Edge Detection

From Sandlot Science

Today's reading

- Cipolla & Gee on edge detection (available online)

Project 1a

assigned last Friday

due this Friday

Last time: Cross-correlation

Let F be the image, H be the kernel (of size 2k+1 x 2k+1), and G be the output image

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u,j+v]$$

This is called a **cross-correlation** operation:

$$G = H \otimes F$$

Last time: Convolution

Same as cross-correlation, except that the kernel is "flipped" (horizontally and vertically)

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

This is called a **convolution** operation:

$$G = H * F$$

Cross-Correlation

- Not commutative

- Associative

$$F * (G * H) = (F * G) * H$$

- No Identity

Convolution

- Commutative

$$F * G = G * F$$

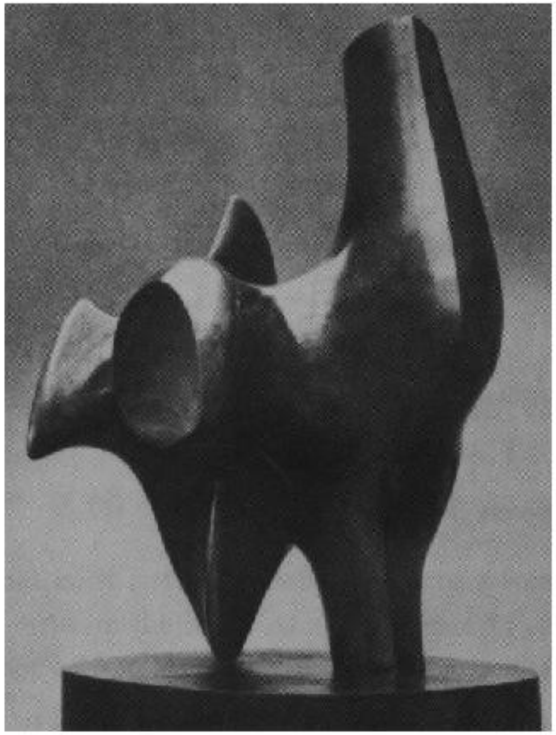
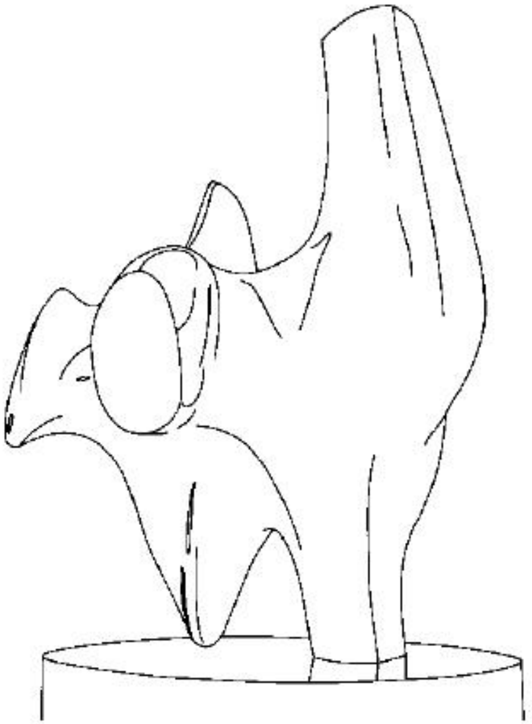
- Associative

