Region Segmentation
Readings: Chapter 10: 10.1
Additional Materials Provided

- K-means Clustering (text)
- EM Clustering (paper)
- Graph Partitioning (text)
- Mean-Shift Clustering (paper)

Image Segmentation

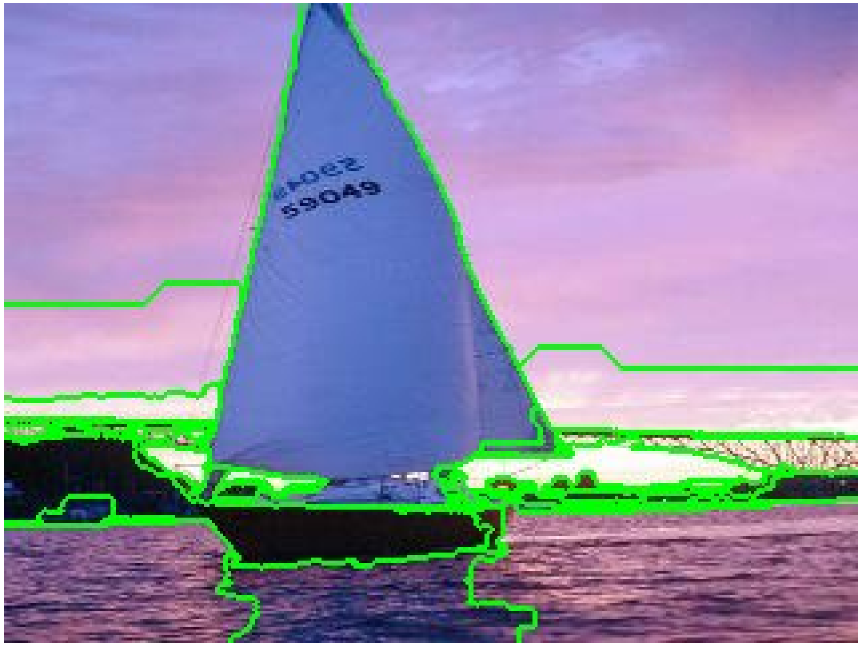
Image segmentation is the operation of partitioning an image into a collection of connected sets of pixels.

1. into regions, which usually cover the image

2. into linear structures, such as
   - line segments
   - curve segments

3. into 2D shapes, such as
   - circles
   - ellipses
   - ribbons (long, symmetric regions)

Example: Regions



Main Methods of Region Segmentation

1. ~~Region Growing~~

2. ~~Split and Merge~~

3. Clustering

Clustering

- There are K clusters $C_1, \ldots, C_K$ with means $m_1, \ldots, m_K$.

- The **least-squares error** is defined as

$$D = \sum_{k=1}^{K} \sum_{x_i \in C_k} \| x_i - m_k \|^2 .$$

- Out of all possible partitions into K clusters, choose the one that minimizes D.

Why don't we just do this?
If we could, would we get meaningful objects?

K-Means Clustering

Form K-means clusters from a set of n-dimensional vectors

1. Set ic (iteration count) to 1

2. Choose randomly a set of K means $m_1(1)$, ..., $m_K(1)$.

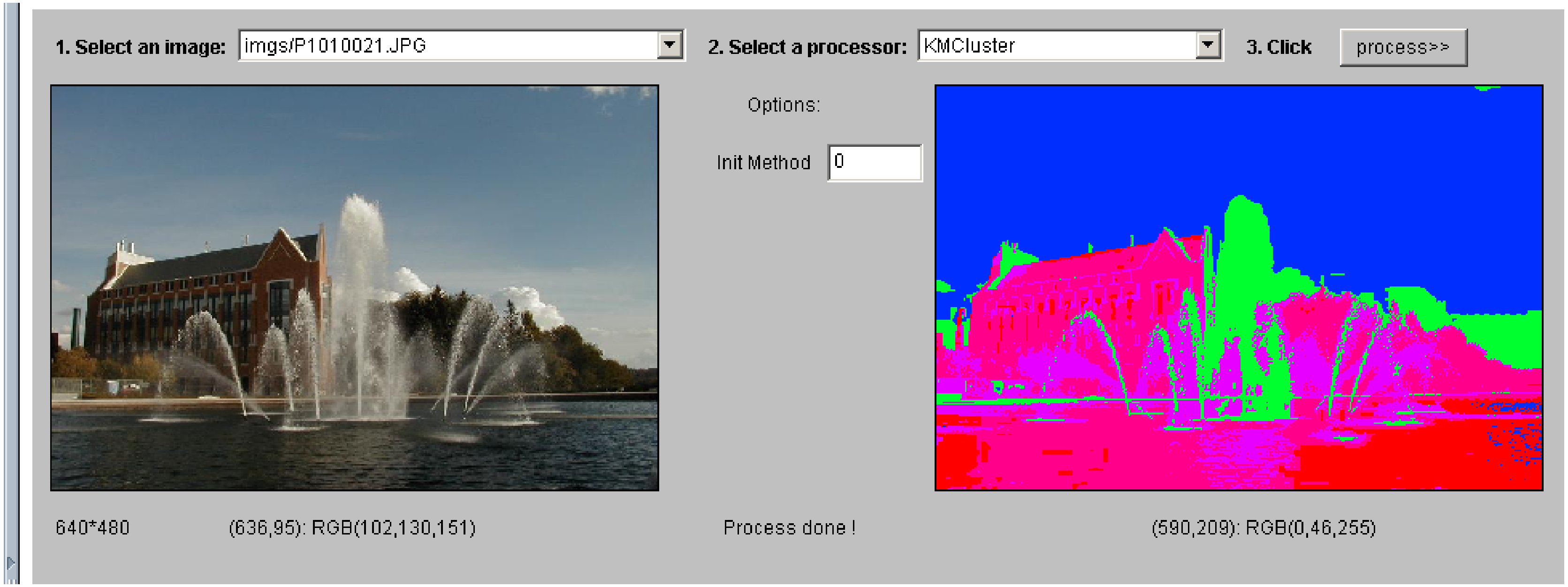
3. For each vector x_i compute $D(x_i , m_k(ic))$, $k=1,...K$
   and assign x_i to the cluster C_j with nearest mean.

