Motion Estimation

http://www.sandlotscience.com/Ambiguous/Barberpole_Illusion.htm

Today's Reading

Trucco & Verri, 8.3 – 8.4 (skip 8.3.3, read only top half of p. 199)

Tracking features

Feature tracking

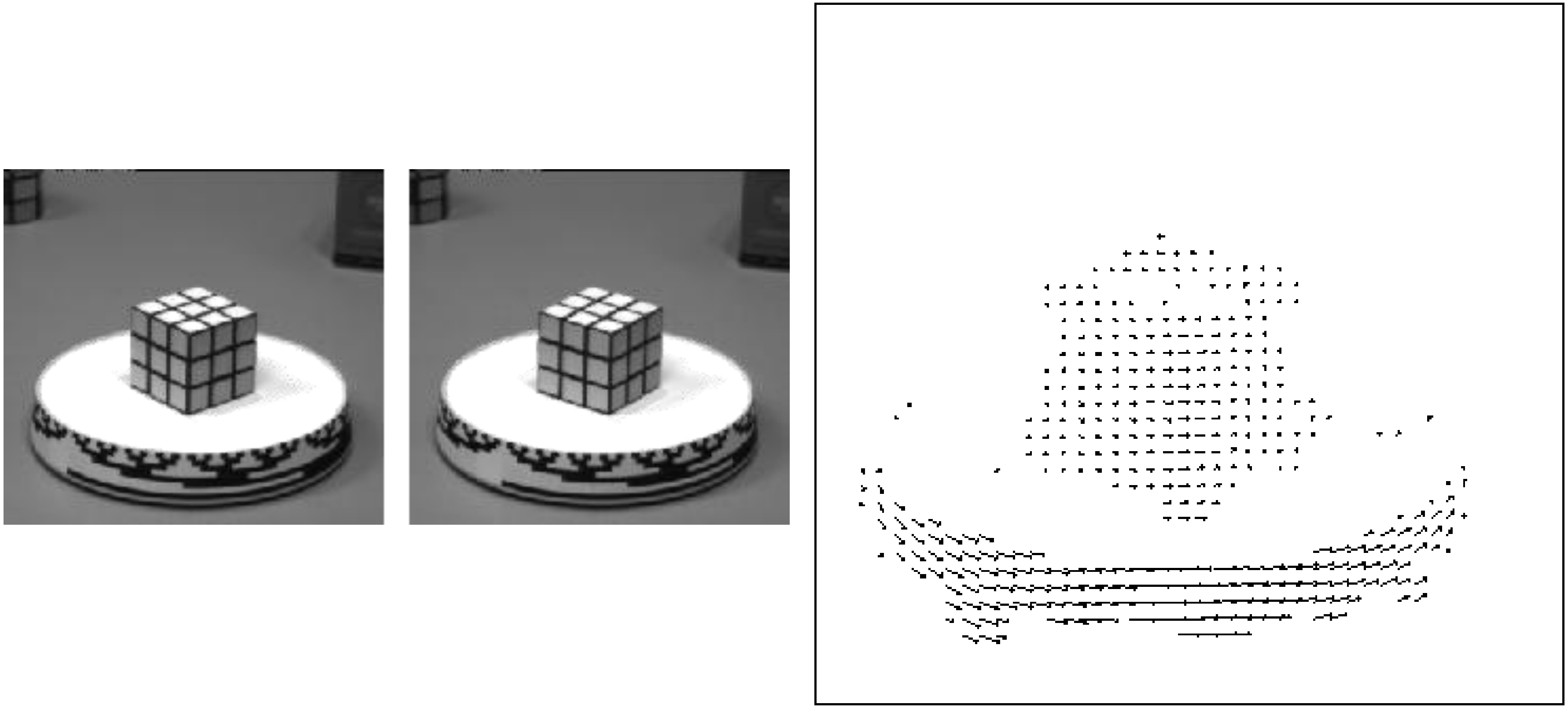
- Find feature correspondence between consecutive H, I
- Chain these together to find long-range correspondences

When will this go wrong?