$$F * (G * H) = (F * G) * H$$

- Identity

$$F * \delta = F$$

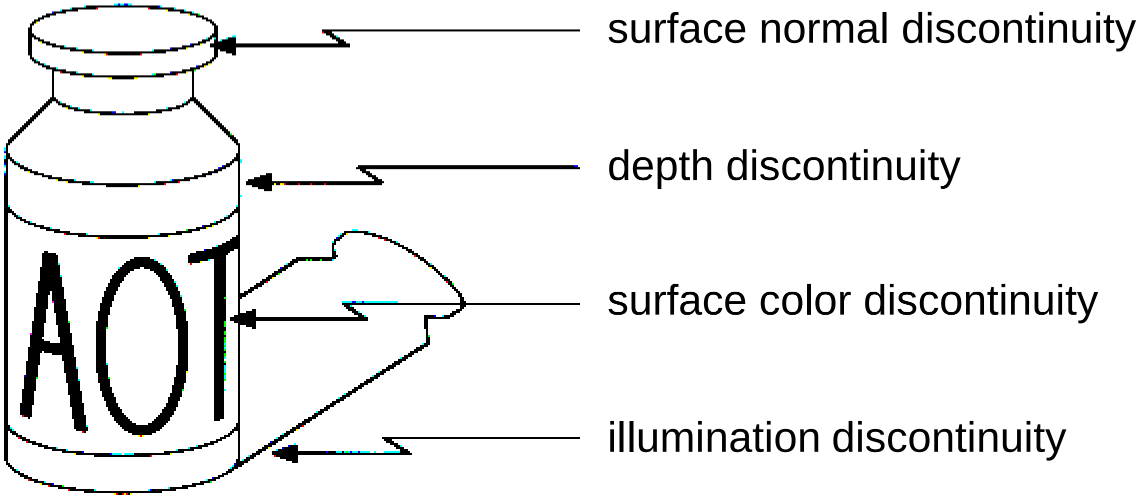
Edge detection

Convert a 2D image into a set of curves

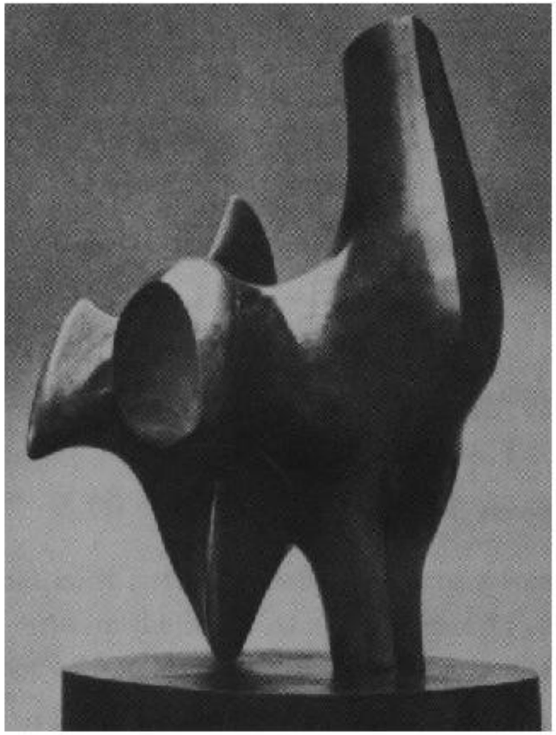
- Extracts salient features of the scene
- More compact than pixels