4. Increment ic by 1, update the means to get $m_1(ic),...,m_K(ic)$.

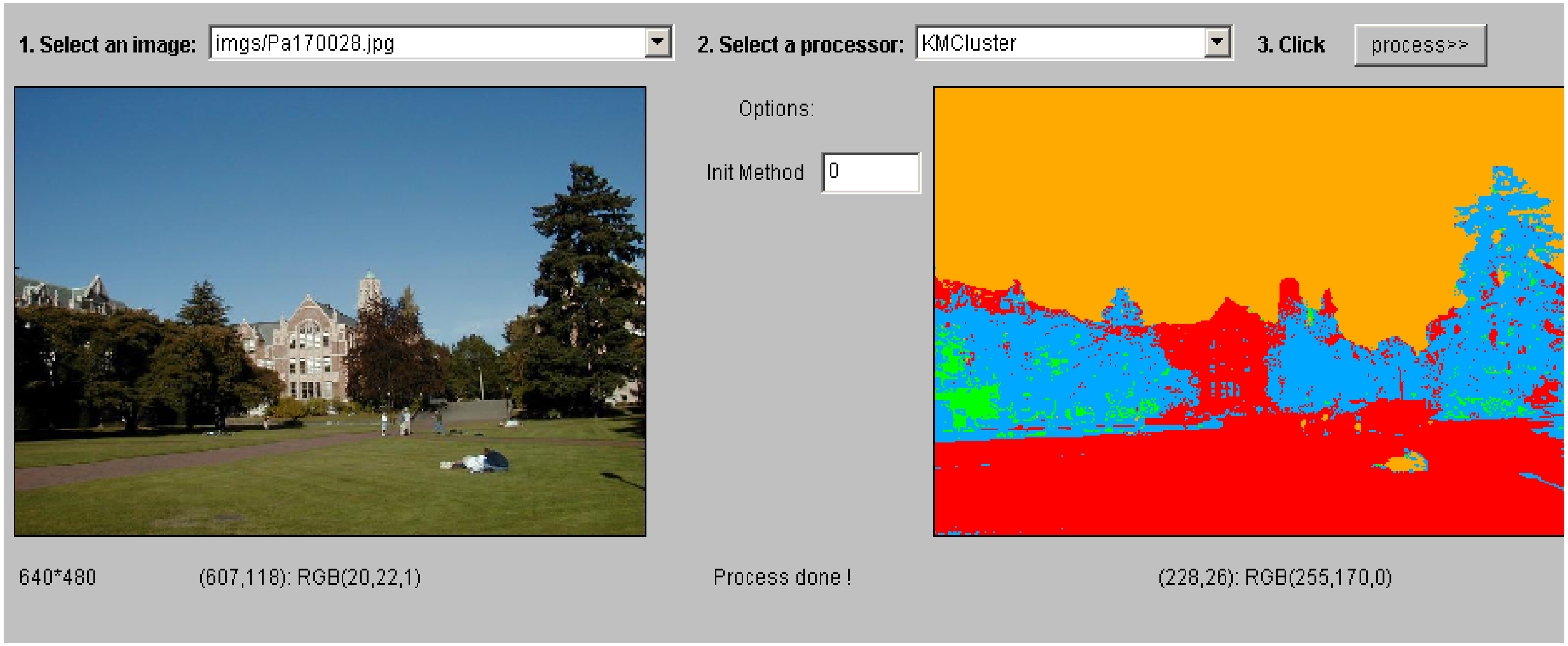
5. Repeat steps 3 and 4 until $C_k(ic) = C_k(ic+1)$ for all k.

K-Means Example 1

K-Means Example 2



K-Means Example 3



K-means Variants

- Different ways to initialize the means
- Different stopping criteria
- Dynamic methods for determining the right number of clusters (K) for a given image

- The EM Algorithm: a probabilistic formulation of K-means

K-Means

- <u>Boot Step</u>:
  - Initialize K clusters: $C_1, \ldots, C_K$

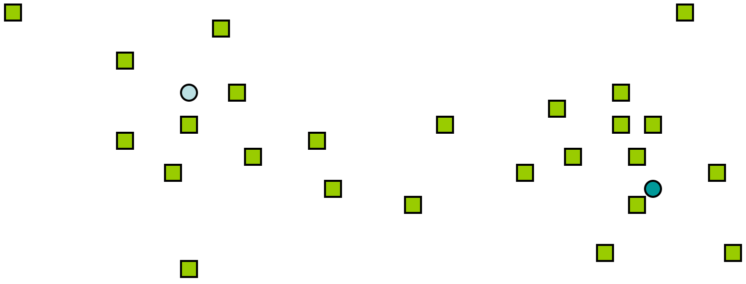
    Each cluster is represented by its mean m_j
- <u>Iteration Step</u>:
  - Estimate the cluster for each data point

$$x_i \Longrightarrow C(x_i)$$

  - Re-estimate the cluster parameters

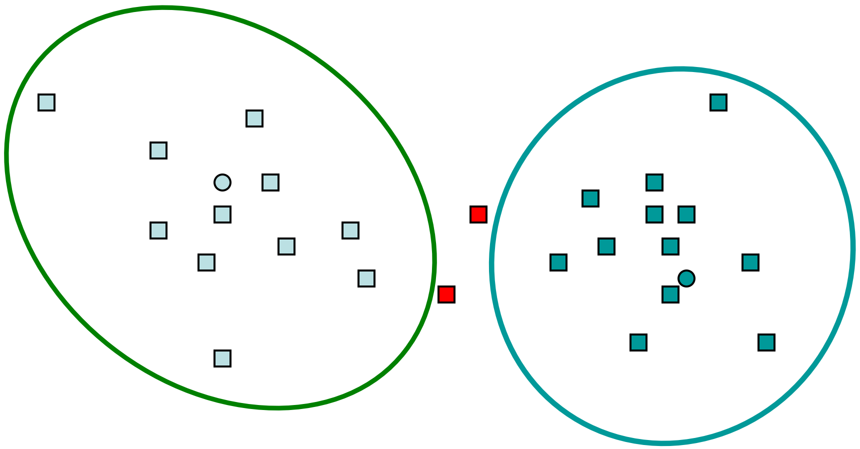
$$m_j = mean\{x_i \mid x_i \in C_j\}$$

K-Means Example



K-Means Example

Where do the red points belong?

