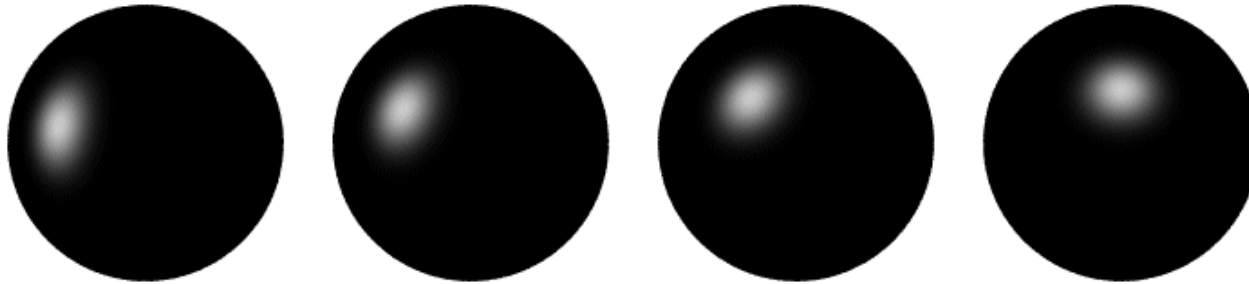


Computing light source directions

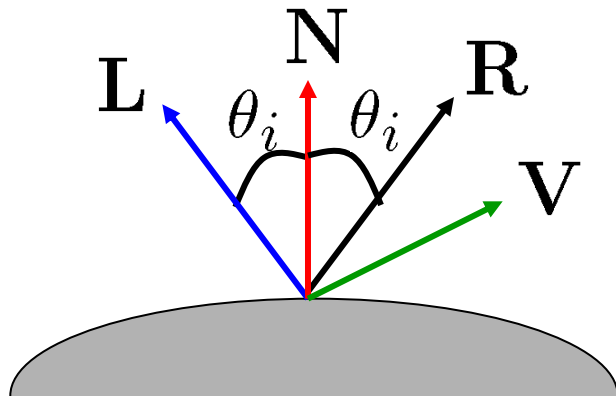
Trick: place a chrome sphere in the scene



- the location of the highlight tells you where the light source is

Recall the rule for specular reflection

For a perfect mirror, light is reflected about **N**



$$I_e = \begin{cases} I_i & \text{if } \mathbf{V} = \mathbf{R} \\ 0 & \text{otherwise} \end{cases}$$

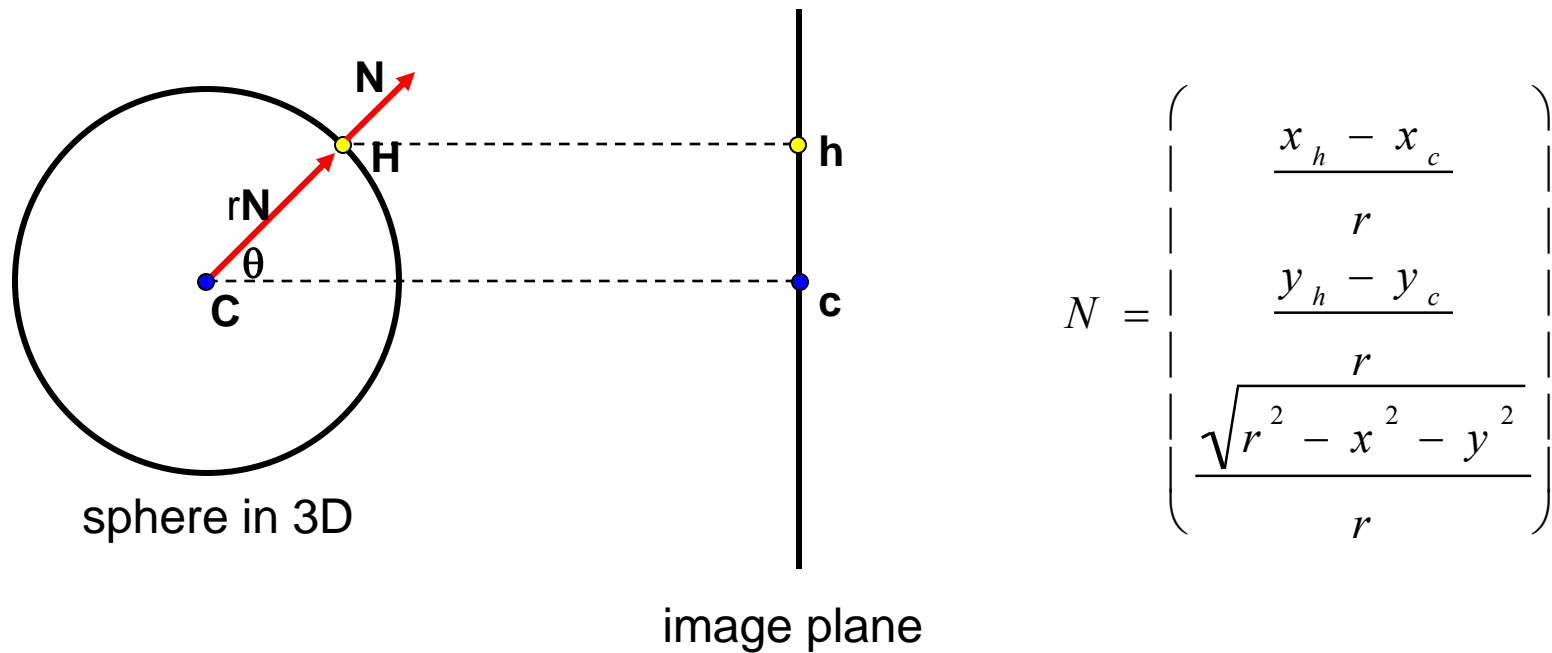
We see a highlight when **V = R**

- then **L** is given as follows:

$$\mathbf{L} = 2(\mathbf{N} \cdot \mathbf{R})\mathbf{N} - \mathbf{R}$$

Computing the light source direction

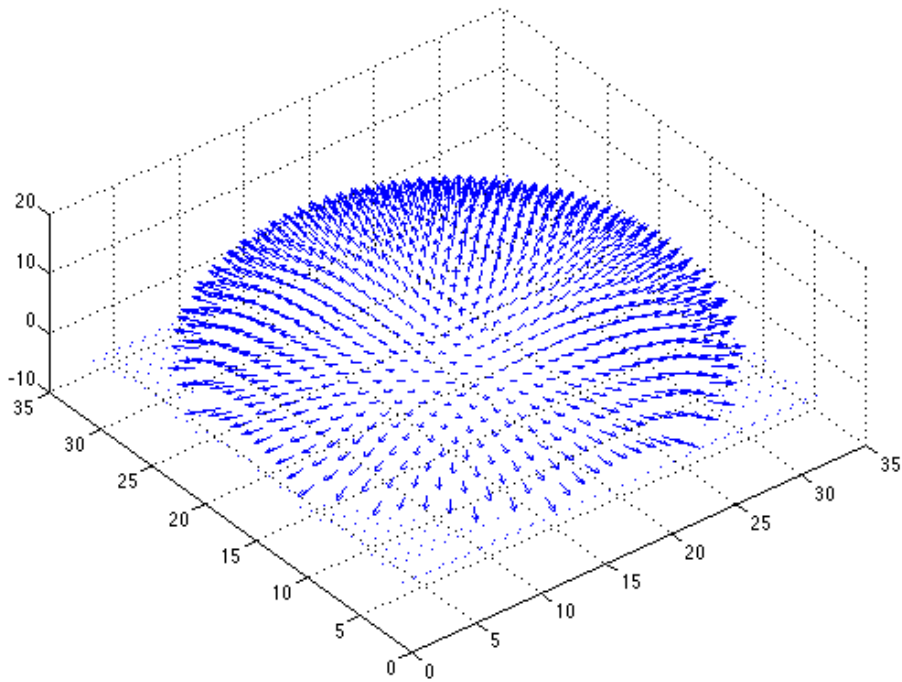
Chrome sphere that has a highlight at position \mathbf{h} in the image