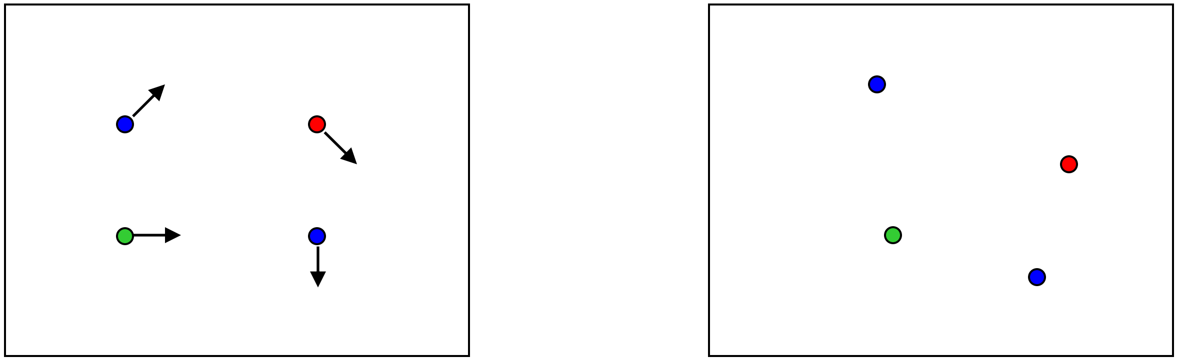
- Occlusions—feature may disappear
  - need mechanism for deleting, adding new features
- Changes in shape, orientation
  - allow the feature to deform
- Changes in color

What if we want the motion at every pixel?

Optical flow



Problem definition: optical flow



$H(x, y)$ $I(x, y)$

How to estimate pixel motion from image H to image I?

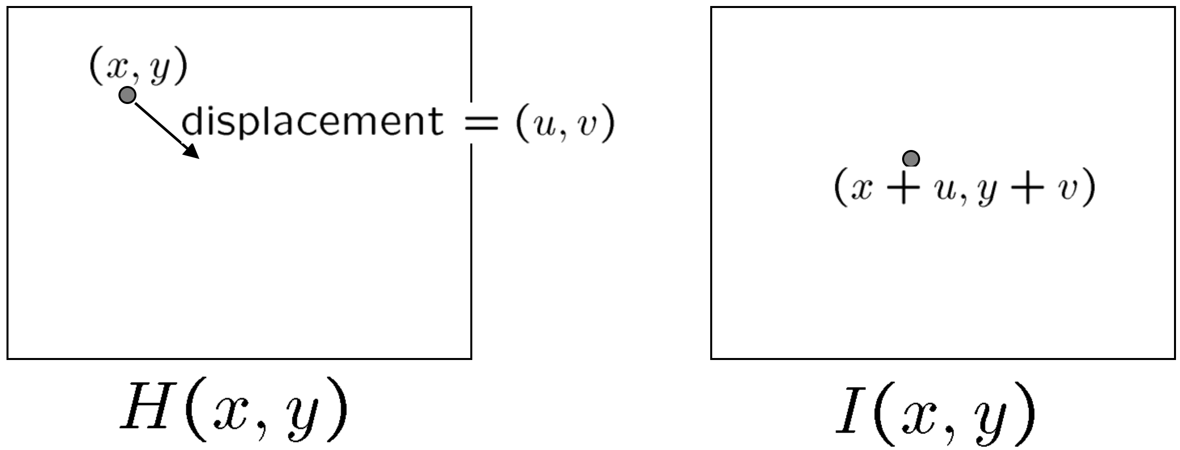
- Solve pixel correspondence problem
  - given a pixel in H, look for nearby pixels of the same color in I

Key assumptions

- **color constancy:** a point in H looks the same in I
  - For grayscale images, this is **brightness constancy**
- **small motion**: points do not move very far

This is called the **optical flow** problem

Optical flow constraints (grayscale images)



Let's look at these constraints more closely

- brightness constancy: Q: what's the equation?

