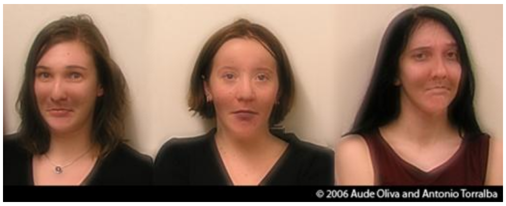
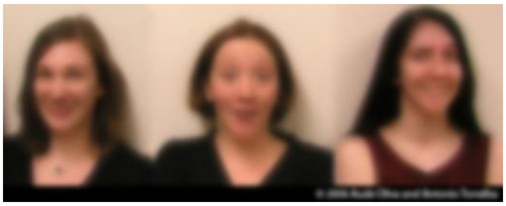
Images & Filtering



Hybrid Images, Oliva et al., http://cvcl.mit.edu/hybridimage.htm

Images & Filtering



Hybrid Images, Oliva et al., http://cvcl.mit.edu/hybridimage.htm

What is an image?

Images as functions

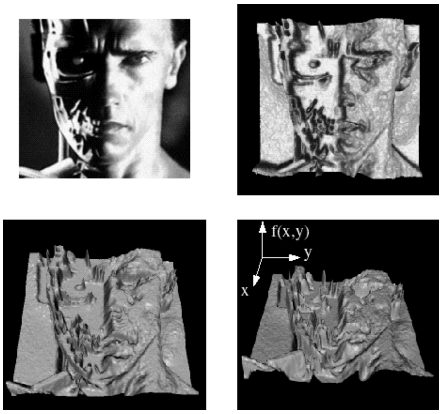
We can think of an **image** as a function, *f*, from $\mathbb{R}^2$ to $\mathbb{R}$:

- $f(x, y)$ gives the **intensity** at position (x, y)
- Realistically, we expect the image only to be defined over a rectangle, with a finite range:
  - $f: [a,b] \times [c,d] \rightarrow [0,1]$

A color image is just three functions pasted together. We can write this as a "vector-valued" function:

$$f(x,y) = \begin{bmatrix} r(x,y) \\ g(x,y) \\ b(x,y) \end{bmatrix}$$

Images as functions



What is a digital image?

In computer vision we usually operate on **digital** (**discrete**) images:

- **Sample** the 2D space on a regular grid
- **Quantize** each sample (round to nearest integer)

If our samples are Δ apart, we can write this as:

f[*i* ,*j*] = Quantize{ *f*(*i* Δ, *j* Δ) }

The image can now be represented as a matrix of integer values

j →, i ↓

62	79	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
166	63	127	17	1	0	99	30

Image processing

An **image processing** operation typically defines a new image *g* in terms of an existing image *f*.

We can transform either the domain or the range of *f*.

Range transformation:

$$g(x,y) = t(f(x,y))$$

What's kinds of operations can this perform?

Image processing

Some operations preserve the range but change the domain of *f* :

$$g(x,y) = f(t_x(x,y), t_y(x,y))$$

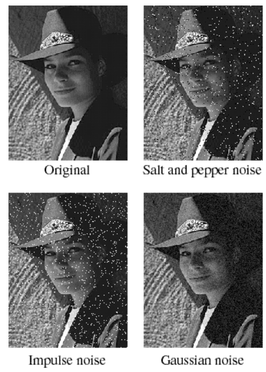
What kinds of operations can this perform?

Image processing

Still other operations operate on both the domain *and* the range of *f* .

Noise

Image processing is useful for noise reduction...



Common types of noise:

- **Salt and pepper noise**: contains random occurrences of black and white pixels
- **Impulse noise:** contains random occurrences of white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution

Ideal noise reduction

Given a camera and a still scene, how can you reduce noise?

Ideal noise reduction

Given a camera and a still scene, how can you reduce noise?

