3D Sensing and Reconstruction

Readings: Ch 12: 12.5-6, Ch 13: 13.1-3, 13.9.4

- Perspective Geometry
- Camera Model
- Stereo Triangulation
- 3D Reconstruction by Space Carving

3D Shape from X means getting 3D coordinates from different methods

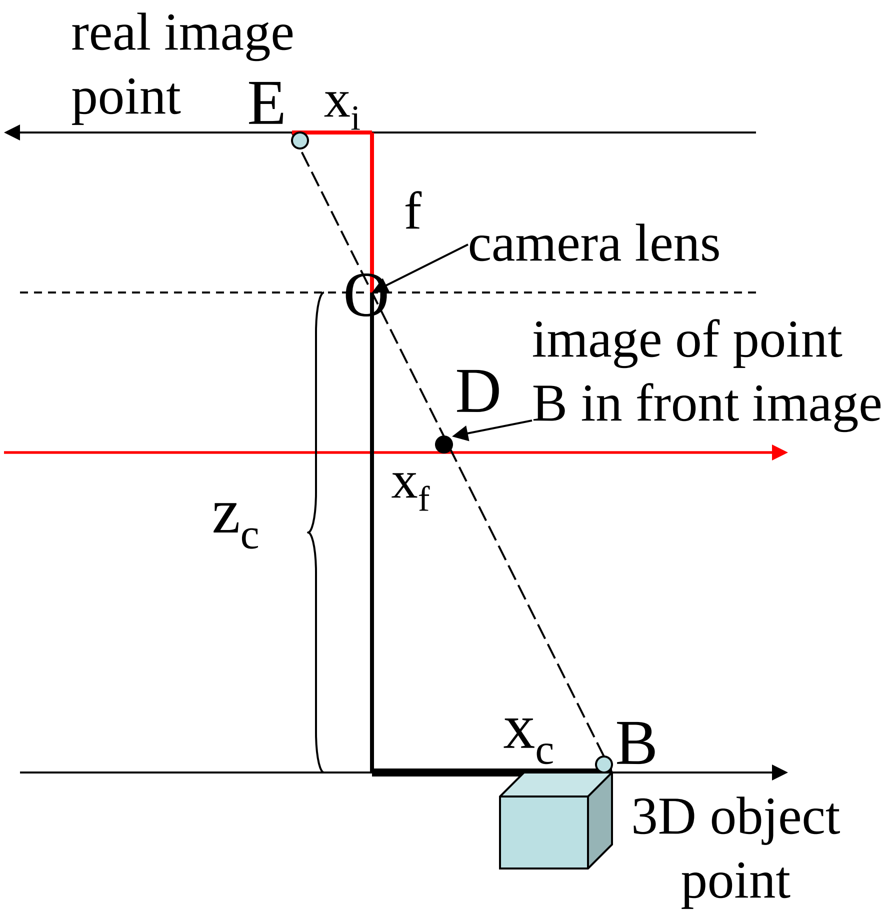
- shading
- silhouette
- texture

mainly research

- stereo
- light striping
- motion

used in practice

Perspective Imaging Model: 1D



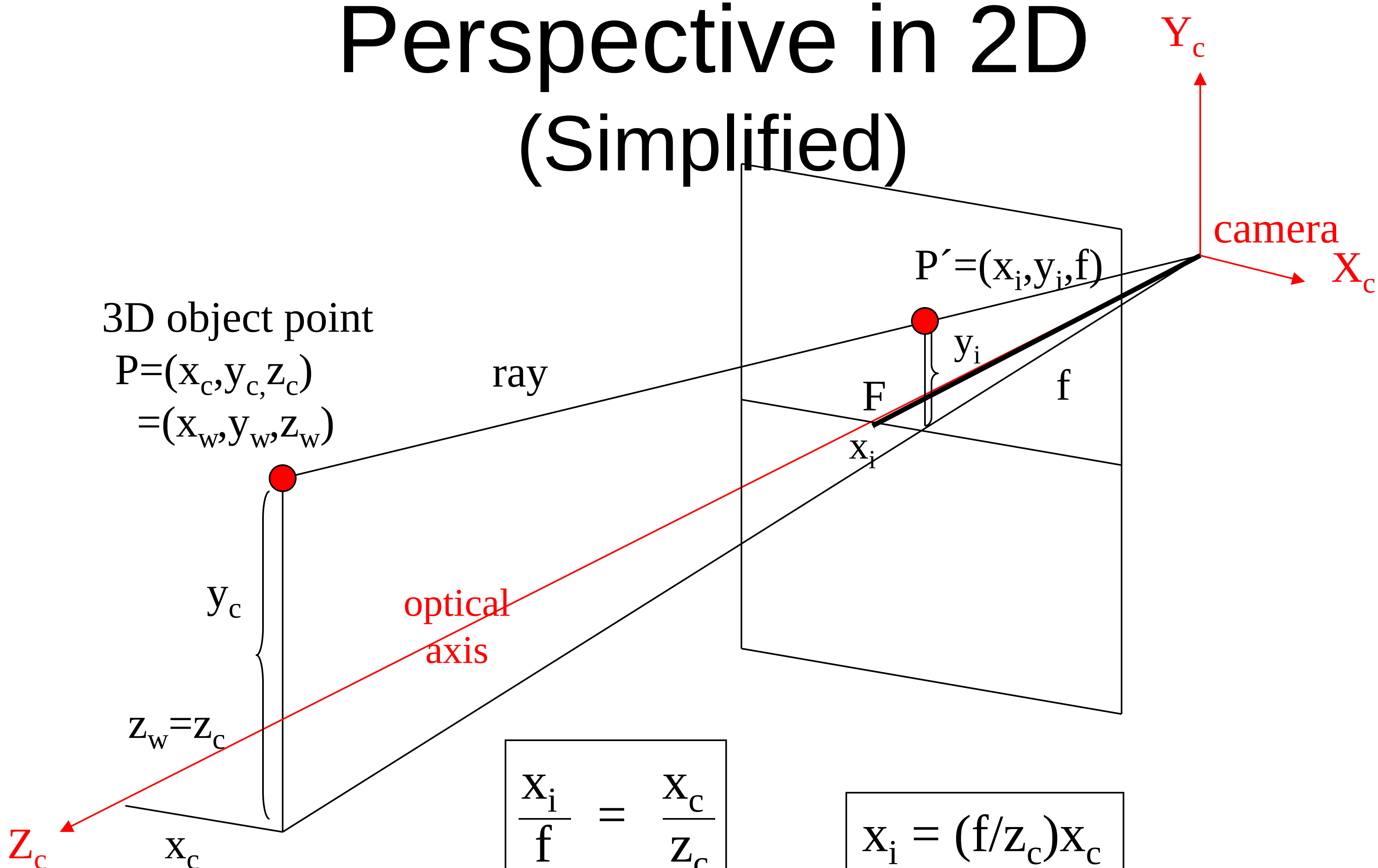
This is the axis of the real image plane.

O is the center of projection.

This is the axis of the front image plane, which we use.

$$\boxed{\frac{x_i}{f} = \frac{x_c}{z_c}}$$

Perspective in 2D
(Simplified)



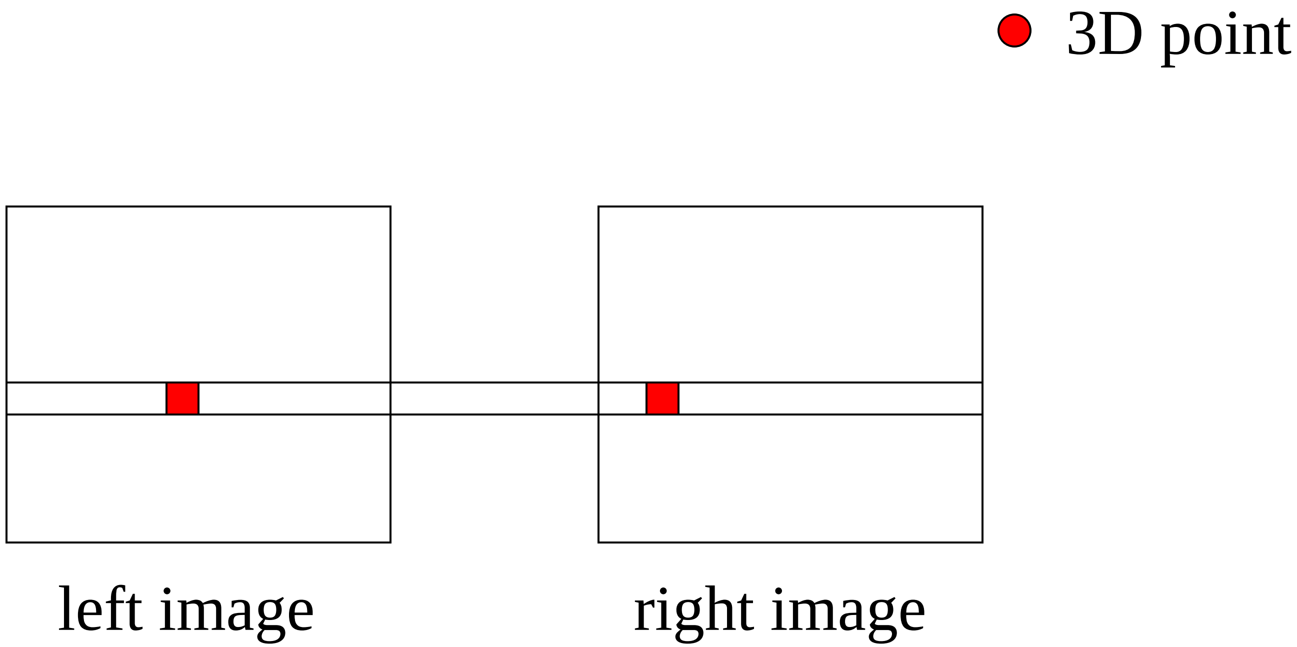
$$\frac{x_i}{f} = \frac{x_c}{z_c}$$

$$\frac{y_i}{f} = \frac{y_c}{z_c}$$

$$x_i = (f/z_c)x_c$$
$$yi = (f/z_c)y_c$$

Here camera coordinates equal world coordinates.

