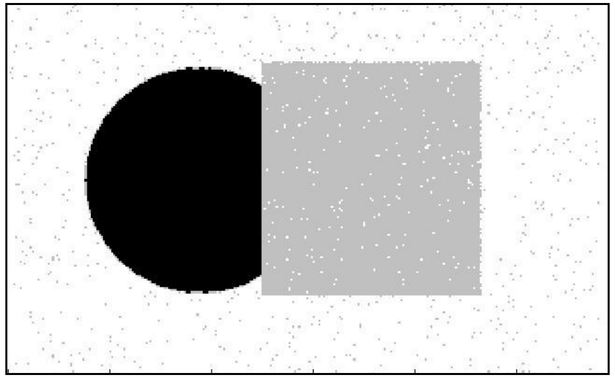
- Feature space: filter bank responses (e.g., 24D)

Smoothing Out Cluster Assignments

- Assigning a cluster label per pixel may yield outliers:

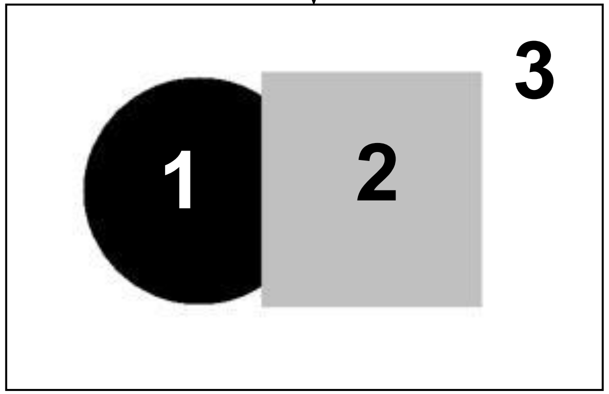


Original



Labeled by cluster center's intensity

?

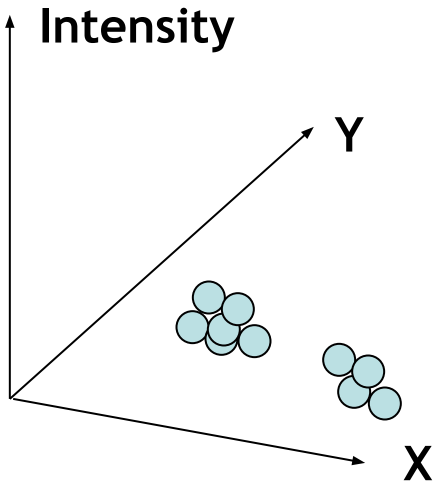


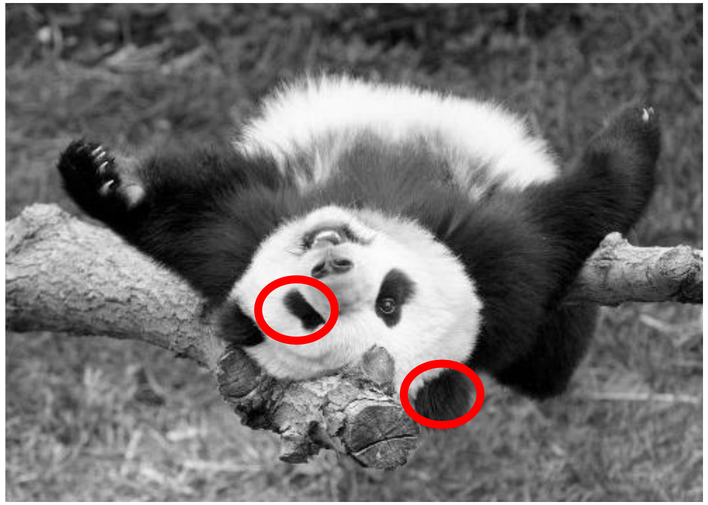
- How can we ensure they are spatially smooth?

Feature Space: RGB + XY location

- Depending on what we choose as the *feature space*, we can group pixels in different ways.

- Grouping pixels based on *intensity+position* similarity





⇒ Way to encode both *similarity* and *proximity.*

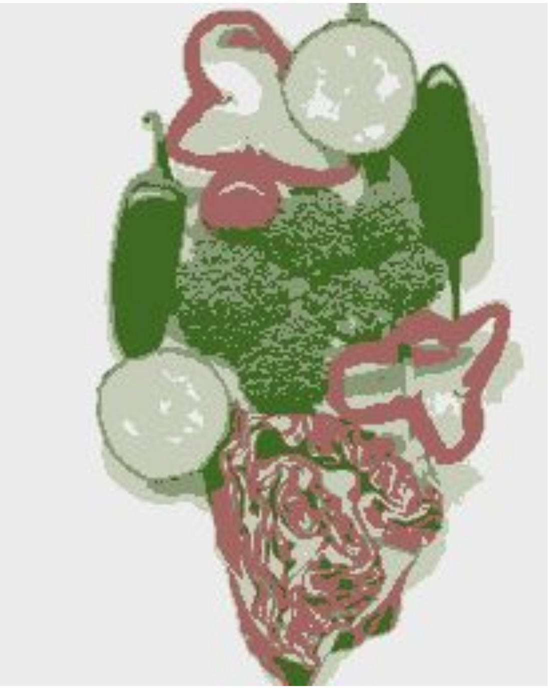
K-Means Clustering Results

- Clusters don’t have to be spatially coherent

Image

grayscale clusters

Color-based clusters



K-Means Clustering Results

- Clustering based on (r,g,b,x,y) values enforces more spatial coherence

How to evaluate clusters?

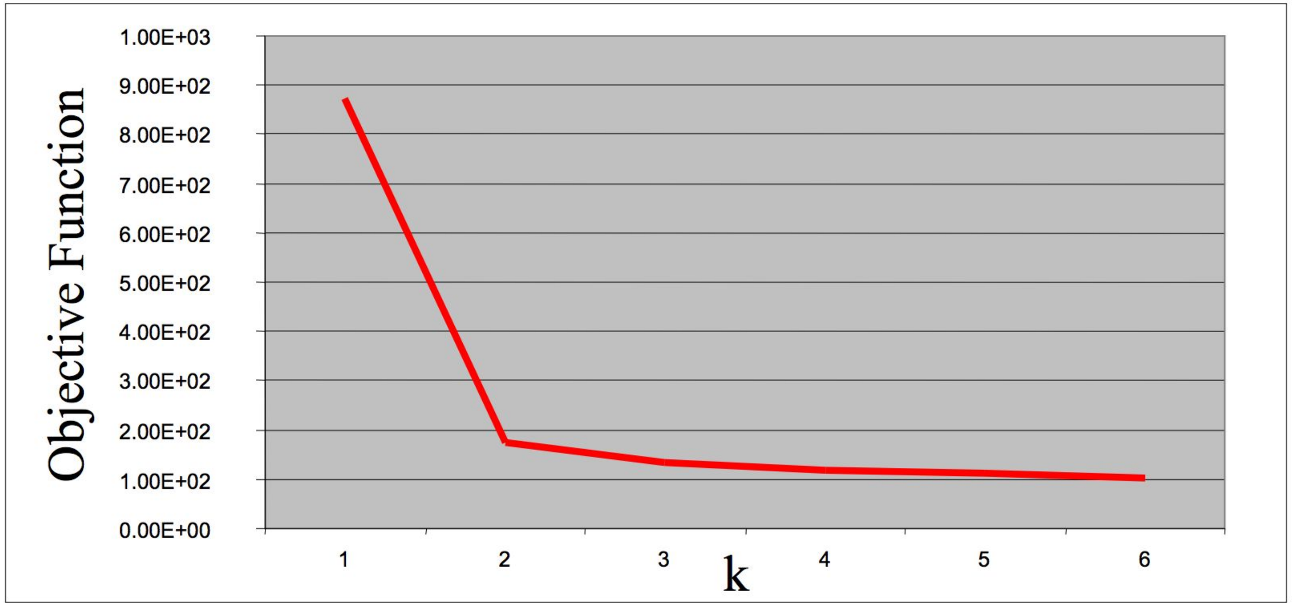
- **Generative**
  - How well are points reconstructed from the clusters?
- **Discriminative**
  - How well do the clusters correspond to labels?
    - Can we correctly classify which pixels belong to the panda?
  - Note: unsupervised clustering does not aim to be discriminative as we don’t have the labels.

How to choose the number of clusters?

Try different numbers of clusters in a validation set and look at performance.