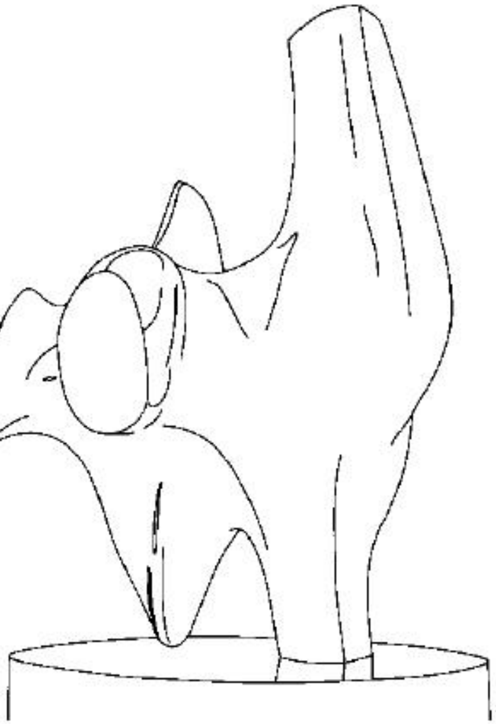
Origin of Edges



Edges are caused by a variety of factors

Edge detection

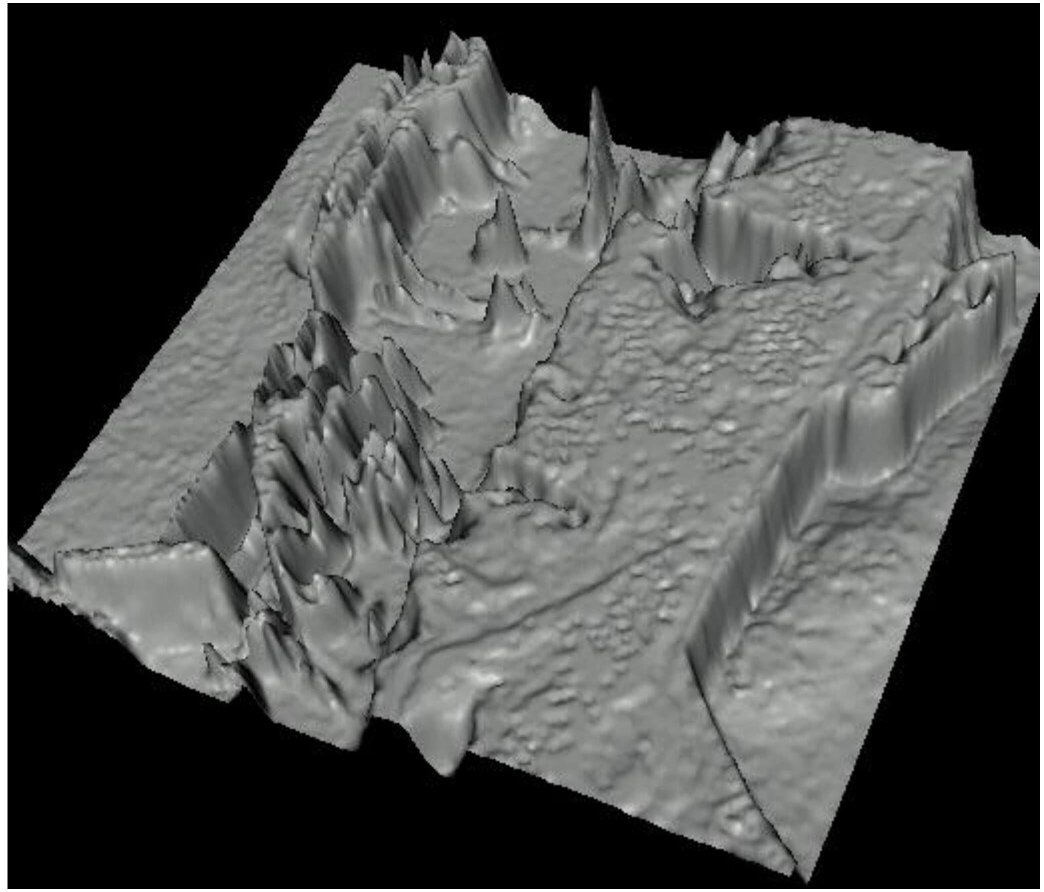




How can you tell that a pixel is on an edge?

snoop demo

Images as functions…

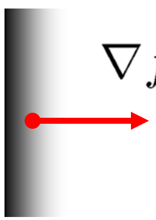


Edges look like steep cliffs

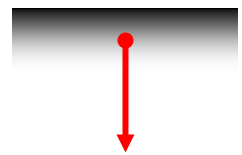
Image gradient

The gradient of an image:

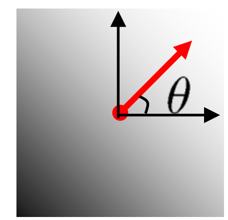
$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$



$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

The gradient points in the direction of most rapid increase in intensity

The gradient direction is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

- how does this relate to the direction of the edge?

The *edge strength* is given by the gradient magnitude

$$||\nabla f|| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

The discrete gradient

How can we differentiate a *digital* image F[x,y]?

$$\frac{\partial F}{\partial x} = \lim_{h \to 0} \frac{F(x+h, y) - F(x, y)}{h}$$

The discrete gradient

How can we differentiate a *digital* image F[x,y]?

- Option 1: reconstruct a continuous image, then take gradient
- Option 2: take discrete derivative ("finite difference")

$$\frac{\partial f}{\partial x}[x,y] \approx F[x+1,y] - F[x,y]$$

How would you implement this as a cross-correlation?

0	0	0
1/2	0	-1/2
0	0	0

H

filter demo

The Sobel operator

Better approximations of the derivatives exist

- The *Sobel* operators below are very commonly used

$$\frac{1}{8}\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array} \qquad \frac{1}{8}\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}$$

$$s_x \qquad\qquad s_y$$

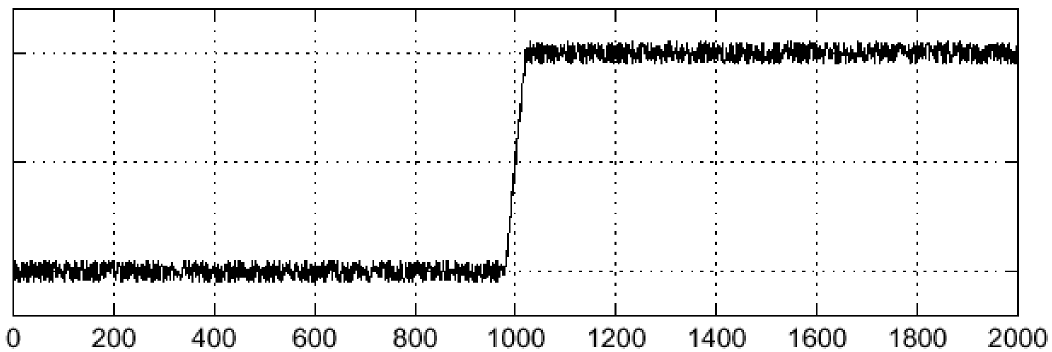
- The standard defn. of the Sobel operator omits the 1/8 term
  - doesn't make a difference for edge detection
  - the 1/8 term **is** needed to get the right gradient value, however