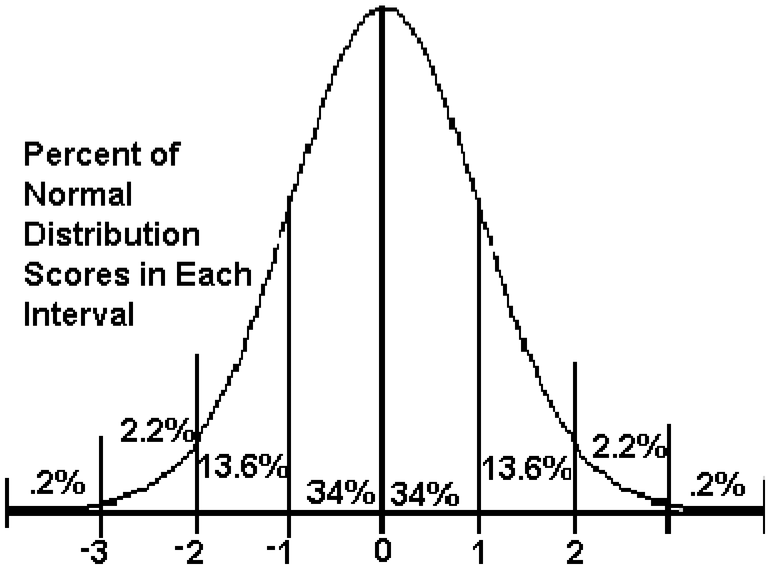


K-means vs. EM

	K-means	EM
Cluster Representation	mean	mean, variance, and weight
Cluster Initialization	randomly select K means	initialize K Gaussian distributions
Expectation	assign each point to closest mean	soft-assign each point to each distribution
Maximization	compute means of current clusters	compute new params of each distribution

Notation

N(μ , σ) is a 1D normal (Gaussian) distribution with mean μ and standard deviation σ (so the variance is σ^2.



$N(\mu, \Sigma)$ is a multivariate Gaussian distribution with mean μ and covariance matrix Σ.

What is a covariance matrix?

$$\begin{array}{c|ccc|} & R & G & B \\ \hline R & \sigma_R^2 & \sigma_{RG} & \sigma_{RB} \\ G & \sigma_{GR} & \sigma_G^2 & \sigma_{GB} \\ B & \sigma_{BR} & \sigma_{BG} & \sigma_B^2 \\ \hline \end{array}$$

variance(X): $\sigma_X^2 = \Sigma\, (x_i - \mu)^2\, (1/N)$

cov(X,Y) $= \Sigma\, (x_i - \mu_x)(y_i - \mu_y)\, (1/N)$

1. Suppose we have a set of clusters: $C_1, C_2, \dots, C_K$ over a set of data points $X = \{x_1, x_2, \dots, x_N\}$.

$P(C_j)$ is the probability or weight of cluster C_j.

$P(C_j \mid x_i)$ is the probability of cluster C_j given point xi.

$P(x_i \mid C_j)$ is the probability of point x_i belonging to cluster C_j.

2. Suppose that a cluster C_j is represented by a Gaussian distribution $N(\mu_j,\sigma_j)$. Then for any point x_i:

$$P(x_i \mid C_j) = \frac{1}{\sqrt{2\pi}\,\sigma_j} e^{-\frac{(x_i - \mu_j)^2}{2\sigma_j^2}}$$

EM: Expectation-Maximization

- Boot Step:
  - Initialize K clusters: $C_1, \ldots, C_K$

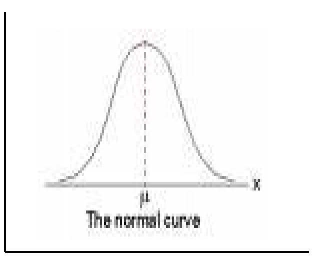
    (μ_j, Σ_j) and $P(C_j)$ for each cluster j.
- Iteration Step:
  - Estimate the cluster of each data point

    $p(C_j \mid x_i)$ ⇨ Expectation
  - Re-estimate the cluster parameters

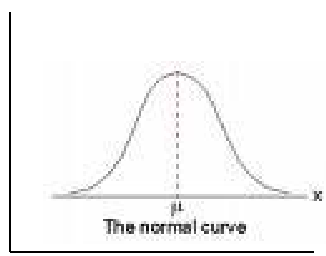
    $(\mu_j, \Sigma_j), p(C_j)$ For each cluster j ⇨ Maximization

1-D EM with Gaussian Distributions

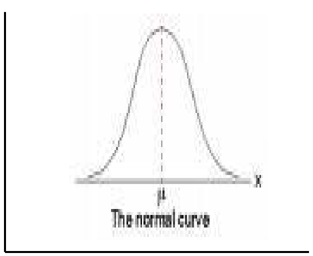
- Each cluster C_j is represented by a Gaussian distribution $N(\mu_j , \sigma_j)$.
- Initialization: For each cluster C_j initialize its mean μ_j , variance σ_j^2, and weight α_j.



$N(\mu_1 , \sigma_1)$
$\alpha_1 = P(C_1)$



$N(\mu_2 , \sigma_2)$
$\alpha_2 = P(C_2)$



$N(\mu_3 , \sigma_3)$
$\alpha_3 = P(C_3)$

Expectation

- For each point x_i and each cluster C_j compute $P(C_j \mid x_i)$.

- $P(C_j \mid x_i) = P(x_i \mid C_j)\, P(C_j) \,/\, P(x_i)$

- $P(x_i) = \sum_j P(x_i \mid C_j)\, P(C_j)$

- Where do we get $P(x_i \mid C_j)$ and $P(C_j)$?