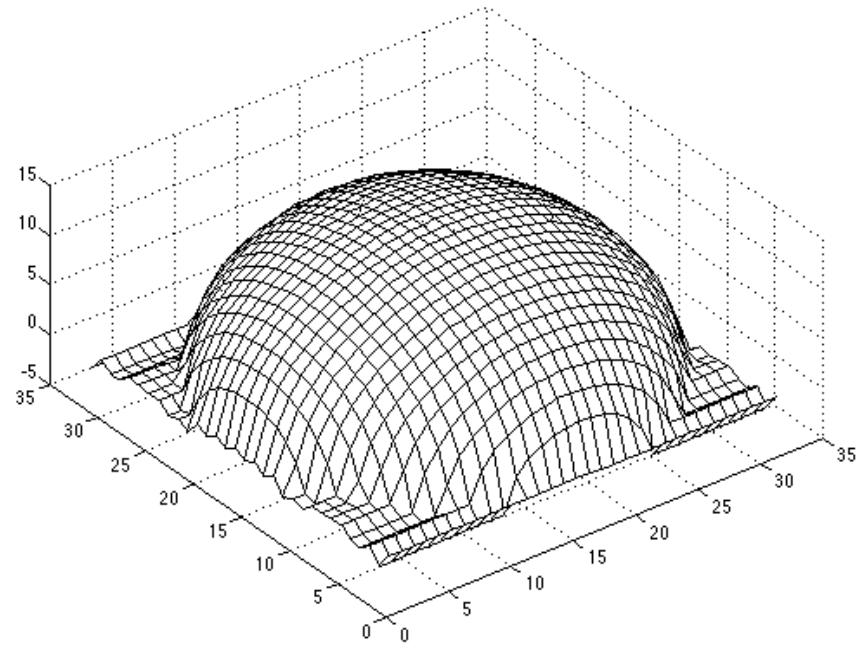
Can compute θ (and hence \mathbf{N}) from this figure

Now just reflect \mathbf{V} about \mathbf{N} to obtain \mathbf{L}

Depth from normals

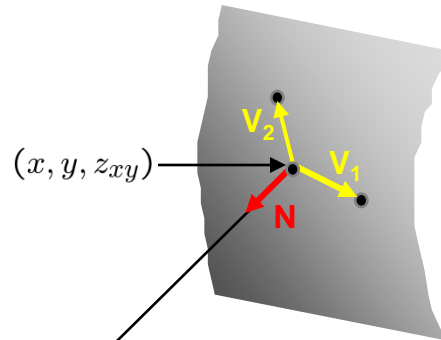


What we have



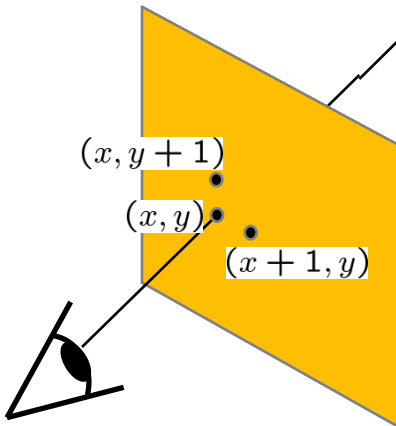
What we want

Depth from normals



$$\begin{aligned} V_1 &= (x + 1, y, z_{x+1,y}) - (x, y, z_{xy}) \\ &= (1, 0, z_{x+1,y} - z_{xy}) \end{aligned}$$

$$\begin{aligned} 0 &= N \cdot V_1 \\ &= (n_x, n_y, n_z) \cdot (1, 0, z_{x+1,y} - z_{xy}) \\ &= n_x + n_z(z_{x+1,y} - z_{xy}) \end{aligned}$$



Get a similar equation for V_2

- Each normal gives us two linear constraints on z
- compute z values by solving a matrix equation
- On the boundary we have only one constraint.

Trick for handling shadows

Weight each equation by the pixel brightness:

$$I_i(I_i) = I_i[k_d \mathbf{N} \cdot \mathbf{L}_i]$$

Gives weighted least-squares matrix equation:

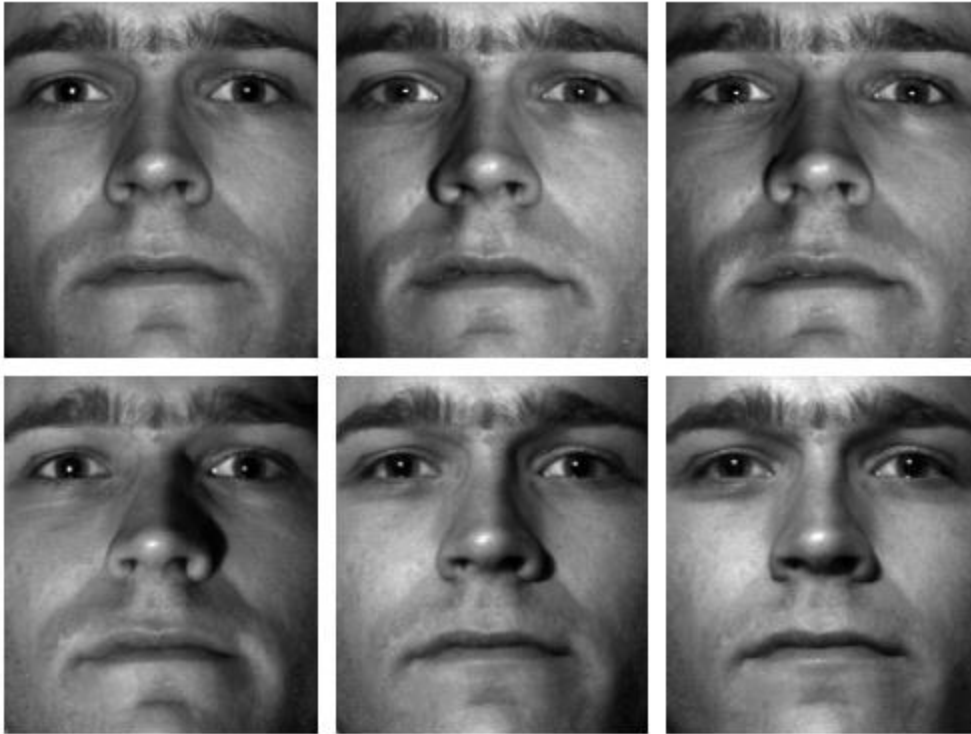
$$\begin{bmatrix} I_1^2 & \dots & I_n^2 \end{bmatrix} = k_d \mathbf{N}^T \begin{bmatrix} I_1 \mathbf{L}_1 & \dots & I_n \mathbf{L}_n \end{bmatrix}$$

Solve for \mathbf{N} , k_d as before

Example



Results...



from Athos Georghiades
<http://cvc.yale.edu/people/Athos.html>

Limitations

- doesn't work for shiny things, semi-translucent things
- shadows, inter-reflections are difficult
- Single light source illumination
- camera and lights have to be distant
- calibration requirements
 - measure light source directions, intensities
 - camera response function