Effects of noise

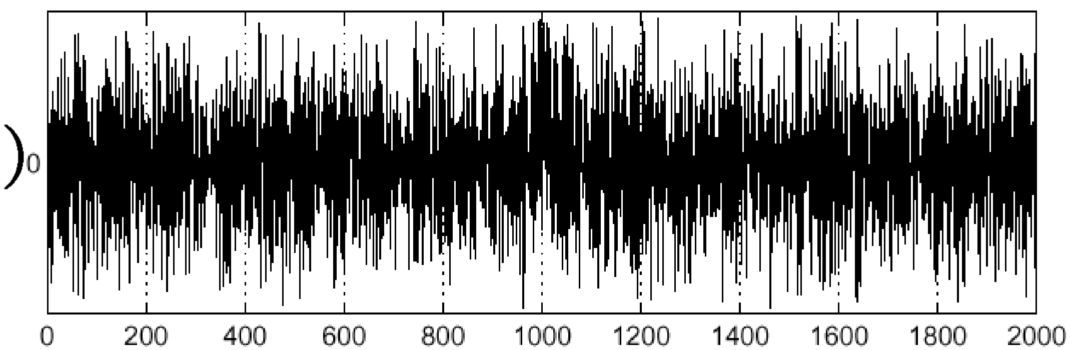
Consider a single row or column of the image