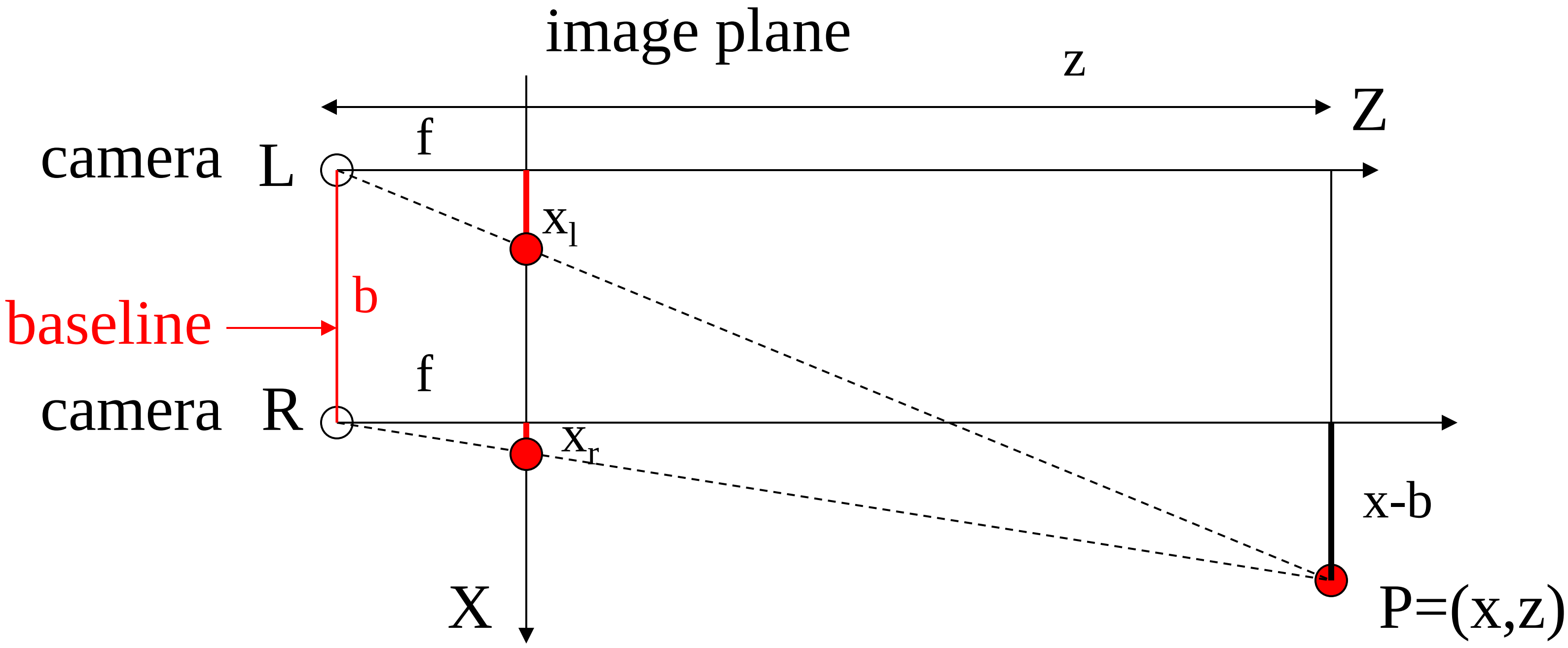
3D from Stereo



disparity: the difference in image location of the same 3D point when projected under perspective to two different cameras.

$$\boxed{d = x_{left} - x_{right}}$$

Depth Perception from Stereo
Simple Model: Parallel Optic Axes



$$\frac{z}{f} = \frac{x}{x_l}$$

$$\frac{z}{f} = \frac{x-b}{x_r}$$

$$\frac{z}{f} = \frac{y}{y_l} = \frac{y}{y_r}$$

y-axis is perpendicular to the page.

Resultant Depth Calculation

For stereo cameras with parallel optical axes, focal length f, baseline b, corresponding image points (x_l,y_l) and (x_r,y_r) with disparity d:

$$z = f*b / (x_l - x_r) = f*b/d$$

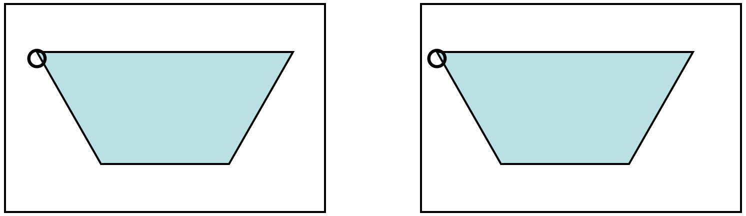
$$x = x_l*z/f \quad \text{or} \quad b + x_r*z/f$$

$$y = y_l*z/f \quad \text{or} \quad y_r*z/f$$

This method of determining depth from disparity is called **triangulation**.

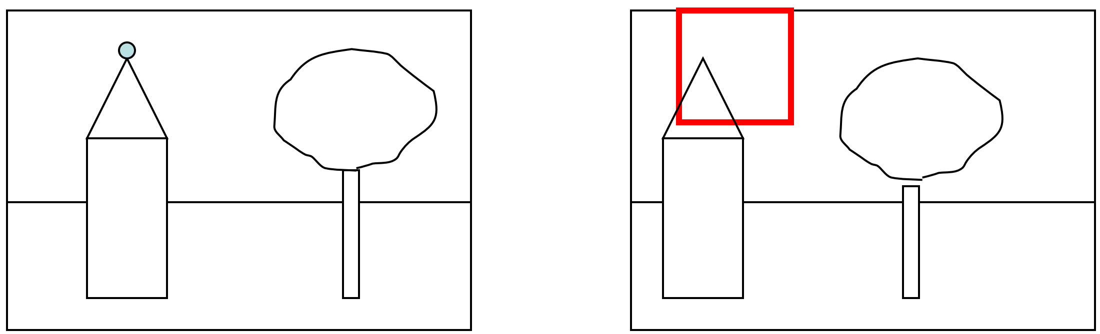
Finding Correspondences

- If the correspondence is correct, triangulation works **VERY** well.
- But correspondence finding is not perfectly solved.
  (What methods have we studied?)
- For some very specific applications, it can be solved for those specific kind of images, e.g. windshield of a car.



