Lecture 2

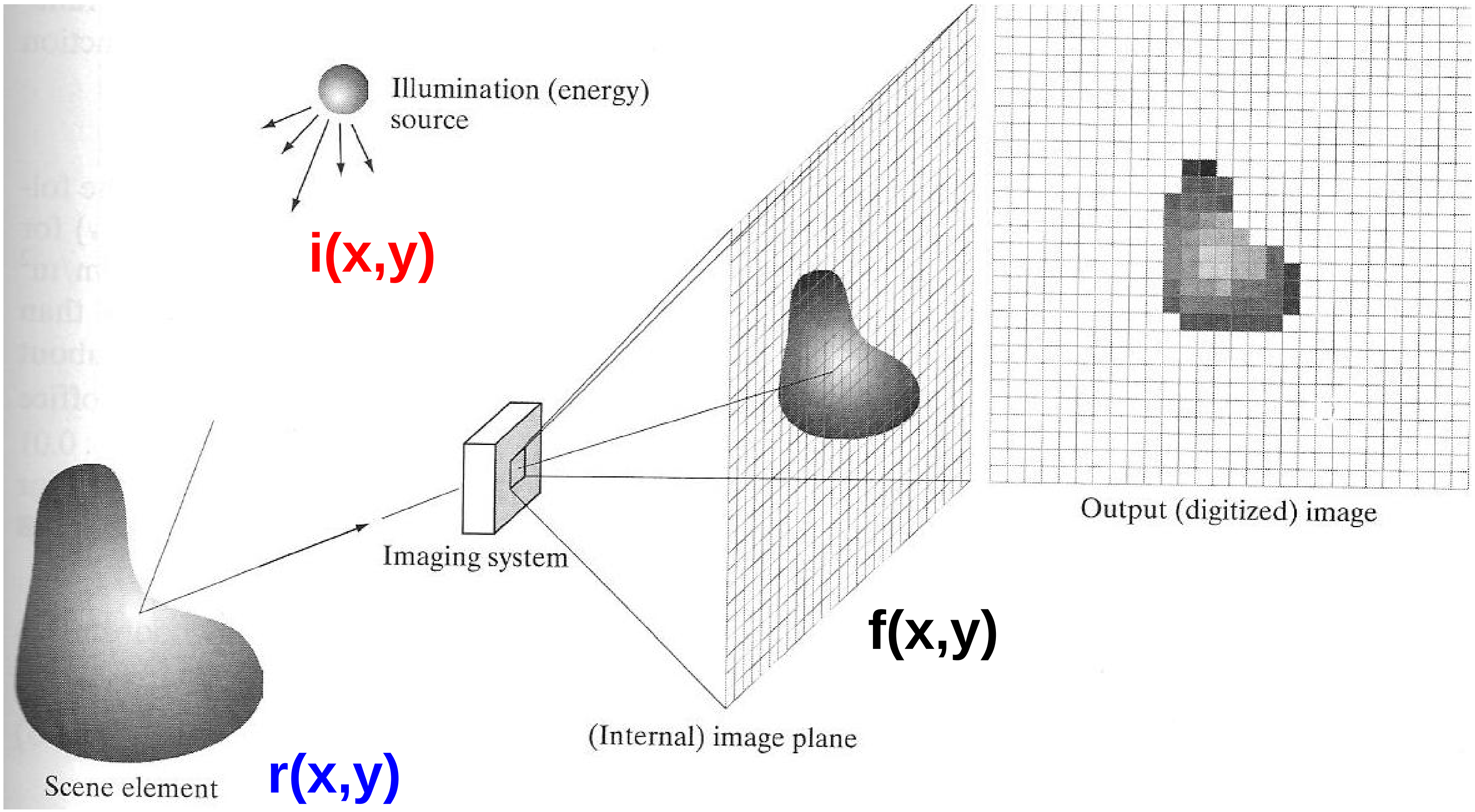
Image Processing and Filtering

"Your x-ray showed a broken rib, but we fixed it with Photoshop."

What’s on our plate today?

- Image formation
- Image sampling and quantization
- Image interpolation
- Domain transformations
  - Affine image transformations
- Range (intensity) transformations
  - Noise reduction through spatial filtering
  - Filtering as cross-correlation
  - Convolution
  - Nonlinear (median) filtering

Image Formation: Basics



(from Gonzalez & Woods, 2008)

Image Formation: Basics

Image f(x,y) is characterized by 2 components

1. **Illumination i(x,y)** = Amount of source illumination incident on scene
2. **Reflectance r(x,y)** = Amount of illumination reflected by objects in the scene

$$f(x,y) = i(x,y)r(x,y)$$

where

$$0 < i(x,y) < \infty \text{ and } 0 < r(x,y) < 1$$

r(x,y) depends on object properties

r = 0 means total absorption and 1 means total reflectance

Image Formation: Basics

$$f(x, y) = i(x, y)r(x, y)$$

where

$$0 < i(x, y) < \infty \text{ and } 0 < r(x, y) < 1$$

Typical values of i(x,y):

- Sun on a clear day: 90,000 lm/m^2
- Cloudy day: 10,000 lm/m^2
- Inside an office: 1000 lm/m^2

Typical values of r(x,y)