Plot of SSD versus values of k

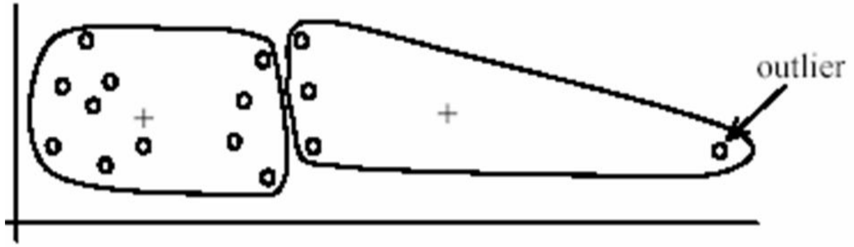
abrupt change at k=2 is suggestive of two clusters in the data

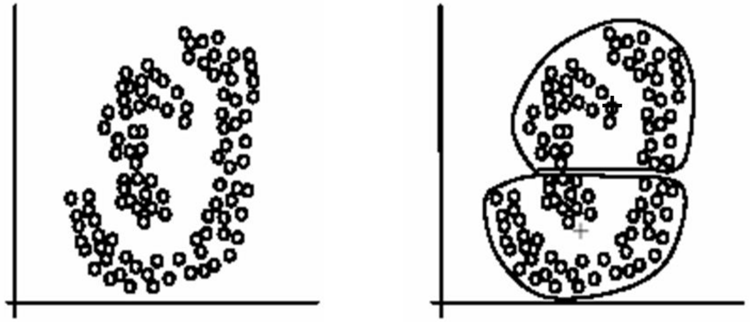


K-Means pros and cons

- **Pros**
  - Good representation of data
  - Simple and fast, Easy to implement
- **Cons**
  - Need to choose K
  - Sensitive to outliers
  - Prone to local minima
  - All clusters have the same parameters (e.g., distance measure is non-adaptive)
  - Can still be slow: each iteration is O(KNd) for N d-dimensional pixels

(B): Ideal clusters





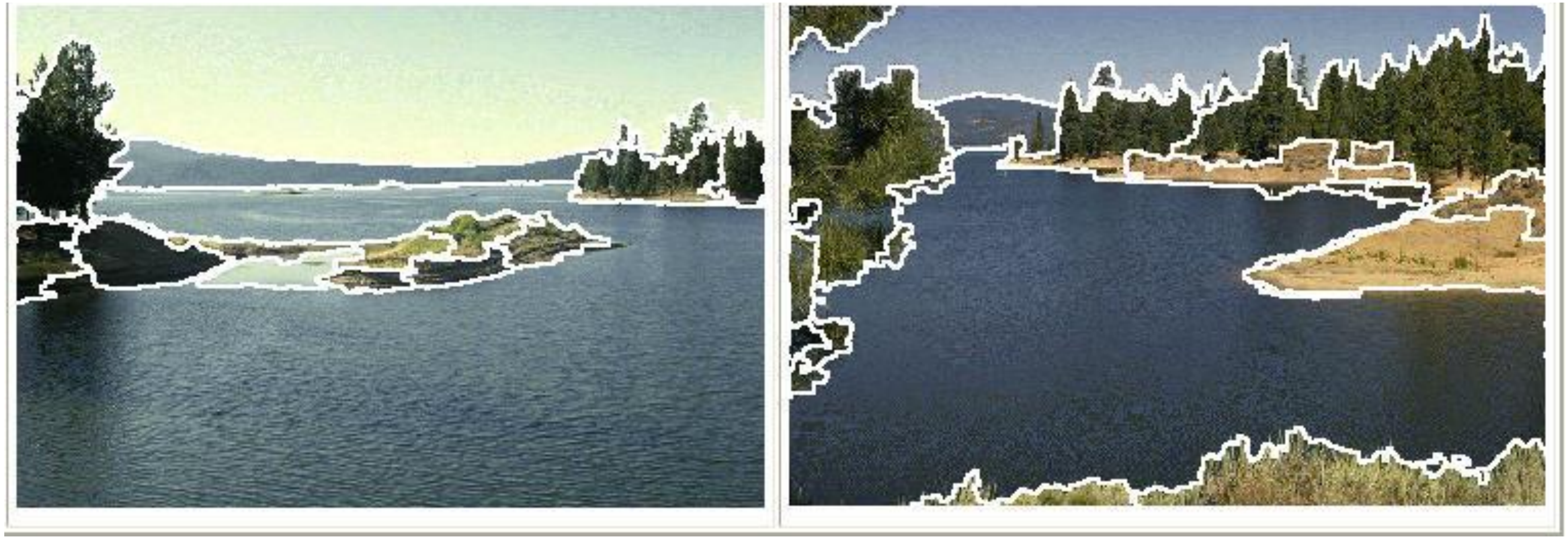
(A): Two natural clusters (B): *k*-means clusters

What will we learn today?

- Introduction to segmentation and clustering
- Gestalt theory for perceptual grouping
- Graph-based oversegmentation
- Agglomerative clustering
- K-means clustering
- Mean-shift clustering

Mean-Shift Segmentation

- An advanced and versatile technique for clustering-based segmentation



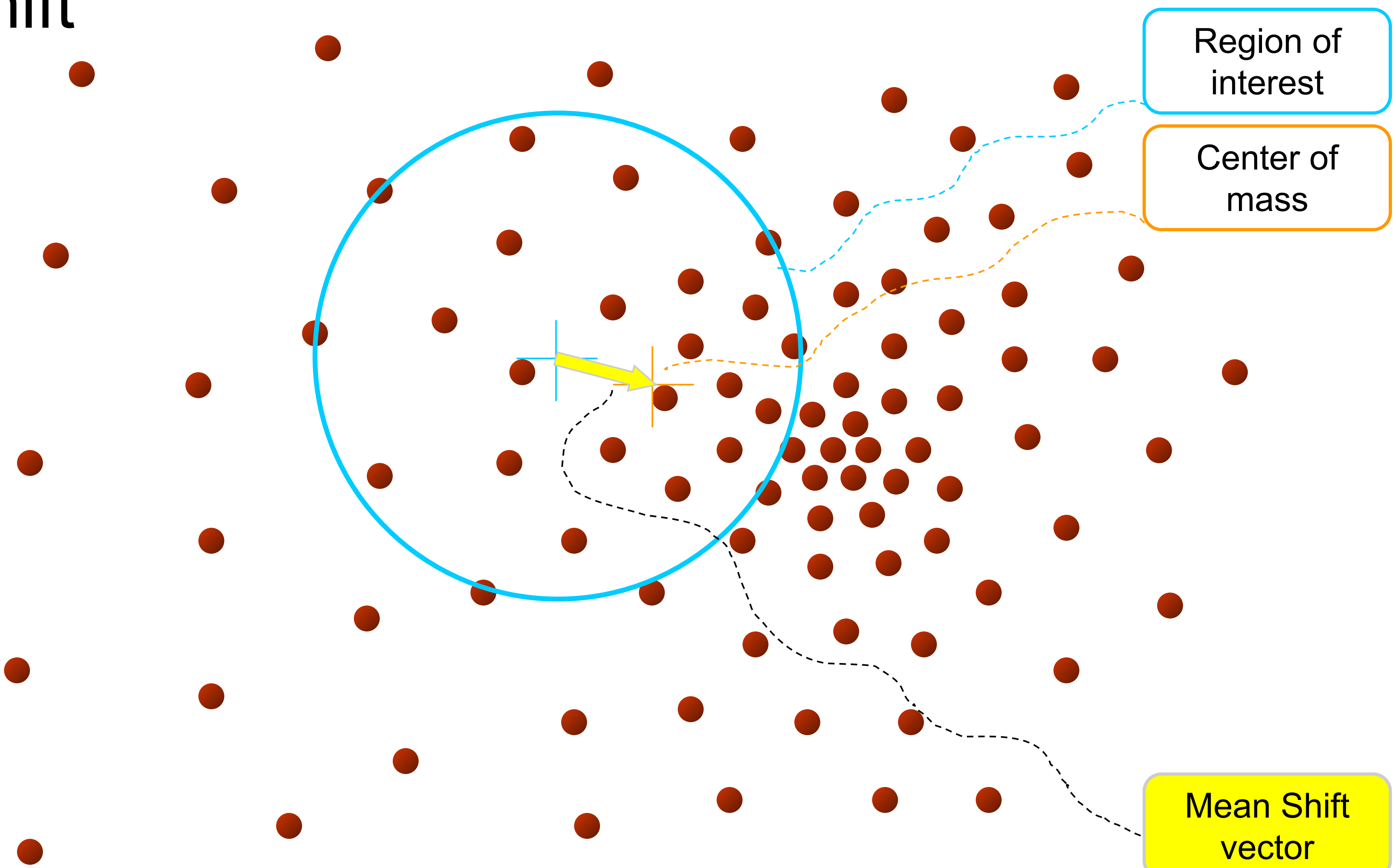
http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html

D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.