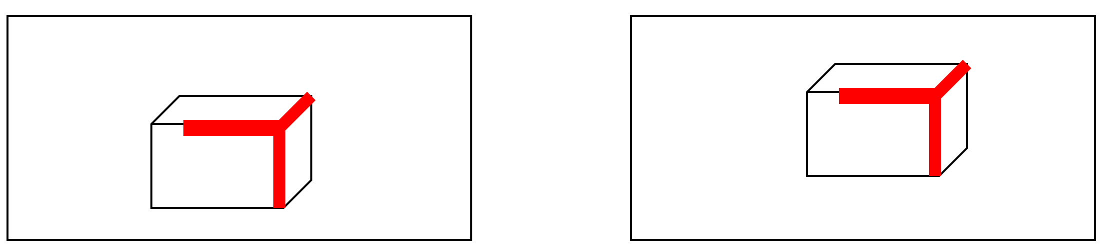
3 Main Matching Methods

1. Cross correlation using small windows.



dense

2. Symbolic feature matching, usually using segments/corners.



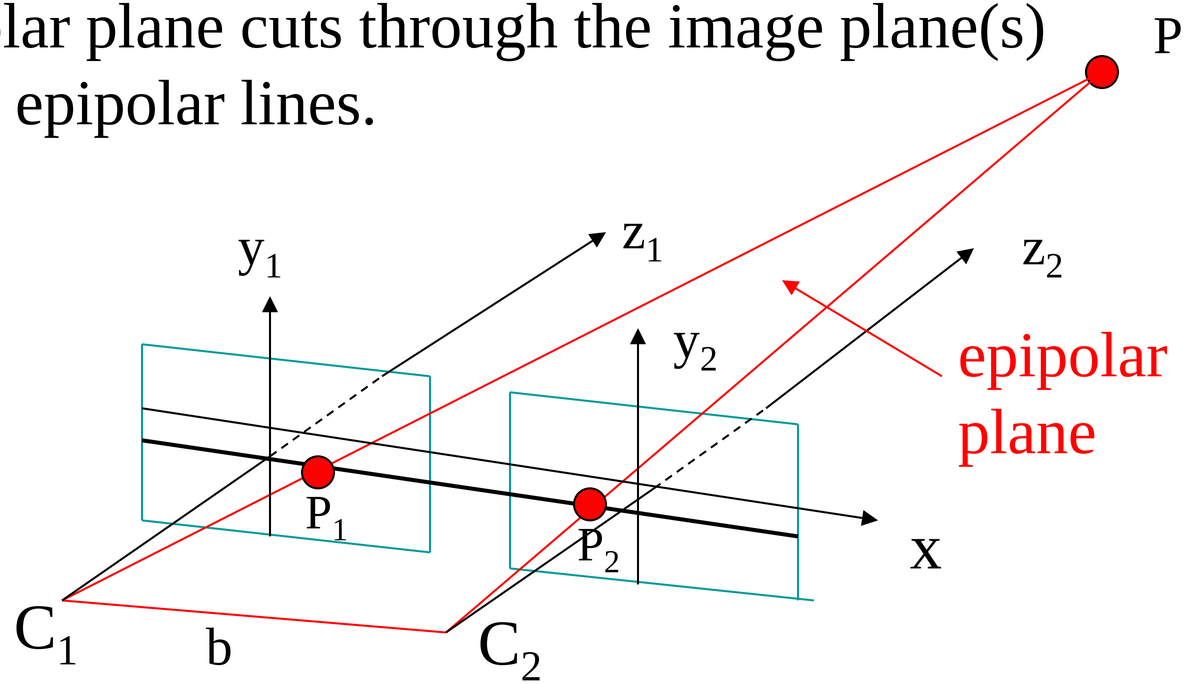
sparse

3. Use the newer interest operators, ie. SIFT.

sparse

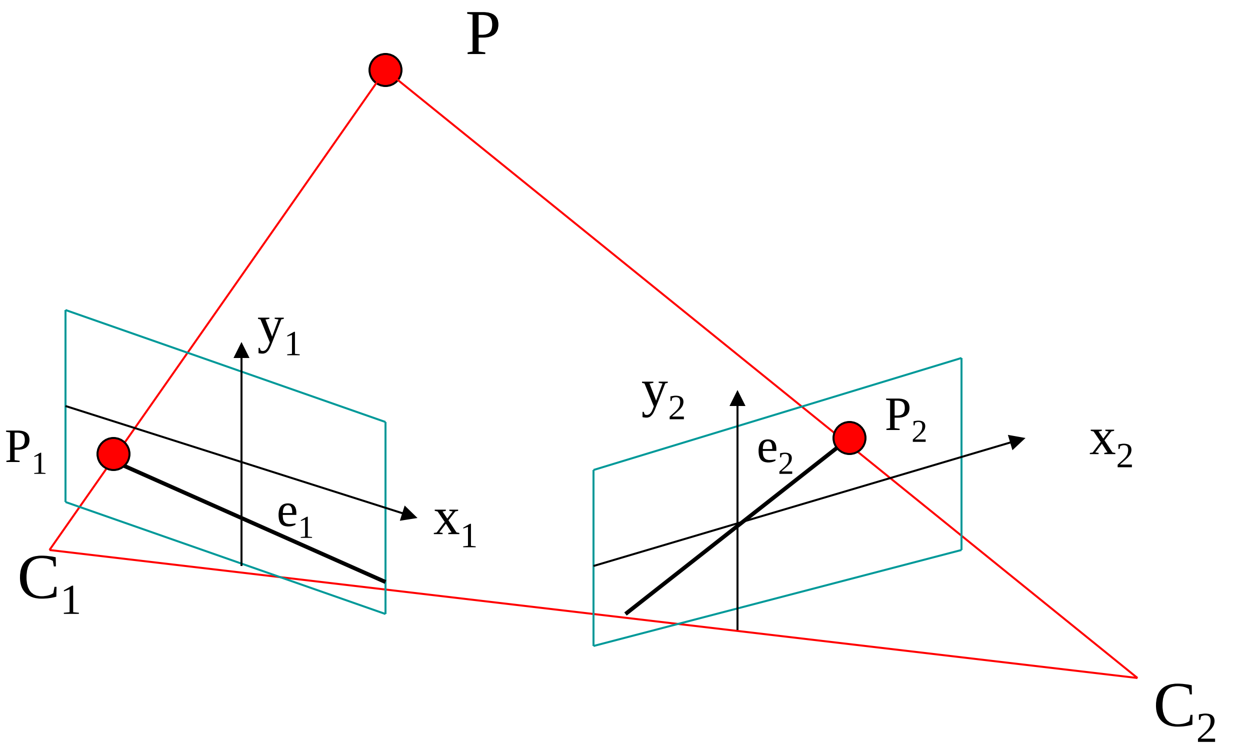
Epipolar Geometry Constraint:
1. Normal Pair of Images

The epipolar plane cuts through the image plane(s) forming 2 epipolar lines.



The match for P_1 (or P_2) in the other image, must lie on the same epipolar line.

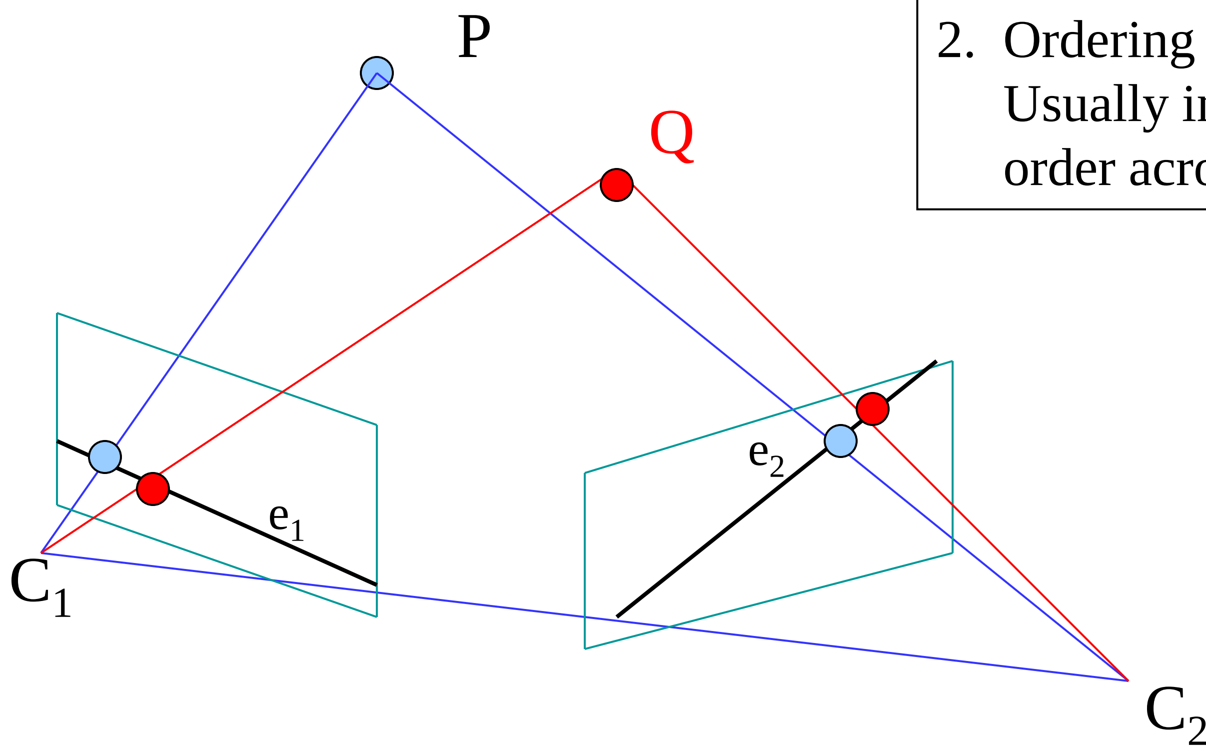
Epipolar Geometry: General Case



Constraints

1. Epipolar Constraint: Matching points lie on corresponding epipolar lines.

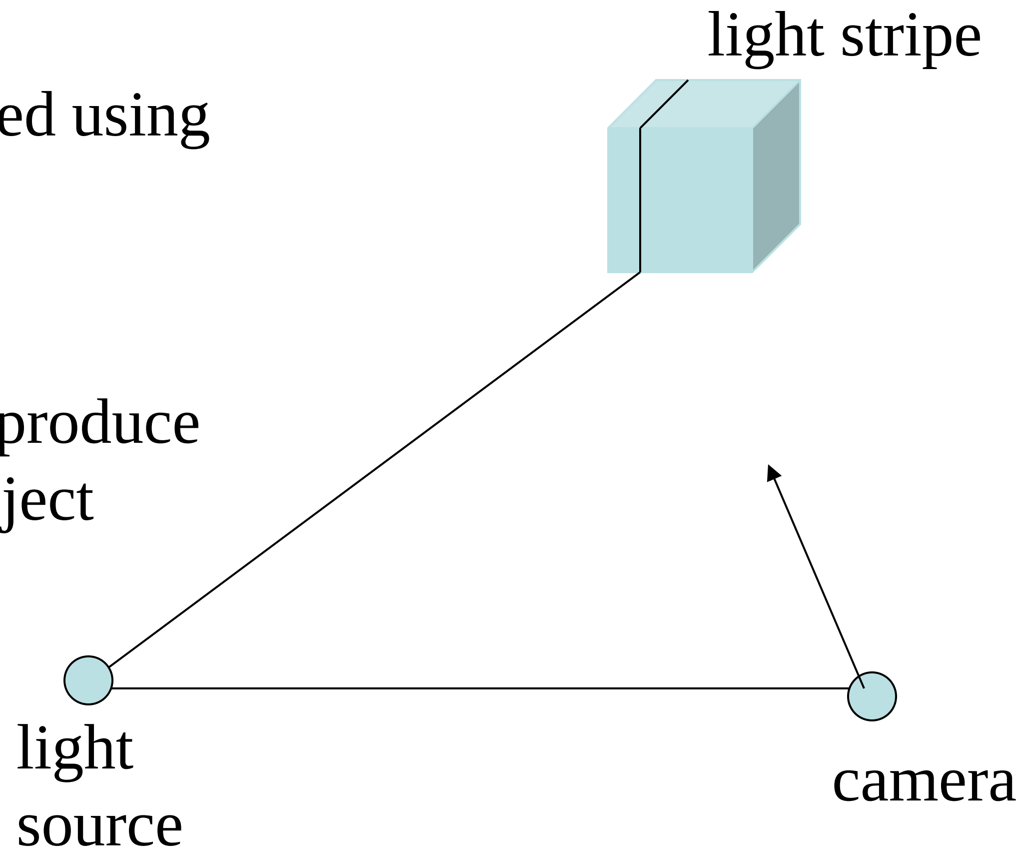
2. Ordering Constraint: Usually in the same order across the lines.



