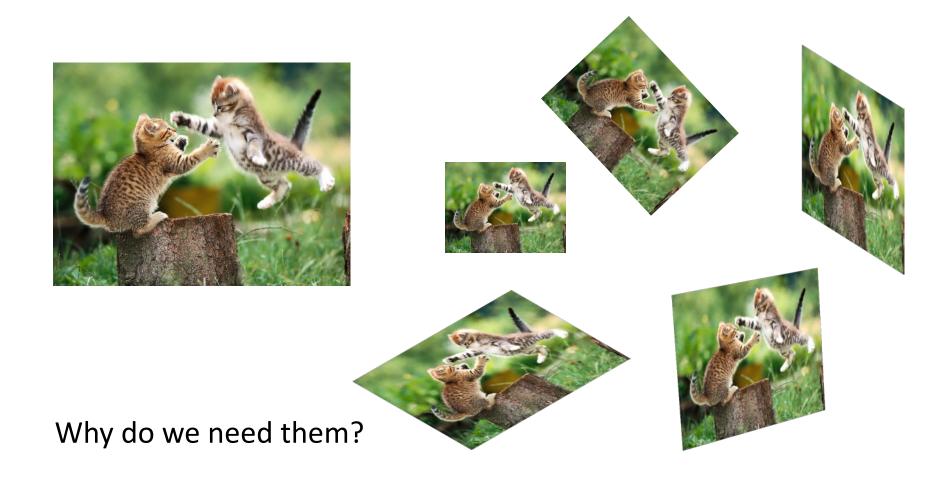
### Geometric Transformations

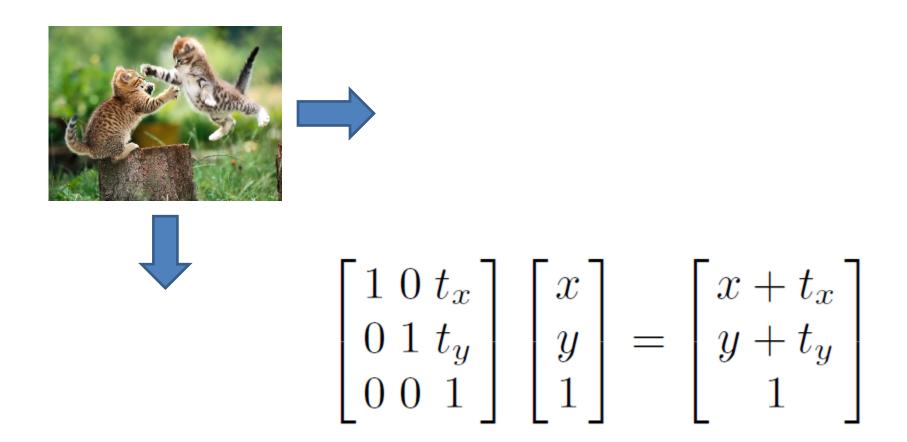
**CSE 455** 

Linda Shapiro

# What are geometric transformations?

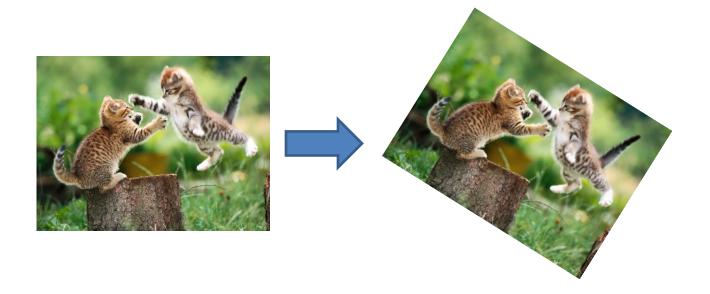


#### **Translation**



Preserves: Orientation

#### Translation and rotation



$$\begin{bmatrix} \cos(\theta) - \sin(\theta) \ t_x \\ \sin(\theta) \ \cos(\theta) \ t_y \\ 0 \ 0 \ 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