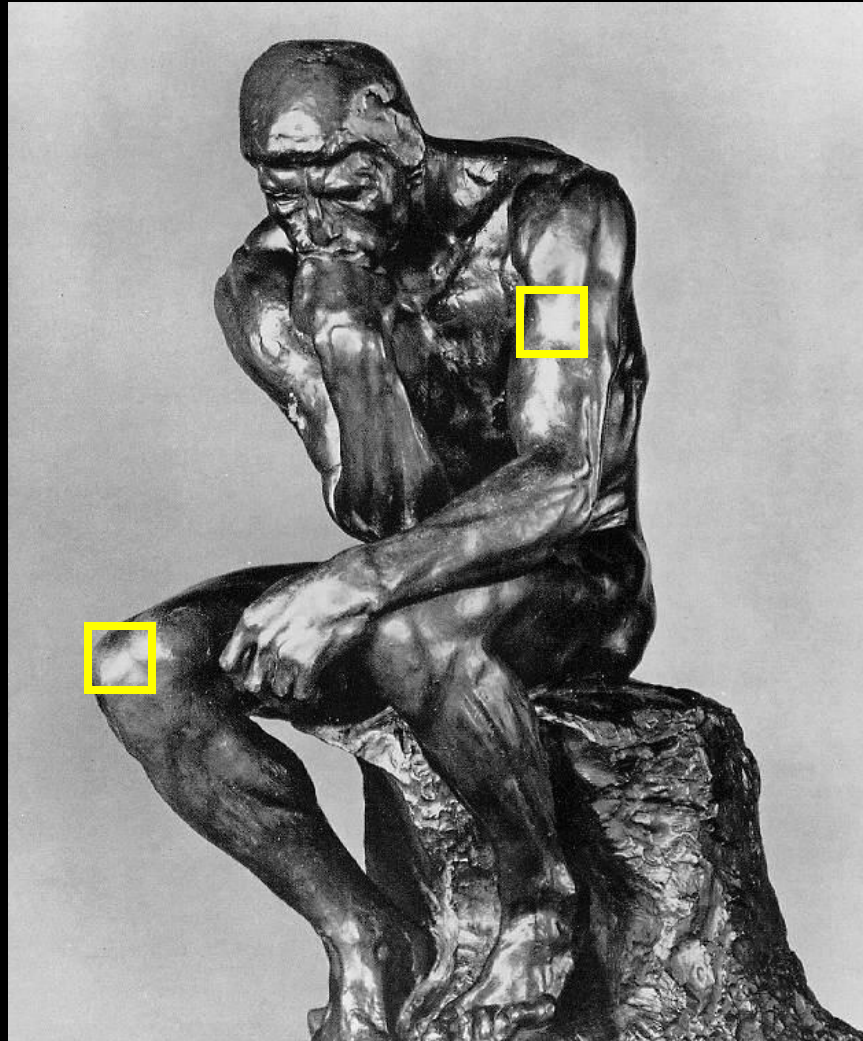
Newer work addresses some of these issues

Some pointers for further reading:

- Zickler, Belhumeur, and Kriegman, "[*Helmholtz Stereopsis: Exploiting Reciprocity for Surface Reconstruction*](#)." IJCV, Vol. 49 No. 2/3, pp 215-227.
- Hertzmann & Seitz, "[*Example-Based Photometric Stereo: Shape Reconstruction with General, Varying BRDFs*](#)." IEEE Trans. PAMI 2005
- Basri, Jacobs and Kemelmacher "[*Photometric Stereo with General Unknown Lighting*](#), International Journal of Computer Vision (IJCV) 2007

Hertzmann & Seitz, *Example-Based
Photometric Stereo: Shape
Reconstruction with General, Varying
BRDFs.* IEEE Trans. PAMI 2005

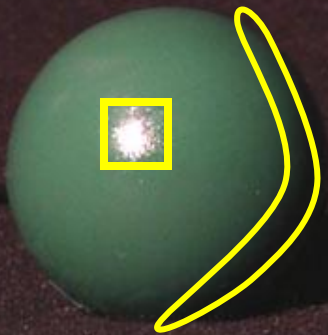
Shiny things



“Orientation consistency”

same surface normal



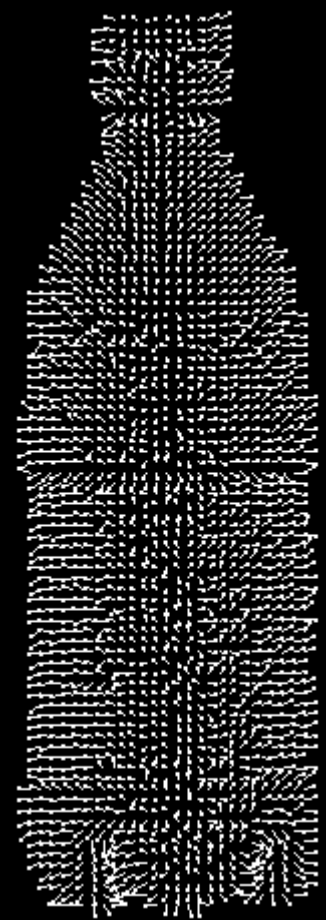
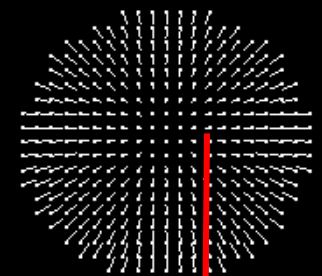












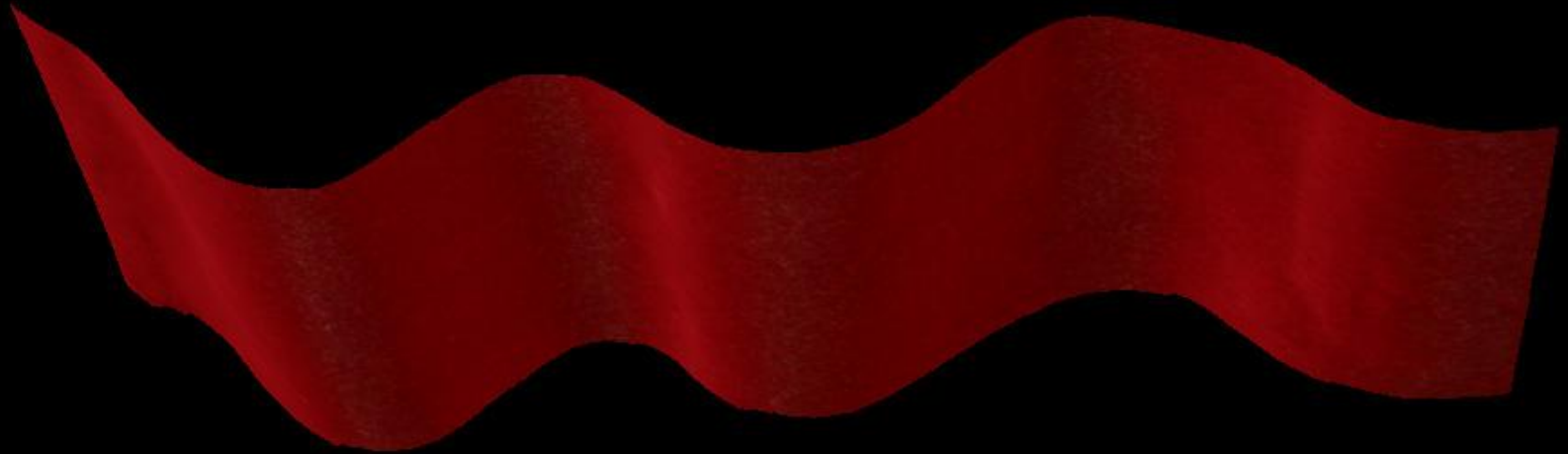
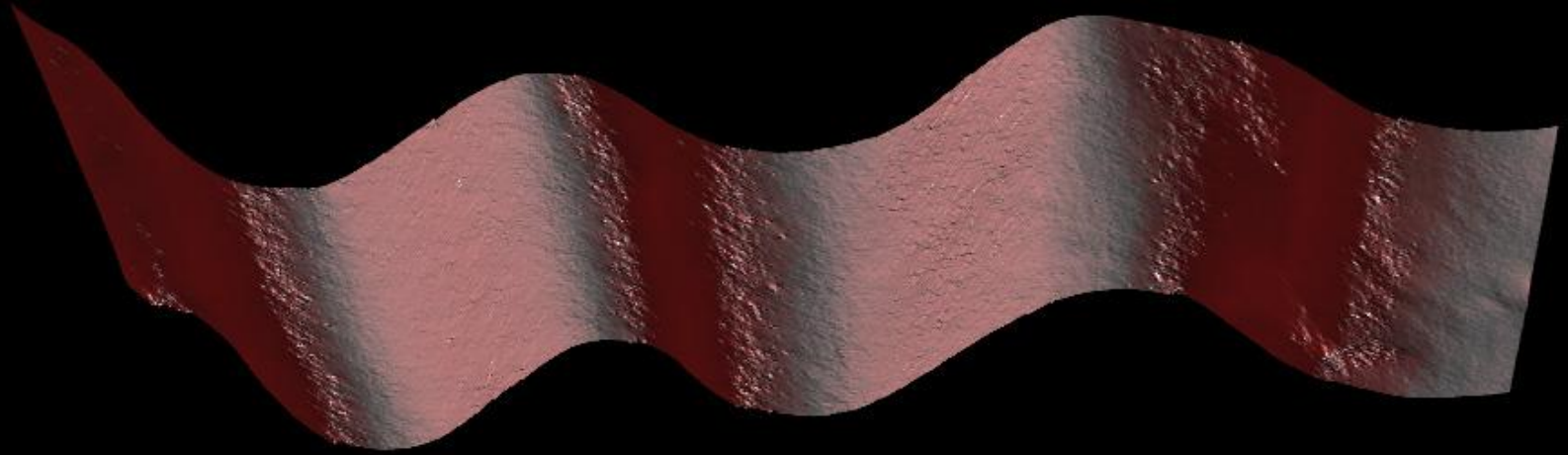
Virtual views