Structured Light

3D data can also be derived using

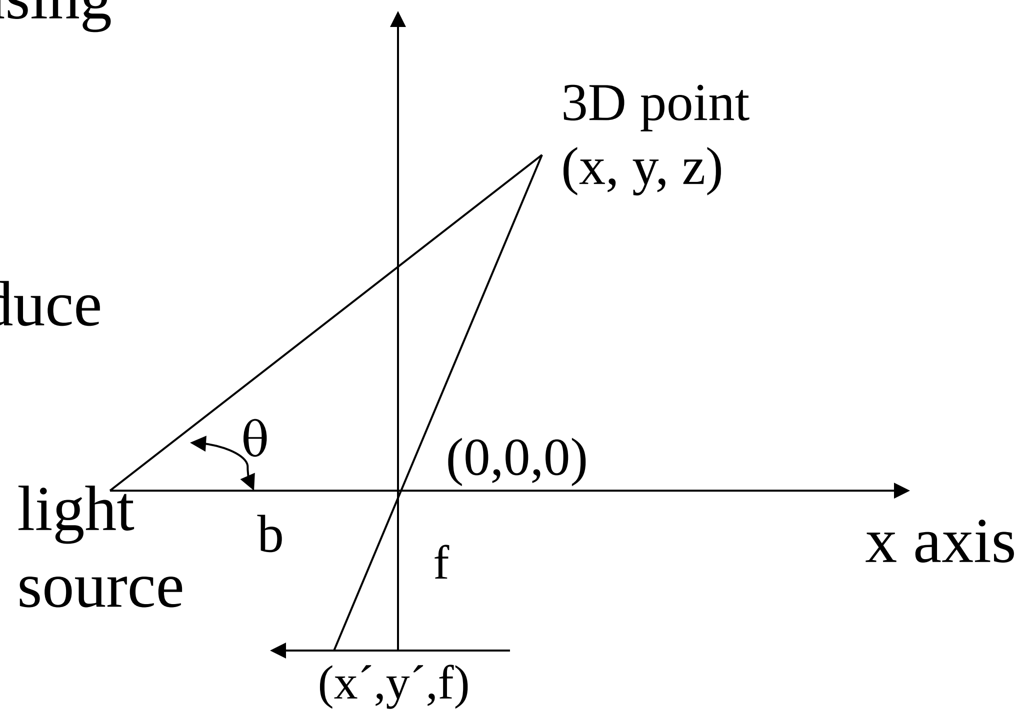
- a single camera
- a light source that can produce stripe(s) on the 3D object



Structured Light
3D Computation

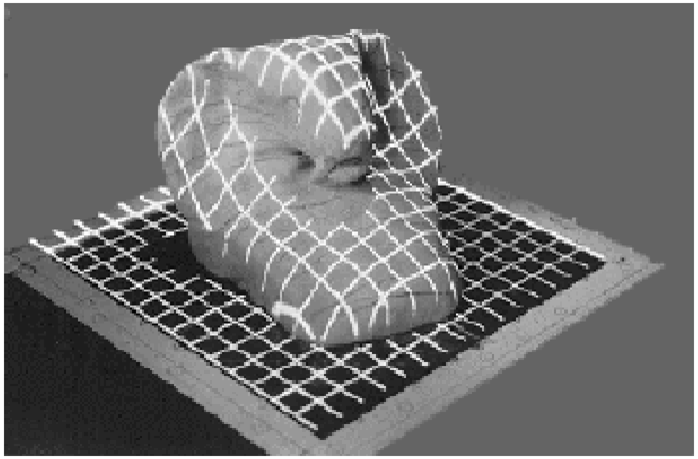
3D data can also be derived using

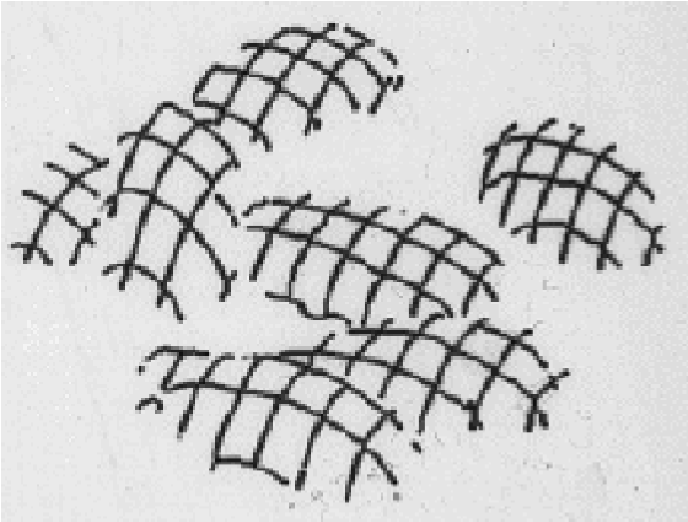
- a single camera
- a light source that can produce stripe(s) on the 3D object



$$\underset{\text{3D}}{[x\ y\ z]} = \frac{b}{f \cot\theta - x'} \underset{\text{image}}{[x'\ y'\ f]}$$

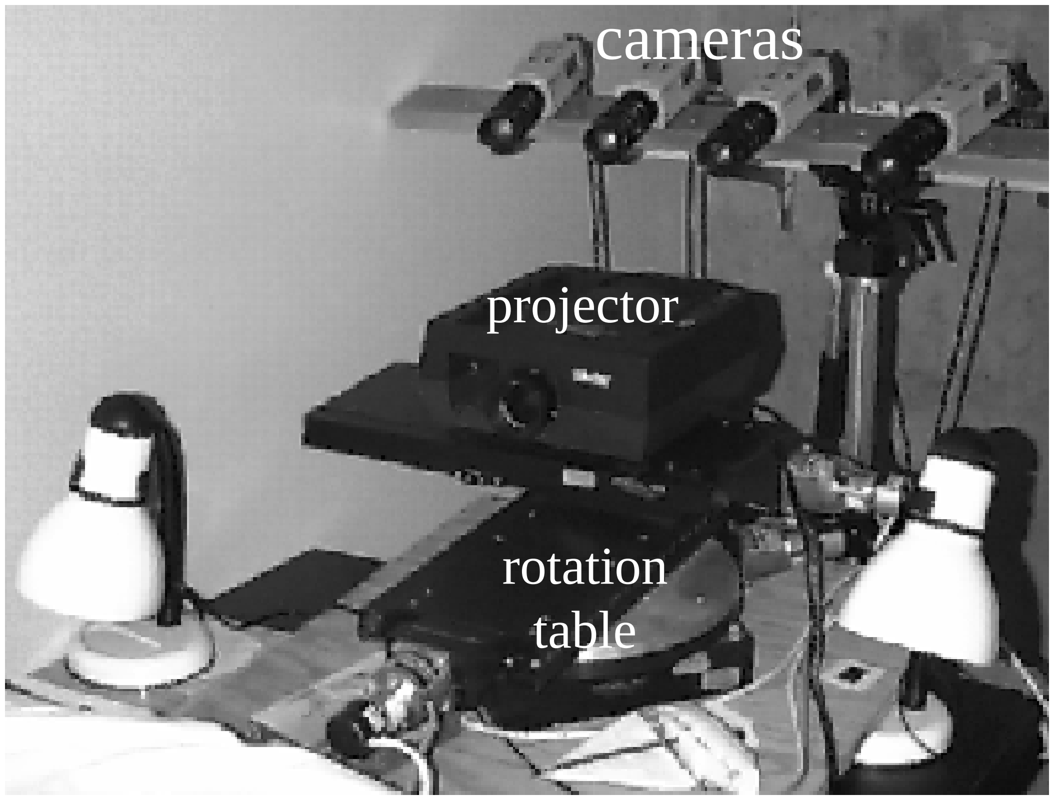
Depth from Multiple Light Stripes

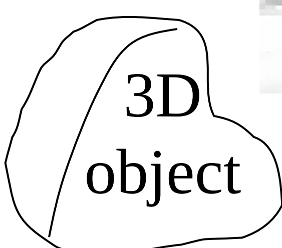




What are these objects?

