

## Cameras

CSE 455, Winter 2010
January 25, 2010

## Announcements

- New Lecturer!


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- Project 1b (seam carving) was due on Friday the $22^{\text {nd }}$
- Project 2 (eigenfaces) went out on Friday the 22 nd
- to be done individually


## Cameras are Everywhere



## Camera Trends



## First Known Photograph



View from the Window at le Gras, Joseph Nicéphore Niépce 1826

## What is an image?

## Images as functions

-We can think of an image as a function, $f$, from $\mathrm{R}^{2}$ to R :

- $f(x, y)$ gives the intensity at position ( $x, y$ )
- Realistically, we expect the image only to be defined over a rectangle, with a finite range:

$$
\text { - } f:[a, b] \times[c, d] \rightarrow[0,1]
$$

-A color image is just three functions pasted together. We can write this as a "vector-valued" function:

$$
f(x, y)=\left[\begin{array}{l}
r(x, y) \\
g(x, y) \\
b(x, y)
\end{array}\right]
$$

Images as functions


## What is a digital image?

- In computer vision we usually operate on digital (discrete) images:
- Sample the 2D space on a regular grid
- Quantize each sample (round to nearest integer)
-If our samples are $\Delta$ apart, we can write this as:
- $f[i, j]=$ Quantize $\{f(i \Delta, j \Delta)\}$
-The image can now be represented as a matrix of integer values

| $i$ | 62 | 79 | 23 | 119 | 120 | 105 | 4 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 10 | 10 | 9 | 62 | 12 | 78 | 34 | 0 |
|  | 10 | 58 | 197 | 46 | 46 | 0 | 0 | 48 |
|  | 176 | 135 | 5 | 188 | 191 | 68 | 0 | 49 |
|  | 2 | 1 | 1 | 29 | 26 | 37 | 0 | 77 |
|  | 0 | 89 | 144 | 147 | 187 | 102 | 62 | 208 |
|  | 255 | 252 | 0 | 166 | 123 | 62 | 0 | 31 |
|  | 166 | 63 | 127 | 17 | 1 | 0 | 99 | 30 |

## Projection



## Projection



## What is an image?

- 2D pattern of intensity values
- 2D projection of 3D objects


Figure from US Navy Manual of Basic Optics and Optical Instruments, prepared by Bureau of Naval Personnel. Reprinted by Dover Publications, Inc., 1969.

## What is an camera?

## Image formation



- Let's design a camera
- Idea 1: put a piece of film in front of an object
- Do we get a reasonable image?


## Pinhole camera



- Add a barrier to block off most of the rays
- This reduces blurring
- The opening known as the aperture
- How does this transform the image?


## Camera Obscura


illum in rabala per radios Solis, quaim in calo contingir: hoc eft,fi in cello fupgrior pars delhquiâ pariarur, in radis apparebit inferiof deficere,vt ratio exigitoptica.


Sic nos exaetì Anno.1944. Lounnii celipfim Solis eb/cruasimus, inuenimuséq deficere paulò plus $\bar{q}$ dex.

- The first camera
- Known to Aristotle
- According to DaVinci "When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size, in a reversed position, owing to the intersection of the rays".
- How does the aperture size affect the image?


## Shrinking the aperture



- Why not make the aperture as small as possible?
- Less light gets through
- Diffraction effects...


## Shrinking the aperture



## Adding a lens



- A lens focuses light onto the film
- There is a specific distance at which objects are "in focus"
- other points project to a "circle of confusion" in the image
- Changing the shape of the lens changes this distance


## Lenses



- A lens focuses parallel rays onto a single focal point
- focal point at a distance $f$ beyond the plane of the lens
- $f$ is a function of the shape and index of refraction of the lens
- Aperture of diameter D restricts the range of rays
- aperture may be on either side of the lens
- Lenses are typically spherical (easier to produce)


## Thin lenses



- Thin lens equation:

$$
\frac{1}{d_{o}}+\frac{1}{d_{i}}=\frac{1}{f}
$$

- Any object point satisfying this equation is in focus
- What is the shape of the focus region?
- How can we change the focus region?
- Thin lens applet: http://www.phy.ntnu.edu.tw/iava/Lens/lens e.html (by Fu-Kwun Hwang )


## Depth of field


$f / 5.6$

f/32

- Changing the aperture size affects depth of field
- A smaller aperture increases the range in which the object is approximately in focus