Velvet



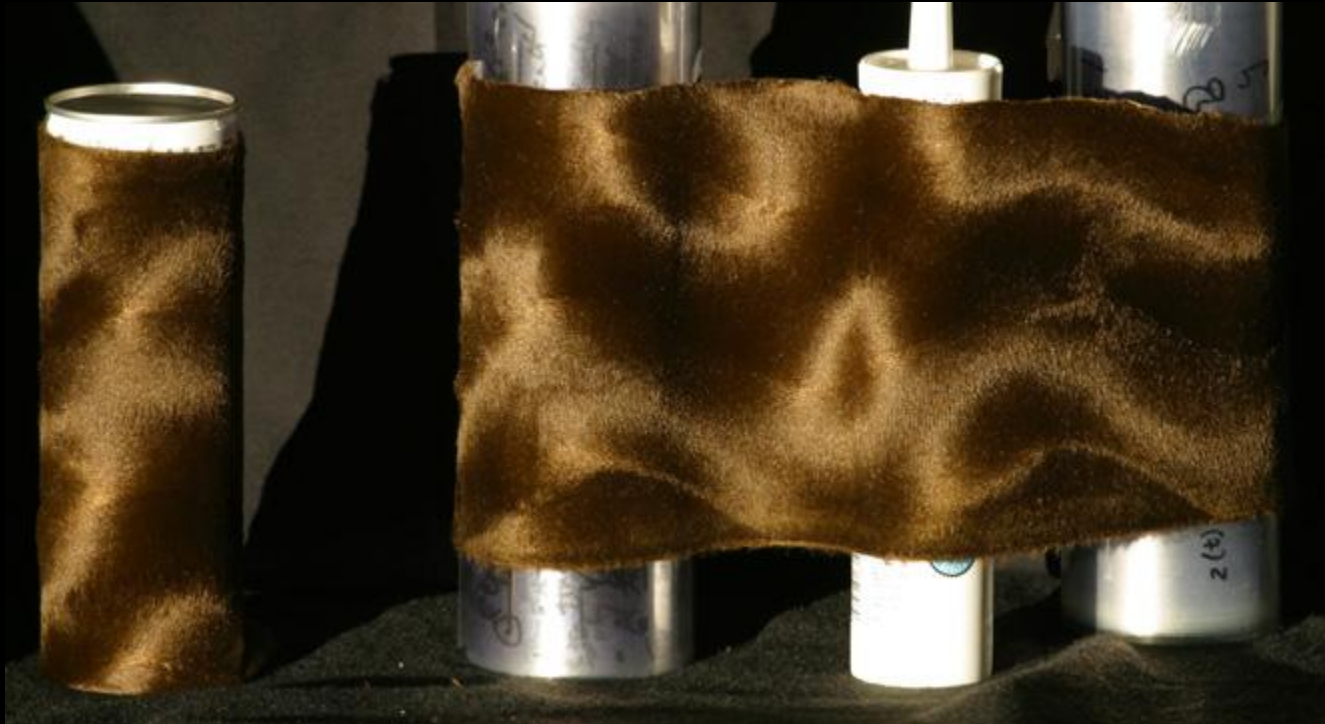
Virtual Views



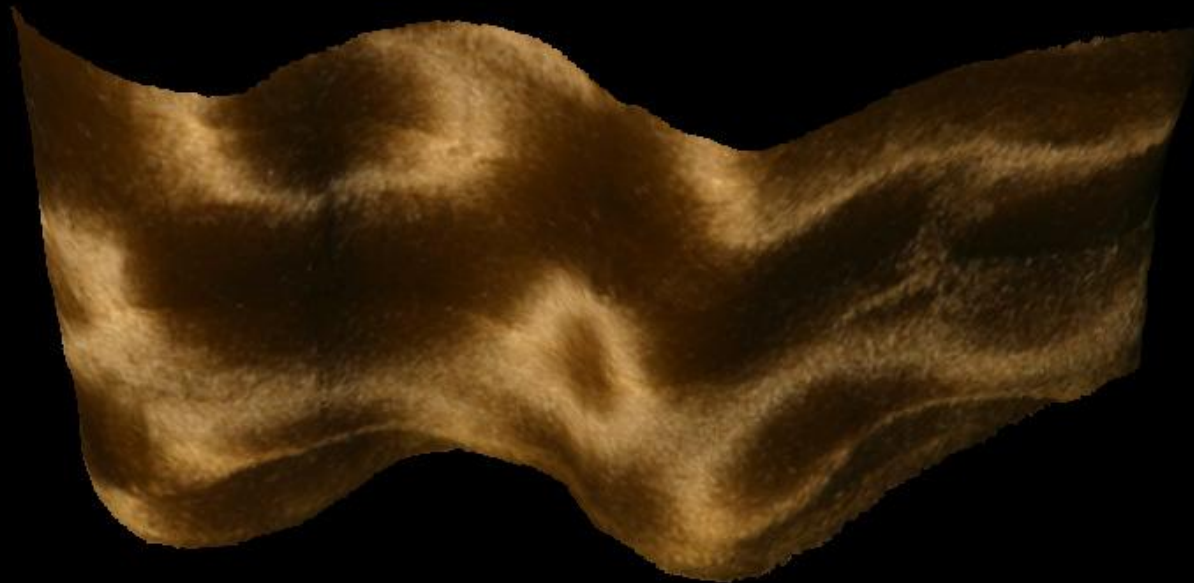
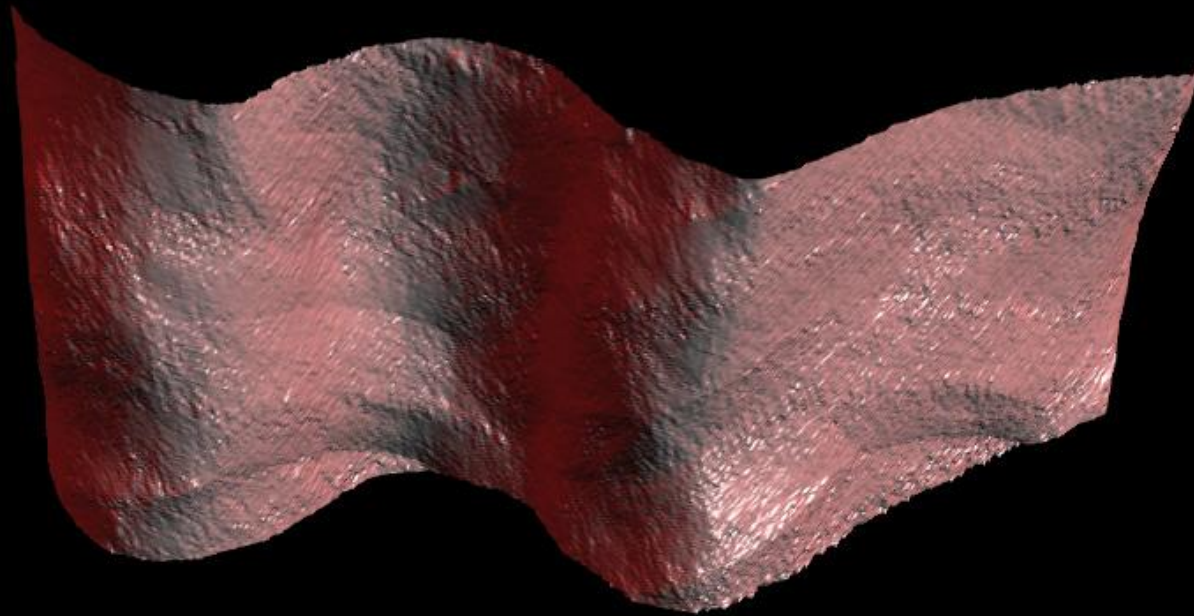
Brushed Fur