- small motion: (u and v are less than 1 pixel)
  - suppose we take the Taylor series expansion of I:

$$I(x+u, y+v) = I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

$$\approx I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v$$

Optical flow equation

Combining these two equations

shorthand: $I_x = \frac{\partial I}{\partial x}$

$$
\begin{aligned}
0 &= I(x+u, y+v) - H(x,y) \\
&\approx I(x,y) + I_x u + I_y v - H(x,y) \\
&\approx (I(x,y) - H(x,y)) + I_x u + I_y v \\
&\approx I_t + I_x u + I_y v \\
&\approx I_t + \nabla I \cdot [u \;\; v]
\end{aligned}
$$

In the limit as u and v go to zero, this becomes exact

$$0 = I_t + \nabla I \cdot [\tfrac{\partial x}{\partial t} \;\; \tfrac{\partial y}{\partial t}]$$

Optical flow equation

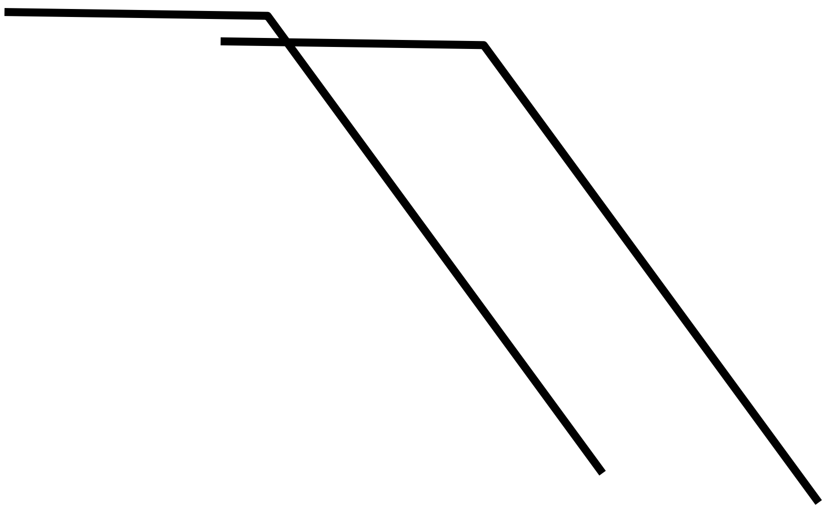
$$0 = I_t + \nabla I \cdot [u \;\; v]$$

Q: how many unknowns and equations per pixel?