1. Use the pdf for a normal distribution:

$$P(x_i \mid C_j) = \frac{1}{\sqrt{2\pi}\,\sigma_j} e^{-\frac{(x_i - \mu_j)^2}{2\sigma_j^2}}$$

2. Use $\alpha_j = P(C_j)$ from the current parameters of cluster C_j.

Maximization

- Having computed $P(C_j \mid x_i)$ for each point x_i and each cluster C_j, use them to compute new mean, variance, and weight for each cluster.

$$\mu_j = \frac{\sum_i p(C_j \mid x_i) \cdot x_i}{\sum_i p(C_j \mid x_i)}$$

$$\sigma_j^2 = \Sigma_j = \frac{\sum_i p(C_j \mid x_i) \cdot (x_i - \mu_j) \cdot (x_i - \mu_j)^T}{\sum_i p(C_j \mid x_i)}$$

$$p(C_j) = \frac{\sum_i p(C_j \mid x_i)}{N}$$

Multi-Dimensional Expectation Step for Color Image Segmentation

Input (Known)		Input (Estimation)		Output
$x_1=\{r_1, g_1, b_1\}$ $x_2=\{r_2, g_2, b_2\}$ … $x_i=\{r_i, g_i, b_i\}$ …	+	Cluster Parameters (μ_1,Σ_1), $p(C_1)$ for C_1 (μ_2,Σ_2), $p(C_2)$ for C_2 … (μ_k,Σ_k), $p(C_k)$ for C_k	⇒	Classification Results $p(C_1\|x_1)$ $p(C_j\|x_2)$ … $p(C_j\|x_i)$ …

$$p(C_j \mid x_i) = \frac{p(x_i \mid C_j)\cdot p(C_j)}{p(x_i)} = \frac{p(x_i \mid C_j)\cdot p(C_j)}{\sum_j p(x_i \mid C_j)\cdot p(C_j)}$$

Multi-dimensional Maximization Step for Color Image Segmentation

Input (Known)

$x_1=\{r_1, g_1, b_1\}$

$x_2=\{r_2, g_2, b_2\}$

…

$x_i=\{r_i, g_i, b_i\}$

…

+

Input (Estimation)

Classification Results

$p(C_1|x_1)$

$p(C_j|x_2)$

…

$p(C_j|x_i)$

…

Output

Cluster Parameters

(μ_1,Σ_1), $p(C_1)$ for C_1

(μ_2,Σ_2), $p(C_2)$ for C_2

…

(μ_k,Σ_k), $p(C_k)$ for C_k

$$\mu_j = \frac{\sum_i p(C_j \mid x_i)\cdot x_i}{\sum_i p(C_j \mid x_i)} \qquad \Sigma_j = \frac{\sum_i p(C_j \mid x_i)\cdot(x_i-\mu_j)\cdot(x_i-\mu_j)^T}{\sum_i p(C_j \mid x_i)} \qquad p(C_j) = \frac{\sum_i p(C_j \mid x_i)}{N}$$

Full EM Algorithm Multi-Dimensional

- Boot Step:
  - Initialize K clusters: $C_1, \ldots, C_K$

    $(\mu_{j,}\ \Sigma_j)$ and $P(C_j)$ for each cluster j.

- Iteration Step:
  - Expectation Step

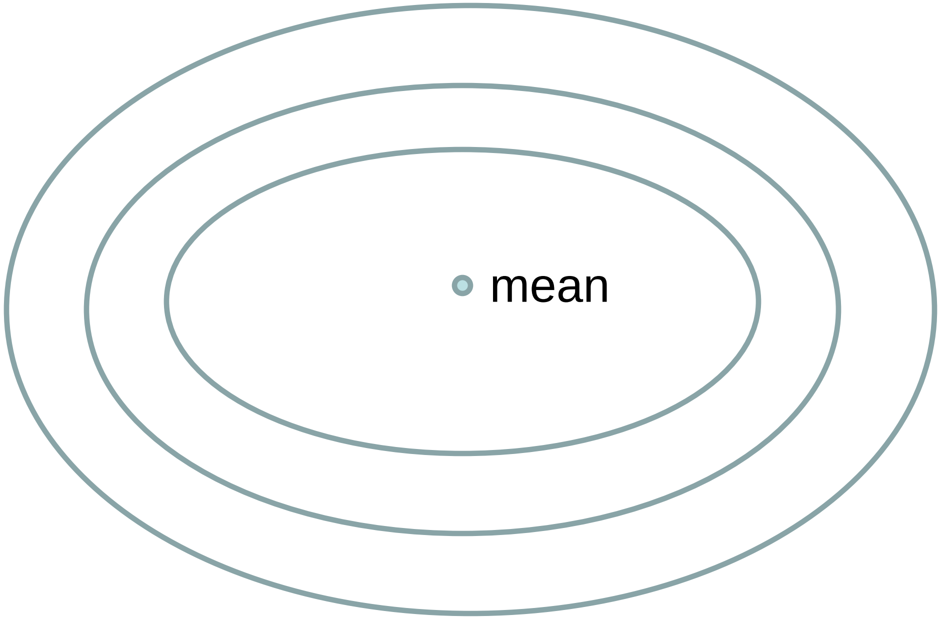
$$p(C_j \mid x_i) = \frac{p(x_i \mid C_j) \cdot p(C_j)}{p(x_i)} = \frac{p(x_i \mid C_j) \cdot p(C_j)}{\sum_j p(x_i \mid C_j) \cdot p(C_j)}$$

  - Maximization Step

$$\mu_j = \frac{\sum_i p(C_j \mid x_i) \cdot x_i}{\sum_i p(C_j \mid x_i)} \qquad \Sigma_j = \frac{\sum_i p(C_j \mid x_i) \cdot (x_i - \mu_j) \cdot (x_i - \mu_j)^T}{\sum_i p(C_j \mid x_i)} \qquad p(C_j) = \frac{\sum_i p(C_j \mid x_i)}{N}$$