Brushed Fur

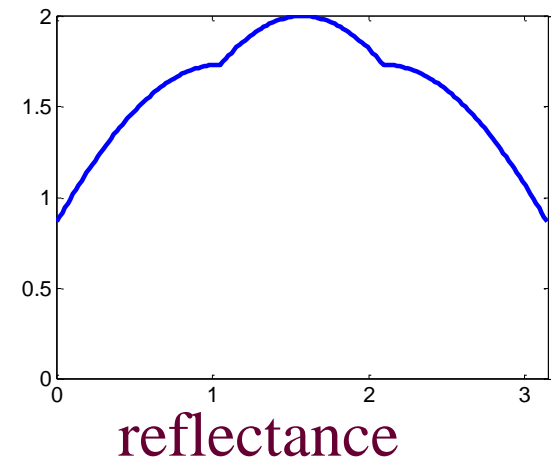
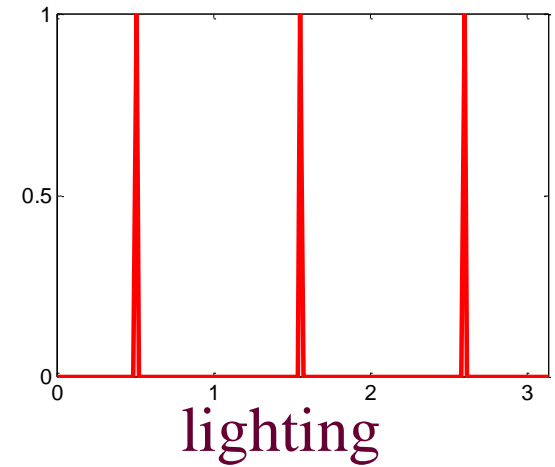
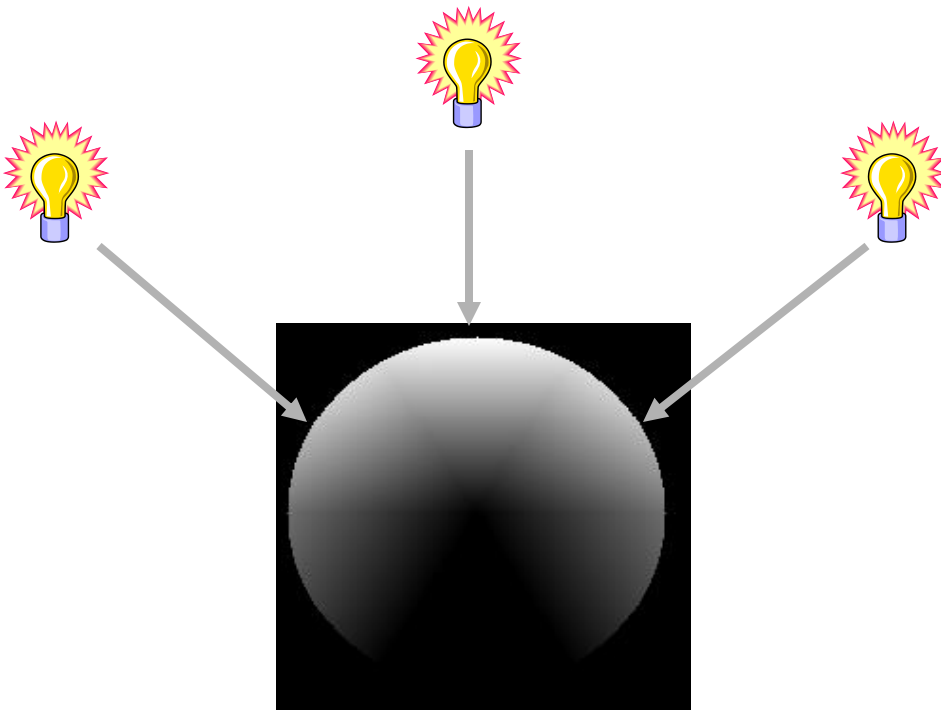


Virtual Views

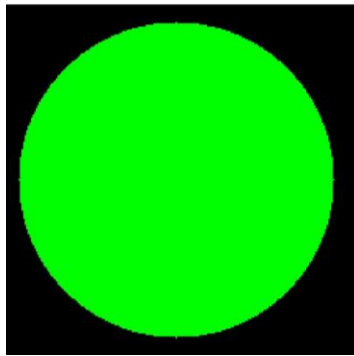


-
- Basri, Jacobs and Kemelmacher
Photometric Stereo with General Unknown
Lighting, International Journal of Computer
Vision (IJCV) 2007