- Plotting intensity as a function of position gives a signal

$f(x)$

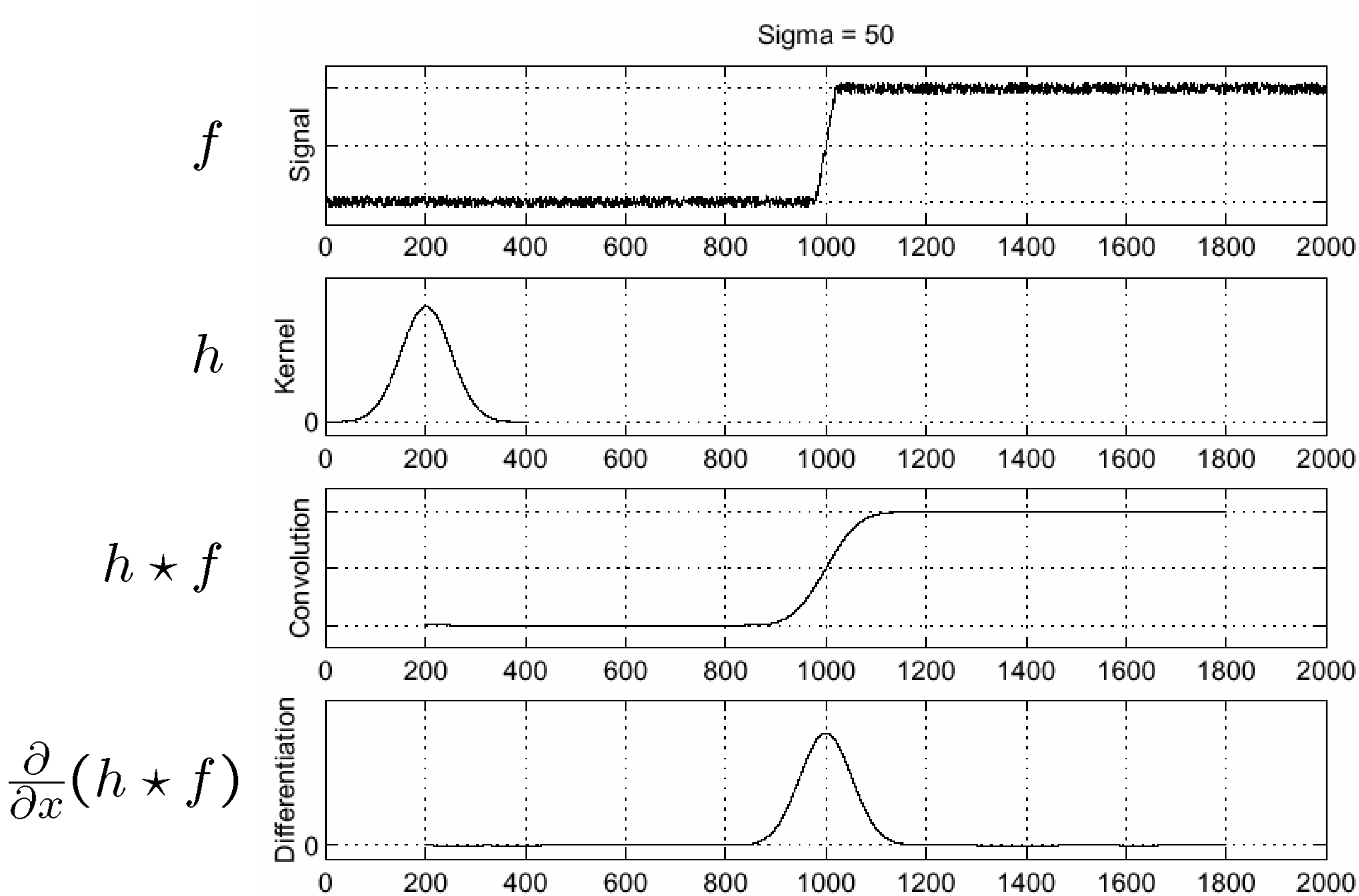


$\frac{d}{dx}f(x)$



Where is the edge?

Solution: smooth first



Where is the edge? Look for peaks in $\frac{\partial}{\partial x}(h \star f)$