Visualizing EM Clusters

ellipses show one, two, and three standard deviations

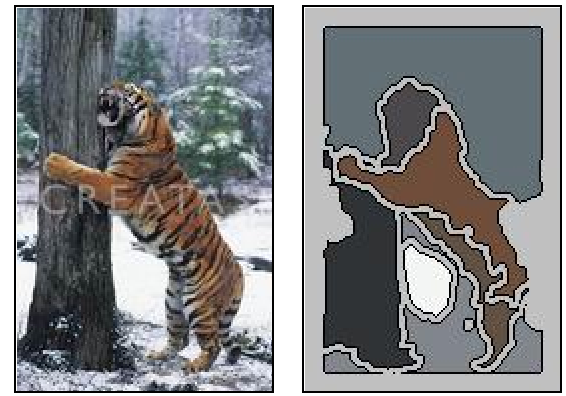


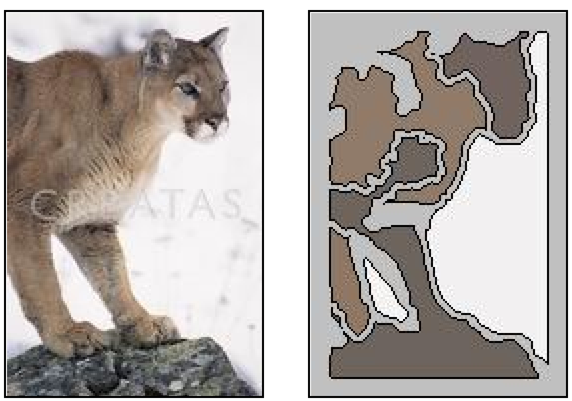
http://en.wikipedia.org/wiki/Expectation%E2%80%93maximization_algorithm

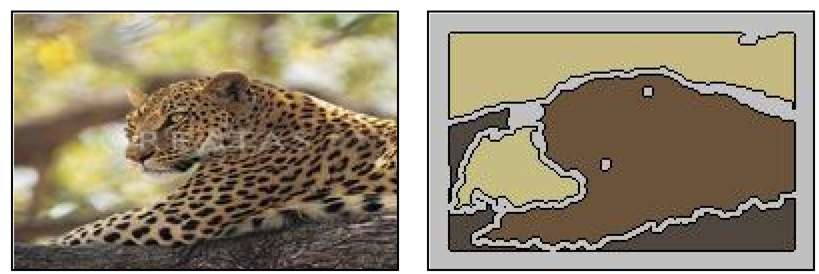
EM Applications

- Blobworld: Image segmentation using Expectation-Maximization and its application to image querying
- Yi’s Generative/Discriminative Learning of object classes in color images

Blobworld: Sample Results



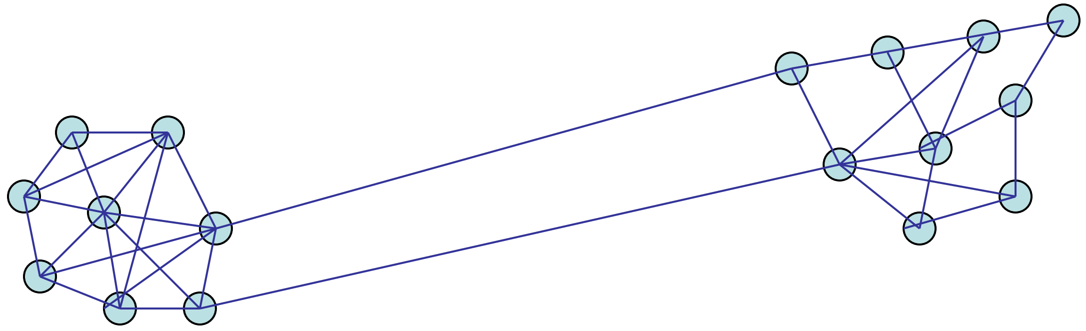




Jianbo Shi's Graph-Partitioning

- An image is represented by a graph whose nodes are pixels or small groups of pixels.

- The goal is to partition the vertices into disjoint sets so that the similarity within each set is high and across different sets is low.



Minimal Cuts

- Let $G = (V,E)$ be a graph. Each edge (u,v) has a weight $w(u,v)$ that represents the similarity between u and v.

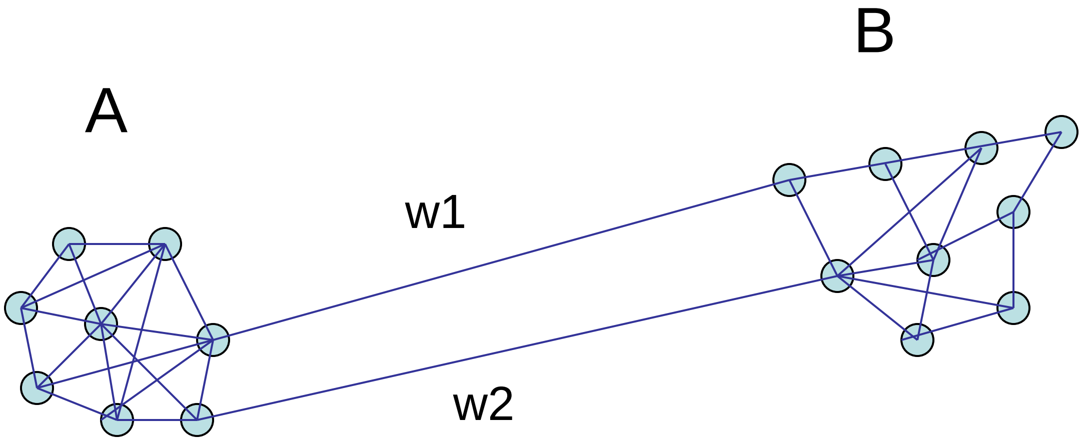
- Graph G can be broken into 2 disjoint graphs with node sets A and B by removing edges that connect these sets.

- Let $\mathrm{cut}(A,B) = \sum_{u \varepsilon A,\ v \varepsilon B} w(u,v)$.

- One way to segment G is to find the minimal cut.