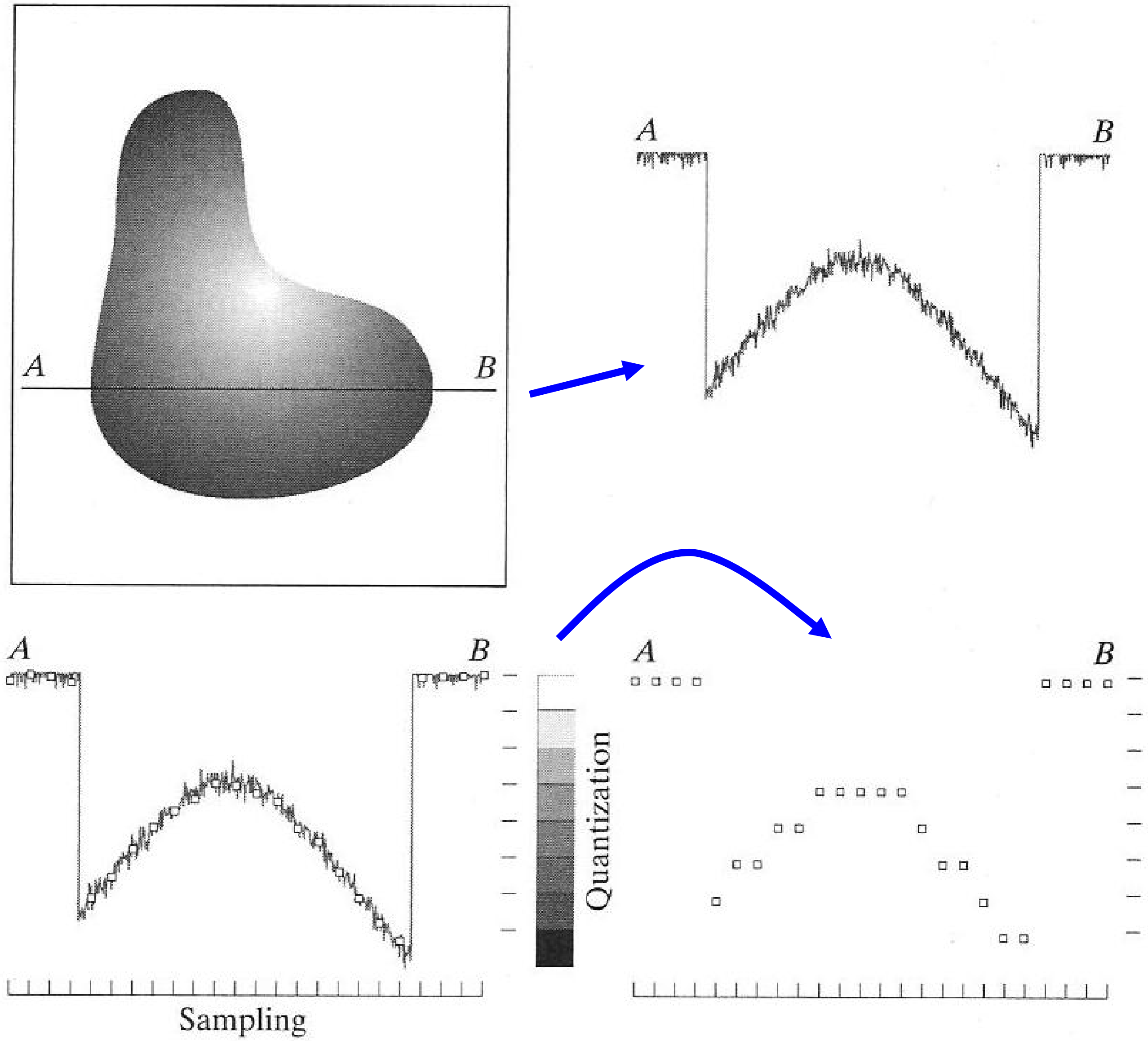
- Black velvet: 0.01, Stainless steel: 0.65, Snow: 0.93

Typical limits of f(x,y) in an office environment

- 10 < f(x,y) < 1000
- Shifted to gray scale [0, L-1]; 0 = black, L-1 = 255 = white