Derivative theorem of convolution

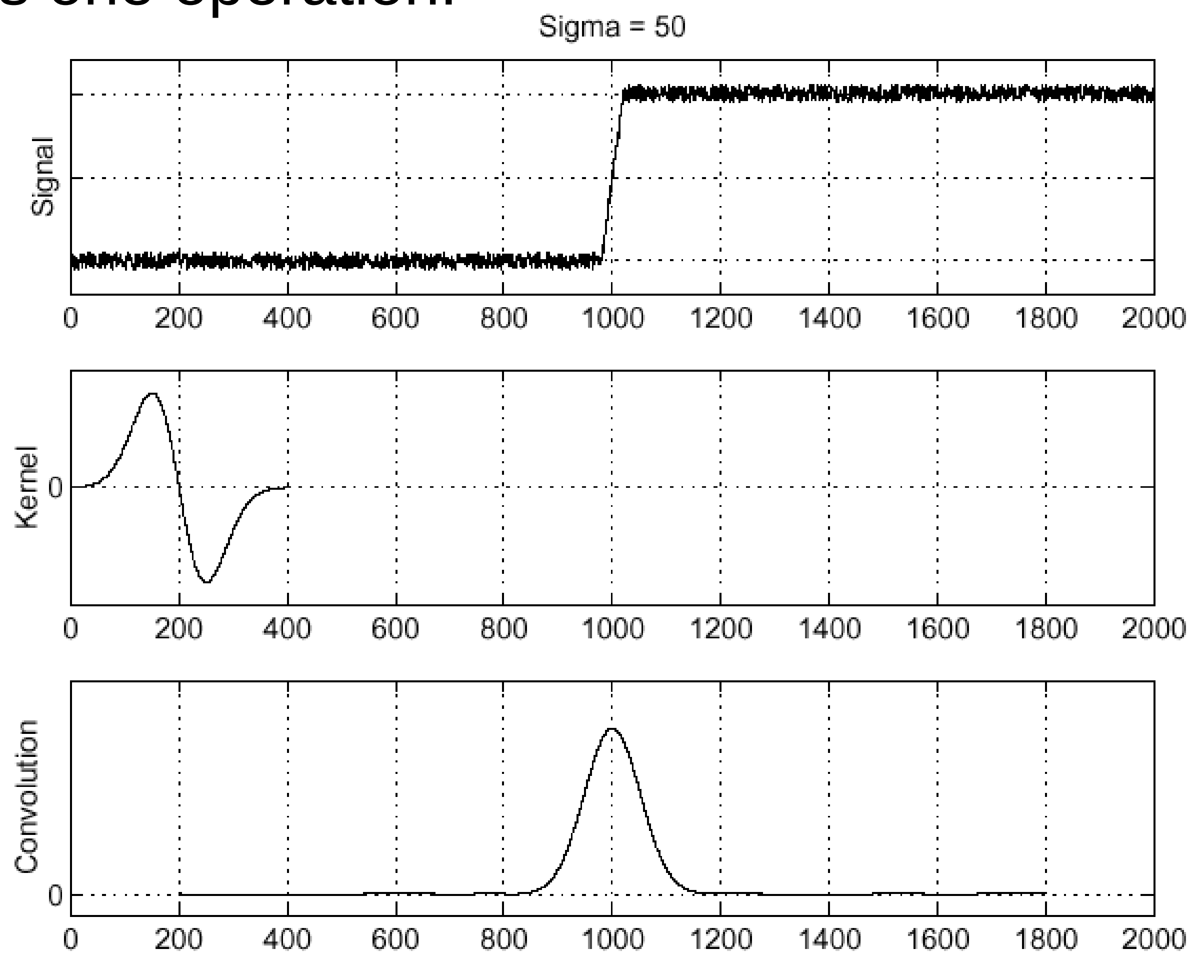
$$\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$$

This saves us one operation:

f

$\frac{\partial}{\partial x}h$

$(\frac{\partial}{\partial x}h) \star f$



How can we find (local) maxima of a function?