Cut(A,B)

$$\mathrm{cut}(A,B) = \sum_{u \in A,\, v \in B} w(u,v)$$



Normalized Cut

Minimal cut favors cutting off small node groups, so Shi proposed the **normalized cut.**

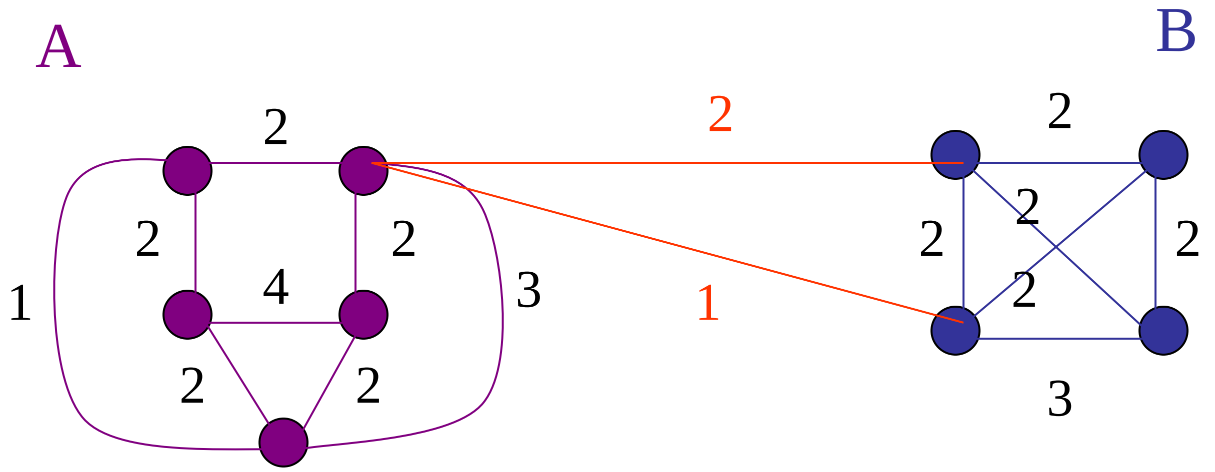
$$\boxed{\mathrm{Ncut}(A,B) = \frac{\mathrm{cut}(A, B)}{\mathrm{asso}(A,V)} + \frac{\mathrm{cut}(A,B)}{\mathrm{asso}(B,V)}}$$

normalized cut

$$\boxed{\mathrm{asso}(A,V) = \sum_{u\in A,\, t\in V} w(u,t)}$$

How much is A connected to the graph as a whole.

Example Normalized Cut



$$\text{Ncut}(A,B) = \frac{3}{21} + \frac{3}{16}$$

Shi turned graph cuts into an eigenvector/eigenvalue problem.

- Set up a weighted graph $G=(V,E)$
  - V is the set of (N) pixels
  - E is a set of weighted edges (weight w_{ij} gives the similarity between nodes i and j)
  - Length N vector d: d_i is the sum of the weights from node i to all other nodes
  - N x N matrix D: D is a diagonal matrix with d on its diagonal
  - N x N symmetric matrix W: $W_{ij} = w_{ij}$

- Let x be a characteristic vector of a set A of nodes
  - x_i = 1 if node i is in a set A
  - x_i = -1 otherwise
- Let y be a continuous approximation to x

$$y = (1 + x) - \frac{\sum_{x_i > 0} d_i}{\sum_{x_i < 0} d_i}(1 - x).$$

- Solve the system of equations

  $(D – W)\, y = \lambda\, D\, y$

  for the eigenvectors y and eigenvalues λ
- Use the eigenvector y with second smallest eigenvalue to bipartition the graph (y => x => A)
- If further subdivision is merited, repeat recursively

How Shi used the procedure

Shi defined the edge weights w(i,j) by

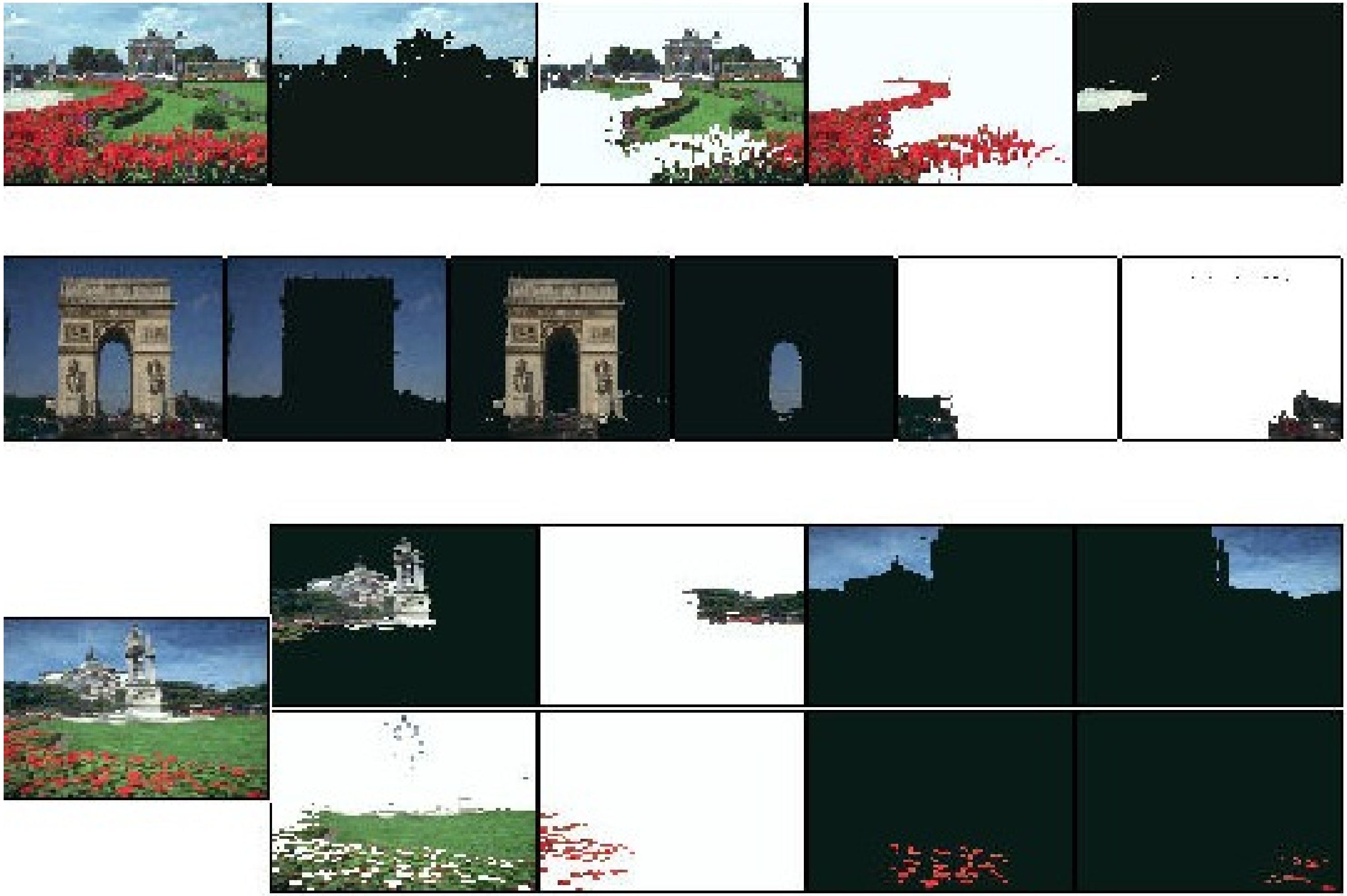
$$w(i,j) = e^{-\|F(i)-F(j)\|_2 / \sigma I} * \begin{cases} e^{-\|X(i)-X(j)\|_2 / \sigma X} & \text{if } \|X(i)-X(j)\|_2 < r \\ 0 & \text{otherwise} \end{cases}$$