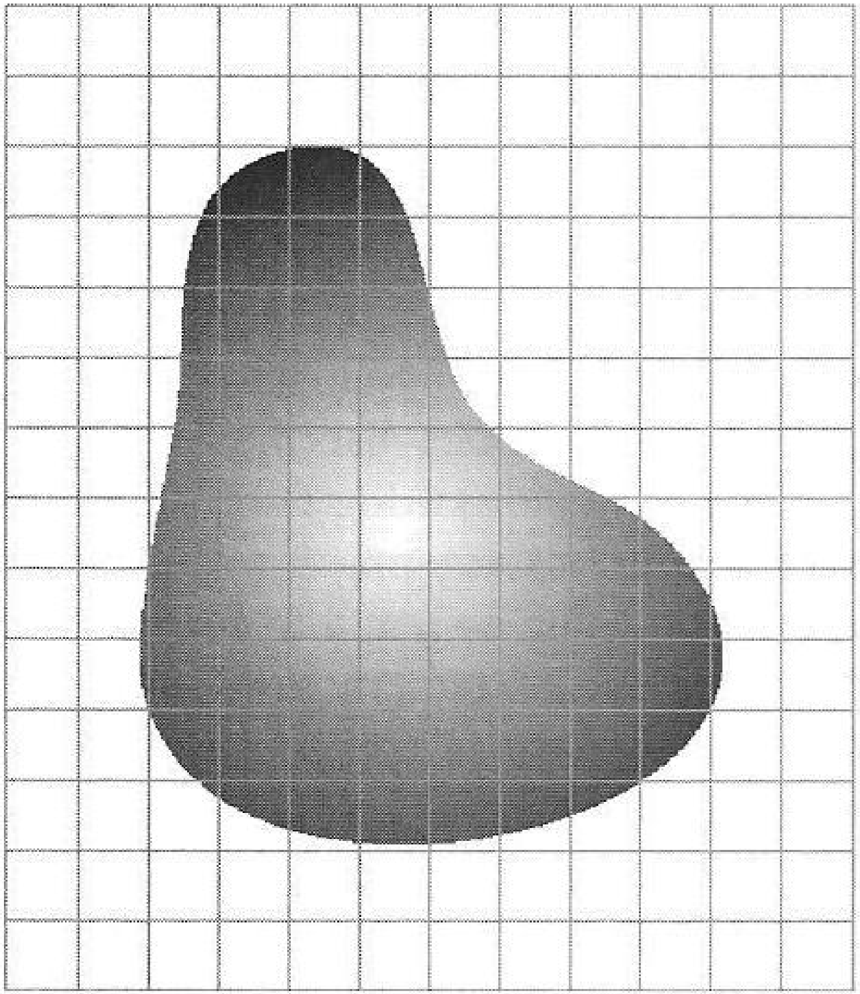
r=1

Sampling and Quantization Process

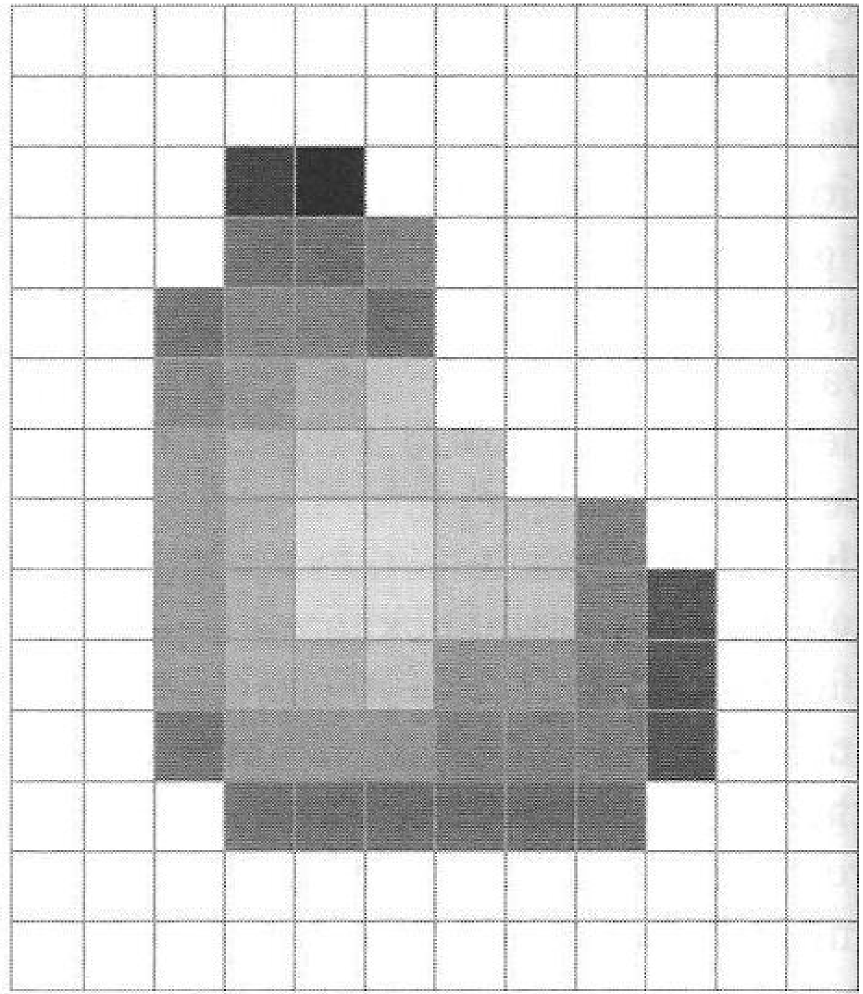


(from Gonzalez & Woods, 2008)

Example of a Quantized 2D Image



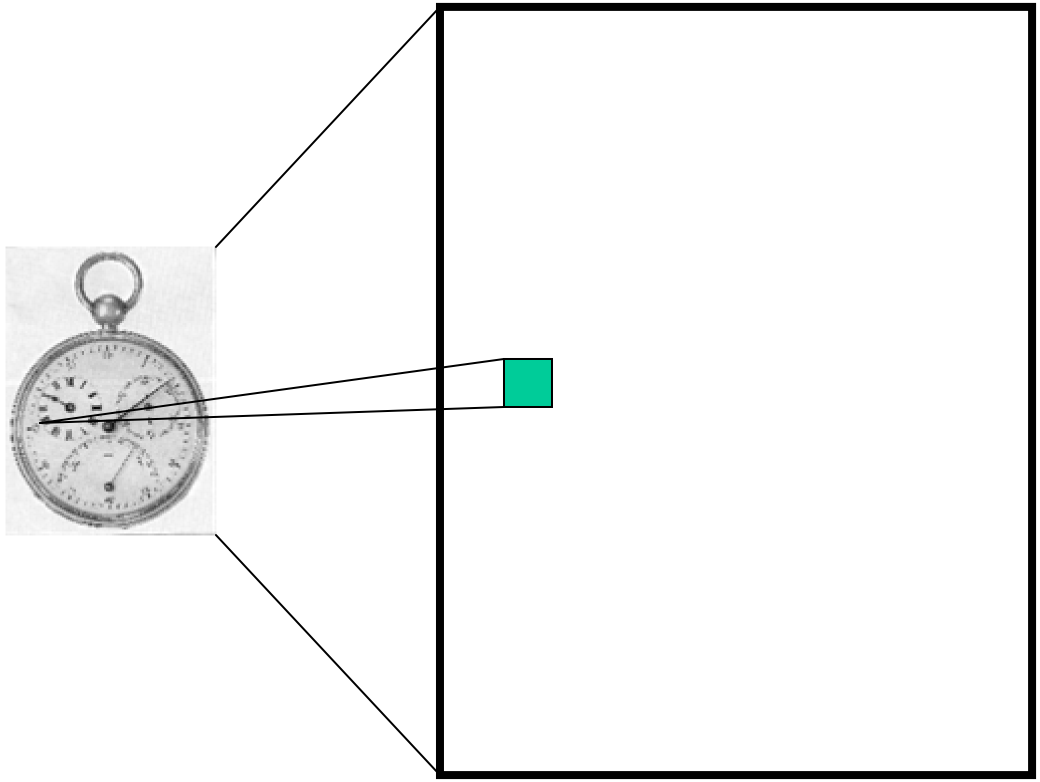
Continuous image projected onto sensor array



Result of sampling and quantization

(from Gonzalez & Woods, 2008)

