Announcements

Project 1
• due Tuesday at 11:59pm (artifact Wednesday at 11:59pm)
• grading session Wednesday (signup!)

Project 2
• choose partners by Wednesday
  – signup:

Readings
• Nalwa 2.1

Projection

Readings
• Nalwa 2.1

Müller-Lyer Illusion

Readings
• Nalwa 2.1

http://www.julianbeever.net/pave.htm

http://www.michaelbach.de/ot/sze_muelue/index.html
Let’s design a camera
  • Idea 1: put a piece of film in front of an object
  • Do we get a reasonable image?

Add a barrier to block off most of the rays
  • This reduces blurring
  • The opening known as the aperture
  • How does this transform the image?

Sun “shadows” during a solar eclipse
by Henrik von Wendt
http://www.flickr.com/photos/hvw/2724969199/

Sun “shadows” during a solar eclipse
http://www.flickr.com/photos/73860948@N08/6678331997/
Pinhole cameras everywhere

Tree shadow during a solar eclipse
photo credit: Nils van der Burg
http://www.physicsatogo.org/index.cfm

Camera Obscura

The first camera
• Known to Aristotle
• How does the aperture size affect the image?

Shrinking the aperture

Why not make the aperture as small as possible?
• Less light gets through
• Diffraction effects...

Shrinking the aperture
Adding a lens

A lens focuses light onto the film
- There is a specific distance at which objects are “in focus”
  - other points project to a “circle of confusion” in the image
- Changing the shape of the lens changes this distance

Lenses

A lens focuses parallel rays onto a single focal point
- focal point at a distance \( f \) beyond the plane of the lens
  - \( f \) is a function of the shape and index of refraction of the lens
- Aperture of diameter \( D \) restricts the range of rays
  - aperture may be on either side of the lens
- Lenses are typically spherical (easier to produce)

Thin lenses

Thin lens equation:
\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
\]
- Any object point satisfying this equation is in focus
- What is the shape of the focus region?
- How can we change the focus region?
- Thin lens applet: [http://www.phy.ntnu.edu.tw/java/Lens/lens_e.html](http://www.phy.ntnu.edu.tw/java/Lens/lens_e.html) (by Fu-Kwun Hwang)

Depth of field

Changing the aperture size affects depth of field
- A smaller aperture increases the range in which the object is approximately in focus

The eye

The human eye is a camera
- Iris - colored annulus with radial muscles
- Pupil - the hole (aperture) whose size is controlled by the iris
- What's the "film"?
  - photoreceptor cells (rods and cones) in the retina

Digital camera

A digital camera replaces film with a sensor array
- Each cell in the array is a Charge Coupled Device
  - light-sensitive diode that converts photons to electrons
  - other variants exist: CMOS is becoming more popular

Issues with digital cameras

Noise
- big difference between consumer vs. SLR-style cameras
- low light is where you most notice noise

Compression
- creates artifacts except in uncompressed formats (tiff, raw)

Color
- color fringing artifacts from Bayer patterns

Blooming
- charge overflowing into neighboring pixels

In-camera processing
- oversharpening can produce halos

Interlaced vs. progressive scan video
- even/odd rows from different exposures

Are more megapixels better?
- requires higher quality lens
- noise issues

Stabilization
- compensate for camera shake (mechanical vs. electronic)

More info online, e.g.,
- http://www.dpreview.com/

Modeling projection

The coordinate system
- We will use the pin-hole model as an approximation
- Put the optical center (Center Of Projection) at the origin
- Put the image plane (Projection Plane) in front of the COP
  - Why?
- The camera looks down the negative z axis
  - we need this if we want right-handed-coordinates
Modeling projection

Projection equations
- Compute intersection with PP of ray from (x,y,z) to COP
- Derived using similar triangles (on board)
  \[(x, y, z) \rightarrow (\frac{-d^x}{z}, \frac{-d^y}{z}, -d)\]
- We get the projection by throwing out the last coordinate:
  \[(x, y, z) \rightarrow (\frac{-d^x}{z}, \frac{-d^y}{z})\]