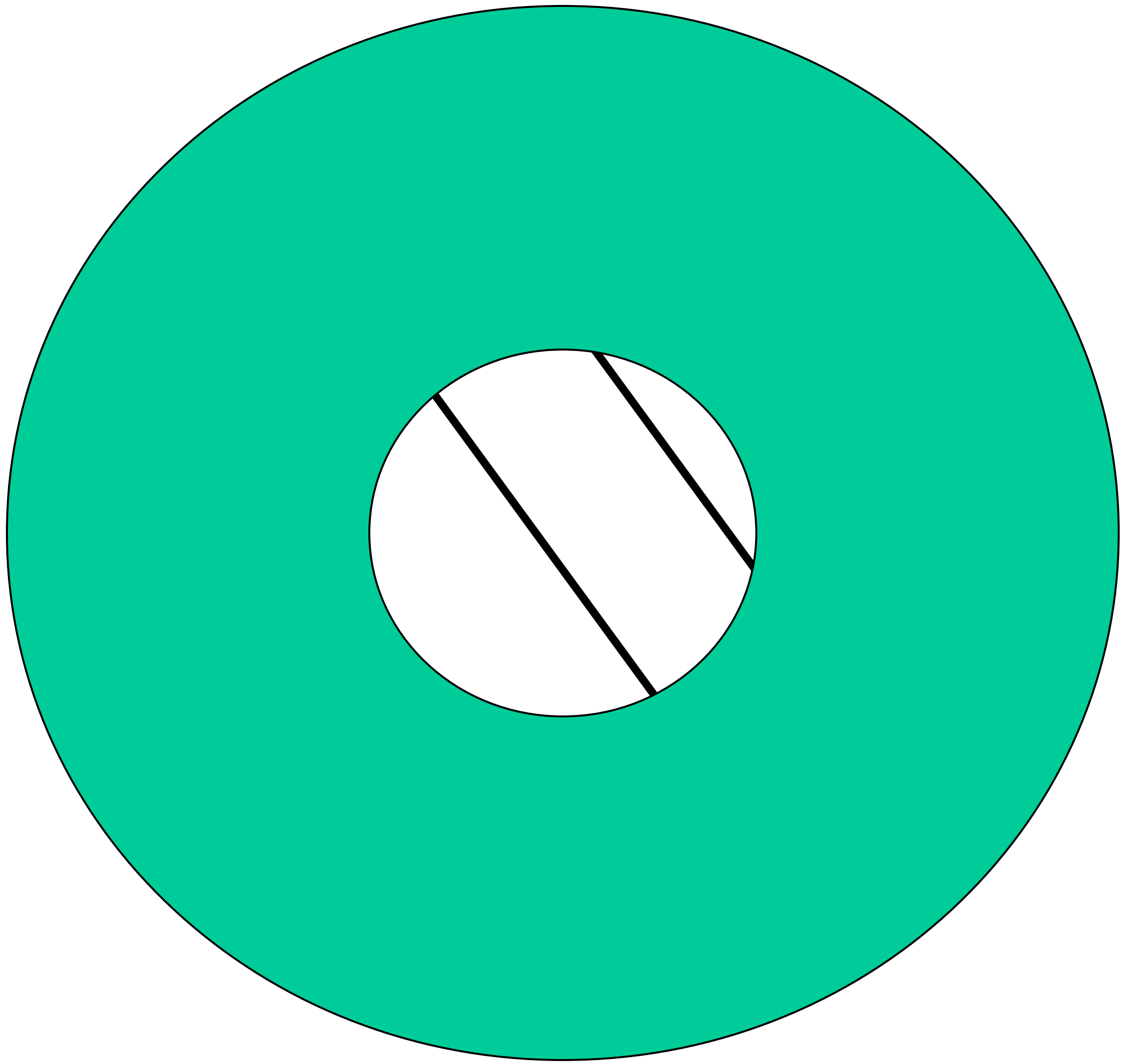
Intuitively, what does this constraint mean?

- The component of the flow in the gradient direction is determined
- The component of the flow parallel to an edge is unknown

Aperture problem



Aperture problem



Solving the aperture problem

How to get more equations for a pixel?

- Basic idea: impose additional constraints
  - most common is to assume that the flow field is smooth locally
  - one method: pretend the pixel's neighbors have the same (u,v)
    - » If we use a 5x5 window, that gives us 25 equations per pixel!

$$0 = I_t(\mathbf{p_i}) + \nabla I(\mathbf{p_i}) \cdot [u \;\; v]$$

$$\underset{\substack{A \\ 25\times2}}{\begin{bmatrix} I_x(\mathbf{p_1}) & I_y(\mathbf{p_1}) \\ I_x(\mathbf{p_2}) & I_y(\mathbf{p_2}) \\ \vdots & \vdots \\ I_x(\mathbf{p_{25}}) & I_y(\mathbf{p_{25}}) \end{bmatrix}} \underset{\substack{d \\ 2\times1}}{\begin{bmatrix} u \\ v \end{bmatrix}} = - \underset{\substack{b \\ 25\times1}}{\begin{bmatrix} I_t(\mathbf{p_1}) \\ I_t(\mathbf{p_2}) \\ \vdots \\ I_t(\mathbf{p_{25}}) \end{bmatrix}}$$

Lucas-Kanade flow

Prob: we have more equations than unknowns

$$\underset{25\times2}{A}\ \underset{2\times1}{d} = \underset{25\times1}{b} \longrightarrow \text{minimize } \|Ad - b\|^2$$

Solution: solve least squares problem

- minimum least squares solution given by solution (in d) of:

$$\underset{2\times2}{(A^T A)}\ \underset{2\times1}{d} = \underset{2\times1}{A^T b}$$

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

- The summations are over all pixels in the K x K window
- This technique was first proposed by Lucas & Kanade (1981)

- $A^T A$ should look familiar…

Conditions for solvability

- Optimal (u, v) satisfies Lucas-Kanade equation

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

$$A^T A \qquad\qquad\qquad\qquad A^T b$$

When is this solvable?