where X(i) is the spatial location of node i
F(i) is the feature vector for node I
which can be intensity, color, texture, motion…

The formula is set up so that w(i,j) is 0 for nodes that are too far apart.

Examples of Shi Clustering

See Shi's Web Page
http://www.cis.upenn.edu/~jshi/



Problems with Graph Cuts

- Need to know when to stop
- Can be slow.

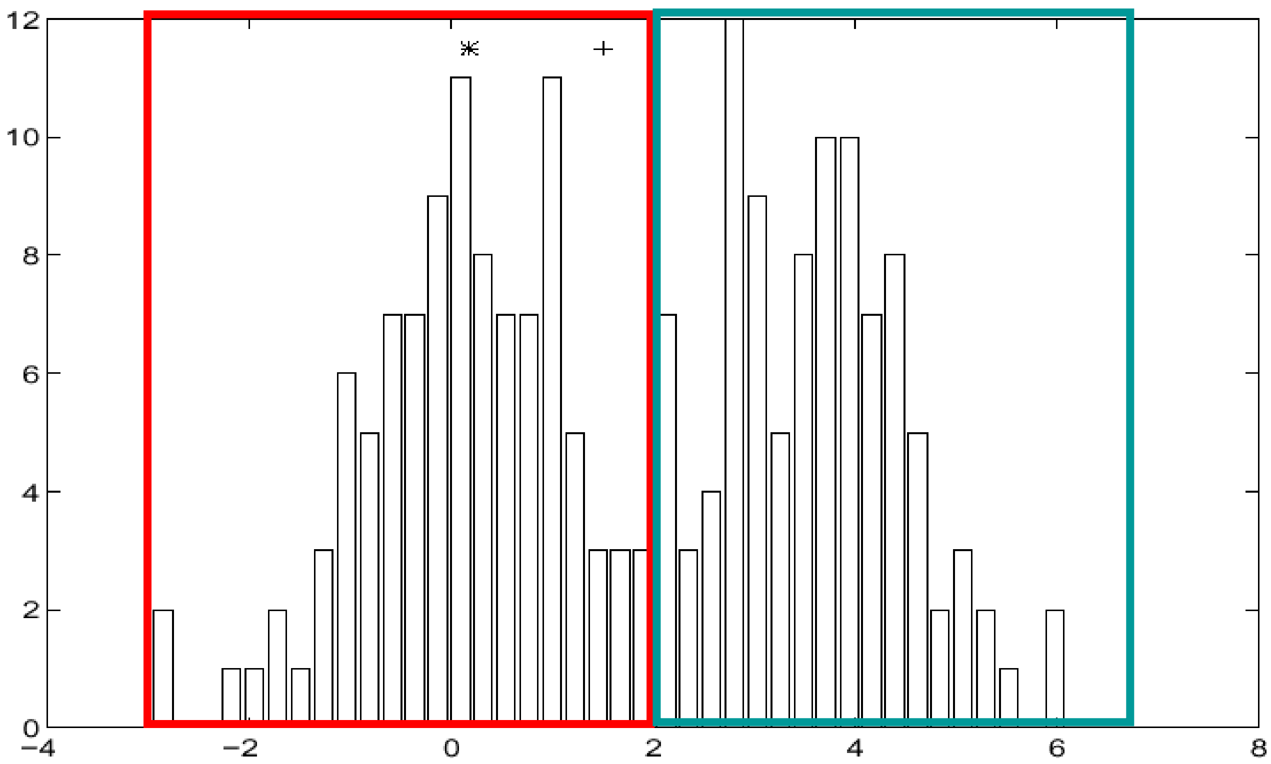
Problems with EM

- Local minima
- Need to know number of segments
- Need to choose generative model