Mean-Shift Algorithm

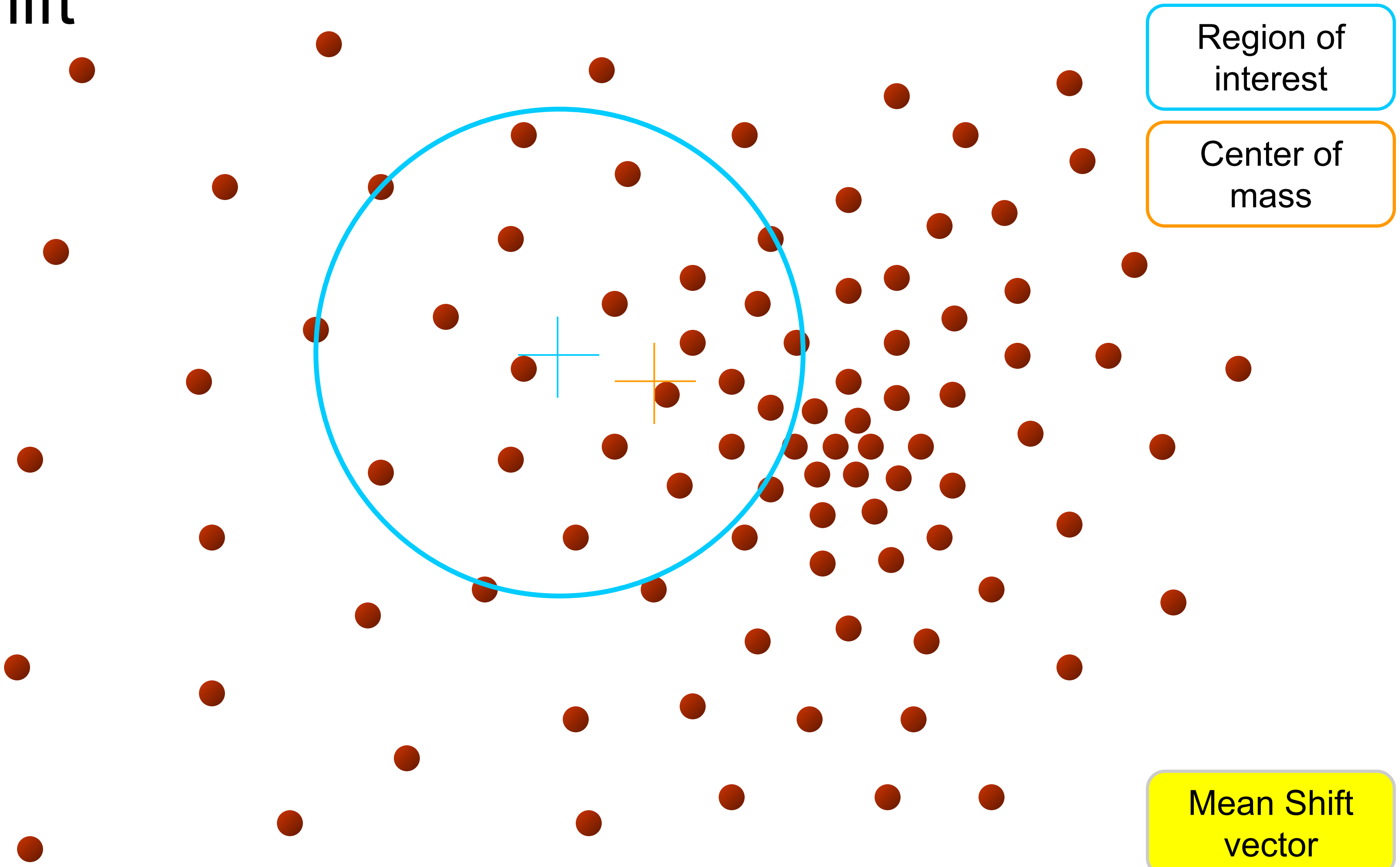
1. Represent each pixel *i* using some feature vector v_i
2. Generate a window **W** as a random pixel feature v_w
3. Identify all the pixels within a radius *r* of v_w
4. Calculate the mean ("center of gravity") amongst the neighbors of **W**
5. Translate the window **W** to the mean feature location
6. Repeat Step 2 until convergence