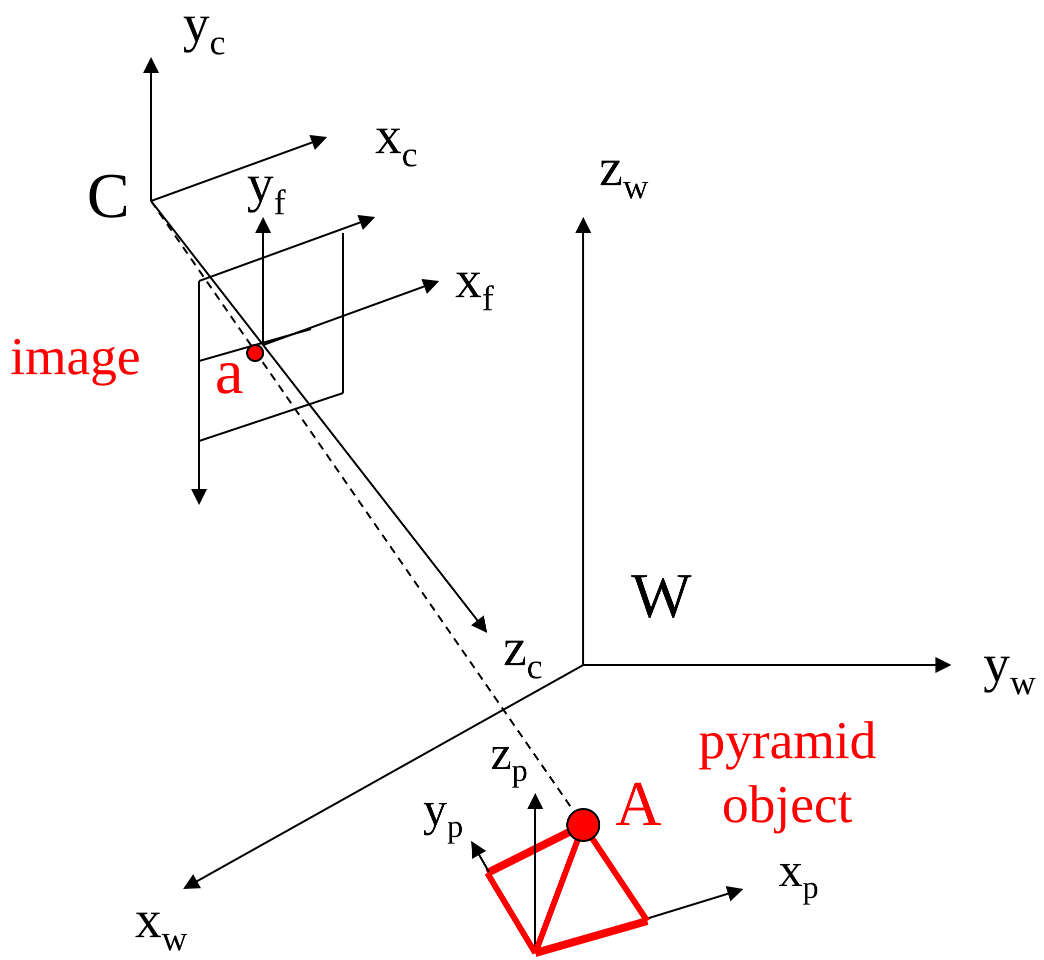
Our (former) System
4-camera light-striping stereo





Camera Model: Recall there are 5 Different Frames of Reference

- **Object**
- **World**
- **Camera**
- **Real Image**
- **Pixel Image**



The Camera Model

How do we get an image point IP from a world point P?

$$\begin{pmatrix} s\,IP_r \\ s\,IP_c \\ s \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & 1 \end{pmatrix} \begin{pmatrix} P_x \\ P_y \\ P_z \\ 1 \end{pmatrix}$$

image point

camera matrix C

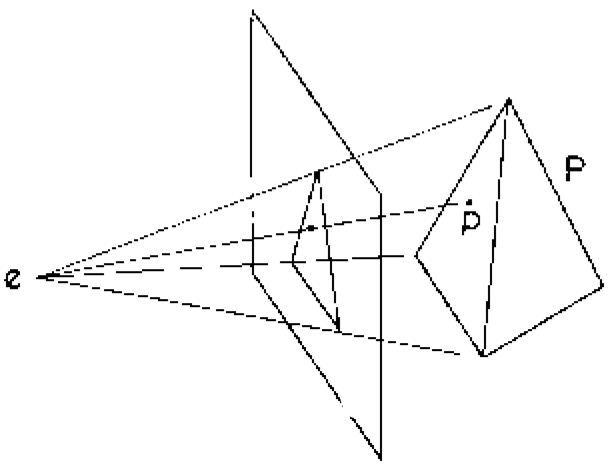
world point

What's in C?

The camera model handles the rigid body transformation from world coordinates to camera coordinates plus the perspective transformation to image coordinates.

1.	CP	=	T R WP
2.	FP	=	π(f) CP

Why is there not a scale factor here?



$$\begin{pmatrix} s\,FP_x \\ s\,FP_y \\ s\,FP_z \\ s \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/f & 0 \end{pmatrix} \begin{pmatrix} CP_x \\ CP_y \\ CP_z \\ 1 \end{pmatrix}$$

image point

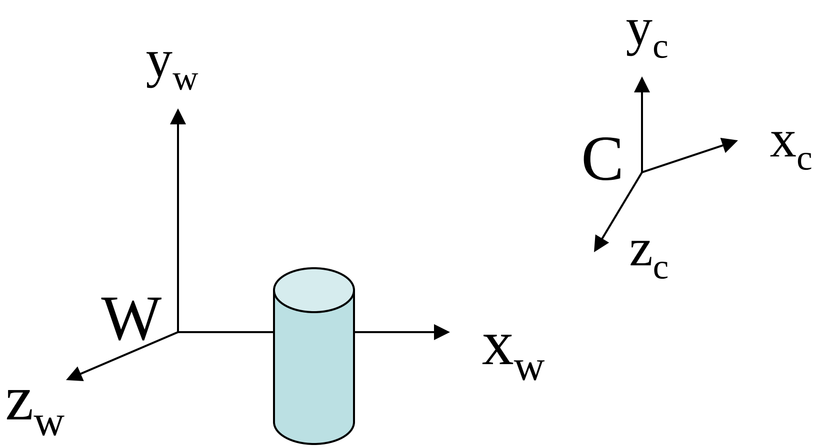
perspective transformation

3D point in camera coordinates

Camera Calibration

- In order work in 3D, we need to know the parameters of the particular camera setup.

- Solving for the camera parameters is called calibration.

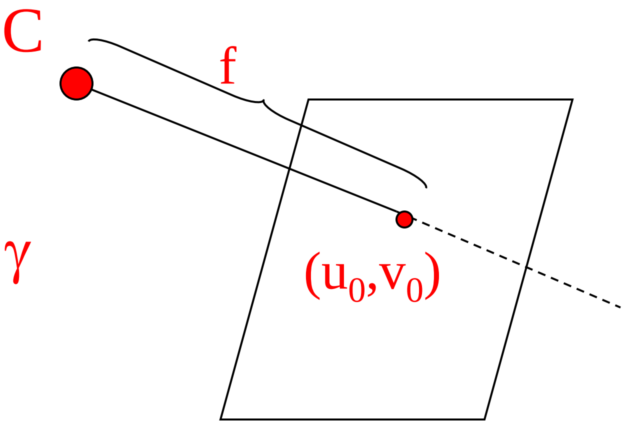


- intrinsic parameters are of the camera device

- extrinsic parameters are where the camera sits in the world

Intrinsic Parameters

- principal point (u_0,v_0)
- scale factors (d_x,d_y)
- aspect ratio distortion factor γ
- focal length f
- lens distortion factor κ
  (models radial lens distortion)



Extrinsic Parameters

- translation parameters

  $t = [t_x \;\; t_y \;\; t_z]$

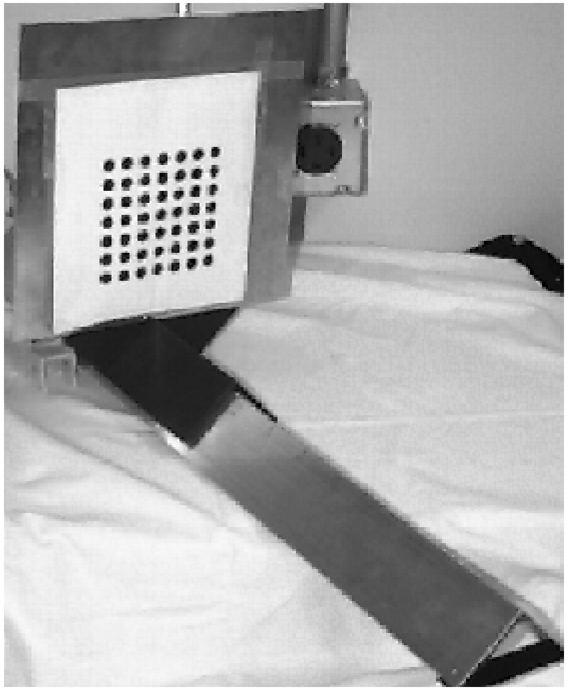
- rotation matrix

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Are there really nine parameters?

Calibration Object

The idea is to snap images at different depths and get a lot of 2D-3D point correspondences.



The Tsai Procedure

- The Tsai procedure was developed by Roger Tsai at IBM Research and is most widely used.

- Several images are taken of the calibration object yielding point correspondences at different distances.