- **A^TA** should be invertible
- **A^TA** entries should not be too small (noise)
- **A^TA** should be well-conditioned
  - λ_1 / λ_2 should not be too large (λ_1 = larger eigenvalue)
  - Closely related to the Harris operator…

Errors in Lucas-Kanade

What are the potential causes of errors in this procedure?

- Suppose A^TA is easily invertible
- Suppose there is not much noise in the image

When our assumptions are violated

- Brightness constancy is **not** satisfied
- The motion is **not** small
- A point does **not** move like its neighbors
  - window size is too large
  - what is the ideal window size?

Improving accuracy

Recall our small motion assumption

$$0 = I(x+u, y+v) - H(x,y)$$

$$\approx I(x,y) + I_x u + I_y v - H(x,y)$$

This is not exact

- To do better, we need to add higher order terms back in:

$$= I(x,y) + I_x u + I_y v + \text{higher order terms} - H(x,y)$$

This is a polynomial root finding problem

- Can solve using **Newton's method**

- Approach so far does one iteration of Newton's method
  - Better results are obtained via more iterations

Iterative Refinement

Iterative Lucas-Kanade Algorithm

1. Estimate velocity at each pixel by solving Lucas-Kanade equations
2. Warp H towards I using the estimated flow field

   - use image warping techniques

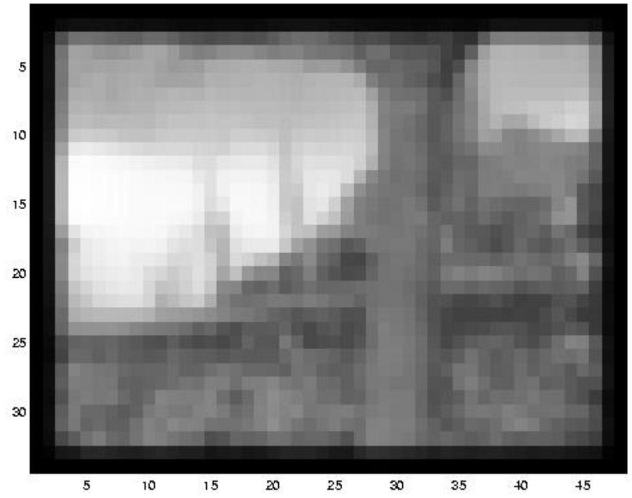
3. Repeat until convergence

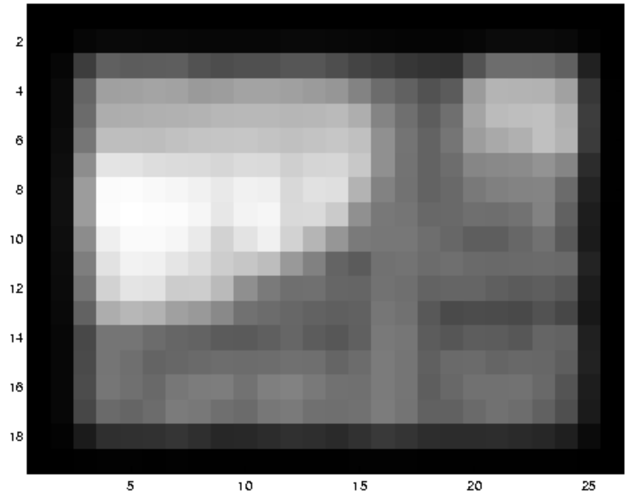
Revisiting the small motion assumption

Is this motion small enough?