Mean-Shift



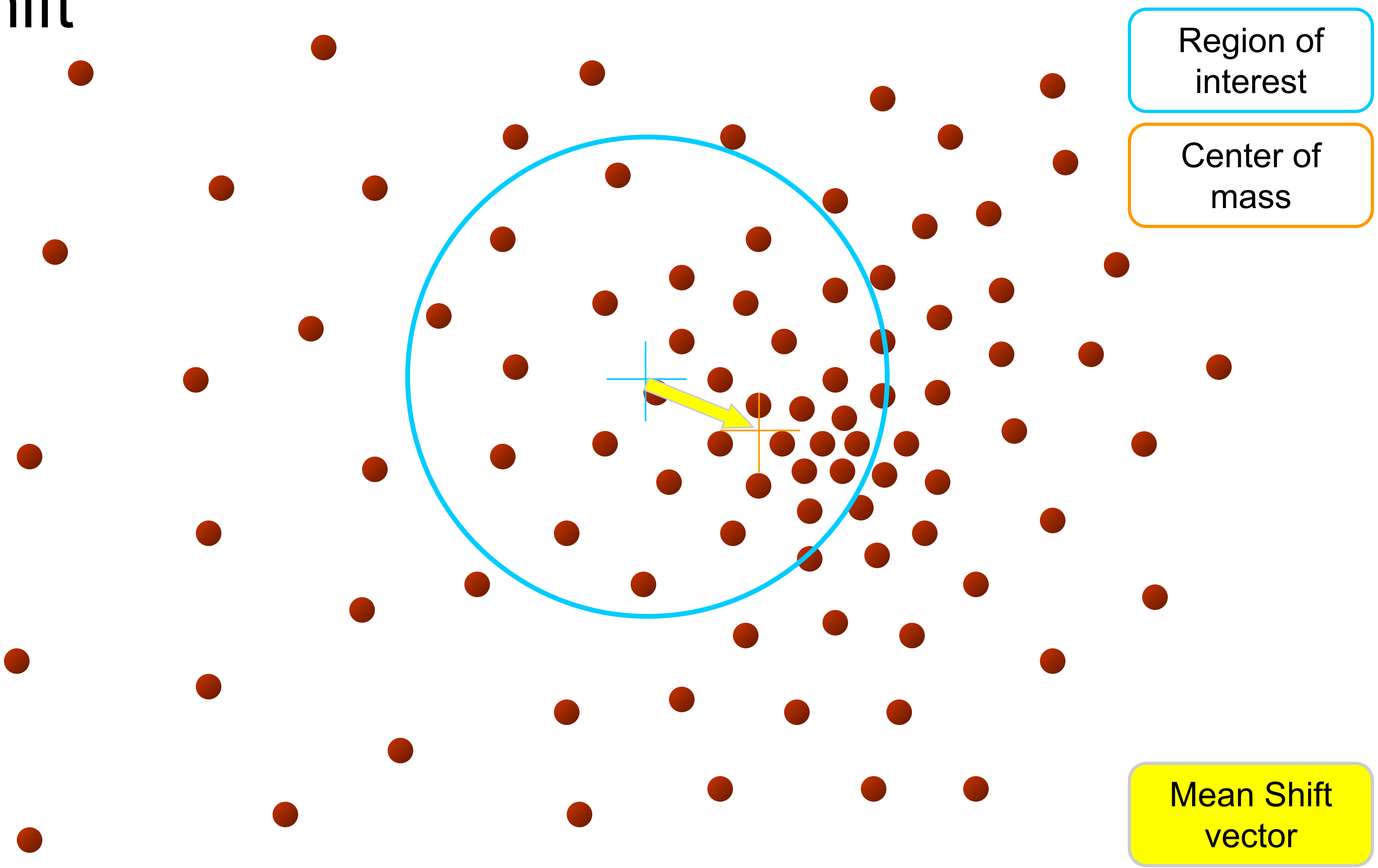
Slide by Y. Ukrainitz & B. Sarel

Mean-Shift



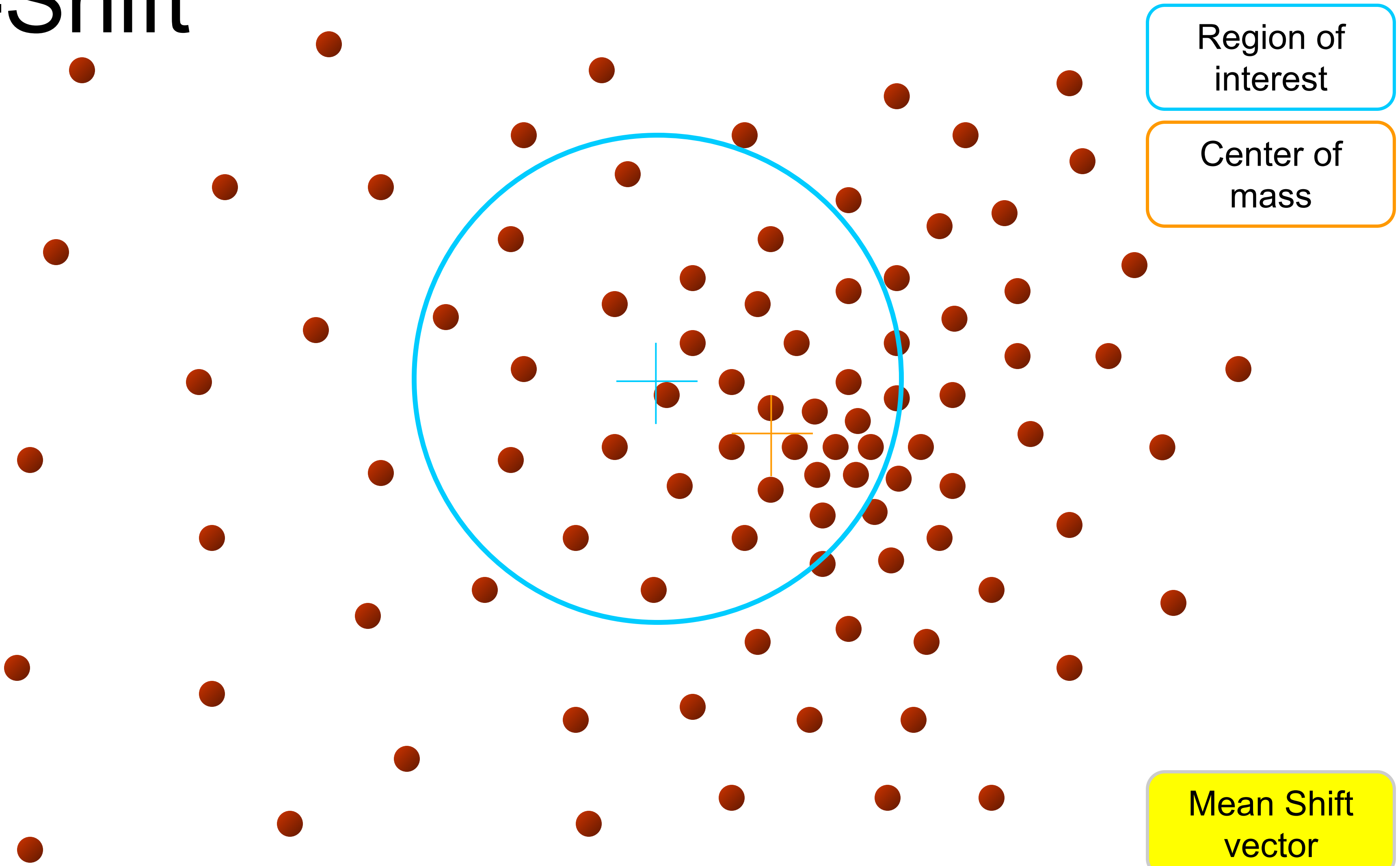
Slide by Y. Ukrainitz & B. Sarel

Mean-Shift



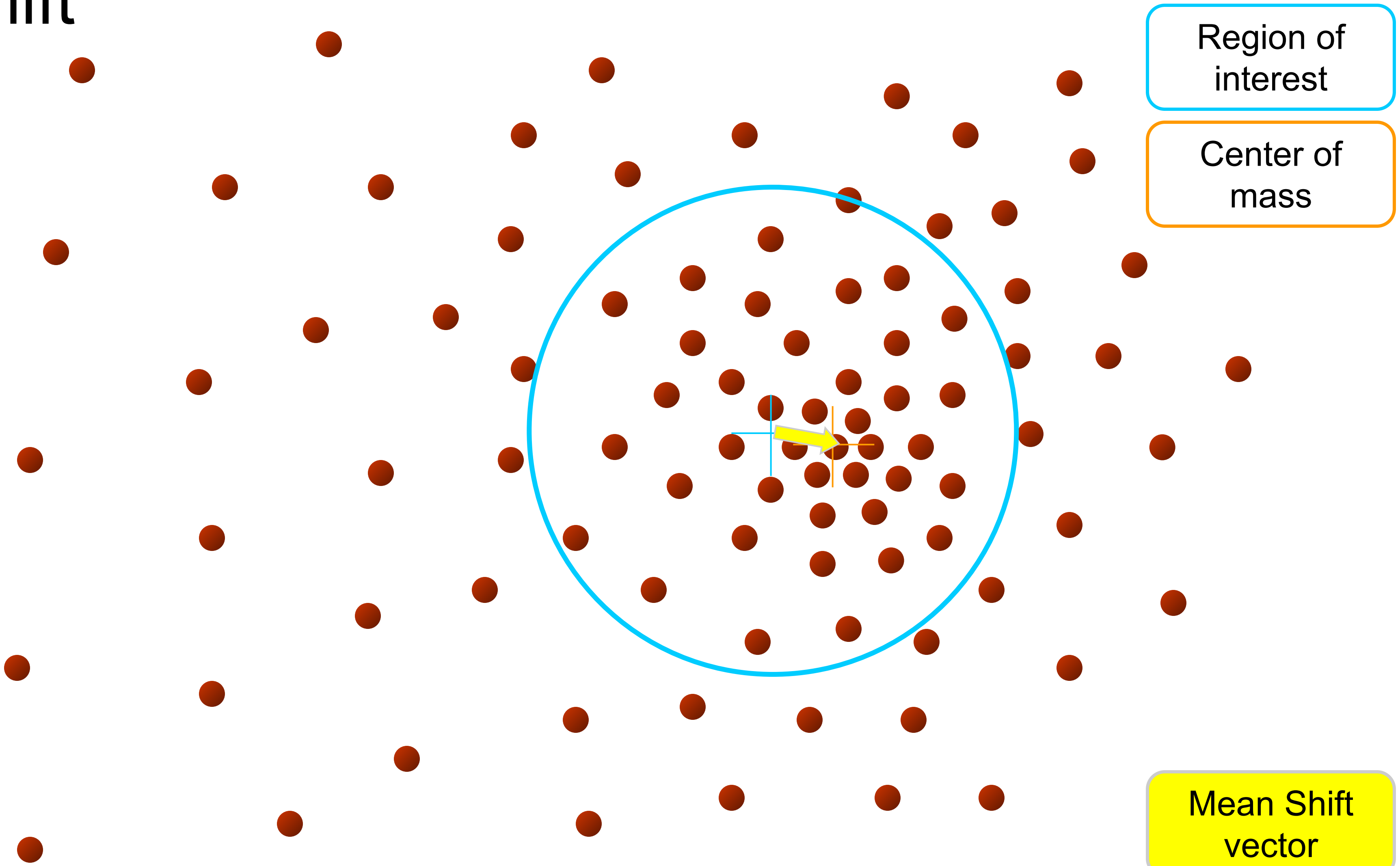
Slide by Y. Ukrainitz & B. Sarel

Mean-Shift



Slide by Y. Ukrainitz & B. Sarel

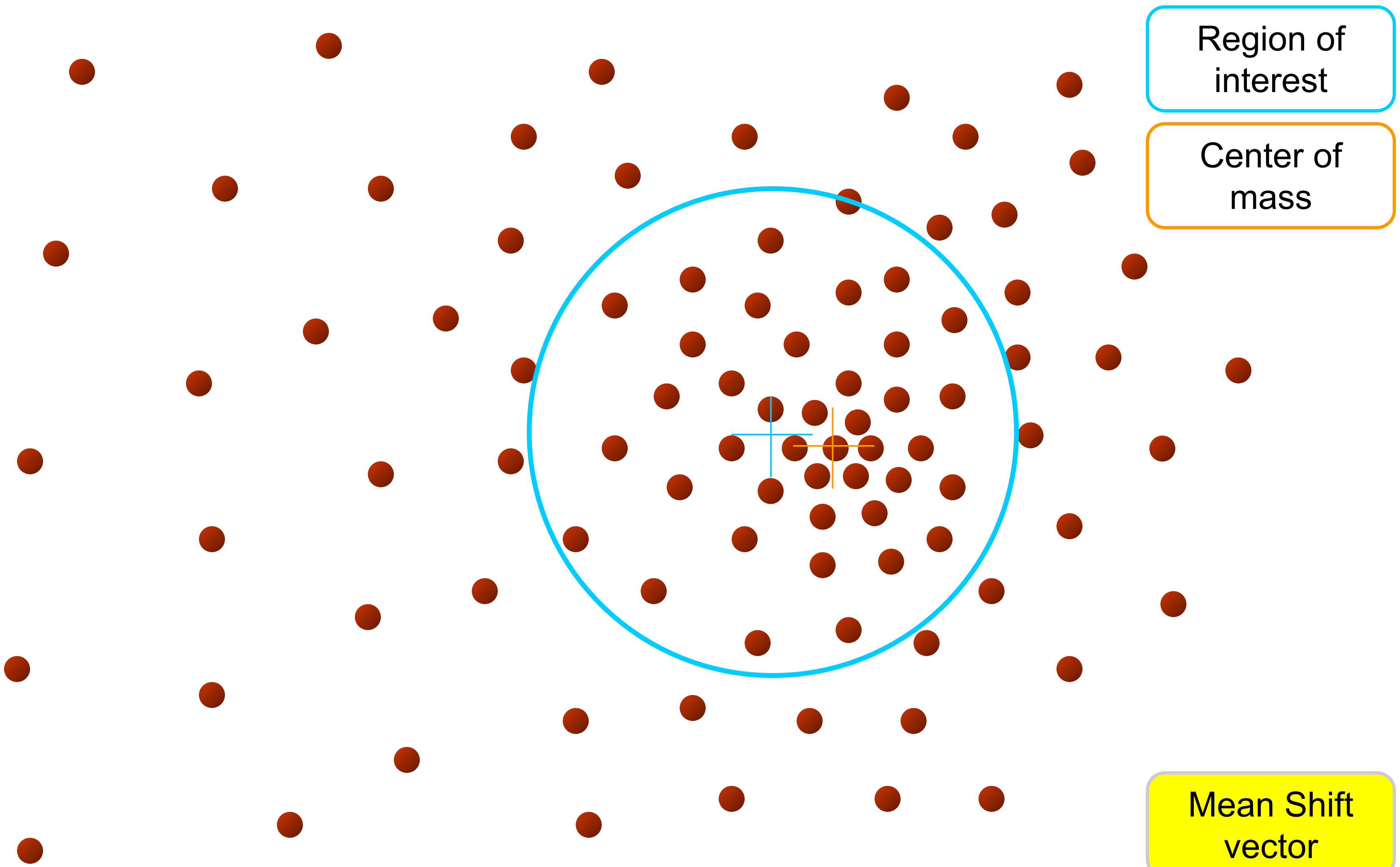
Mean-Shift



Slide by Y. Ukrainitz & B. Sarel