Illumination Modeling



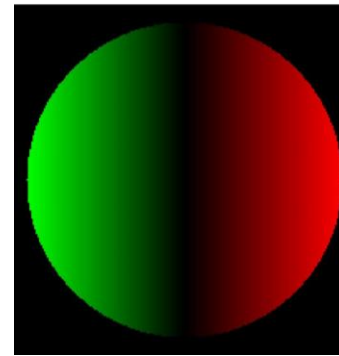
Spherical Harmonics



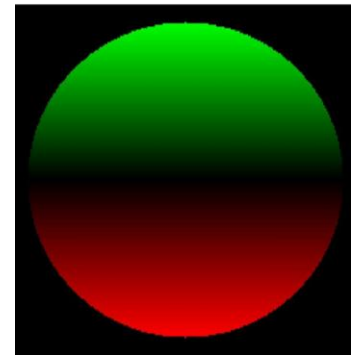
1



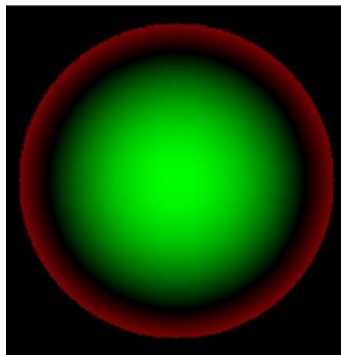
Z



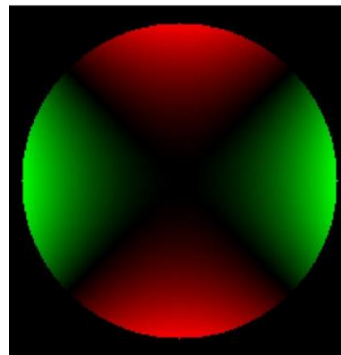
X



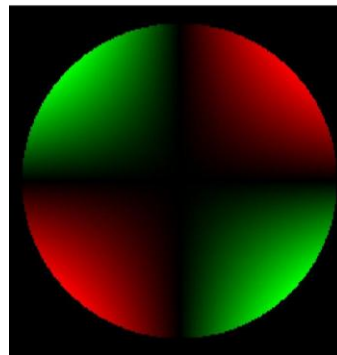
Y



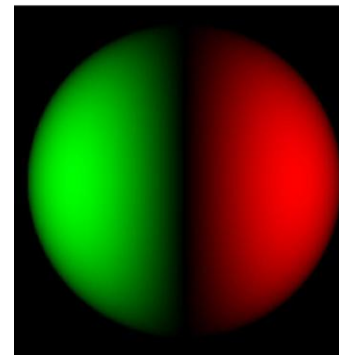
$3Z^2 - 1$



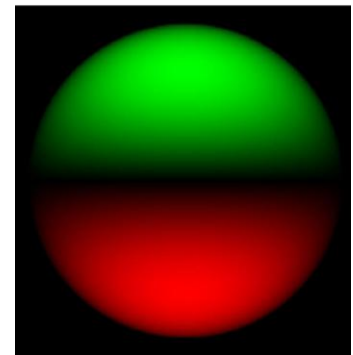
$X^2 - Y^2$



XY

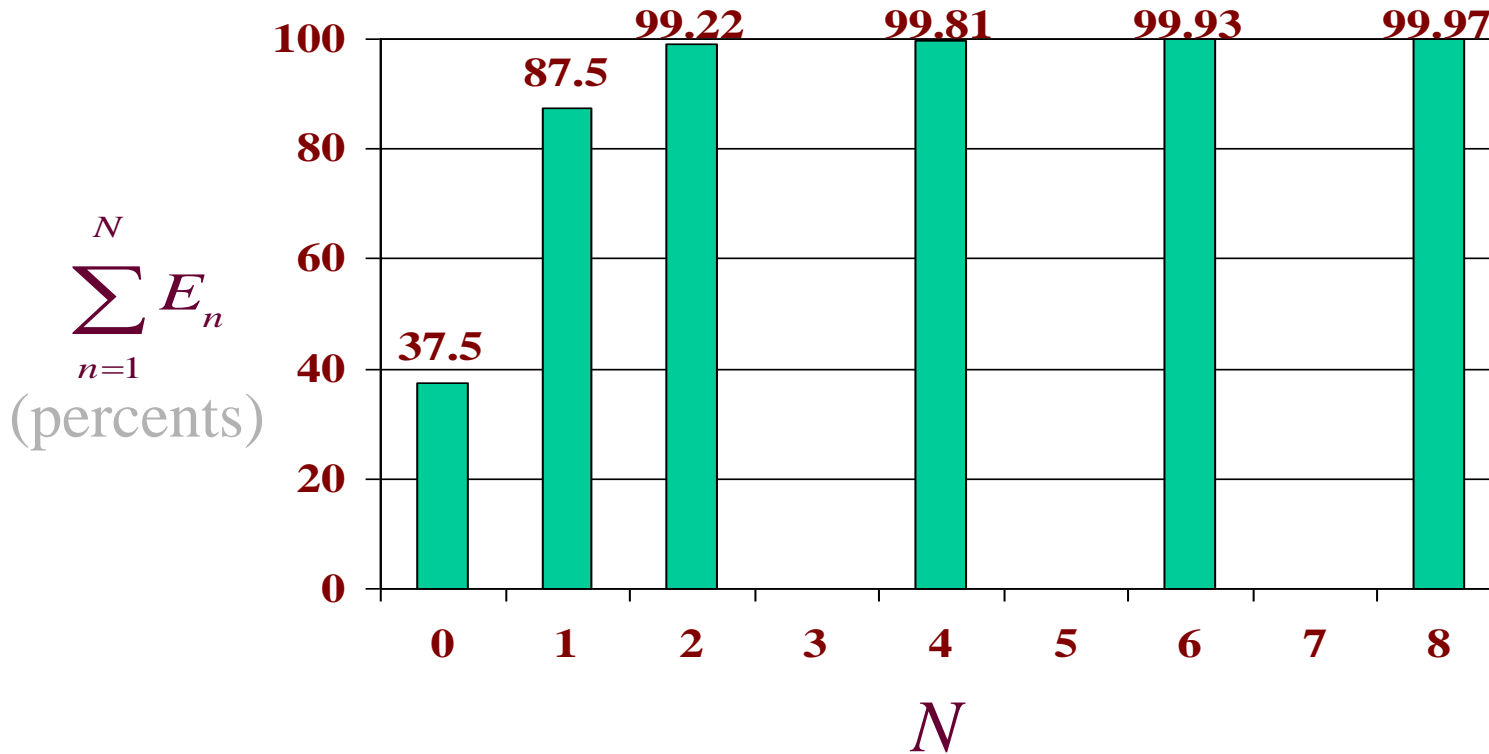


XZ



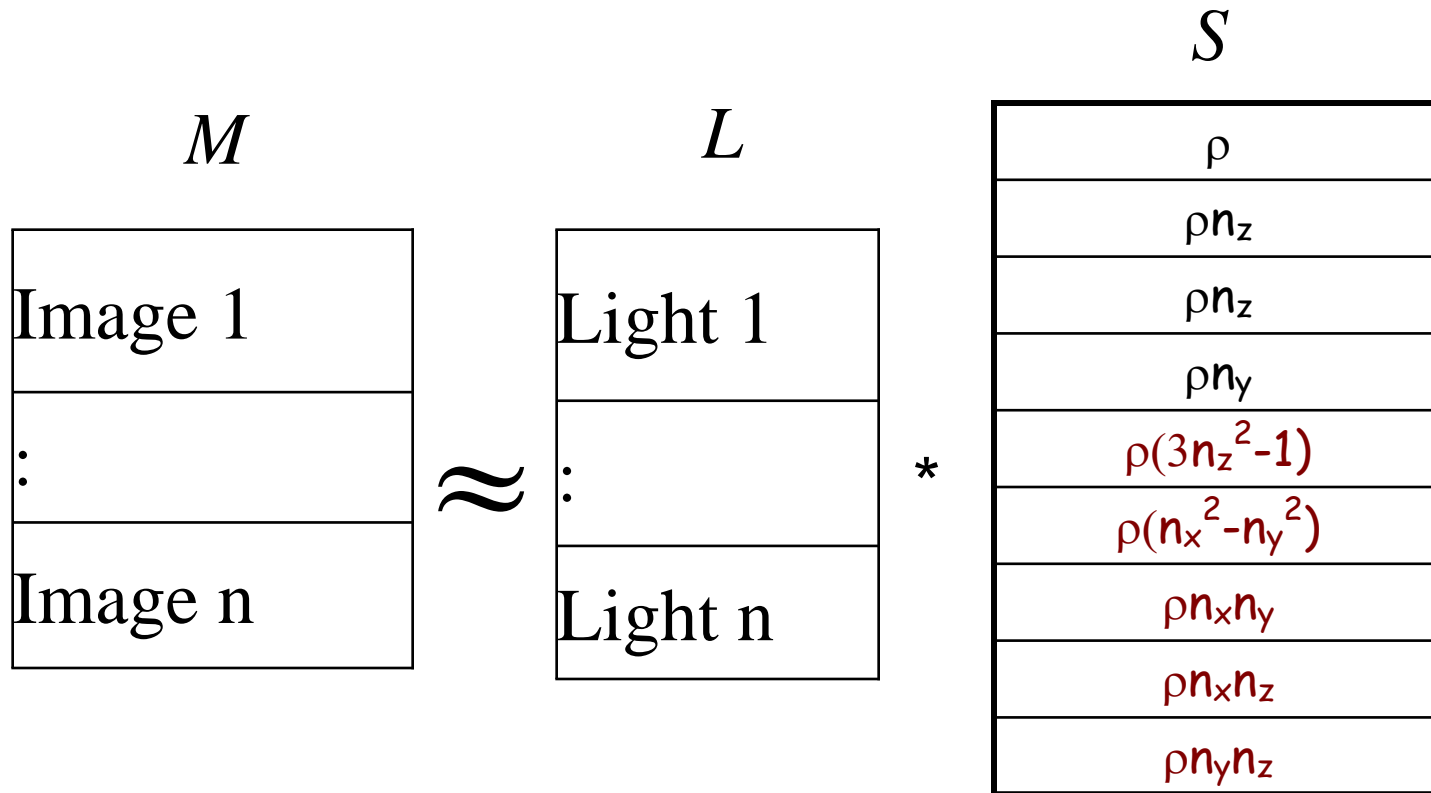
YZ

Cumulative Energy



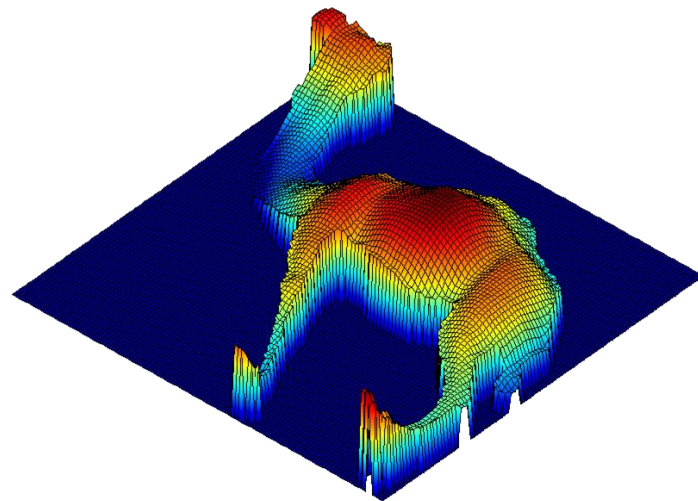
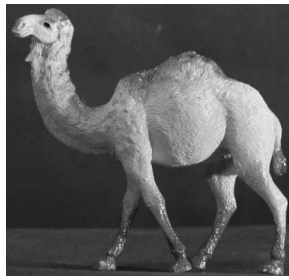
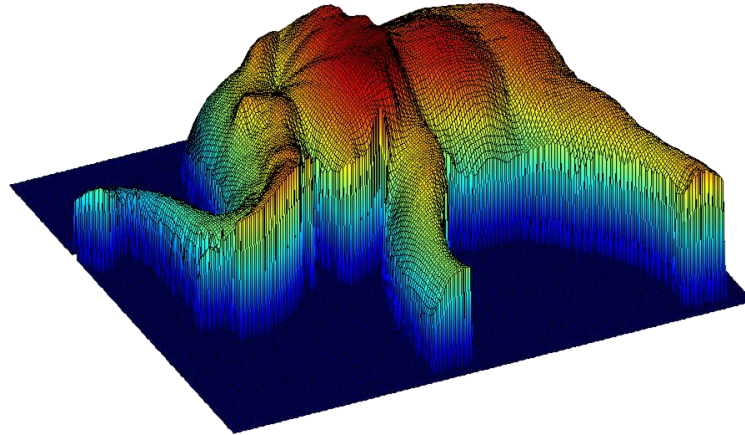
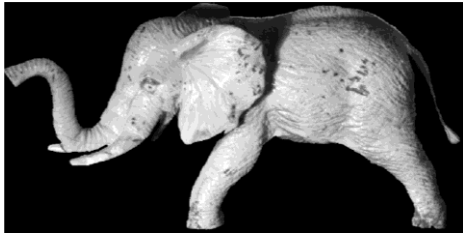
Photometric Stereo

Factorization to Shape and Light



SVD recovers L and S up to an ambiguity

Results with arbitrary unknown lighting

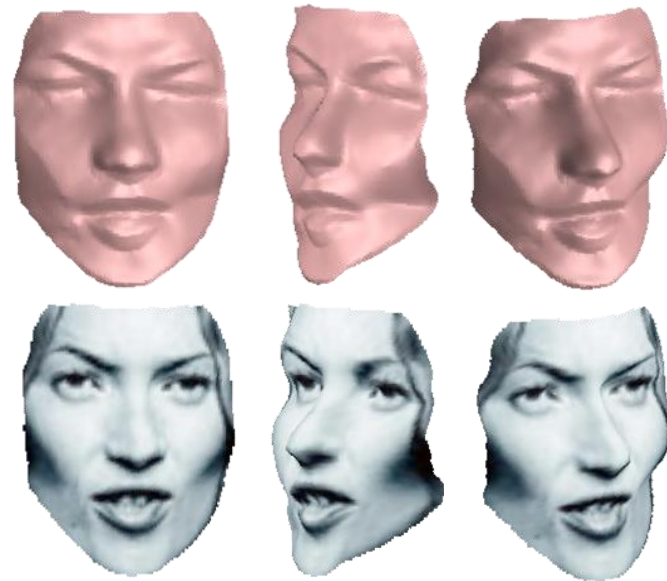


3D Face Shape Reconstruction

Input: Single Image,
general lighting, pose



Output: 3D shape



Kemelmacher and Basri [Molding Face Shapes by Example](#)
IEEE Trans. PAMI

One approach