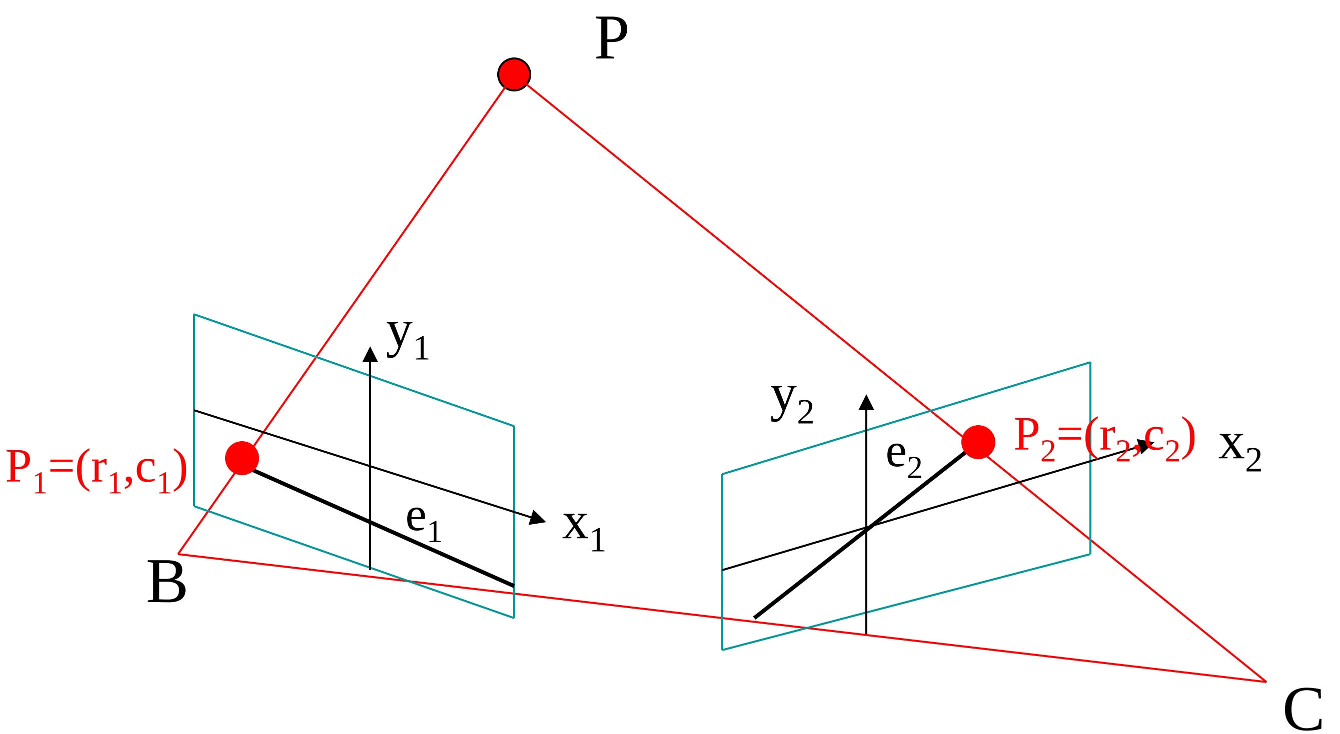
- Tsai's algorithm requires $n > 5$ correspondences

$$\{(x_i, y_i, z_i), (u_i, v_i)) \mid i = 1,\ldots,n\}$$

between (real) image points and 3D points.

- Lots of details in Chapter 13.

We use the camera parameters of each camera for general stereo.



For a correspondence (r_1, c_1) in image 1 to (r2,c2) in image 2:

1. Both cameras were calibrated. Both camera matrices are then known. From the two camera equations B and C we get

   4 linear equations in 3 unknowns.

$$r_1 = (b_{11} - b_{31}*r_1)\mathbf{x} + (b_{12} - b_{32}*r_1)\mathbf{y} + (b_{13}-b_{33}*r_1)\mathbf{z}$$
$$c_1 = (b_{21} - b_{31}*c1)\mathbf{x} + (b_{22} - b_{32}*c_1)\mathbf{y} + (b_{23}-b_{33}*c_1)\mathbf{z}$$

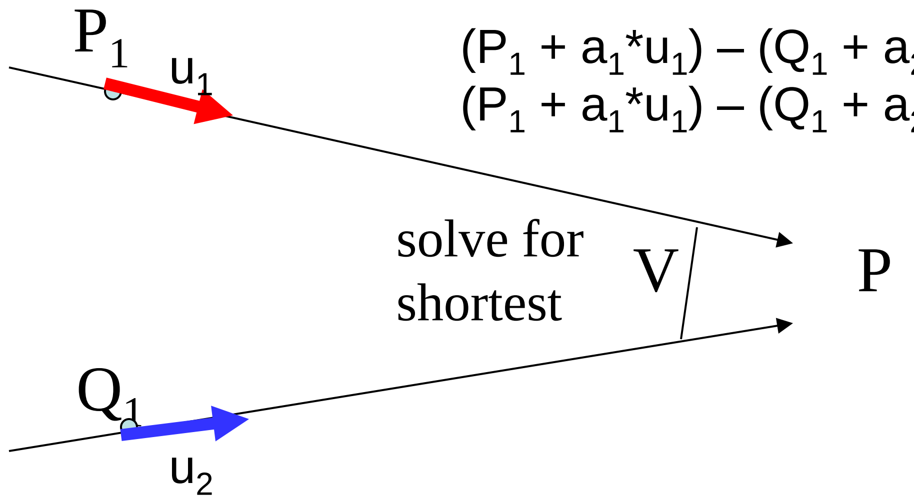
$$r_2 = (c_{11} - c_{31}*r_{2)}\mathbf{x} + (c_{12} - c_{32}*r_2)\mathbf{y} + (c_{13} - c_{33}*r_2)\mathbf{z}$$
$$c_2 = (c_{21} - c_{31}*c_2)\mathbf{x} + (c_{22} - c_{32}*c_2)\mathbf{y} + (c_{23} - c_{33}*c_2)\mathbf{z}$$

Direct solution uses 3 equations, won't give reliable results.

Solve by computing the closest approach of the two skew rays.

$$V = (P_1 + a_1 * u_1) - (Q_1 + a_2 * u_2)$$

$$(P_1 + a_1 * u_1) - (Q_1 + a_2 * u_2) \cdot u_1 = 0$$
$$(P_1 + a_1 * u_1) - (Q_1 + a_2 * u_2) \cdot u_2 = 0$$



If the rays intersected perfectly in 3D, the intersection would be P. Instead, we solve for the shortest line segment connecting the two rays and let P be its midpoint.

Surface Modeling and Display from Range and Color Data

Kari	Pulli	UW
Michael	Cohen	MSR
Tom	Duchamp	UW
Hugues	Hoppe	MSR
John	McDonald	UW
Linda	Shapiro	UW
Werner	Stuetzle	UW

UW = University of Washington
Seattle, WA USA

MSR = Microsoft Research
Redmond, WA USA

Introduction

Goal

- develop robust algorithms for constructing 3D models from range & color data
- use those models to produce realistic renderings of the scanned objects



Surface Reconstuction

Step 1: Data acquisition

Obtain range data that covers the object. Filter, remove background.

Step 2: Registration

Register the range maps into a common coordinate system.

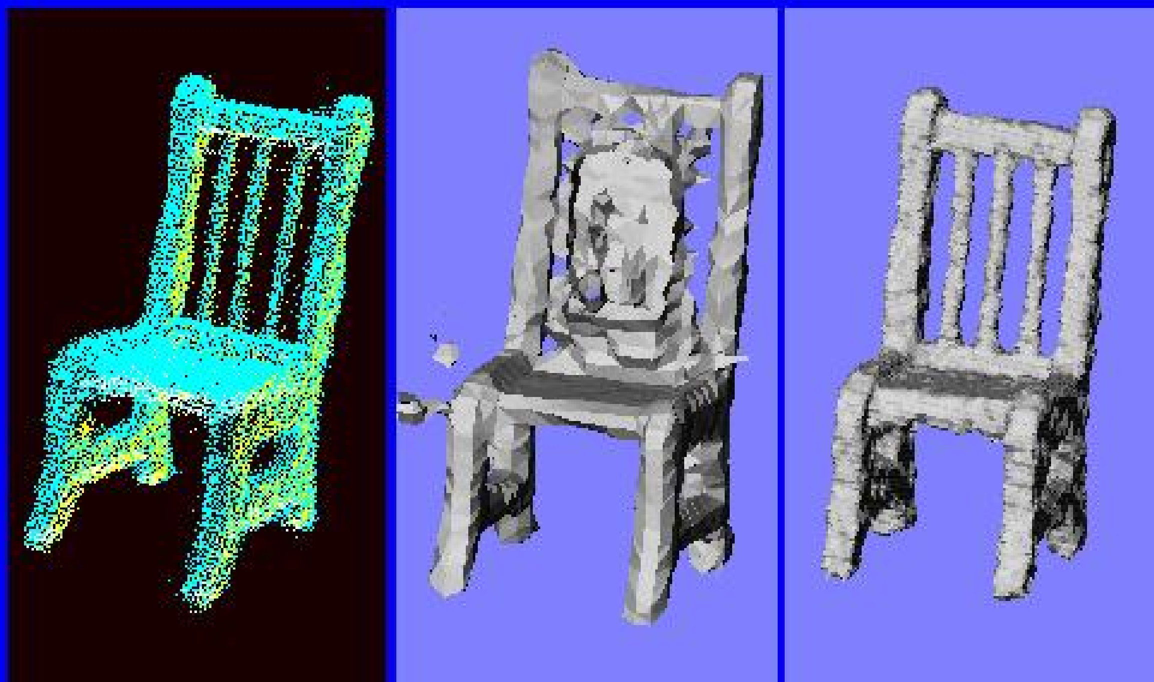
Step 3: Integration

Integrate the registered range data into a single surface representation.