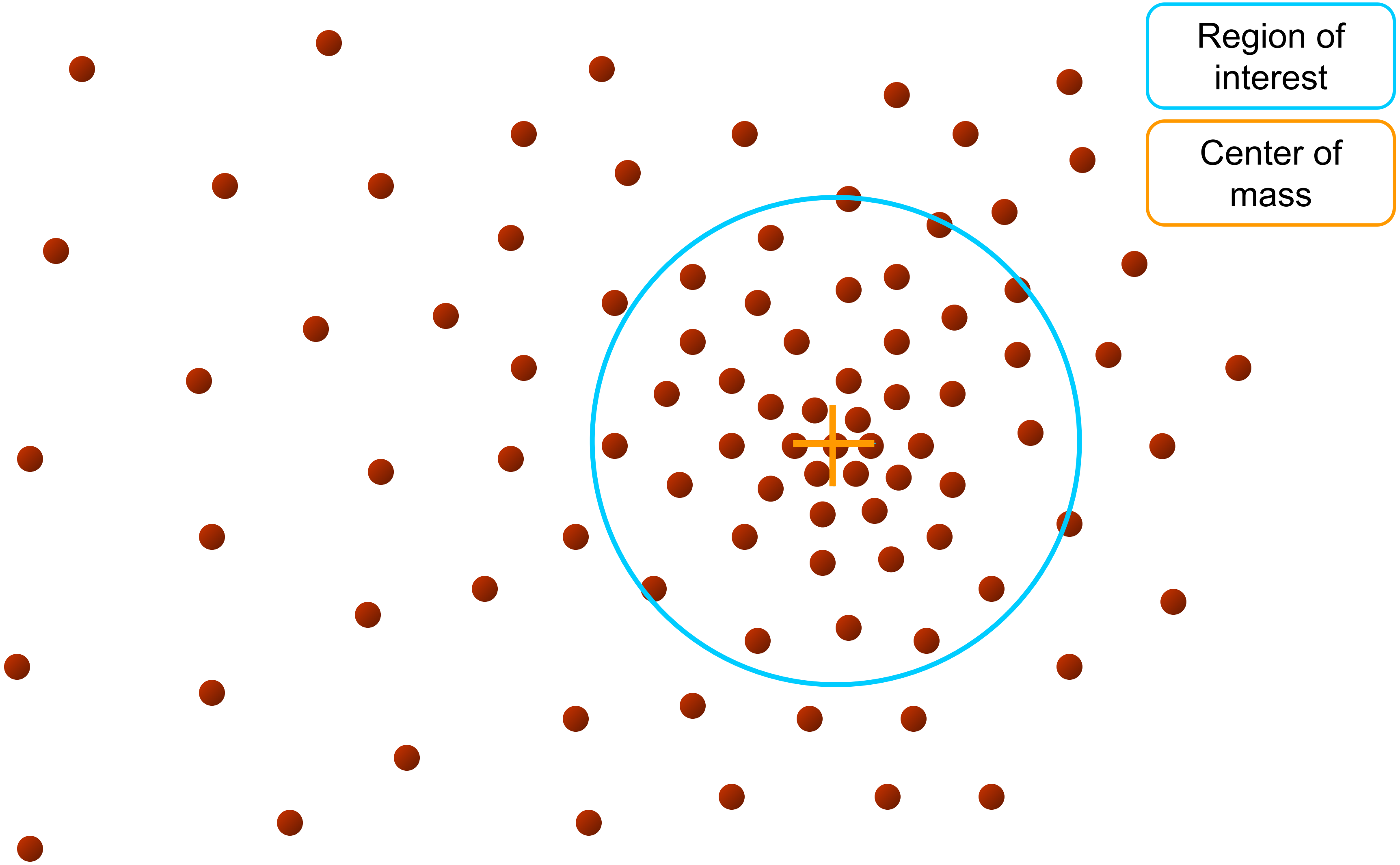
26-Oct-17

Mean-Shift



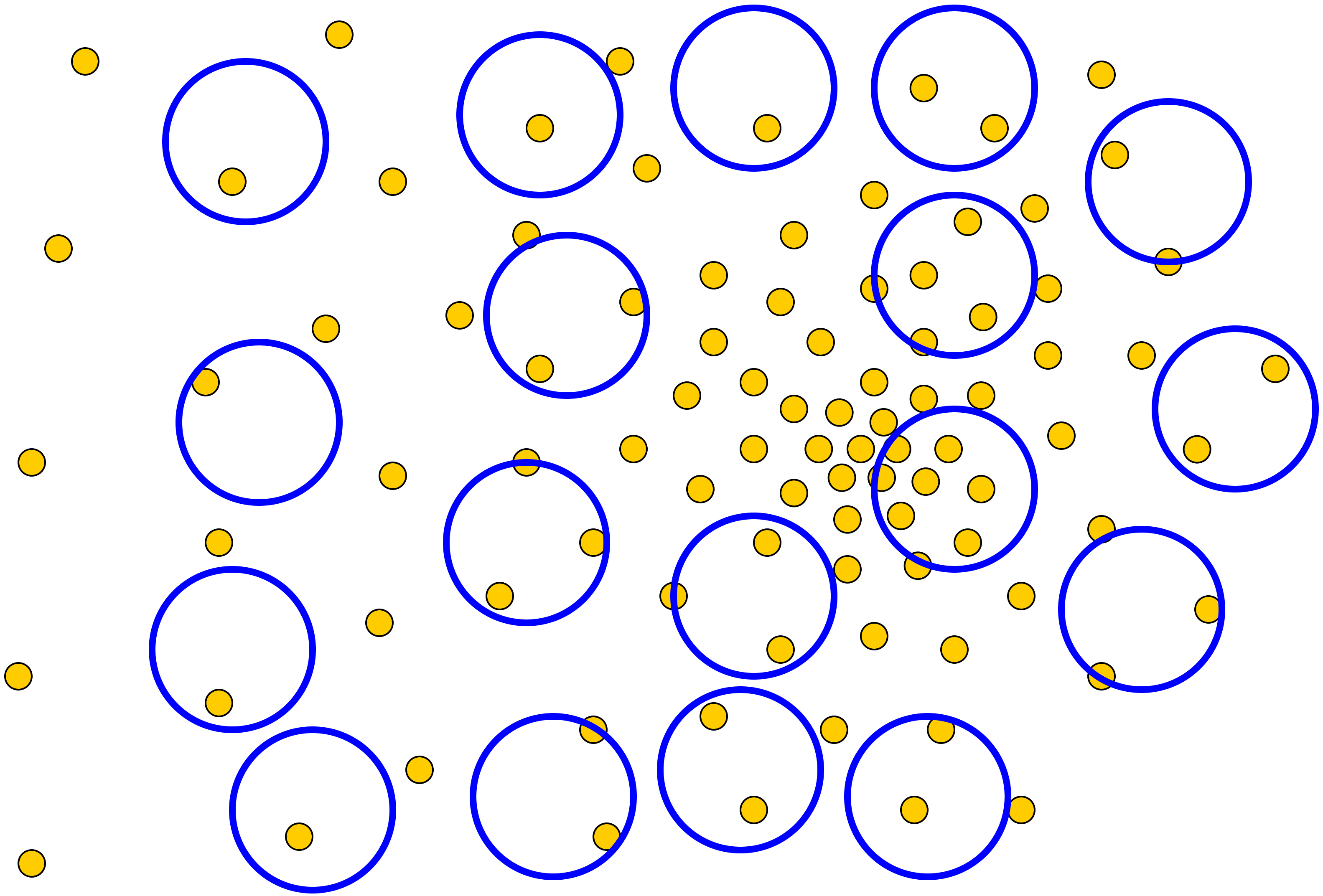
Slide by Y. Ukrainitz & B. Sarel

Mean-Shift



Slide by Y. Ukrainitz & B. Sarel

Real Modality Analysis

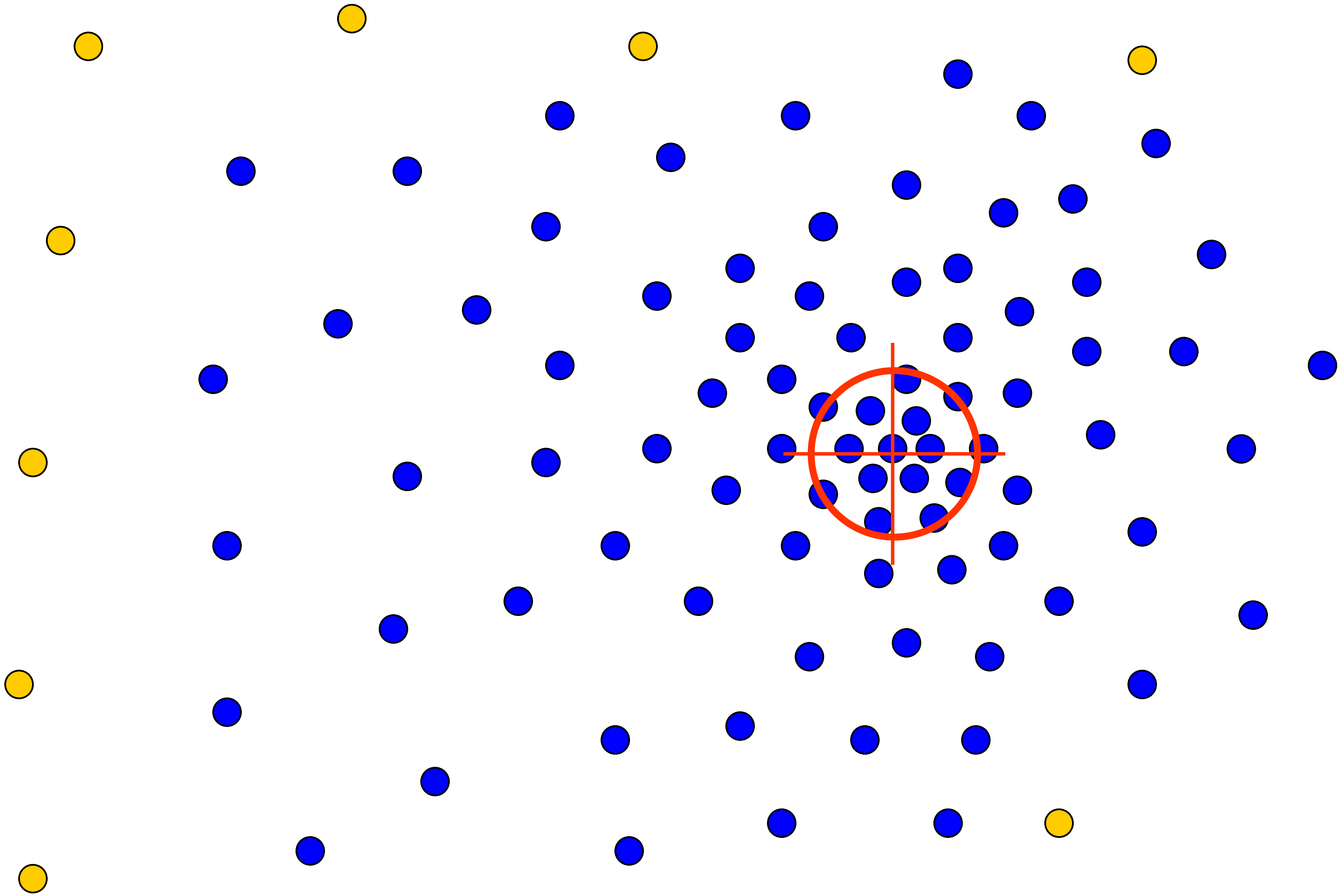


Tessellate the space with windows

Run the procedure in parallel

Slide by Y. Ukrainitz & B. Sarel

Real Modality Analysis