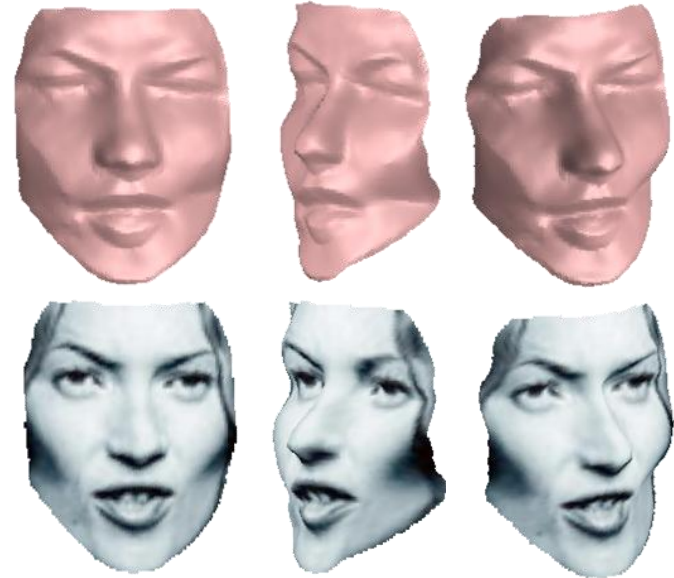
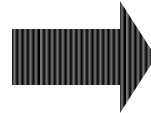
- Represent input face as a combination of hundreds of 3D face in a database:



Our approach

Input
image

Single
reference
model



Input image is a guide to “mold” a single reference model.

unknown lighting
unknown boundaries
unknown albedo

significant differences in shape including

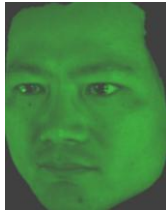
- expression
- gender
- race

Harmonic representation of lighting

ρ albedo
 \mathbf{n} surface normal
 $\mathbf{n} = (n_x, n_y, n_z)$

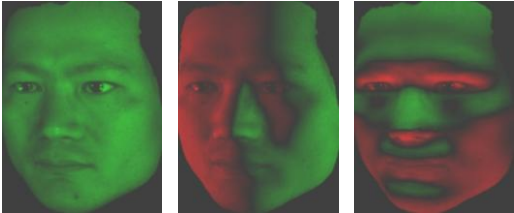
■ Positive values
■ Negative values

$$I(x, y) = R(\mathbf{n}; \rho, \mathbf{l}) \approx \rho \sum_{i=0}^{K-1} l_i Y_i(\mathbf{n})$$



37.6%

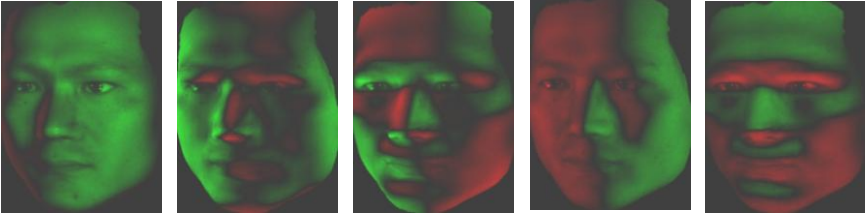
ρ



87.5%

In practice
~95% accuracy

ρn_z ρn_x ρn_y



99.2%

$\rho(3n_z^2 - 1)$ $\rho(n_x^2 - n_y^2)$ $\rho n_x n_y$ $\rho n_x n_z$ $\rho n_y n_z$

- Lambertian reflectance
- Accounts for attached shadows
- Arbitrary illumination (point sources, diffuse, combinations...)