Flower images from Wikipedia http://en.wikipedia.org/wiki/Depth of field

## Back to Project: Müller-Lyer Illusion



## Which line is longer?


http://www.michaelbach.de/ot/sze muelue/index.html

## Modeling projection



- The coordinate system
- We will use the pin-hole model as an approximation
- Put the optical center (Center Of Projection) at the origin
- Put the image plane (Projection Plane) in front of the COP
- Why?
- The camera looks down the negative $z$ axis
- we need this if we want right-handed-coordinates


## Modeling projection



- Projection equations
- Compute intersection with PP of ray from ( $x, y, z$ ) to COP
- Derived using similar triangles (on board)

$$
(x, y, z) \rightarrow\left(-d \frac{x}{z},-d \frac{y}{z},-d\right)
$$

- We get the projection by throwing out the last coordinate:

$$
(x, y, z) \rightarrow\left(-d \frac{x}{z},-d \frac{y}{z}\right)
$$

## Homogeneous coordinates

- Is this a linear transformation?
- no-division by $z$ is nonlinear

Trick: add one more coordinate:

$$
(x, y) \Rightarrow\left[\begin{array}{l}
x \\
y \\
1
\end{array}\right]
$$

homogeneous image coordinates

homogeneous scene coordinates

Converting from homogeneous coordinates

$$
\left[\begin{array}{c}
x \\
y \\
w
\end{array}\right] \Rightarrow(x / w, y / w) \quad\left[\begin{array}{l}
x \\
y \\
z \\
w
\end{array}\right] \Rightarrow(x / w, y / w, z / w)
$$

## Perspective Projection

- Projection is a matrix multiply using homogeneous coordinates:

$$
\begin{aligned}
& {\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 / d & 0
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z \\
1
\end{array}\right]=} {\left[\begin{array}{c}
x \\
y \\
-z / d
\end{array}\right] \Rightarrow\left(-d \frac{x}{z},-d \frac{y}{z}\right) } \\
& \text { divide by third coordinate }
\end{aligned}
$$

- This is known as perspective projection
- The matrix is the projection matrix
- Can also formulate as a 4x4 (today's reading does this)

$$
\begin{gathered}
{\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & -1 / d & 0
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z \\
1
\end{array}\right]=\left[\begin{array}{c}
x \\
y \\
z \\
-z / d
\end{array}\right] \Rightarrow\left(-d \frac{x}{z},-d \frac{y}{z}\right)} \\
\text { divide by fourth coordinate }
\end{gathered}
$$

## Perspective Projection

- How does scaling the projection matrix change the transformation?

$$
\left.\left.\begin{array}{l}
{\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 / d & 0
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z \\
1
\end{array}\right]=\left[\begin{array}{c}
x \\
y \\
-z / d
\end{array}\right] \Rightarrow\left(-d \frac{x}{z},-d \frac{y}{z}\right)} \\
{\left[\begin{array}{cccc}
-d & 0 & 0 & 0 \\
0 & -d & 0 & 0 \\
0 & 0 & 1 & 0
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z \\
1
\end{array}\right]=\left[\begin{array}{c}
-d x \\
-d y \\
z
\end{array}\right] \Rightarrow\left(-d \frac{x}{z},\right.}
\end{array}\right]=d \frac{y}{z}\right) .
$$

- Projection matrix is defined "up to a scale"


## Geometric properties of perspective projection

- Geometric properties of perspective projection
- Points go to points
- Lines go to lines
- Planes go to whole image or half-plane
- Polygons go to polygons
- Angles \& distances not preserved
- Degenerate cases:
- line through focal point yields point
- plane through focal point yields line


## Orthographic projection

- Special case of perspective projection
- Distance from the COP to the PP is infinite

- Good approximation for telephoto optics
- Also called "parallel projection": $(x, y, z) \rightarrow(x, y)$
- What's the projection matrix?

$$
\left[\begin{array}{llll}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z \\
1
\end{array}\right]=\left[\begin{array}{l}
x \\
y \\
1
\end{array}\right] \Rightarrow(x, y)
$$

## Other types of projection

- Scaled orthographic
- Also called "weak perspective"

$$
\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 / d
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z \\
1
\end{array}\right]=\left[\begin{array}{c}
x \\
y \\
1 / d
\end{array}\right] \Rightarrow(d x, d y)
$$

- Affine projection
- Also called "paraperspective"

$$
\left[\begin{array}{llll}
a & b & c & d \\
e & f & g & h \\
0 & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z \\
1
\end{array}\right]
$$

## Changes in Perspective

"Dolly Zoom" Effect (Popularized by Alfred Hitchcock)