#### Scale



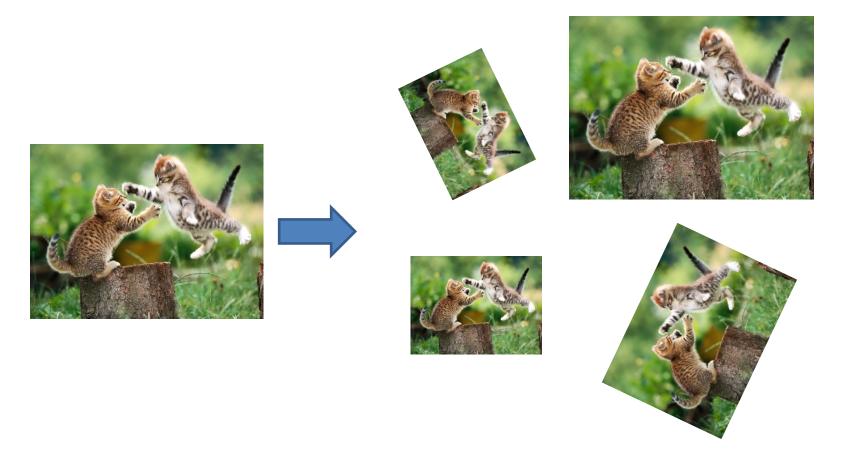




$$\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

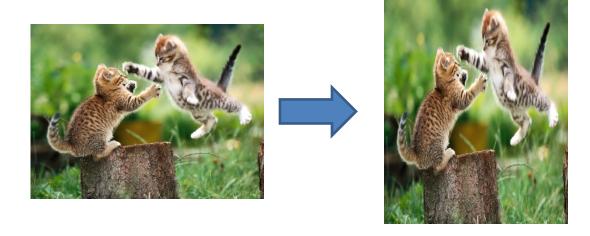
### Similarity transformations



Similarity transform (4 DoF) = translation + rotation + scale

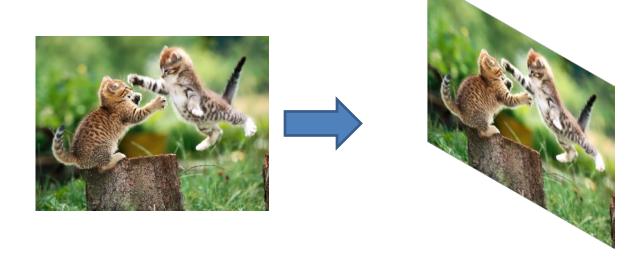
Preserves: Angles

### Aspect ratio



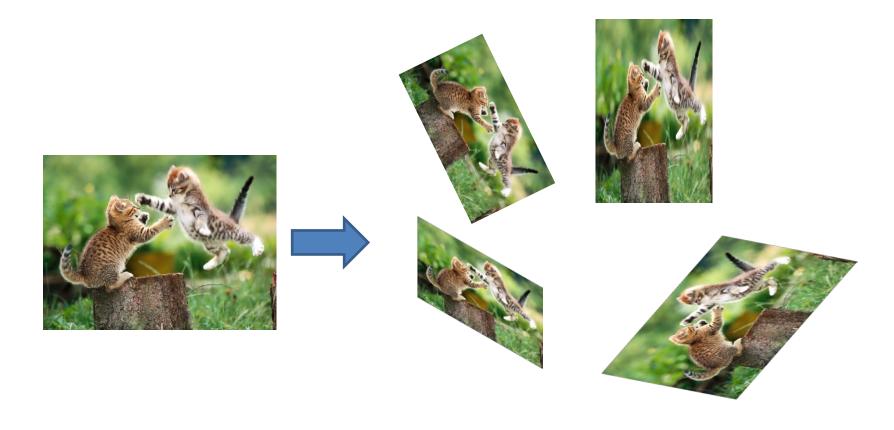
$$\begin{bmatrix} a & 0 & 0 \\ 0 & \frac{1}{a} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

#### Shear



$$\begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

#### Affine transformations



Affine transform (6 DoF) = translation + rotation + scale + aspect ratio + shear

Preserves: Parallelism

## What is missing?



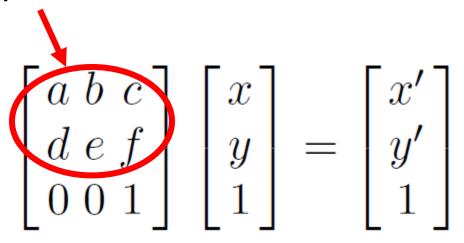


Canaletto

Are there any other planar transformations?

#### General affine

We already used these



How do we compute projective transformations?

### Homogeneous coordinates

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

#### One extra step:

$$x' = u/w$$
$$y' = v/w$$

#### Projective transformations

a.k.a. Homographies

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \qquad \begin{aligned} x' &= u/w \\ y' &= v/w \end{aligned}$$

"keystone" distortions









Preserves: Straight Lines

### Finding the transformation

```
Translation = 2 degrees of freedom
Similarity = 4 degrees of freedom
Affine = 6 degrees of freedom
Homography = 8 degrees of freedom
```

How many corresponding points do we need to solve?

### Finding the transformation





- How can we find the transformation between these images?
- How many corresponding points do we need to solve?

# What can I use homographies for?





# For one thing: Panoramas