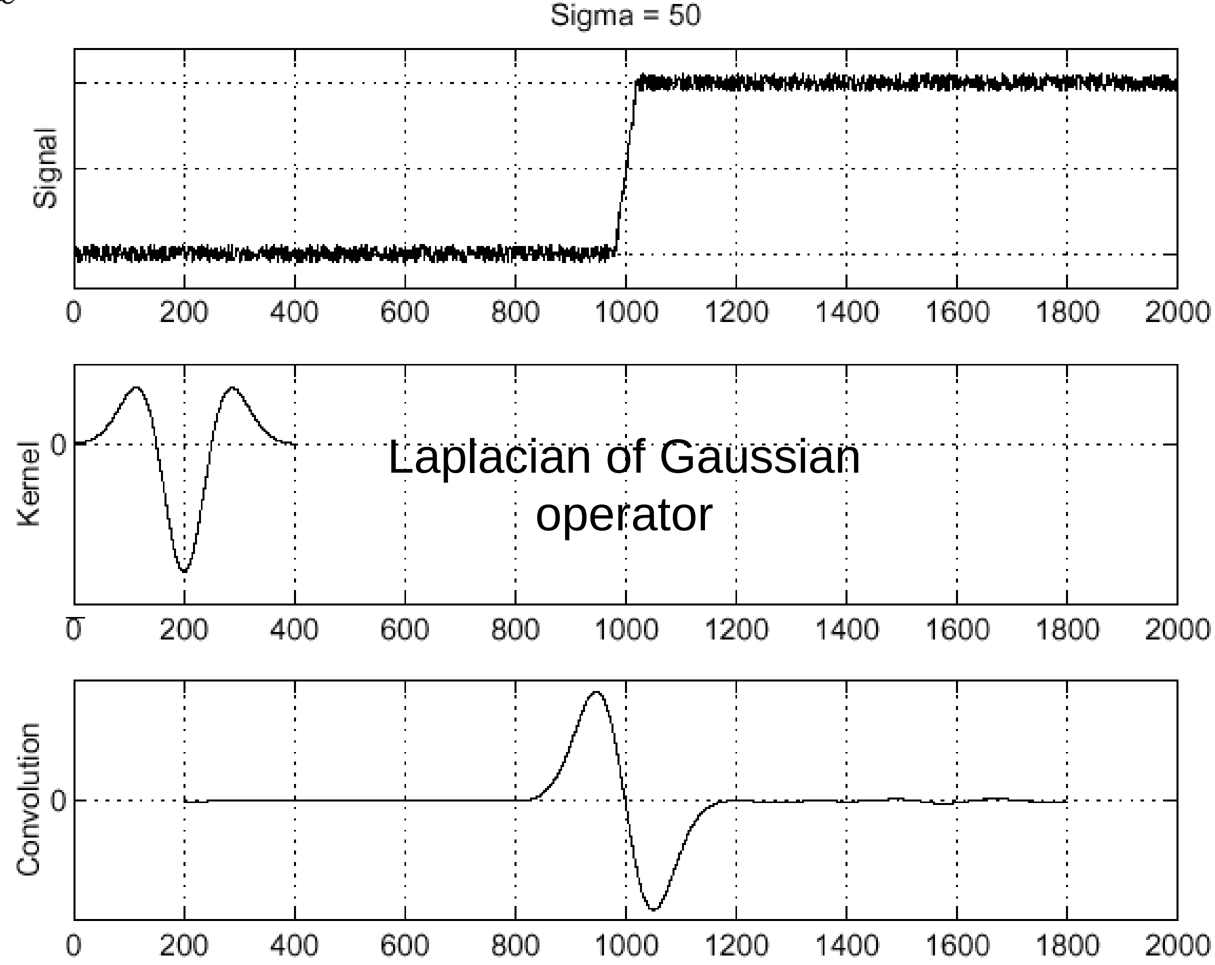
Laplacian of Gaussian

Consider $\frac{\partial^2}{\partial x^2}(h \star f)$

f

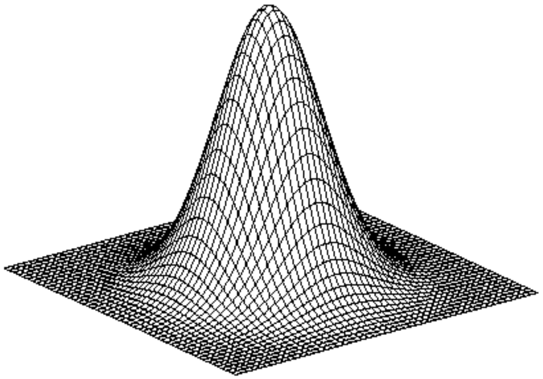
$\frac{\partial^2}{\partial x^2}h$

$(\frac{\partial^2}{\partial x^2}h) \star f$



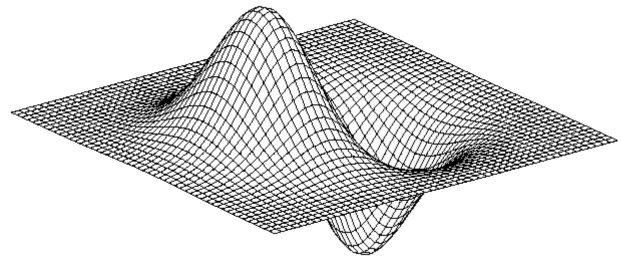
Where is the edge? Zero-crossings of bottom graph

2D edge detection filters



Gaussian

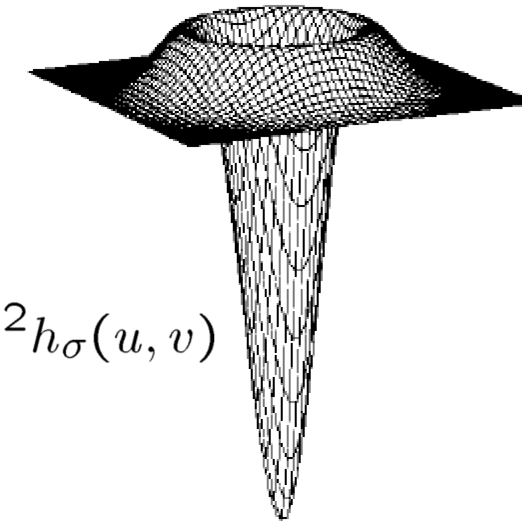
$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



derivative of Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

Laplacian of Gaussian



$$\nabla^2 h_\sigma(u,v)$$

∇^2 is the **Laplacian** operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

filter demo

The Canny edge detector

original image (Lena)