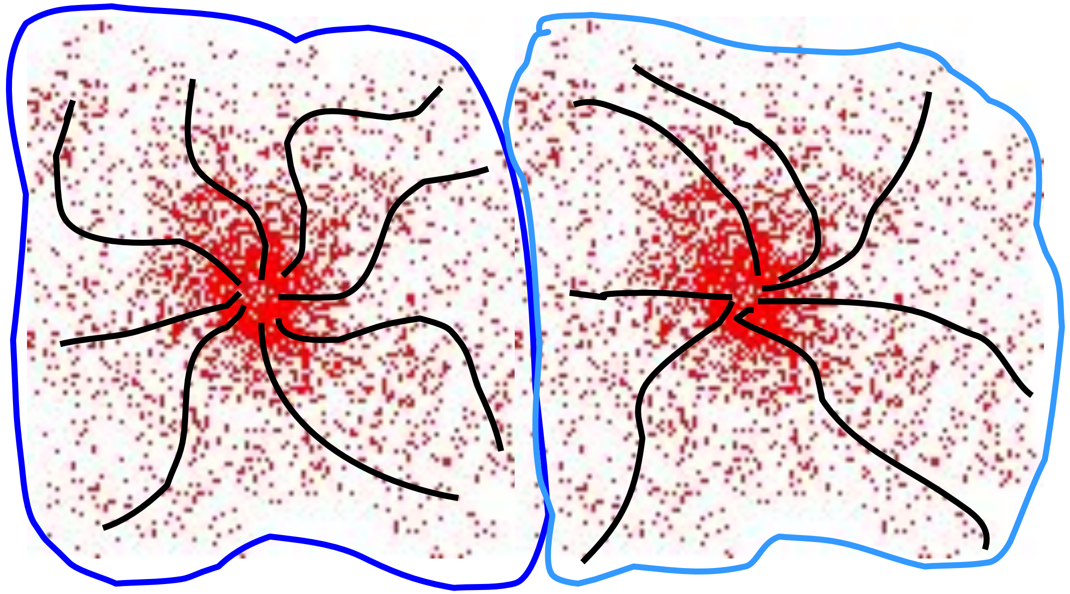


The blue data points were traversed by the windows towards the mode.

Slide by Y. Ukrainitz & B. Sarel

Mean-Shift Clustering

- Initialize not just 1 window but a multiple windows at random
- All pixels that end up in the same location belong to the same **cluster**
- **Attraction basin**: the feature region for which all windows end up in the same location