Suppose we want to zoom an image



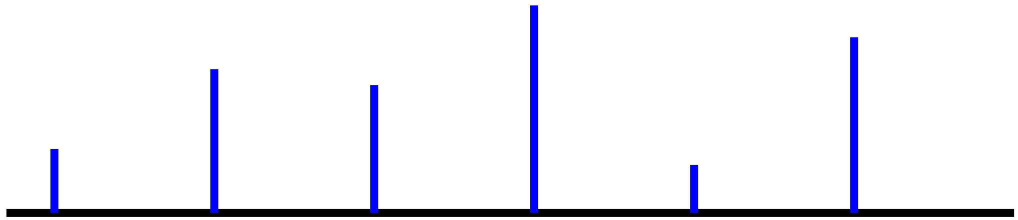
Original image

Need to fill in values for new pixels

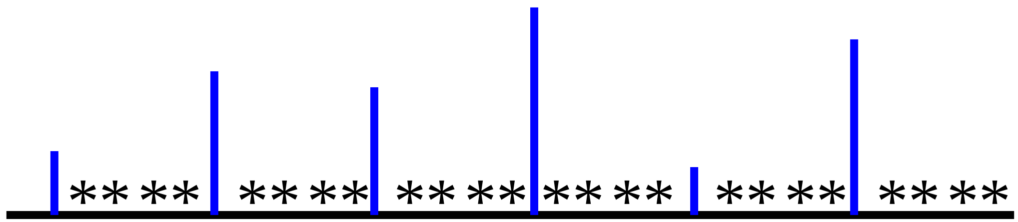
Zoomed image

Interpolation

Original

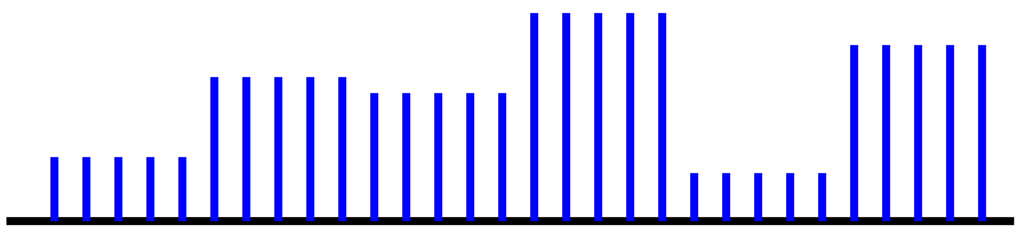


Zoomed



Need to fill in missing values *

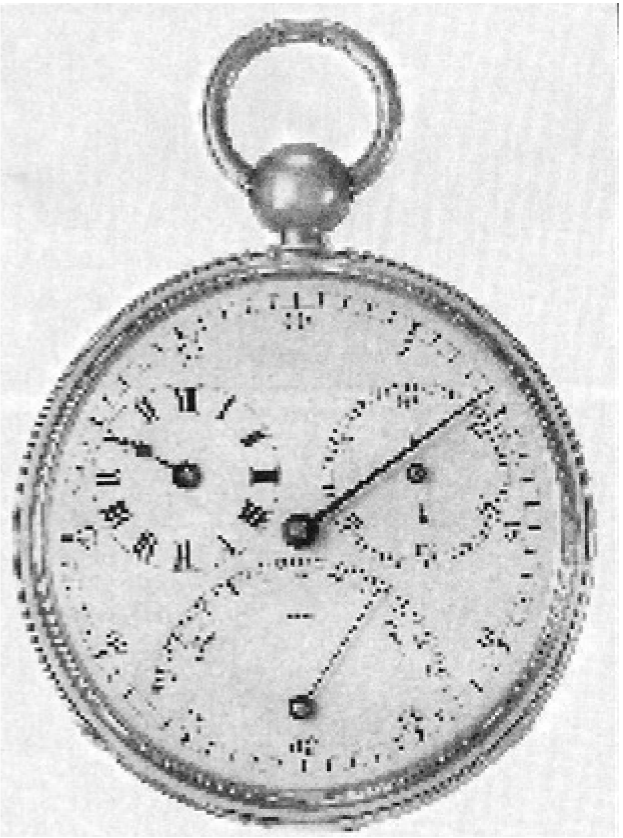
Nearest Neighbor Interpolation



For each new pixel, copy nearest value

Neared Neighbor Interpolation

Original image



Zoomed image

Can we do better?

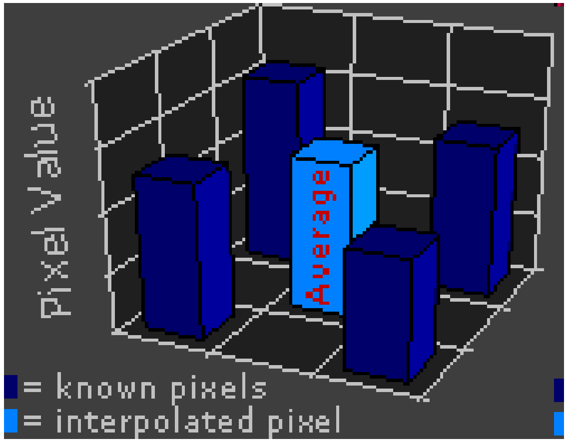
Other image interpolation techniques

Bilinear interpolation:

Compute pixel value v(x,y) as:

$$v(x, y) = ax + by + cxy + d$$

a, b, c, d determined from **four nearest neighbors** of (x,y)

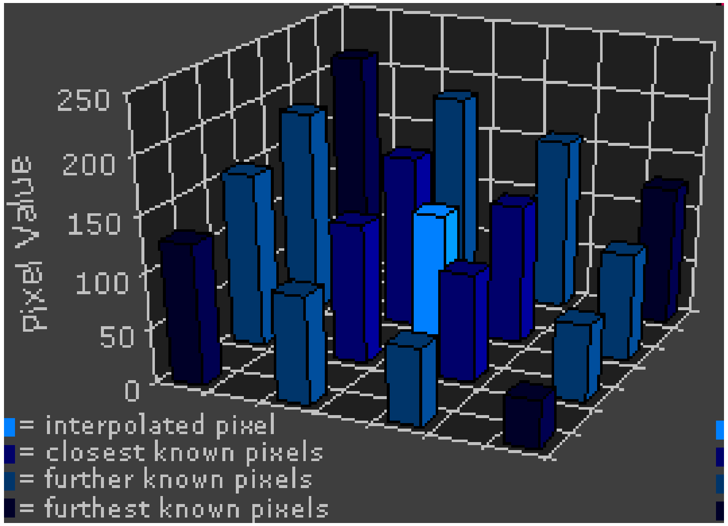


Bicubic interpolation:

(Used in most commercial image editing programs, e.g., Photoshop)

$$v(x, y) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} x^i y^j$$

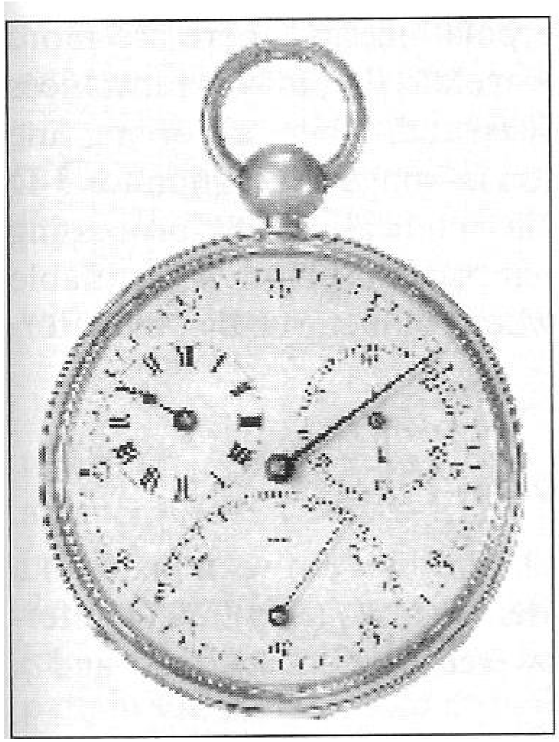
a_{ij} determined from **16 nearest neighbors** of (x,y)