## Camera parameters

A camera is described by several parameters

- Translation T of the optical center from the origin of world coords
- Rotation R of the image plane
- focal length $f$, principle point $\left(x_{c}^{\prime}, y^{\prime}{ }_{c}\right)$, pixel size $\left(s_{x}, s_{y}\right)$
- blue parameters are called "extrinsics," red are "intrinsics"


## Projection equation

$$
\mathbf{X}=\left[\begin{array}{c}
s x \\
s y \\
s
\end{array}\right]=\left[\begin{array}{llll}
* & * & * & * \\
* & * & * & * \\
* & * & * & *
\end{array}\right]\left[\begin{array}{c}
X \\
Y \\
Z \\
1
\end{array}\right]=\mathbf{\Pi X}
$$



- The projection matrix models the cumulative effect of all parameters
- Useful to decompose into a series of operations

$$
\begin{gathered}
\boldsymbol{\Pi}=\left[\begin{array}{ccc}
-f s_{x} & 0 & x_{c}^{\prime} c \\
0 & -f s_{y} & y_{c}^{\prime} \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{array}\right] \\
\text { intrinsics }
\end{gathered} \underset{\text { projection }}{\left[\begin{array}{cc}
\mathbf{R}_{3 \times 3} & \mathbf{0}_{3 \times 1} \\
\mathbf{0}_{1 \times 3} & 1
\end{array}\right]} \underset{\text { rotation }}{\text { translation }}\left[\begin{array}{cc}
\mathbf{T}_{3 \times 3} & \mathbf{T}_{3 \times 1} \\
\mathbf{0}_{1 \times 3} & 1
\end{array}\right]
$$

- The definitions of these parameters are not completely standardized
- especially intrinsics - varies from one book to another


## Distortion



No distortion


Pin cushion


Barrel

- Radial distortion of the image
- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens


## Correcting radial distortion


from Helmut Dersch

## Distortion



## Modeling distortion

Project ( $\hat{x}, \hat{y}, \hat{z}$ ) $\quad x_{n}^{\prime}=\hat{x} / \widehat{z}$
to "normalized"
image coordinates
$y_{n}^{\prime}=\widehat{y} / \widehat{z}$

$$
r^{2}=x_{n}^{\prime 2}+y_{n}^{\prime 2}
$$

Apply radial distortion

$$
\begin{aligned}
x_{d}^{\prime} & =x_{n}^{\prime}\left(1+\kappa_{1} r^{2}+\kappa_{2} r^{4}\right) \\
y_{d}^{\prime} & =y_{n}^{\prime}\left(1+\kappa_{1} r^{2}+\kappa_{2} r^{4}\right)
\end{aligned}
$$

Apply focal length translate image center

$$
\begin{aligned}
x^{\prime} & =f x_{d}^{\prime}+x_{c} \\
y^{\prime} & =f y_{d}^{\prime}+y_{c}
\end{aligned}
$$

- To model lens distortion
- Use above projection operation instead of standard projection matrix multiplication


## Chromatic Aberration

## Rays of different wavelength focus in different planes



Axial chromatic aderration


Magnification chromatio aderration
cannot be removed completely


The image is blurred and appears colored at the fringe.

## Vignetting



- Some light misses the lens or is otherwise blocked by parts of the lens


## Other types of lenses/cameras



Tilt-shift images from Vincent Laforet
More examples: http://www.smashingmagazine.com/2008/11/16/beautiful-examples-of-tilt-shift-photography/

## "Human Camera" (The eye)



- The human eye is a camera
- Iris - colored annulus with radial muscles
- Pupil - the hole (aperture) whose size is controlled by the iris
- What's the "film"?
- photoreceptor cells (rods and cones) in the retina


## Digital Camera



- A digital camera replaces film with a sensor array
- Each cell in the array is a Charge Coupled Device
- light-sensitive diode that converts photons to electrons
- other variants exist: CMOS is becoming more popular
- http://electronics.howstuffworks.com/digital-camera.htm


## How do they work?

- Basic process:
- photons hit a detector
- the detector becomes charged
- the charge is read out as brightness
- Sensor types:
- CCD (charge-coupled device)
- CMOS



## Issues with digital cameras

- Noise
- big difference between consumer vs. SLR-style cameras
- low light is where you most notice noise
- Compression
- creates artifacts except in uncompressed formats (tiff, raw)
- Color
- color fringing artifacts from Bayer patterns
- Blooming
- charge overflowing into neighboring pixels
- In-camera processing
- oversharpening can produce halos
- Interlaced vs. progressive scan video
- even/odd rows from different exposures
- Are more megapixels better?
- requires higher quality lens
- noise issues

More info online, e.g.,
http://electronics.howstuffworks.com/digitalcamera.htm
http://www.dpreview.com/