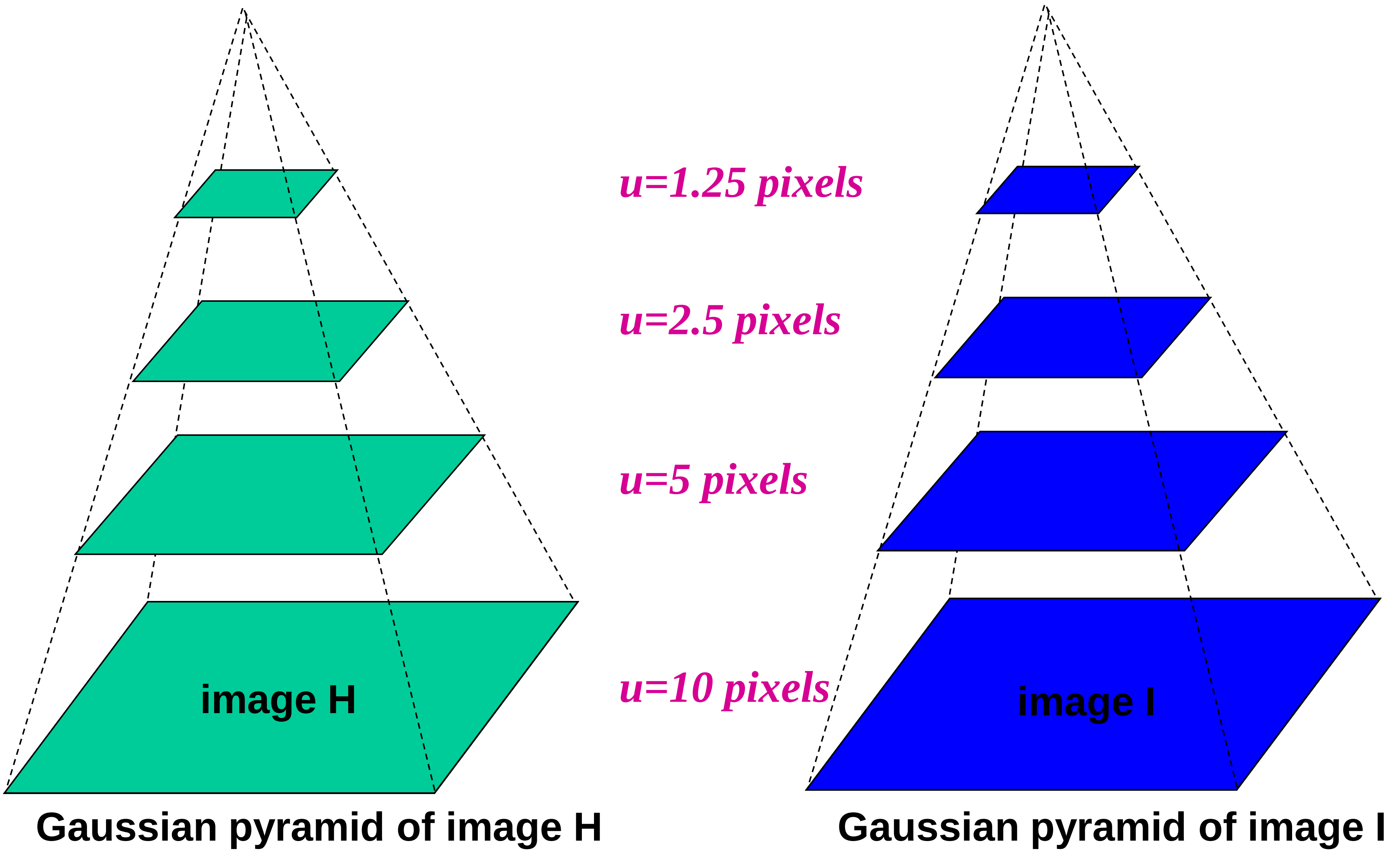
- Probably not—it's much larger than one pixel (2^{nd} order terms dominate)
- How might we solve this problem?

Reduce the resolution!





Coarse-to-fine optical flow estimation



Coarse-to-fine optical flow estimation