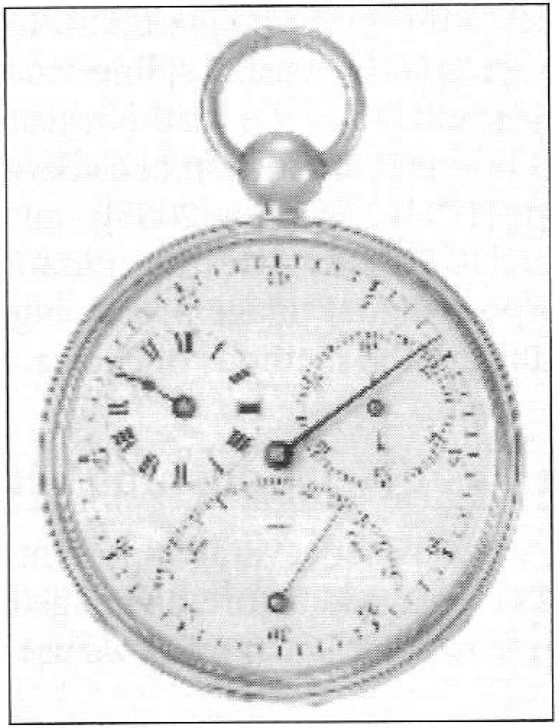
(from http://www.cambridgeincolour.com/tutorials/image-interpolation.htm)

See also http://en.wikipedia.org/wiki/Bilinear_interpolation

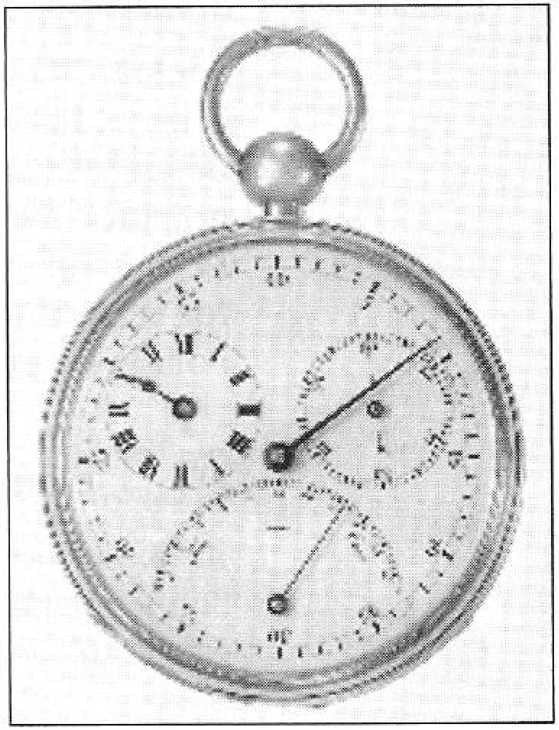
Comparison of Interpolation Techniques



Nearest Neighbor



Bilinear



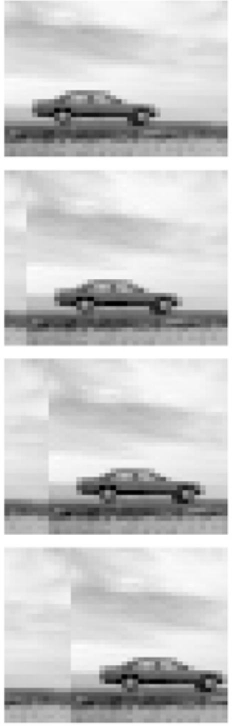
Bicubic

Recall from Last Time

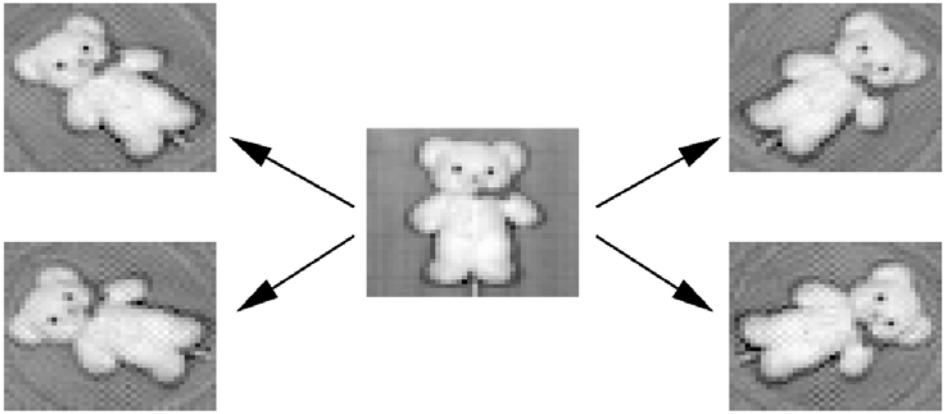
Domain transformation: $g(x, y) = f(t_x(x, y), t_y(x, y))$

(What is an example?)

Translation



Rotation



How are these done?

Geometric spatial transformations of images

Two steps:

1. Spatial transformation of coordinates (x,y)
2. Interpolation of intensity value at new coordinates

We already know how to do (2), so focus on (1)

Example: What does the transformation

$(x,y) = T((v,w)) = (v/2,w/2)$ do?

[Shrinks original image in half in both directions]

Affine Spatial Transformations

- Most commonly used set of transformations
- General form:

$$[x \quad y \quad 1] = [v \quad w \quad 1]\mathbf{T} = [v \quad w \quad 1]\begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}$$

- [x y 1] are called homogenous coordinates
- Can translate, rotate, scale, or shear based on values t_{ij}
- Multiple transformations can be concatenated by multiplying them to form new matrix $\mathbf{T}'$

Example: Translation

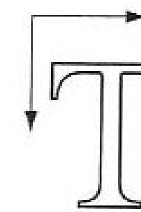
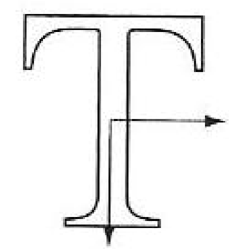
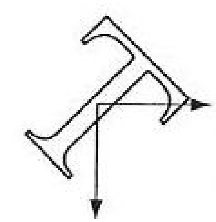
$$\begin{bmatrix} x & y & 1 \end{bmatrix} = \begin{bmatrix} v & w & 1 \end{bmatrix} \mathbf{T} = \begin{bmatrix} v & w & 1 \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}$$

What does **T** look like for translation?