The Canny edge detector

norm of the gradient

The Canny edge detector

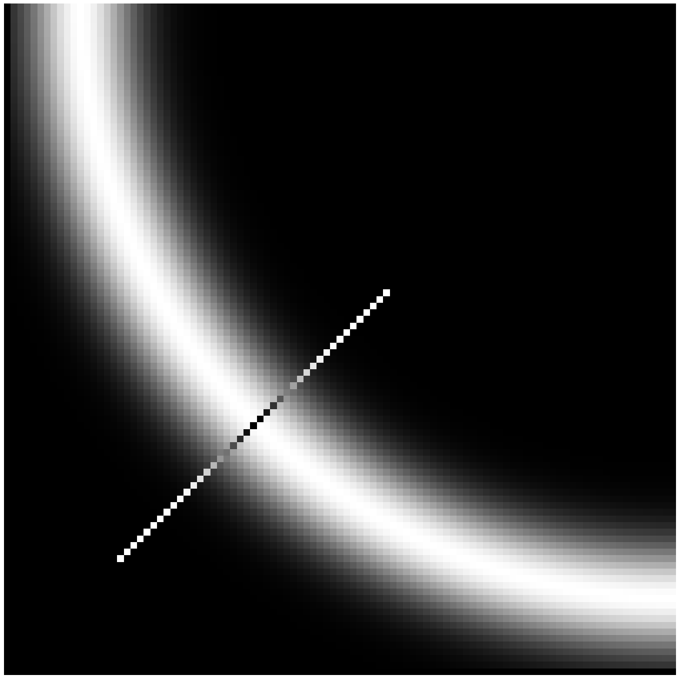
thresholding

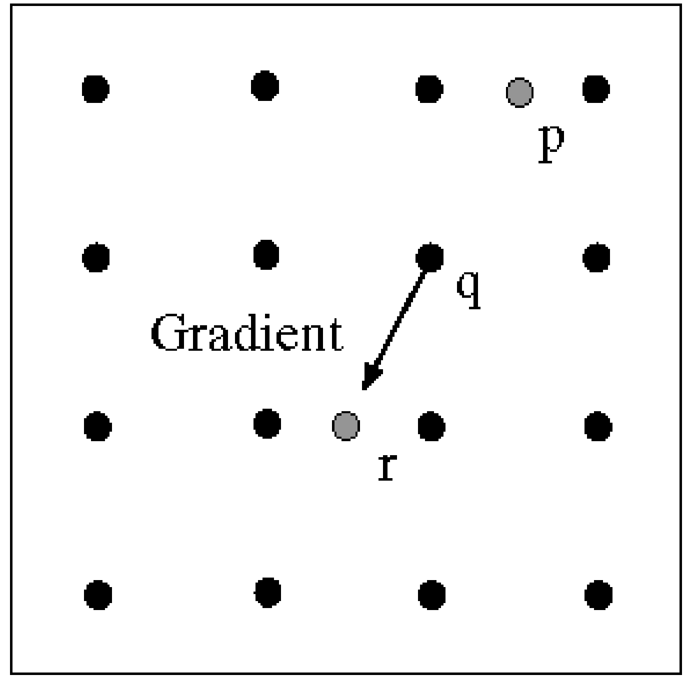
The Canny edge detector

thinning

(non-maximum suppression)

Non-maximum suppression





Check if pixel is local maximum along gradient direction

- requires checking interpolated pixels p and r

Effect of σ (Gaussian kernel spread/size)

original   Canny with $\sigma = 1$ Canny with $\sigma = 2$

The choice of σ depends on desired behavior

- large σ detects large scale edges
- small σ detects fine features

Edge detection by subtraction

original

Edge detection by subtraction

smoothed (5x5 Gaussian)

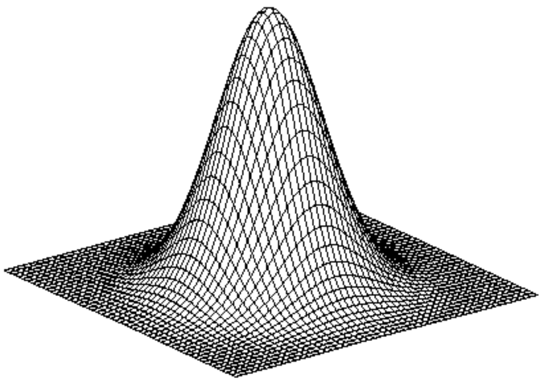
Edge detection by subtraction

smoothed – original
(scaled by 4, offset +128)

Why does this work?

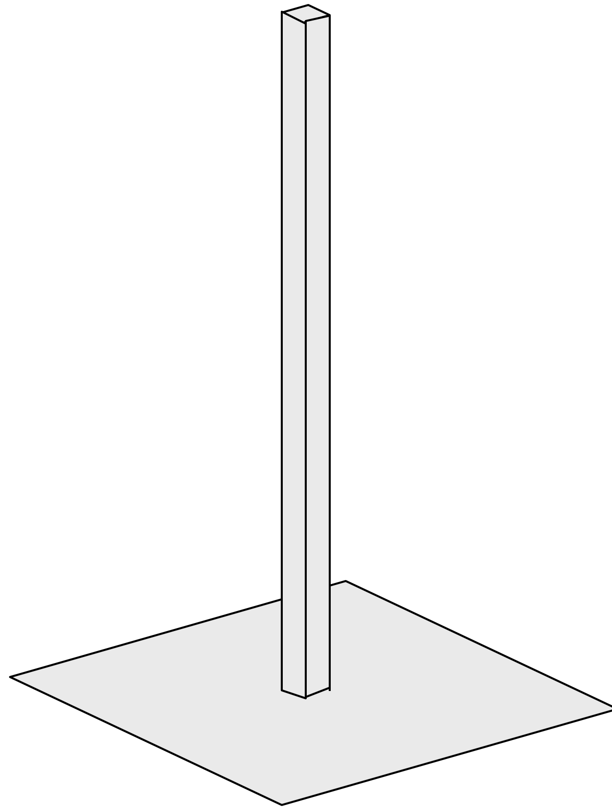
filter demo

Gaussian - image filter



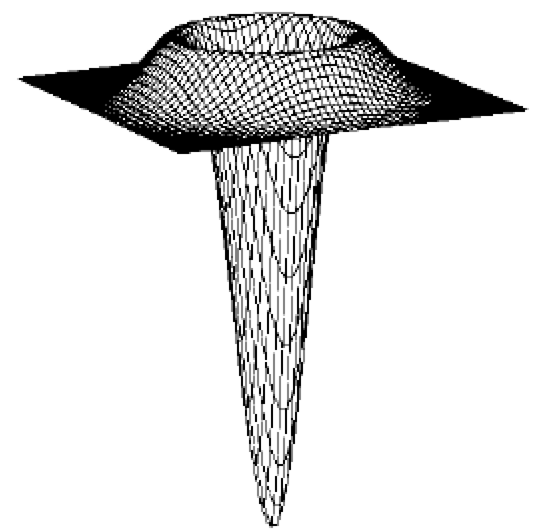
Gaussian

$-$



delta function

$\approx$



Laplacian of Gaussian