Formulation

$$I(x, y) = R(\mathbf{n}; \rho, \mathbf{l}) \approx \rho \sum_{i=0}^{K-1} l_i Y_i(\mathbf{n})$$

Unknown surface normal:

$$\mathbf{n}(x, y) = \frac{\mathbf{1}}{\sqrt{p^2 + q^2 + 1}} (p, q, -\mathbf{1})^T$$

$$p(x, y) = \partial z / \partial x$$

$$q(x, y) = \partial z / \partial y$$

Unknown harmonic coeff. $\mathbf{l} = (l_0 \dots l_{K-1})$

Unknown albedo $\rho(x, y)$

Known image:



Known reference model:



$$\min_{l, z, \rho} \int_{\Omega} \left(I(x, y) - \rho \sum_{i=0}^{K-1} l_i Y_i(\mathbf{n}) \right)^2 + \lambda_1 (\Delta G * d_z(x, y))^2 + \lambda_2 (\Delta G * d_\rho(x, y))^2 dx dy$$

$$d_z(x, y) = z(x, y) - z_{\text{ref}}(x, y) \quad d_\rho(x, y) = \rho(x, y) - \rho_{\text{ref}}(x, y)$$

Reconstruction steps

$$\min_{l, z, \rho} \int_{\Omega} \left(I(x, y) - \rho \sum_{i=0}^{K-1} l_i Y_i(\mathbf{n}) \right)^2 + \lambda_1 (\Delta G * d_z(x, y))^2 + \lambda_2 (\Delta G * d_\rho(x, y))^2 dx dy$$

Solve for lighting:

$$\min_l \sum_{(x, y) \in \Omega} \left(I(x, y) - \rho_{ref} \sum_{i=0}^{K-1} l_i Y_i(\mathbf{n}_{ref}) \right)^2$$

Different faces experience the same lighting in similar way. So use reference face to fill in the missing data.

Solve for depth:

$$I(x, y) = \rho_{ref} l_0 + \frac{\rho_{ref}}{N_{ref}} (l_1 p + l_2 q - l_3)$$

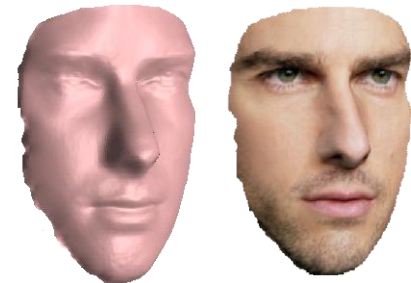
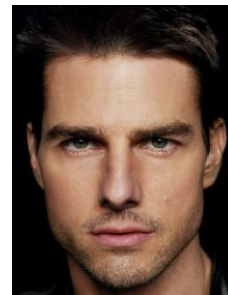
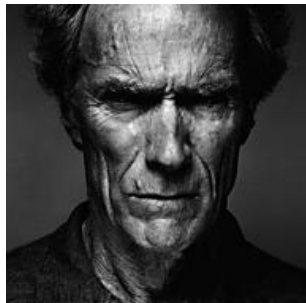
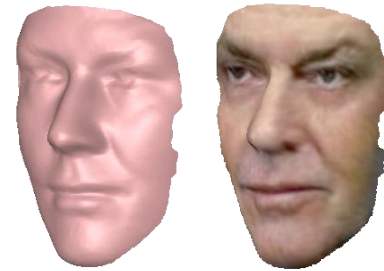
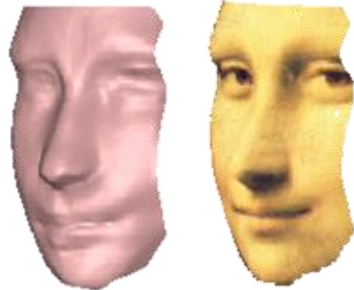
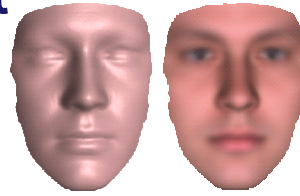
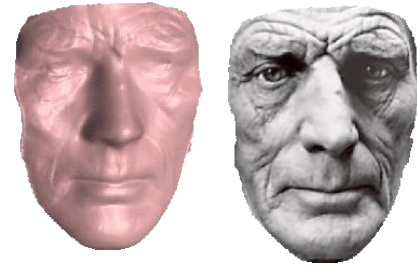
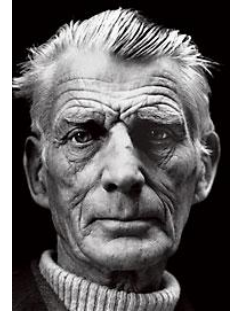
$$N = \sqrt{p^2 + q^2 + 1}$$

Linear PDE in $z(x, y)$

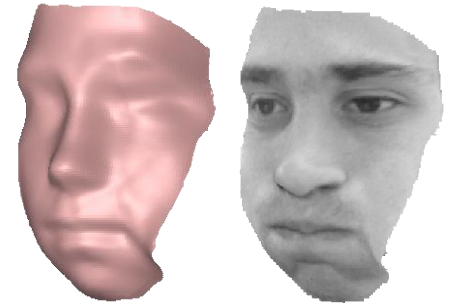
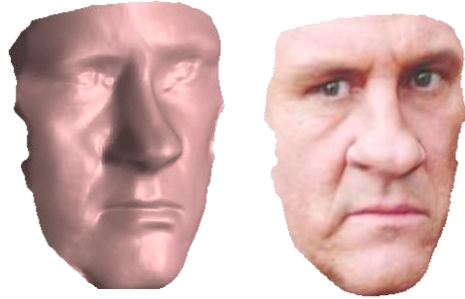
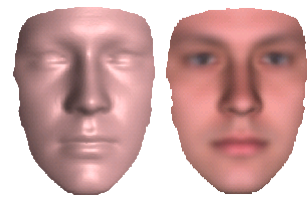
Boundaries: We let partial derivatives across boundaries to vanish

Results

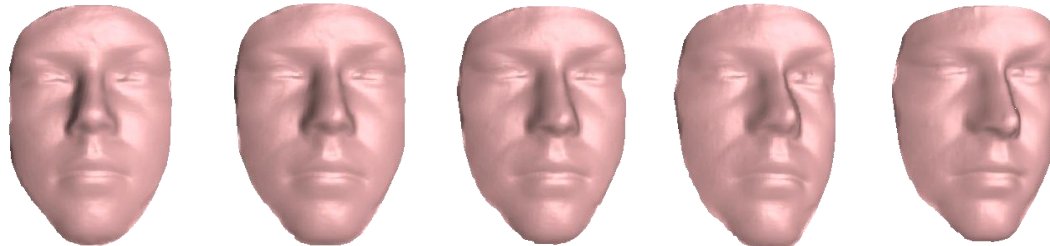
on images downloaded from the internet



Expressions



Results: unknown pose



$8.1 \pm 5.7\%$

$6.9 \pm 5.3\%$

$7.7 \pm 5.7\%$

$7.3 \pm 6.1\%$

$8 \pm 7.2\%$