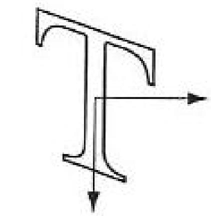
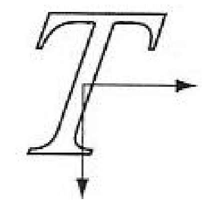
$x = v + t_x$

$y = w + t_y$

Affine Transformations

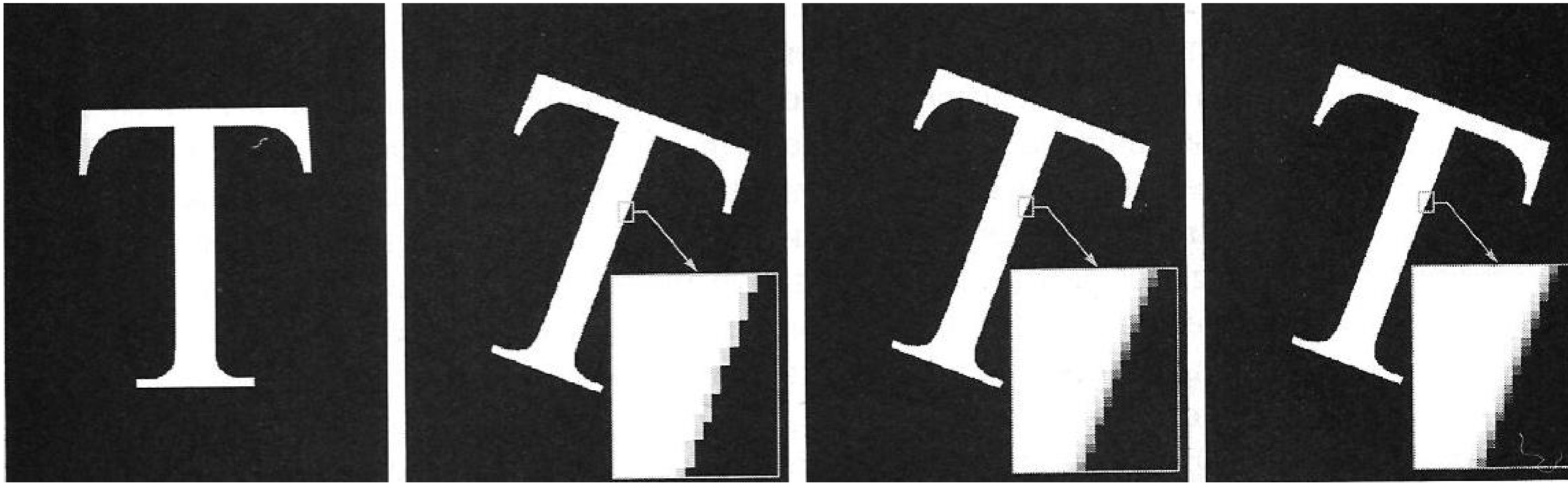
Transformation	Affine Matrix T	Coordinate Equations	Example
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v\cos\theta - w\sin\theta$ $y = v\sin\theta + w\cos\theta$	

Affine Transformations (cont.)

Transformation	Affine Matrix T	Coordinate Equations	Example
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	

Example of Affine Transformation

Image rotated 21 degrees



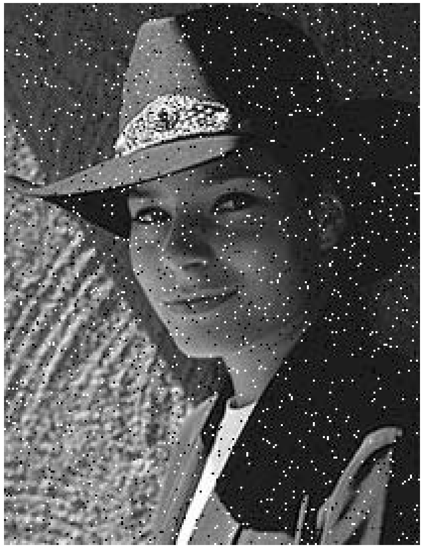
Nearest Neighbor | Bilinear | Bicubic

(from Gonzalez & Woods, 2008)

Recall from last time

Range transformation: $g(x,y) = t(f(x,y))$

(What is an example?)



Noise filtering

Image processing for noise reduction

Common types of noise:

- **Salt and pepper noise**: contains random occurrences of black and white pixels
- **Impulse noise:** contains random occurrences of white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution

Original

Salt and pepper noise

Impulse noise

Gaussian noise

How do we reduce the effects of noise?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

$G[x, y]$

How do we reduce the effects of noise?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

				80					

$G[x, y]$

How do we reduce the effects of noise?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

				80					
		10							

$G[x, y]$

Idea: Compute mean value for each pixel from neighbors

Mean filtering

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

$G[x, y]$

Filtering as cross-correlation

If the averaging window is (2k+1)x(2k+1):

$$G[i,j] = \frac{1}{(2k+1)^2} \sum_{u=-k}^{k} \sum_{v=-k}^{k} F[i+u, j+v]$$

In our example in previous slide, k = 1 for a 3x3 averaging window

Filtering as cross-correlation

Can generalize this by allowing *different weights for different neighboring pixels*:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i+u, j+v]$$

This is called **cross-correlation**, denoted by:

$$G = H \otimes F$$

H is called the “filter,” “kernel,” or “mask.”