Step 4: Optimization

Fit the surface more accurately to the data, simplify the representation.

Problem

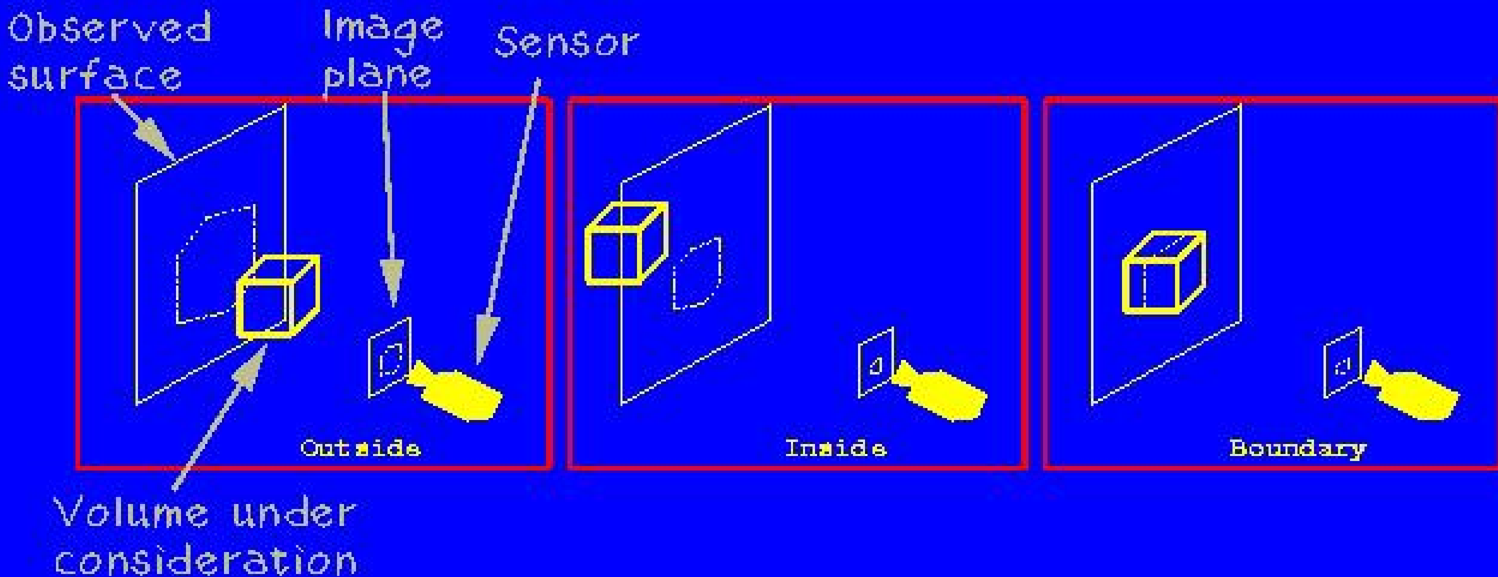


Noisy registered data

Signed distance fn & marching cubes

Hierarchical & directional space carving

Carve space in cubes



Label cubes

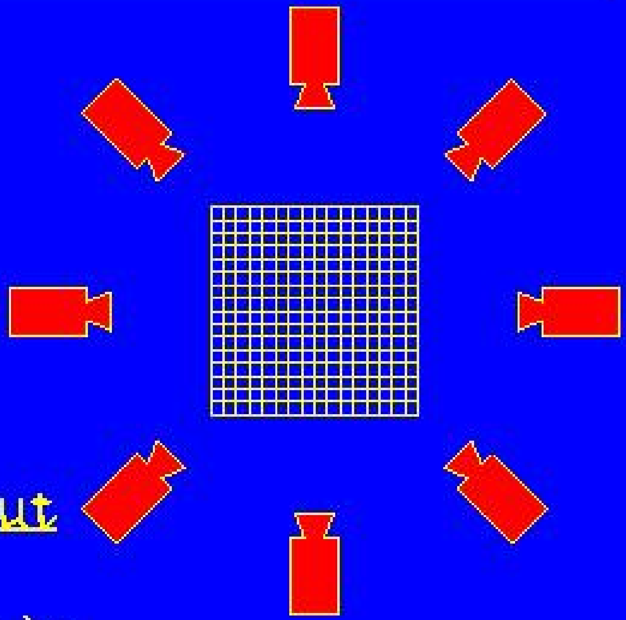
- Project cube to image plane (hexagon)
- Test against data in the hexagon

Several views

Processing order:

FOR EACH cube

FOR EACH view



Rules:

<u>any</u> view thinks cube's <u>out</u>

=> it's <u>out</u>

<u>every</u> view thinks cube's <u>in</u>

=> it's <u>in</u>

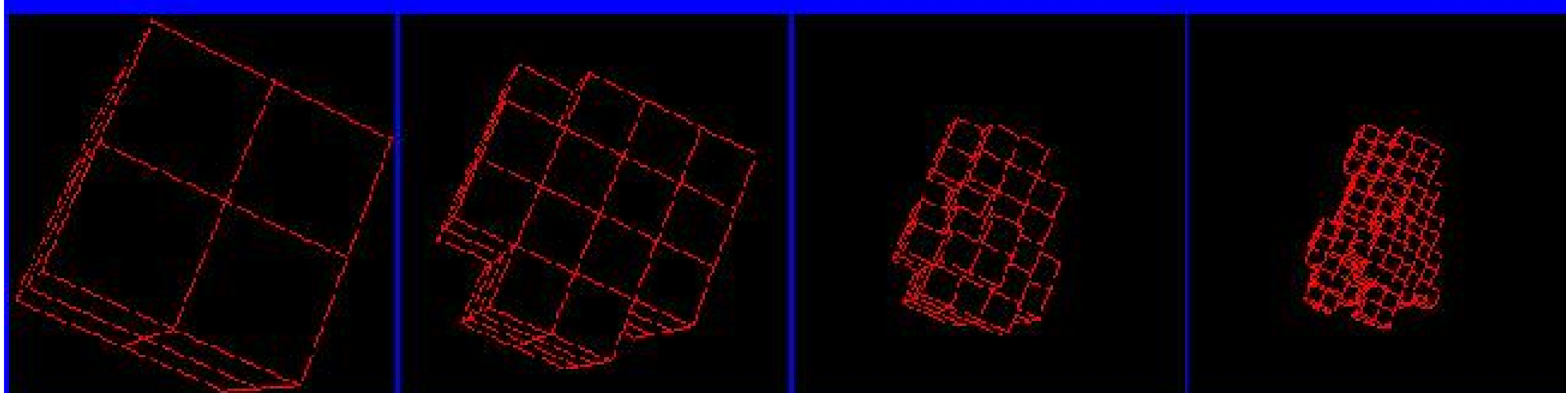
else

=> it's at boundary

Hierarchical space carving

- Big cubes => fast, poor results
- Small cubes => slow, more accurate results
- Combination = octrees