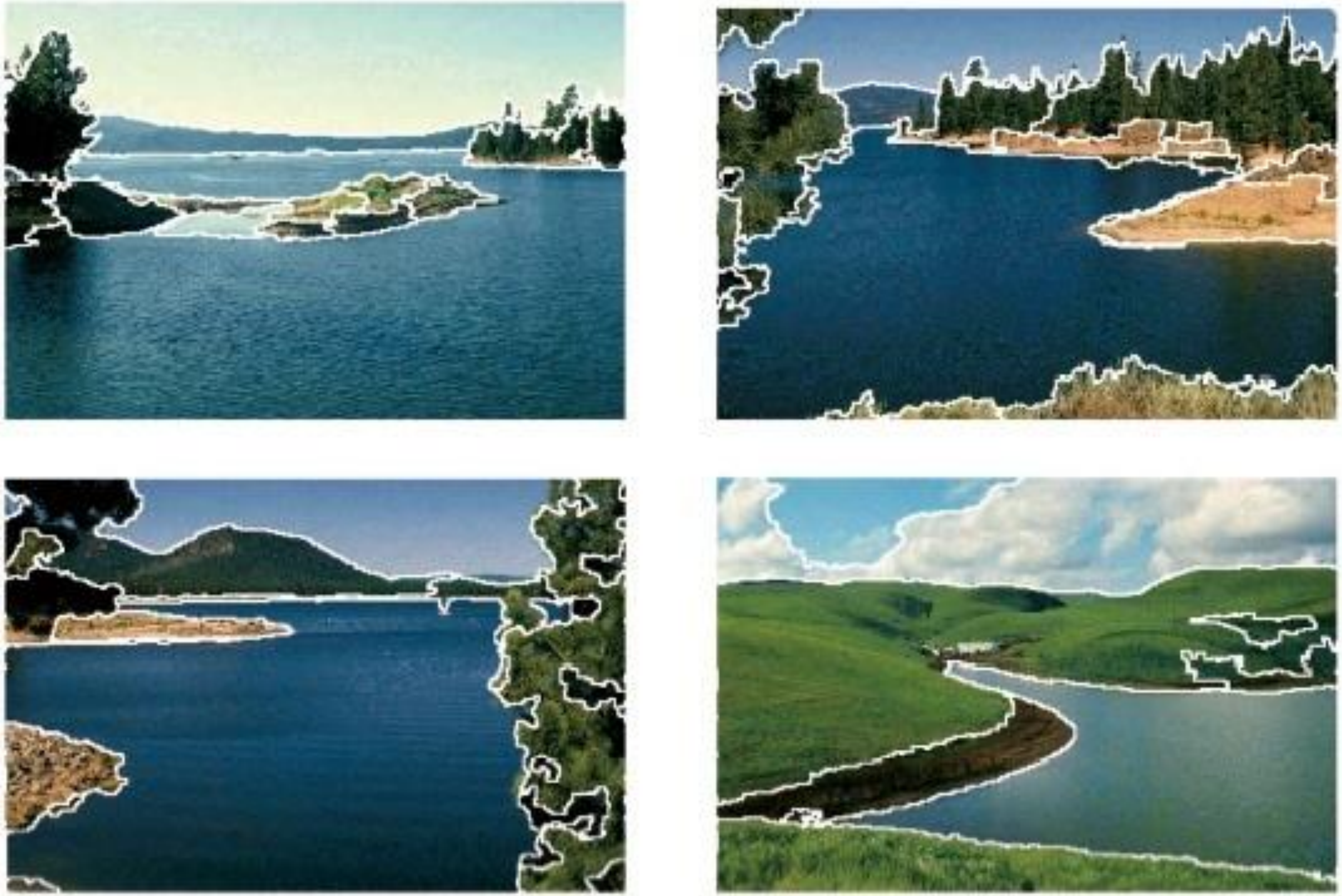


Mean-Shift Segmentation Results

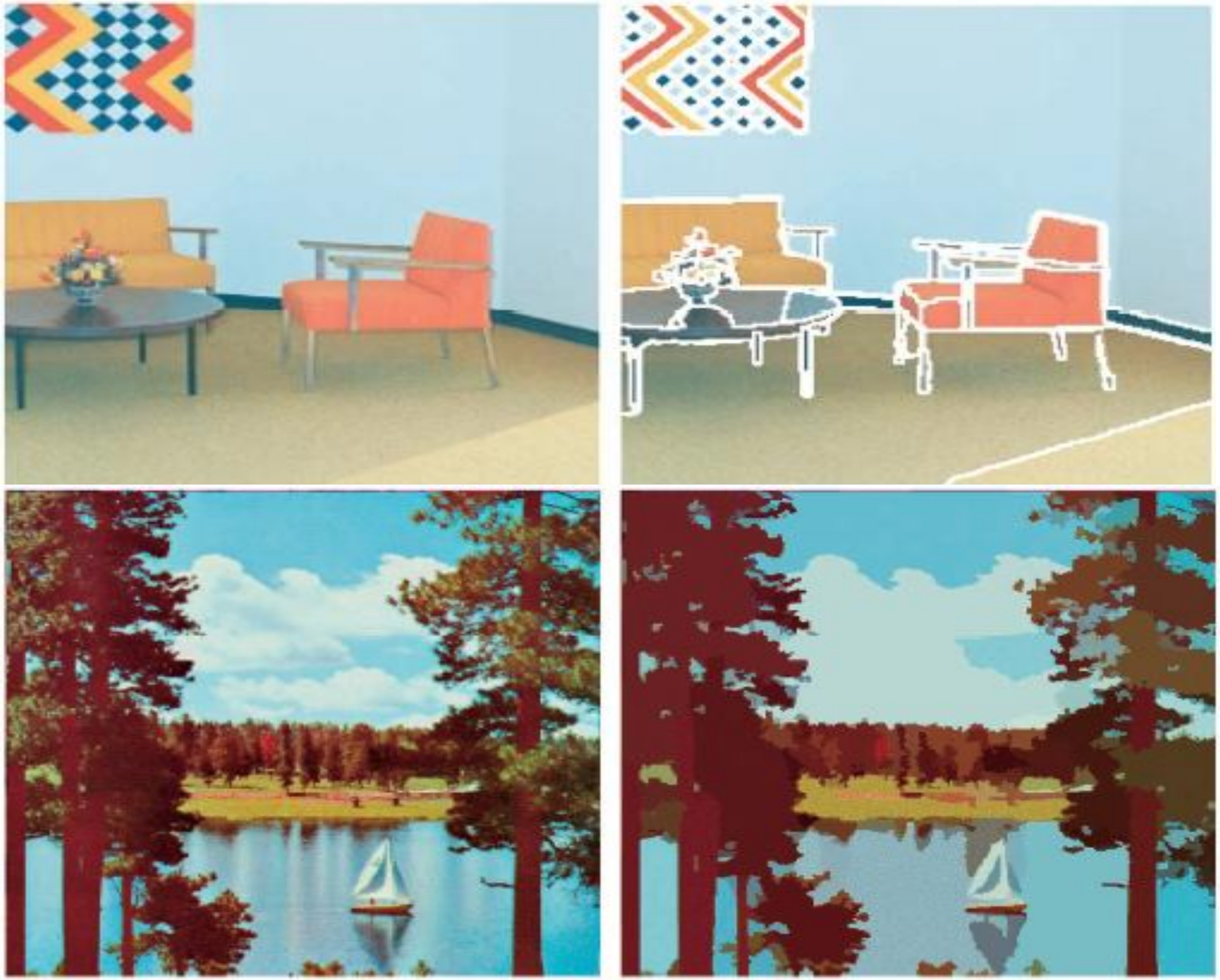



http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html

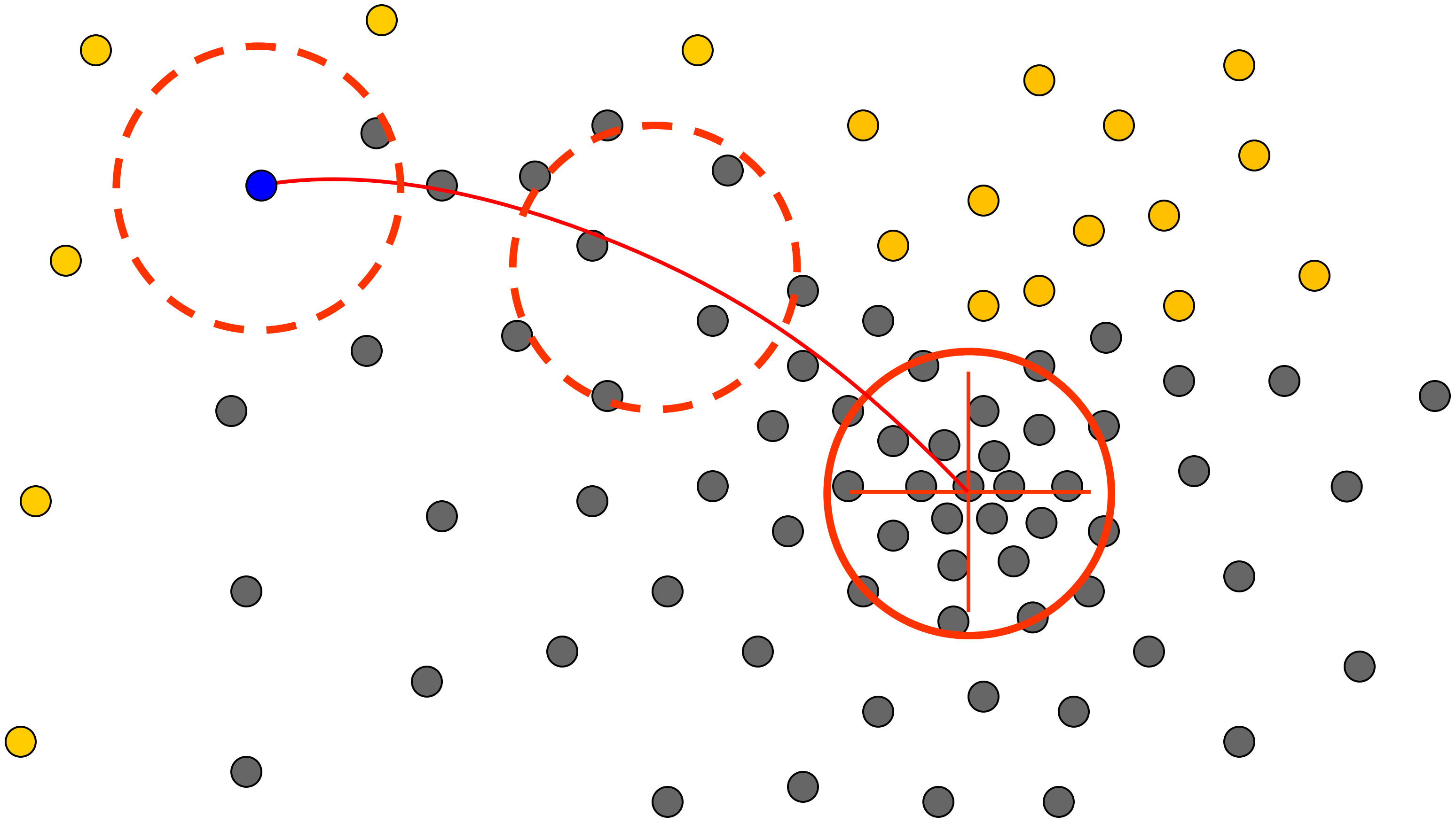
More Results



More Results



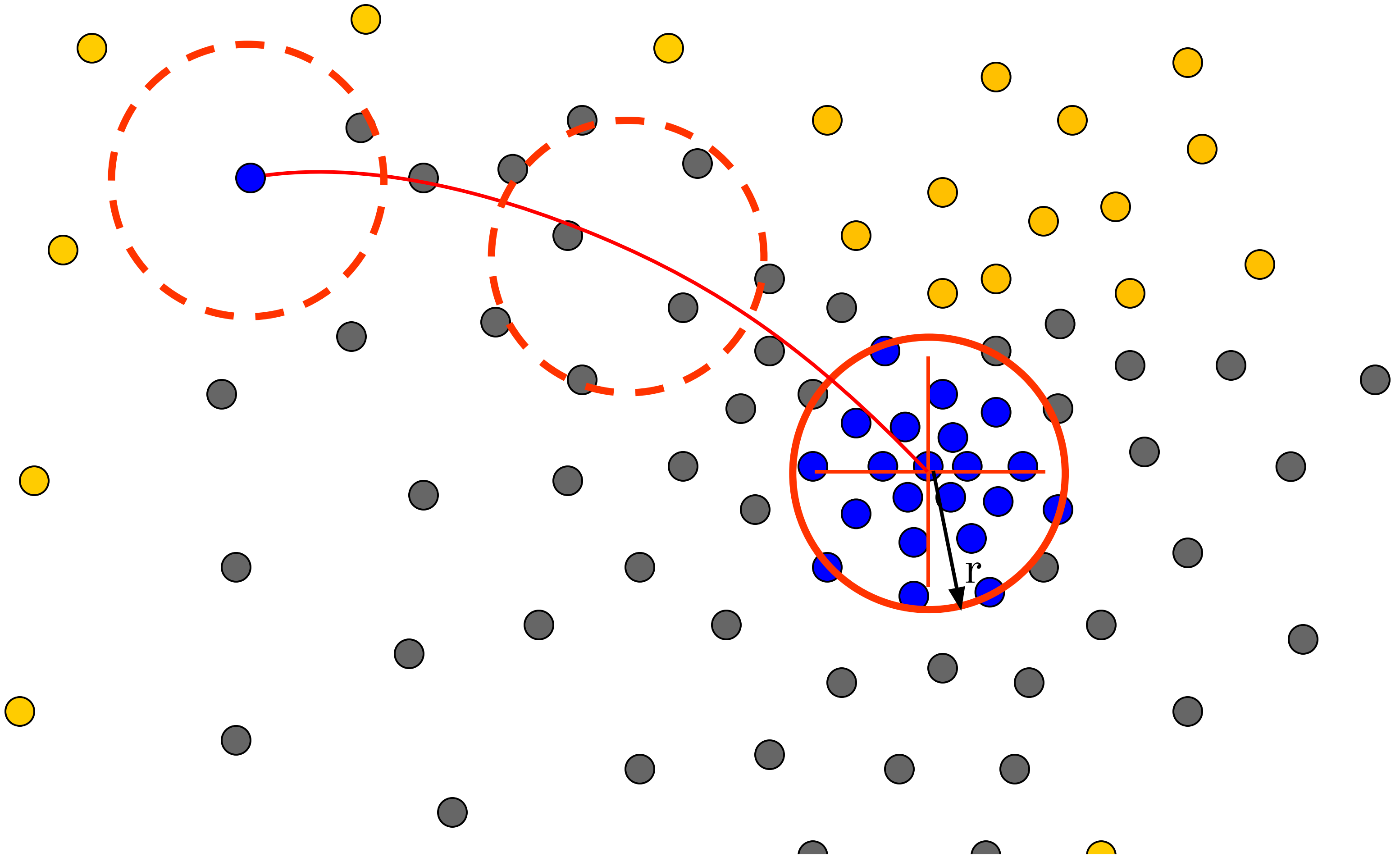
Problem: Computational Complexity



- Need to shift one window for every pixel
- Many computations will be redundant.