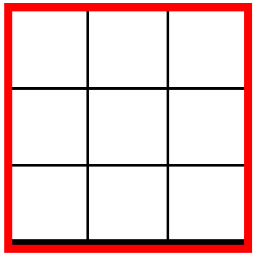
Note: During implementation, we avoid the negative filter indices by using H[u+k,v+k] instead of H[u,v]

Kernel for mean filtering

What is the kernel for a 3x3 mean filter?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$



$H[u, v]$

Kernel for mean filtering

What is the kernel for a 3x3 mean filter?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

1/9

1	1	1
1	1	1
1	1	1

$H[u, v]$

Example of mean filtering

Input image

Salt and pepper noise

Filtered Images

3 x 3

5 x 5

7 x 7

Kernel size

Gaussian Filtering

A Gaussian kernel gives less weight to pixels further from the center of the window

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

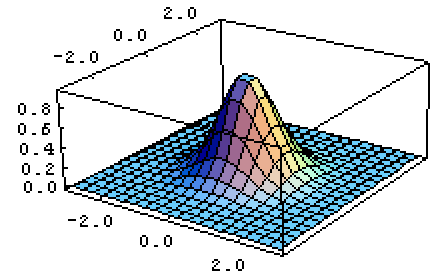
$F[x, y]$

$\frac{1}{16}$

1	2	1
2	4	2
1	2	1

$H[u, v]$

Kernel approximates Gaussian function:



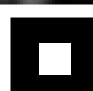
$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$

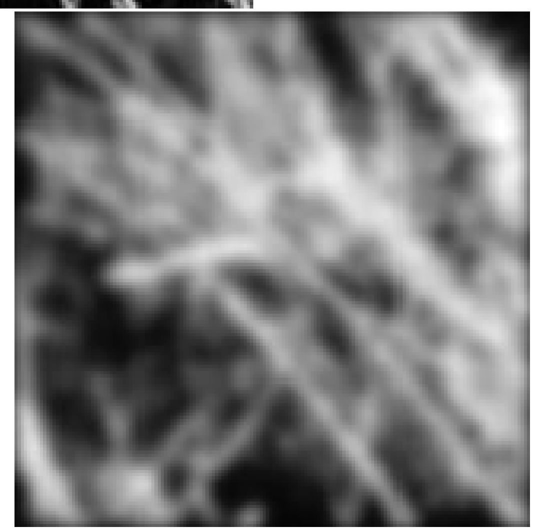
What happens if you increase σ ?

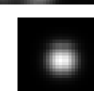
Mean versus Gaussian filtering

Input Image

Mean filtered







Gaussian filtered

Filtering an impulse

Impulse signal

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$F[x, y]$

Kernel

a	b	c
d	e	f
g	h	i

$H[u, v]$

$$G = H \otimes F$$

Output = ?

$G[x, y]$

Filtering an impulse

Impulse signal

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$F[x,y]$

Filter Kernel

a	b	c
d	e	f
g	h	i

$H[u,v]$

$$G = H \otimes F$$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	**i**	**h**	**g**	0	0
0	0	**f**	**e**	**d**	0	0
0	0	**c**	**b**	**a**	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$G[x,y]$

Output is equal to filter kernel flipped horizontally & vertically

What if we want to get an output that looks exactly like the filter kernel?

Flipping kernels

Impulse signal

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$F[x, y]$

Filter Kernel

a	b	c
d	e	f
g	h	i

Flipped Kernel

i	h	g
f	e	d
c	b	a

$$G = H \otimes F$$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	**a**	**b**	**c**	0	0
0	0	**d**	**e**	**f**	0	0
0	0	**g**	**h**	**i**	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$G[x, y]$

Output is equal to filter kernel!

Convolution

A **convolution** is a cross-correlation where the *filter is flipped both horizontally and vertically* before being applied to the image:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

Written as: $G = H \star F$

Compare with cross-correlation:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u,j+v]$$

If H is a Gaussian or mean kernel, how does convolution differ from cross-correlation?

Why convolution?

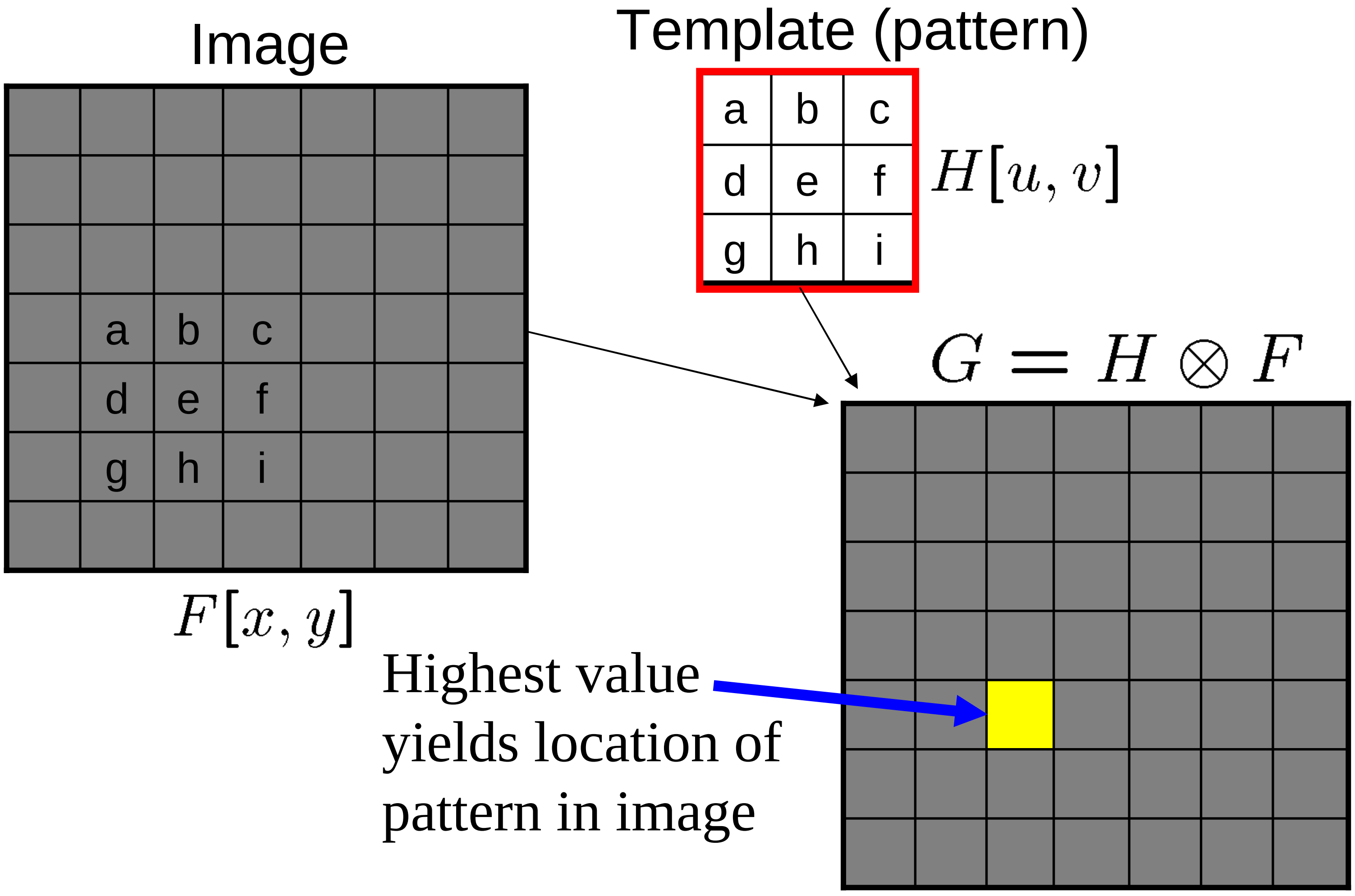
- Convolution is associative (cross-corr. is not):