RULES:
- cube's out => done
- cube's in => done
- else => recurse



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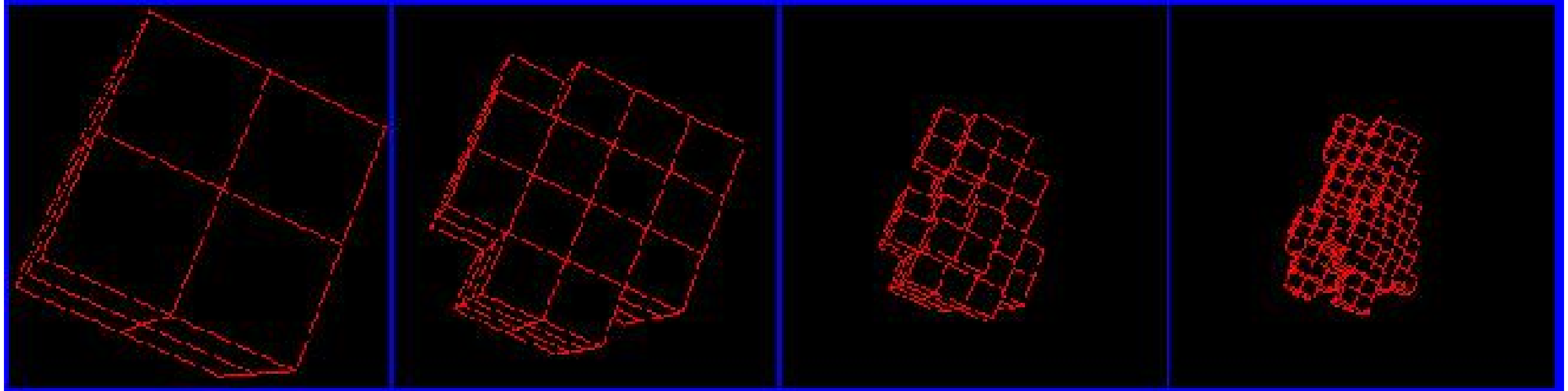
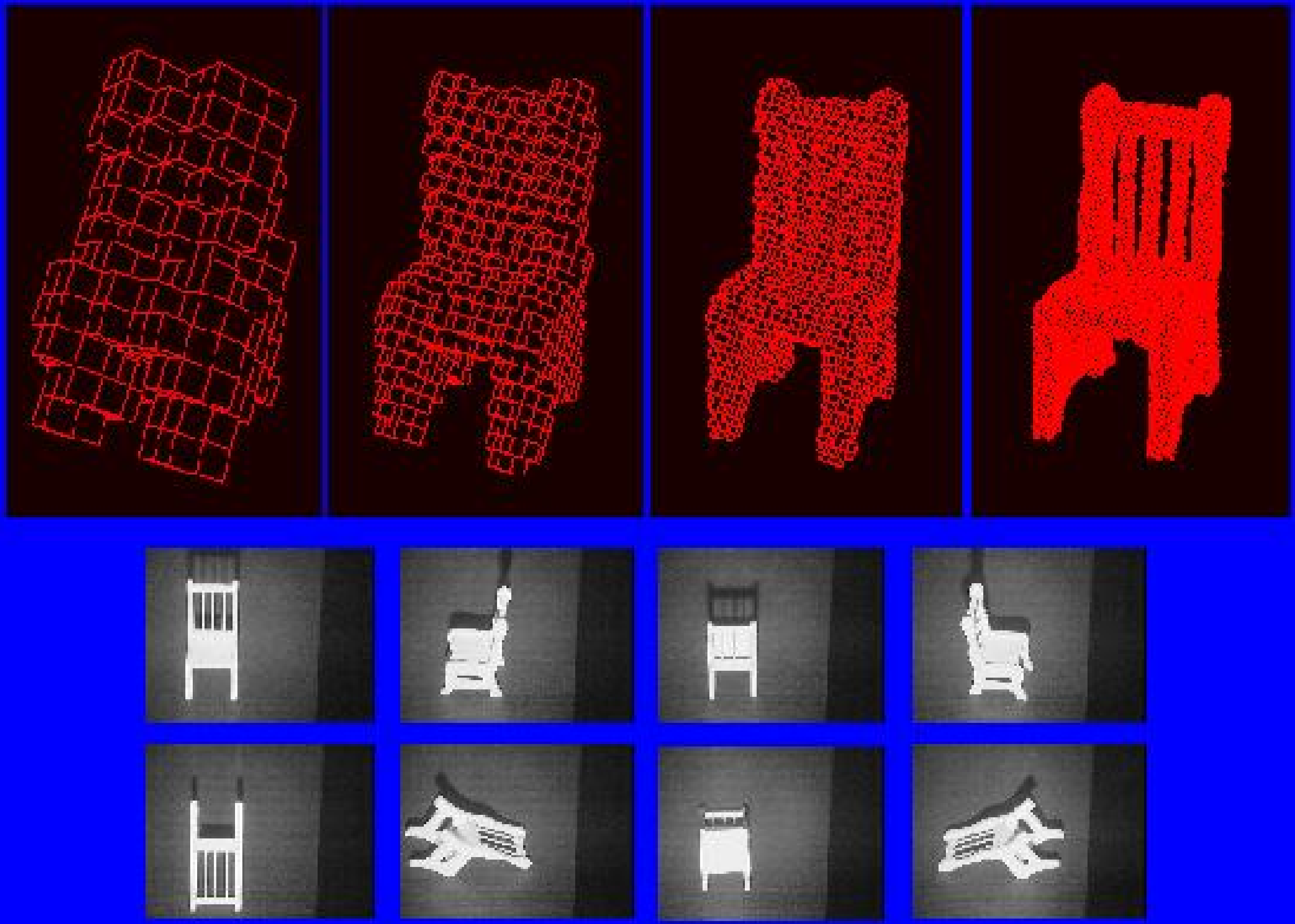
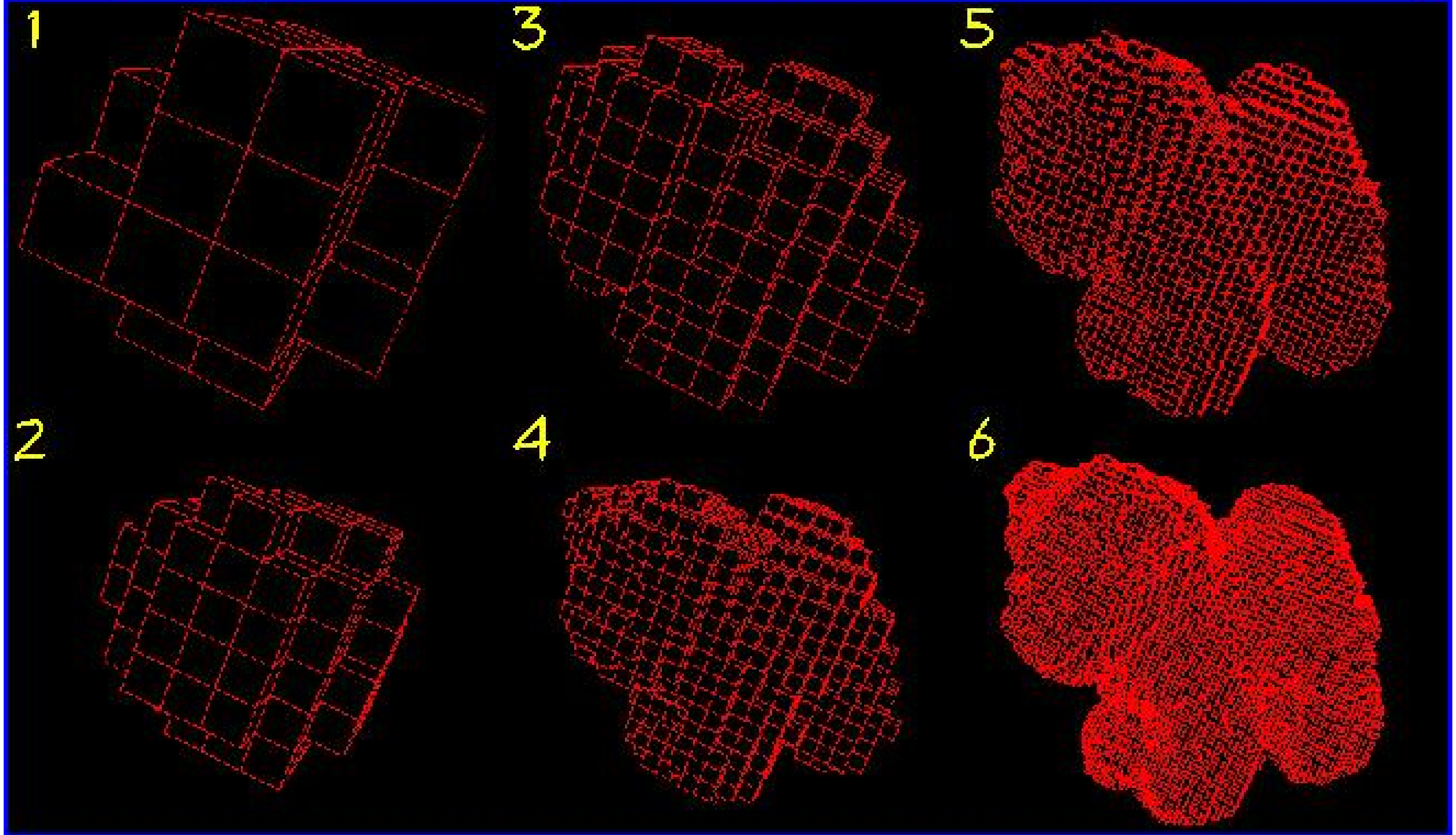


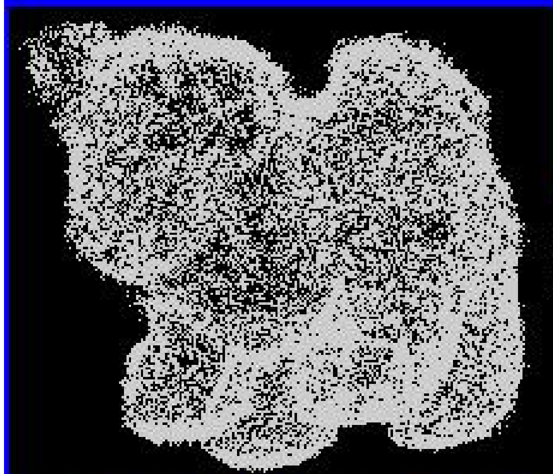
The rest of the chair



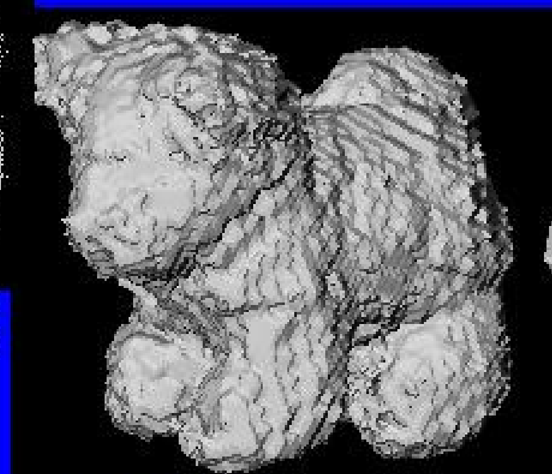
Same for a husky pup



Optimizing the dog mesh



Registered points



Initial mesh

Optimized mesh

View dependent texturing

Our viewer