Homogeneous coordinates

Is this a linear transformation?
- no—division by z is nonlinear

Trick: add one more coordinate:
  \[ (x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad \quad (x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \]

Converting from homogeneous coordinates

\[ \begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w) \quad \quad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w) \]

Perspective Projection

Projection equations
- Compute intersection with PP of ray from (x,y,z) to COP
- Derived using similar triangles (on board)

\[ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z/d \\ 1 \end{bmatrix} \Rightarrow (\frac{-d^x}{z}, \frac{-d^y}{z}) \]

This is known as **perspective projection**
- The matrix is the **projection matrix**
- Can also formulate as a 4x4 (today's reading does this)

\[ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z/d \\ 1 \end{bmatrix} \Rightarrow (\frac{-d^x}{z}, \frac{-d^y}{z}) \]

\[ \begin{bmatrix} -d & 0 & 0 & 0 \\ 0 & -d & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} -dx \\ -dy \\ z \\ 1 \end{bmatrix} \Rightarrow (\frac{-d^x}{z}, \frac{-d^y}{z}) \]
Orthographic projection

Special case of perspective projection
- Distance from the COP to the PP is infinite
- Good approximation for telephoto optics
- Also called “parallel projection”: \((x, y, z) \rightarrow (x, y)\)
- What’s the projection matrix?

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
= 
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
\Rightarrow (x, y)
\]

Camera parameters

A camera is described by several parameters
- Translation \(T\) of the optical center from the origin of world coords
- Rotation \(R\) of the image plane
- focal length \(f\), principle point \((x_c', y_c')\), pixel size \((s_x, s_y)\)
- blue parameters are called “extrinsics,” red are “intrinsics”

Projection equation

\[
x = \begin{bmatrix}
w x  \\
w y  \\
w
\end{bmatrix}
= \begin{bmatrix}
\ast & \ast & \ast & \ast \\
\ast & \ast & \ast & \ast \\
\ast & \ast & \ast & \ast \\
\ast & \ast & \ast & \ast
\end{bmatrix}
\begin{bmatrix}
X  \\
Y \\
Z \\
1
\end{bmatrix}
= \Pi X
\]

- The projection matrix models the cumulative effect of all parameters
- Useful to decompose into a series of operations

\[
\Pi = 
\begin{bmatrix}
-\beta_x & 0 & x_c' & 0 \\
0 & -\beta_y & y_c' & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\mathbf{R} & \mathbf{t} \\
\mathbf{0} & 1
\end{bmatrix}
= \mathbf{I} \mathbf{t} \mathbf{R} \mathbf{t}^{-1}
\]

- The definitions of these parameters are not completely standardized
  - especially intrinsics—varies from one book to another

Distortion

Radial distortion of the image
- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens
Correcting radial distortion

from Helmut Dersch

Modeling distortion

Project \((\tilde{x}, \tilde{y}, \tilde{z})\) to "normalized" image coordinates

\[
x'_n = \frac{\tilde{x}}{\tilde{z}}
\]

\[
y'_n = \frac{\tilde{y}}{\tilde{z}}
\]

\[
r^2 = x'_n^2 + y'_n^2
\]

Apply radial distortion

\[
x'_d = x'_n(1 + \kappa_1 r^2 + \kappa_2 r^4)
\]

\[
y'_d = y'_n(1 + \kappa_1 r^2 + \kappa_2 r^4)
\]

Apply focal length translate image center

\[
x' = f x'_d + x_c
\]

\[
y' = f y'_d + y_c
\]

To model lens distortion

- Use above projection operation instead of standard projection matrix multiplication

Many other types of projection exist...
360 degree field of view…

Basic approach
- Take a photo of a parabolic mirror with an orthographic lens (Nayar)
- Or buy one a lens from a variety of omnicam manufacturers…
  - See http://www.cis.upenn.edu/~kostas/omni.html

Tilt-shift

Tilt-shift images from Olivo Barbieri and Photoshop imitations

Rotating sensor (or object)

Also known as "cyclographs", "peripheral images"