  $F * (G * I) = (F * G) * I$
- Important for efficiency:

  To apply two filters F and G sequentially to incoming images I, <u>pre-compute</u> (F * G) and perform only 1 convolution (with pre-computed filter)
- Convolution also allows effects of filtering to be analyzed using Fourier analysis (will touch on this later)

Cross-correlation and template matching

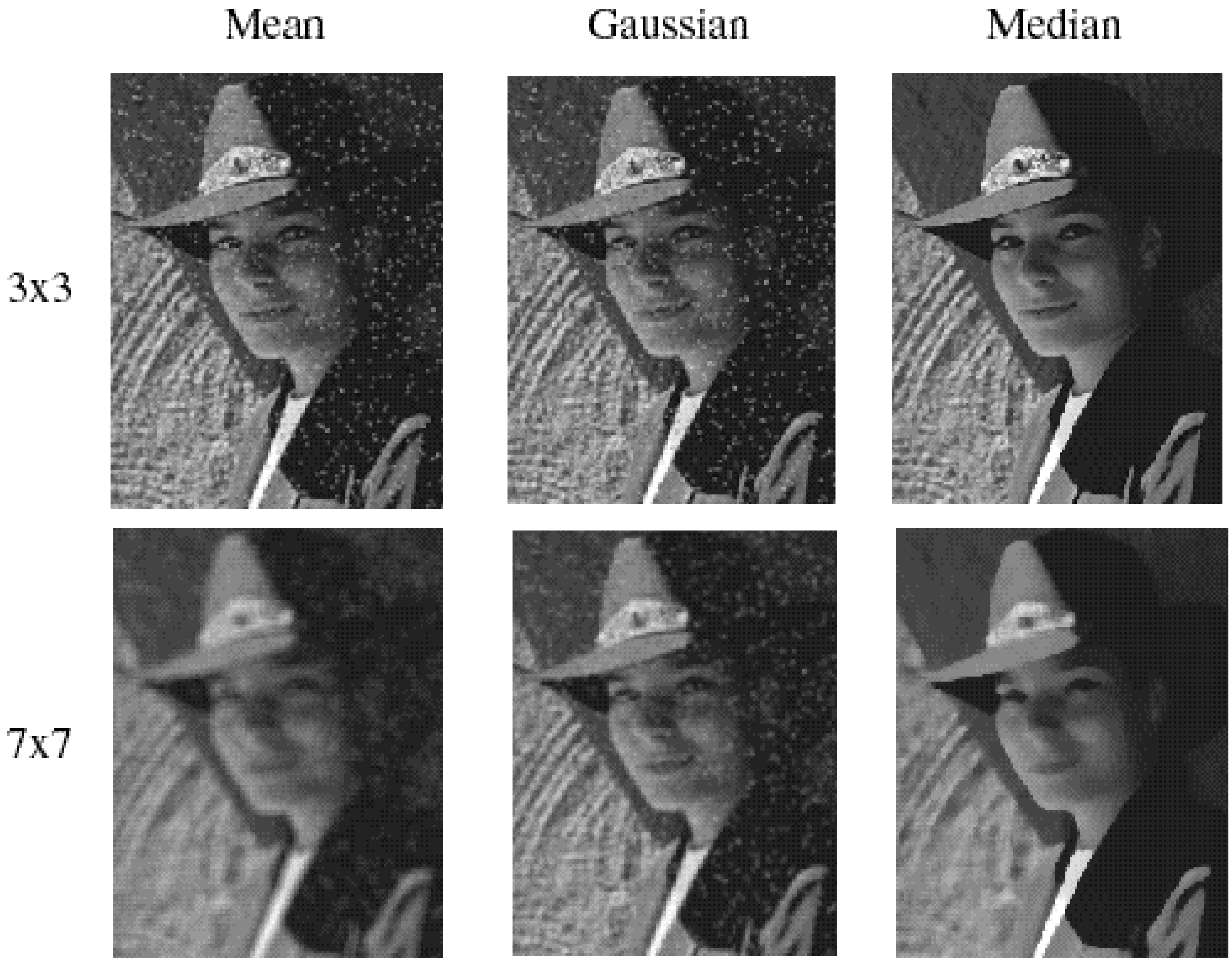
Cross-correlation is useful for *template matching* (locating a given pattern in an image)



Nonlinear filters: Median filter

- A **Median Filter** replaces the value of a pixel by the median of intensity values of neighbors
  - Recall: m is the median of a set of values iff half the values in the set are <= m and half are >= m.
  - Median filtering of image I: For each location (x,y), sort intensity values in its neighborhood, determine median intensity value, and assign that value to I(x,y)
- Is a median filter better than a mean filter?
- Is median filtering a convolution?

Comparison of filters (salt-and-pepper noise)



Comparison of filters (Gaussian noise)

Next Time: Edge detection

- Things to do:
  - Read Chap. 5: Secs. 5.6 - 5.8, 5.11 and online article by Cipolla & Gee on edge detection
  - Mailing list: cse455@cs.washington.edu
    - Did you receive the first message? Otherwise, sign up
  - Prepare for C/C++ programming
  - Visit Vision and Graphics Lab (Sieg 327)
    - Your ID card should open Sieg 327
    - Check to make sure ASAP

Have a good weekend!