Practical noise reduction

How can we “smooth” away noise in a single image?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	100	130	110	120	110	0	0
0	0	0	110	90	100	90	100	0	0
0	0	0	130	100	90	130	110	0	0
0	0	0	120	100	130	110	120	0	0
0	0	0	90	110	80	120	100	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Mean filtering

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

Mean filtering

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

Mean filtering

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

Mean filtering

When does this work?

$$F(x) = I(x) + N(x)$$
$$G(x) = \frac{1}{3}\left(F(x-1) + F(x) + F(x+1)\right)$$
$$G(x) = \frac{1}{3}\left(I(x-1) + I(x) + I(x+1)\right) + 3N(x)$$
$$E(G(x)) = \frac{1}{3}\left(I(x-1) + I(x) + I(x+1)\right)$$

When does $E(G(x)) = I(x)$?

$$3I(x) = I(x-1) + I(x) + I(x+1)$$
$$I(x-1) - 2I(x) + I(x+1) = 0$$

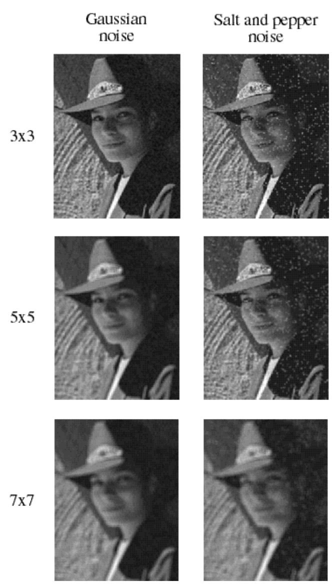
17

Mean filtering

When does this work?

18

Effect of mean filters



19

Cross-correlation filtering

Let’s write this down as an equation. Assume the averaging window is (2k+1)x(2k+1):

$$G[i,j] = \frac{1}{(2k+1)^2} \sum_{u=-k}^{k} \sum_{v=-k}^{k} F[i+u, j+v]$$

We can generalize this idea by allowing different weights for different neighboring pixels:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i+u, j+v]$$

This is called a **cross-correlation** operation and written:

$$G = H \otimes F$$

H is called the “filter,” “kernel,” or “mask.”

The above allows negative filter indices. When you implement need to use: H[u+k,v+k] instead of H[u,v]

20

Mean kernel

What's the kernel for a 3x3 mean filter?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

$H[u, v]$

Gaussian Filtering

A Gaussian kernel gives less weight to pixels further from the center of the window

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

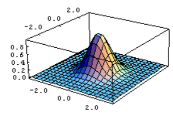
$\frac{1}{16}$

1	2	1
2	4	2
1	2	1

$H[u, v]$

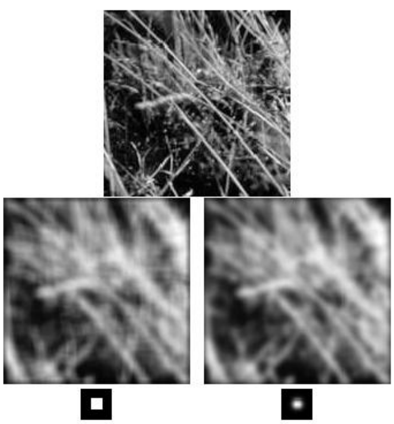
This kernel is an approximation of a Gaussian function:

$$h(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$



What happens if you increase σ ?

Mean vs. Gaussian filtering



Filtering an impulse

a	b	c
d	e	f
g	h	i

$H[u, v]$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$F[x, y]$

$G[x, y]$

Convolution

A **convolution** operation is a cross-correlation where the filter is flipped both horizontally and vertically before being applied to the image:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

It is written: $G = H \star F$

Suppose H is a Gaussian or mean kernel. How does convolution differ from cross-correlation?

Suppose F is an impulse function (previous slide) What will G look like?

Continuous Filters

We can also apply filters to *continuous* images.

In the case of cross correlation: $g = h \otimes f$

$$g(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(u,v) f(x+u, y+v) du dv$$

In the case of convolution: $g = h \star f$

$$g(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(u,v) f(x-u, y-v) du dv$$

Note that the image and filter are infinite.

Median filters

A **Median Filter** operates over a window by selecting the median intensity in the window.

What advantage does a median filter have over a mean filter?

Is a median filter a kind of convolution?

Comparison: salt and pepper noise

Comparison: Gaussian noise