Slide credit: Bastian Leibe

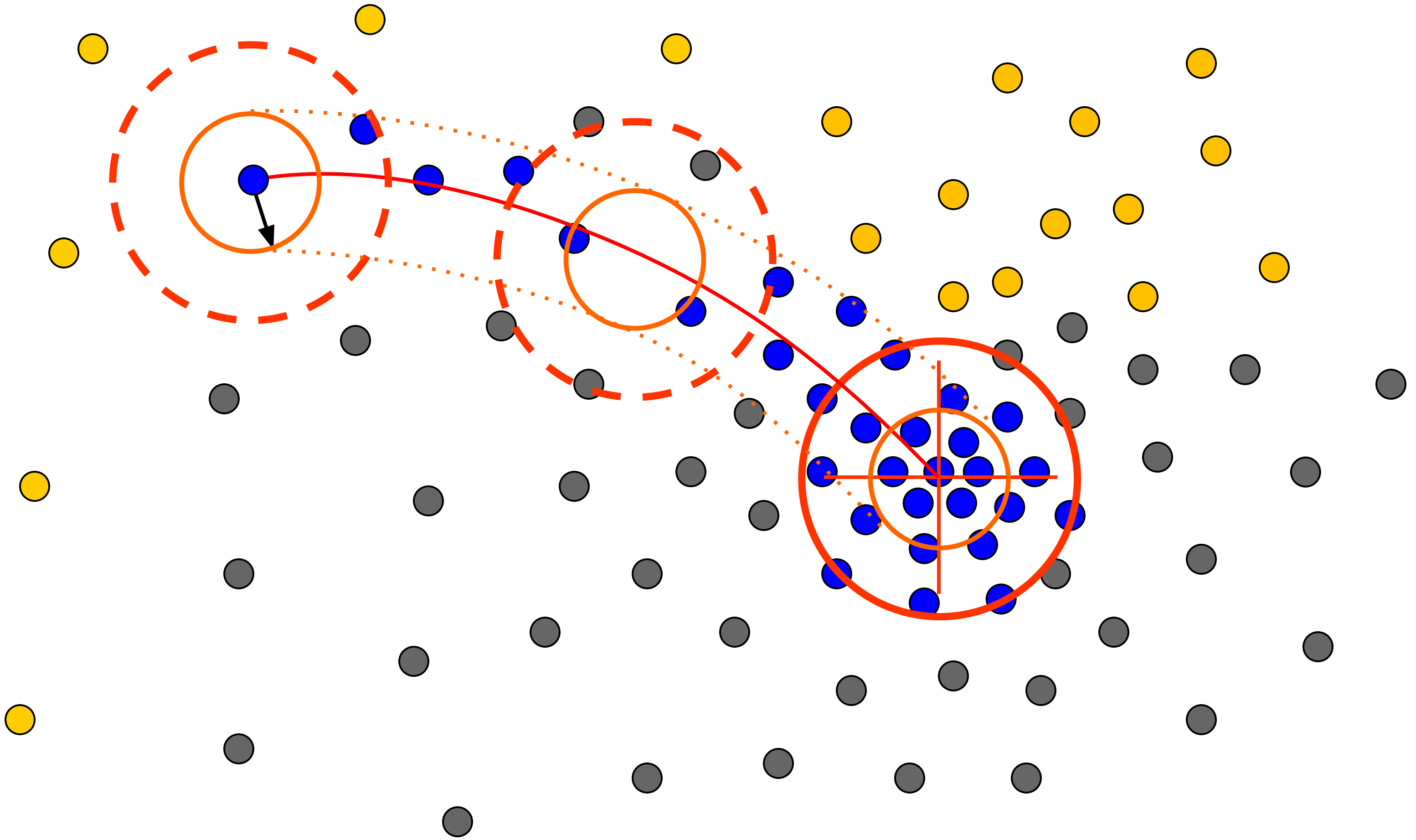
Speedups: Basin of Attraction



1. Assign all points within radius r of end point to the mode.

Slide credit: Bastian Leibe

Speedups

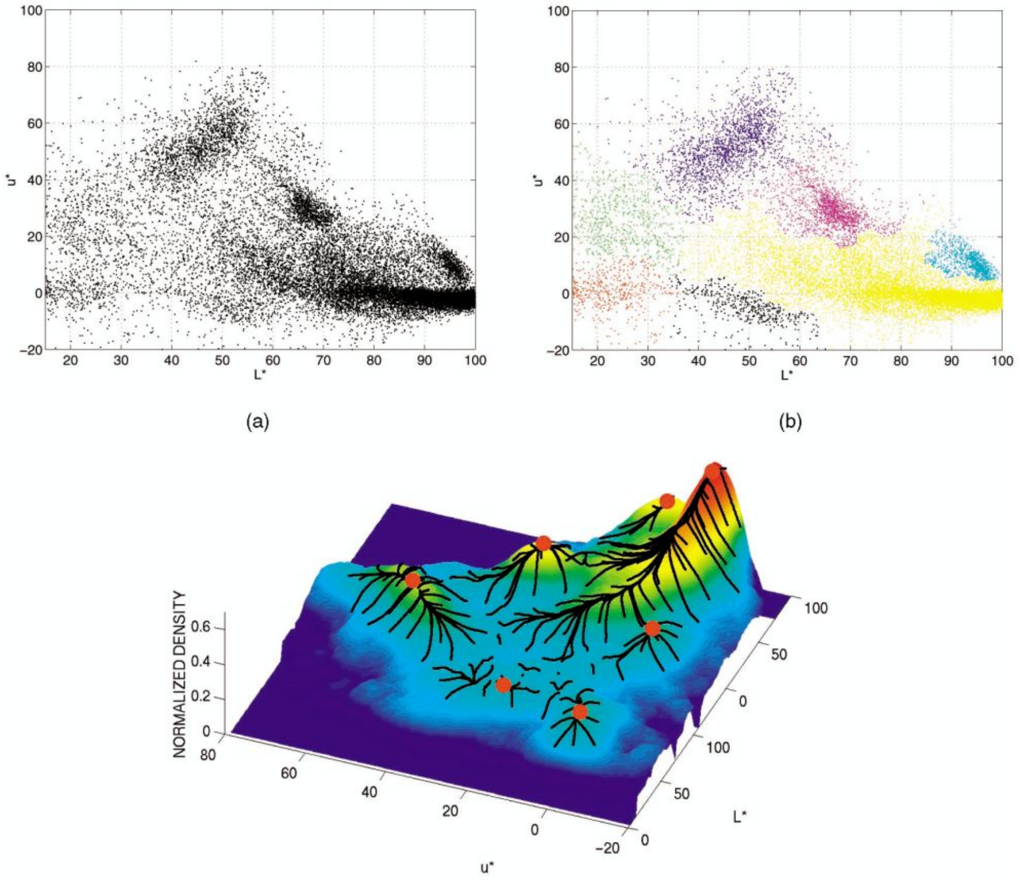


2. Assign all points within radius r/c of the search path to the mode -> reduce the number of data points to search.

Slide credit: Bastian Leibe

Mean-Shift Clustering

- Find features (color, gradients, texture, etc)
- Initialize windows at individual pixel locations
- Perform mean shift for each window until convergence
- At every step, merge windows that have high overlap to reduce computation



Mean-Shift pros and cons

- **Pros**
  - General, application-independent algorithm
  - Model-free, does not assume any prior shape (spherical, elliptical, etc.) of data clusters
  - Just a single parameter (window size *r*)
    - *r* has a physical meaning (unlike k-means)
  - Finds variable number of modes
  - Robust to outliers
- **Cons**
  - Output depends on window size
  - Window size (bandwidth) selection is not easy
  - Computationally (relatively) expensive (~2s/image)
  - Does not scale well with dimension of feature space

Summary

- Introduction to segmentation and clustering
- Gestalt theory for perceptual grouping
- Graph-based oversegmentation
- Agglomerative clustering
- K-means clustering
- Mean-shift clustering

Next time

Cameras and Calibration

Other Kernels

A kernel is a function that satisfies the following requirements :

1. $\int_{R^d} \phi(x) = 1$
2. $\phi(x) \geq 0$

Some examples of kernels include :

1. Rectangular $\phi(x) = \begin{cases} 1 & a \leq x \leq b \\ 0 & else \end{cases}$
2. Gaussian $\phi(x) = e^{-\frac{x^2}{2\sigma^2}}$
3. Epanechnikov $\phi(x) = \begin{cases} \frac{3}{4}(1-x^2) & if\ |x| \leq 1 \\ 0 & else \end{cases}$

source

Technical Details

Taking the derivative of: $\hat{f}_K = \frac{1}{nh^d} \sum_{i=1}^{n} K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h}\right)$

$$\nabla \hat{f}(\mathbf{x}) = \underbrace{\frac{2c_{k,d}}{nh^{d+2}} \left[\sum_{i=1}^{n} g\left(\left\|\frac{\mathbf{x} - \mathbf{x}_i}{h}\right\|^2\right)\right]}_{\text{term 1}} \underbrace{\left[\frac{\sum_{i=1}^{n} \mathbf{x}_i g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)} - \mathbf{x}\right]}_{\text{term 2}}, \qquad (3)$$

where $g(x) = -k'(x)$ denotes the derivative of the selected kernel profile.

- Term1: this is proportional to the density estimate at x (similar to equation 1 from two slides ago).
- Term2: this is the mean-shift vector that points towards the direction of maximum density.

Comaniciu & Meer, 2002

Technical Details

Finally, the mean shift procedure from a given point x_t is:

1. Compute the mean shift vector **m**:

$$\left[\frac{\sum_{i=1}^{n} \mathbf{x}_i g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)} - \mathbf{x}\right]$$

2. Translate the density window:

$$\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \mathbf{m}(\mathbf{x}_i^t).$$

3. Iterate steps 1 and 2 until convergence.

$$\nabla f(\mathbf{x}_i) = 0.$$

Comaniciu & Meer, 2002

Technical Details

Given n data points $\mathbf{x}_i \in \mathbb{R}^d$, the multivariate kernel density estimate using a radially symmetric kernel[1] (e.g., Epanechnikov and Gaussian kernels), $K(\mathbf{x})$, is given by,

$$\hat{f}_K = \frac{1}{nh^d} \sum_{i=1}^{n} K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h}\right), \tag{1}$$

where h (termed the *bandwidth* parameter) defines the radius of kernel. The radially symmetric kernel is defined as,

$$K(\mathbf{x}) = c_k k(\|\mathbf{x}\|^2), \tag{2}$$

where c_k represents a normalization constant.

Comaniciu & Meer, 2002