# Geometric Transformations 

$$
\text { CSE } 455
$$

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## What are geometric transformations?



## Translation



$$
\left[\begin{array}{lll}
1 & 0 & t_{x} \\
0 & 1 & t_{y} \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
1
\end{array}\right]=\left[\begin{array}{c}
x+t_{x} \\
y+t_{y} \\
1
\end{array}\right]
$$

## Translation and rotation



$$
\left[\begin{array}{ccc}
\cos (\theta) & -\sin (\theta) & t_{x} \\
\sin (\theta) & \cos (\theta) & t_{y} \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
1
\end{array}\right]=\left[\begin{array}{l}
x^{\prime} \\
y^{\prime} \\
1
\end{array}\right]
$$

## Scale



$$
\left[\begin{array}{lll}
a & 0 & 0 \\
0 & a & 0 \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
1
\end{array}\right]=\left[\begin{array}{c}
x^{\prime} \\
y^{\prime} \\
1
\end{array}\right]
$$

## Similarity transformations



Similarity transform (4 DoF) = translation + rotation + scale

Preserves: Angles

## Aspect ratio



$$
\left[\begin{array}{lll}
a & 0 & 0 \\
0 & \frac{1}{a} & 0 \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
1
\end{array}\right]=\left[\begin{array}{l}
x^{\prime} \\
y^{\prime} \\
1
\end{array}\right]
$$

## Shear



$$
\left[\begin{array}{lll}
1 & a & 0 \\
b & 1 & 0 \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
1
\end{array}\right]=\left[\begin{array}{l}
x^{\prime} \\
y^{\prime} \\
1
\end{array}\right]
$$

## Affine transformations



Affine transform (6 DoF) = translation + rotation + scale + aspect ratio + shear

## What is missing?



Canaletto

Are there any other planar transformations?

## General affine

We already used these

$$
\left[\begin{array}{lll}
a & b & c \\
d & e & f \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
1
\end{array}\right]=\left[\begin{array}{l}
x^{\prime} \\
y^{\prime} \\
1
\end{array}\right]
$$

How do we compute projective transformations?

## Homogeneous coordinates

$$
\left[\begin{array}{lll}
a & b & c \\
d & e & f \\
g & h & 1
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
1
\end{array}\right]=\left[\begin{array}{l}
u \\
v \\
w
\end{array}\right]
$$

One extra step:

$$
\begin{aligned}
x^{\prime} & =u / w \\
y^{\prime} & =v / w
\end{aligned}
$$

## Projective transformations

a.k.a. Homographies


Preserves: Straight Lines

## Finding the transformation

| Translation | $=2$ degrees of freedom |
| :--- | :--- |
| Similarity | $=4$ degrees of freedom |
| Affine | $=6$ degrees of freedom |
| Homography | $=8$ degrees of freedom |

How many corresponding points do we need to solve?

## Finding the transformation



- How can we find the transformation between these images?
- How many corresponding points do we need to solve?


## What can I use homographies for?



## For one thing: Panoramas