Mean-Shift Clustering

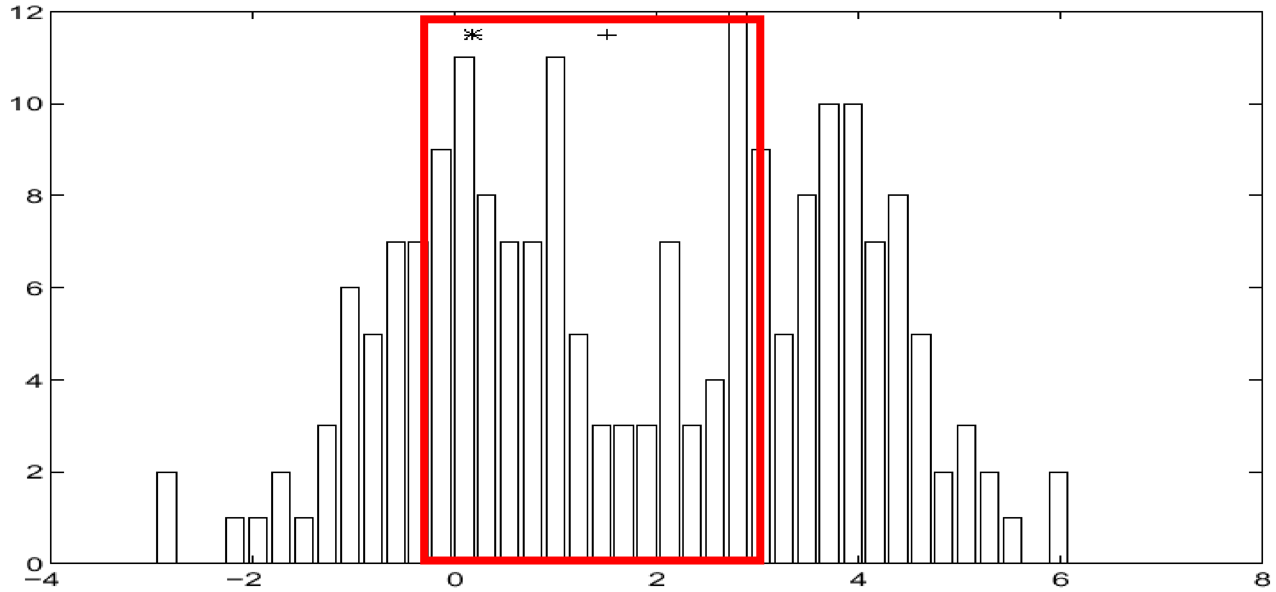
- Simple, like K-means
- But you don't have to select K
- Statistical method
- Guaranteed to converge to a fixed number of clusters.

Finding Modes in a Histogram



- How Many Modes Are There?
  - Easy to see, hard to compute

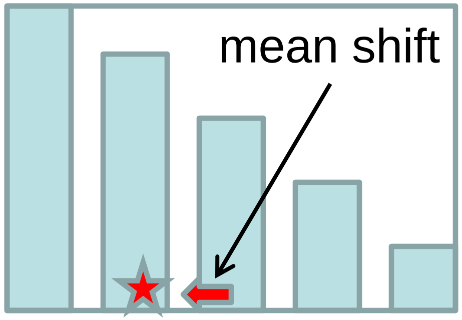
Mean Shift [Comaniciu & Meer]



- Iterative Mode Search
  1. Initialize random seed, and window W
  2. Calculate center of gravity (the "mean") of W: $\sum_{x \in W} x H(x)$
  3. Translate the search window to the mean
  4. Repeat Step 2 until convergence

Numeric Example
Must Use *Normalized* Histogram!

window W centered at 12

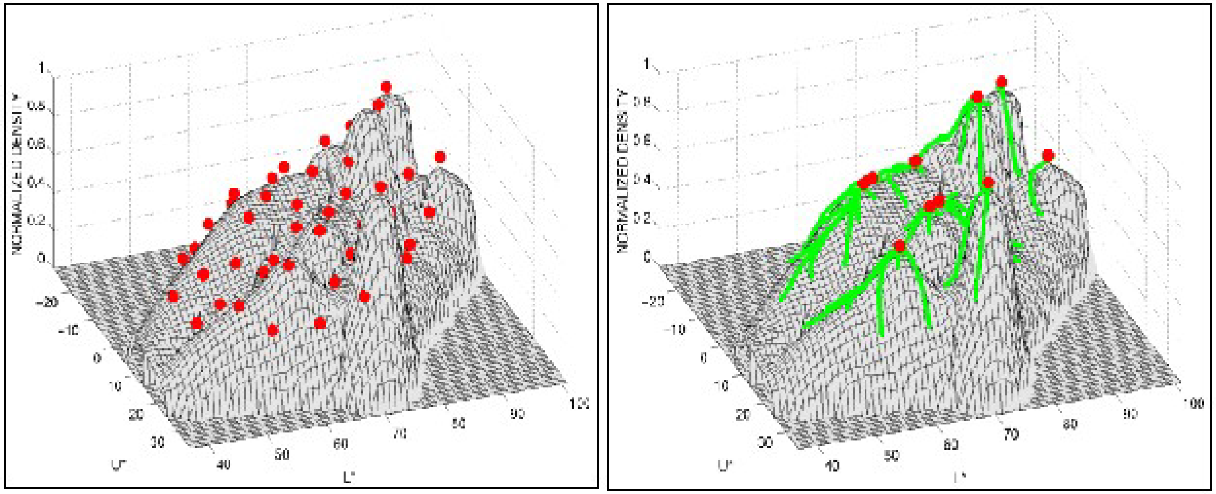


x	10	11	12	13	14
H(x)	5	4	3	2	1
N(x)	5/15	4/15	3/15	2/15	1/15

$$\sum x\, N(x) = 10(5/15)+11(4/15)+12(3/15)+13(2/15)+14(1/15)$$
$$= 11.33$$

Mean Shift Approach

– Initialize a window around each point
– See where it shifts—this determines which segment it's in
– Multiple points will shift to the same segment



Mean shift trajectories

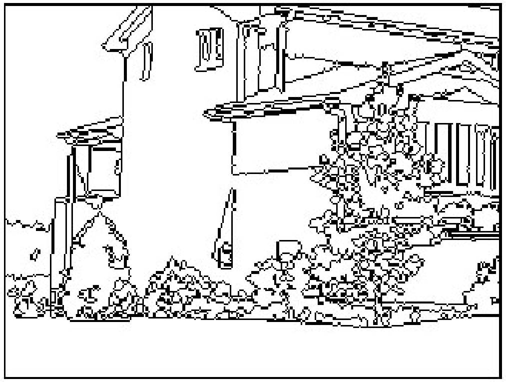
Segmentation Algorithm

- First run the mean shift procedure for each data point x and store its convergence point z.
- Link together all the z's that are closer than .5 from each other to form clusters
- Assign each point to its cluster
- Eliminate small regions

Mean-shift for image segmentation

- Useful to take into account spatial information
  - instead of (R, G, B), run in (R, G, B, x, y) space





References

- Shi and Malik, "Normalized Cuts and Image Segmentation," Proc. CVPR 1997.
- Carson, Belongie, Greenspan and Malik, "Blobworld: Image Segmentation Using Expectation-Maximization and its Application to Image Querying," IEEE PAMI, Vol 24, No. 8, Aug. 2002.
- Comaniciu and Meer, "Mean shift analysis and applications," Proc. *ICCV* 1999.